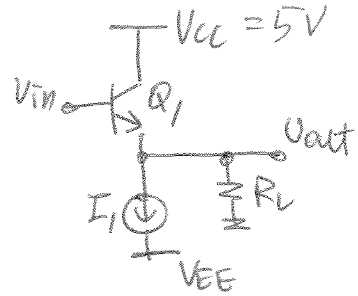
1. $A_v = \dfrac{g_{m1} R_L}{1 + g_{m1} R_L}$



(a) $0.8 = \dfrac{g_{m1}(8\Omega)}{1 + g_{m1}(8\Omega)}$

$\Rightarrow g_{m1} = 0.5 = \dfrac{I_C}{V_T} = \dfrac{I_1}{V_T}$

$\therefore I_1 = 13\ mA$

$P_{LOAD} = 0.5W$

$R_L = 8\Omega$

(Assume Vout biased at $V_{BE(ON),1} \approx 800\ mV$

(b) When $V_{in} = V_p = V_{CC}$, $V_{out} \approx V_{CC} - V_{BE(ON),1}$

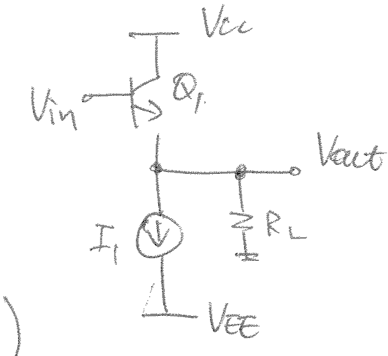
$I_{C1} = I_1 + \dfrac{V_{out}}{R_L} \Rightarrow I_{C1} = I_1 + \dfrac{5-0.8}{8} \approx 0.54\ A$

$\Rightarrow g_{m1} = \dfrac{I_{C1}}{V_T} = \dfrac{0.54A}{0.026V} = 20.8\ S$

$\Rightarrow A_v\Big|_{V_{in}=V_p} = \dfrac{g_{m1} R_L}{1 + g_{m1} R_L} = \dfrac{(20.8S)(8\Omega)}{1 + (20.8S)(8\Omega)} \approx 0.99$

2.

(a) $I_1 = V_p / R_L \qquad V_p \gg V_T$



$$A_V = \frac{I_C R_L}{I_C R_L + V_T}$$

$$= \frac{\frac{I_C}{I_1} V_p}{\frac{I_C}{I_1} V_p + V_T} = \frac{V_p}{V_p + V_T} \quad (\approx 1)$$

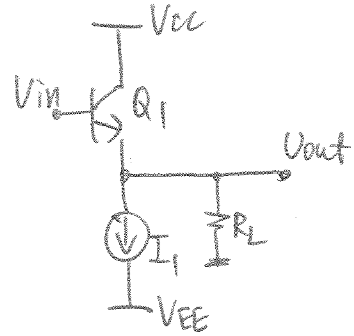
(b) When $V_{out} = V_p$, $I_{C_1} = I_1 + \frac{V_{out}}{R_L} = \frac{V_p}{R_L} + \frac{V_p}{R_L}$

$$= \frac{2V_p}{R_L}$$

$$\therefore A_V = \frac{\left(2\frac{V_p}{R_L}\right) R_L}{\left(\frac{2V_p}{R_L}\right) R_L + V_T} = \frac{2V_p}{2V_p + V_T} \quad \left(\approx \frac{2V_p}{2V_p} = 1.\right)$$

$$\Delta A_V = \frac{\frac{2V_p}{2V_p + V_T} - \frac{V_p}{V_p + V_T}}{\frac{V_p}{V_p + V_T}} = \frac{V_T}{2V_p + V_T} \quad \left(\approx \frac{V_T}{V_p}\right)$$

3. $A_v = 0.7$ $\quad R_L = 4\Omega$



Q_1 shuts off when:

$$I_1 = \frac{V_P}{R_L}$$

- Suppose $V_{out} = V_p \sin\omega t$. $\left(\omega = \frac{2\pi}{T}\right)$

$$P_{R_L,AVG} = \frac{1}{T}\int_0^T \frac{(V_{out})^2}{R_L}\,dt = \frac{1}{T}\int_0^T \frac{V_p^2 \sin^2\omega t}{R_L}\,dt$$

$$= \frac{1}{2} V_P^2 / R_L$$

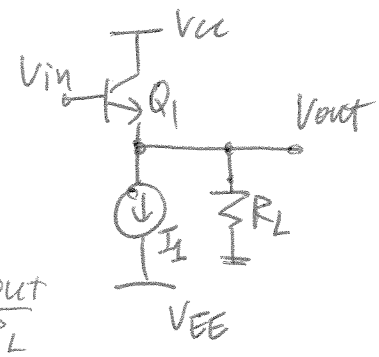
$\therefore$ Largest power (average) $= \dfrac{1}{2}\dfrac{(I_1 R_L)^2}{R_L}$

$$A_v = 0.7 = \frac{g_{m_1} R_L}{1 + g_{m_1} R_L} \Rightarrow g_{m_1} = \frac{A_v}{(1-A_v)R_L} = \frac{0.7}{(1-0.7)(4)} = 0.58\,S$$

$$\Rightarrow I_{C_1}\ (= I_1) = g_{m_1} V_T = 0.015\ A$$

$$\therefore P_{AV,MAX} = \frac{1}{2} I_1^2 R_L = \frac{1}{2}(0.015\,A)^2(4\Omega) = 0.45\ W$$

4. $A_v = \dfrac{g_{m1} R_L}{1 + g_{m1} R_L}$ $\quad \left(g_m = \dfrac{I_C}{V_T} \right)$



- Q_1 shuts off when $I_1 = -\dfrac{V_{out}}{R_L}$

$\Rightarrow V_p = I_1 \times R_L$

$$g_{m1} = \frac{A_v}{(1-A_v)R_L} = \frac{I_{C1}}{V_T} \Rightarrow I_{C1} = \frac{V_T A_v}{R_L(1-A_v)} \quad (= I_1)$$

- Power delivered to R_L:

$$P_{R_L} = \frac{1}{T}\int_0^T \frac{V_{out}^2}{R_L}\,dt = \frac{1}{T}\int_0^T \frac{V_p^2 \sin^2 \omega t}{R_L}\,dt$$

$$= \frac{1}{2}\frac{V_p^2}{R_L}$$

$\therefore$ Maximum power $= \dfrac{1}{2}\dfrac{(I_1 R_L)^2}{R_L}$

$$= \frac{1}{2}\left[\frac{V_T A_v}{(1-A_v)}\right]^2 \cdot \frac{1}{R_L}$$

5.

(a) By KCL,

$$I_1 = I_{S1} \cdot \exp\left(\frac{V_{in}-V_{out}}{V_T}\right) + \frac{V_{cc}-V_{out}}{R_L}$$

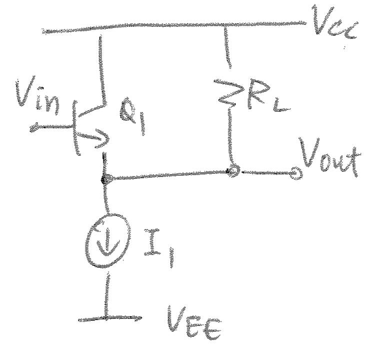
$$\Rightarrow V_{in} = V_{out} + V_T \ln\left(\frac{I_1}{I_{S1}} - \frac{V_{cc}-V_{out}}{I_{S1}R_L}\right)$$

$= 0$ (X) — no solution

$\therefore V_{out} \approx 5 - I_1 R_L = 4.84\,V$

(i.e. Q_1 is off.)

Assume $V_{cc} = 5V$



$I_{S1} = 5 \cdot 10^{-17} A$

$R_L = 8\Omega$

$I_1 = 20mA$.

(b) $(0.01)I_1 = I_1 - \frac{V_{cc}-V_{out}}{R_L}$

$\Rightarrow V_{out} = 4.84\ V$

$$I_{C1} = (0.01)I_1 = I_{S1} \exp\left(\frac{V_{in}-V_{out}}{V_T}\right)$$

$$\Rightarrow V_{in} = V_{out} + V_T \ln\left(0.01 \frac{I_1}{I_S}\right)$$

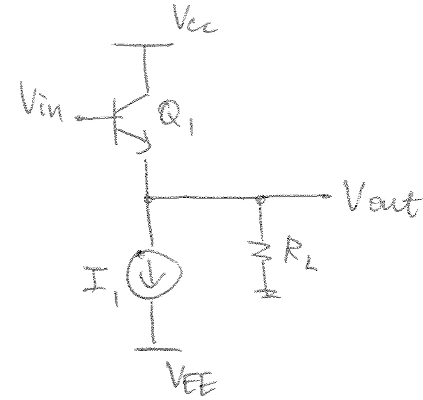
$$= 4.84 + (0.026) \ln\left(0.01 \frac{20mA}{5 \cdot 10^{-17}A}\right)$$

$\approx 5.59\ V$

(exceeds V_{cc})

b.

(a) Calculate V_{BE} for



$I_{S_1} = 6 \cdot 10^{-17} A$

$R_L = 8 \Omega$

$I_1 = 25 mA$

$V_p = 1 V$

$\underline{V_{in} = 1 V:}$

$$I_{C_1} = I_1 + \frac{V_{out}}{R_L}$$

$$\Rightarrow I_{S_1} \cdot \exp\left(\frac{V_{in} - V_{out}}{V_T}\right) = I_1 + \frac{V_{out}}{R_L}$$

Solving for V_{out} gives:

$V_{out} \approx 0.113 V$

$$\therefore V_{BE}\Big|_{V_{in}=1V} = V_{in} - V_{out} = 1 - 0.113 = 0.887 V$$

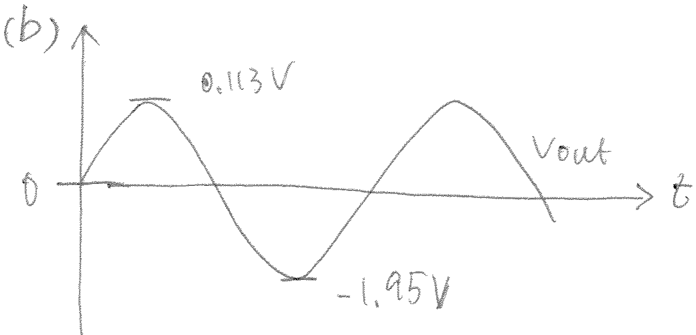
$\underline{V_{in} = -1 V:}$

$$I_{C_1} = I_1 + \frac{-V_{out}}{R_L} \Rightarrow I_{S_1} \exp\left(\frac{V_{in} - V_{out}}{V_T}\right) = I_1 - \frac{V_{out}}{R_L}$$

Solving for V_{out} gives: $V_{out} \approx -1.95 V$

$$\therefore V_{BE}\Big|_{V_{in}=-1V} = V_{in} - V_{out} = -1 - (-1.95V) = 0.95 V$$

(b)



7. Determine V_p such that

$$V_{BE}\Big|_{V_{in}=+V_p} - V_{BE}\Big|_{V_{in}=-V_p} = 10\text{ mV}$$

$$\Rightarrow \left(V_p^+ - V_{out,+}\right) - \left(V_p^- - V_{out,-}\right) = 10\text{ mV}$$

$$I_s \exp\left(\frac{V_p^+ - V_{out,+}}{V_T}\right) = I_1 + \frac{V_{out,+}}{R_L} \quad \text{—— ①}$$

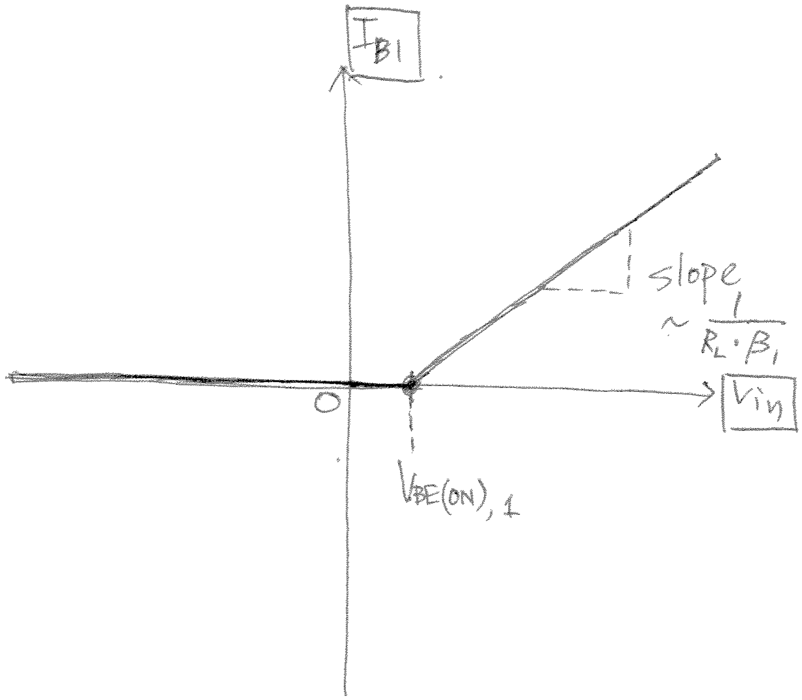
$$I_s \exp\left(\frac{V_p^- - V_{out,-}}{V_T}\right) = I_1 - \frac{V_{out,-}}{R_L} \quad \text{—— ②}$$

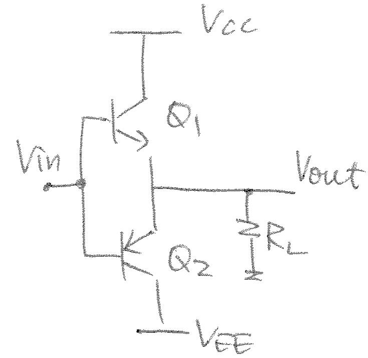
Iterate ① & ②. This gives:

$V_p \approx 0.7$ V

$$\Rightarrow \text{Nonlinearity} = \frac{10\text{ mV}}{0.7 \times 2} = 0.007.$$

8.





β_1 : current gain of Q_1

- Q_1 is on whenever $V_{in} \geq V_{BE(ON),1}$. In this region,

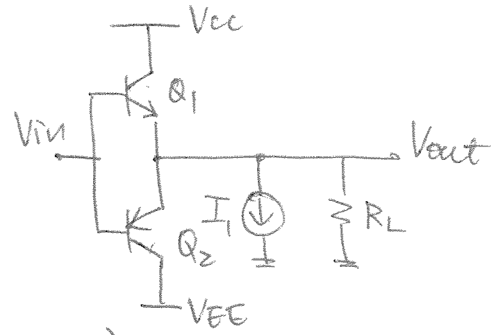
$$V_{out} = V_{in} - V_{BE(ON)1} \qquad I_{C_1} = \frac{V_{out}}{R_L}$$

$$\therefore I_{B_1} = \frac{I_{C1}}{\beta} = \frac{V_{out}}{\beta R_L} = \frac{V_{in} - V_{BE(ON)1}}{\beta R_L}.$$

9.

(a) To guarantee Q_1 on,

- $V_{out} \approx V_{in} - V_{BE(ON)1}$
$= -800\,mV$



$$\Rightarrow I_{C_1} = I_1 + \frac{V_{out}}{R_L} \quad (Q_2 \text{ is off})$$

$I_{S_2} = 6 \cdot 10^{-17} A$

$R_L = 8\,\Omega$

- $I_{C_1} \geq 0 \Rightarrow I_1 + \frac{V_{out}}{R_L} \geq 0$

$$\Rightarrow I_1 + \frac{-800\,mV}{R_L} \geq 0$$

$$\therefore I_1 R_L \geq 800\,mV \quad \text{——— (1)}$$

(b) When Q_2 turns on,

$$-\frac{V_{out}}{R_L} - I_1 = I_{C_2}$$

$$\Rightarrow -\frac{V_{out}}{R_L} - \left(\frac{800\,mV}{R_L}\right) = I_{S_2} \exp\left(\frac{V_{BE2}}{V_T}\right)$$

$$\Rightarrow V_{out} = -R_L I_{S_2} \cdot \exp\left(\frac{V_{BE2}}{V_T}\right) - 0.8$$

$$= -(8\Omega)(6 \times 10^{-17} A) \exp\left(\frac{0.8}{0.026}\right) - 0.8$$

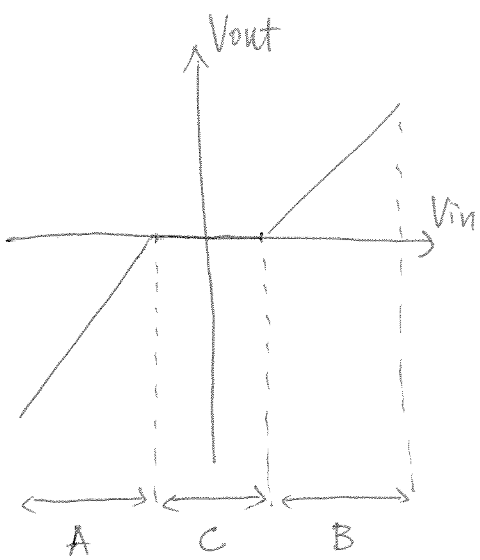
$$\approx -0.81\ V$$

$$\therefore V_{in} = V_{out} - |V_{BE(ON)2}| = -0.81 - 0.8 = -1.61\ V$$

10. Consider two scenarios:

- In gain regions ($|V_{in}| \geq |V_{BE(ON)}|$), V_{out} tracks V_{in}.
- In dead zone, both transistors shut off.

In both cases, V_{out} has an important role. Current source I_1 affects the input/output characteristic by modulating V_{out}:



I/O characteristic of Push-pull stage.

Consider region A:

$$I_{C2} + I_1 = \frac{-V_{out}}{R_L}$$

$\therefore I_1 \uparrow\downarrow \Rightarrow |V_{BE2}| = |V_{out} - V_{in}|$ stays relatively constant.

(Q_2 absorbs/sinks all the currents from I_1 in order to have the same $|V_{BE2}|$)

Consider region B:

$$I_{C1} = I_1 + \frac{V_{out}}{R_L}$$

$\therefore I_1 \uparrow\downarrow \Rightarrow |V_{BE1}| = |V_{in} - V_{out}|$ stays relatively constant.

(Q_1 provides/sources current to I_1 in order to have $|V_{BE1}|$ constant.)

Consider region C: (Dead zone).

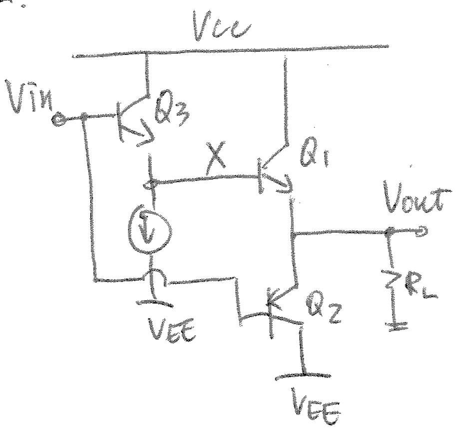
$I_1 = -\frac{V_{out}}{R_L}$ (Both transistors off)

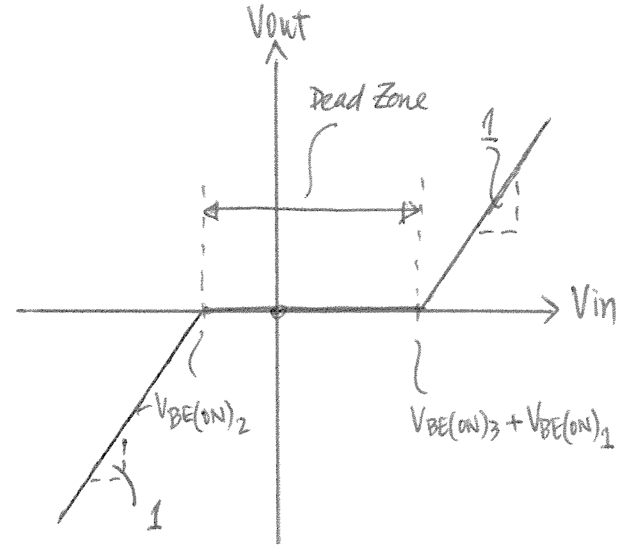
$\therefore I_1 \uparrow \Rightarrow V_{out} \downarrow$

$I_1 \downarrow \Rightarrow V_{out} \uparrow$

i.e. In the dead zone, Vout is predominantly controlled by I_1. One can use this to control Vout and effectively shift the region of dead zone.
($\because V_{out}|_{V_{in}=0} \neq 0$ anymore)

11.





Dead Zone

$= |V_{BE(ON)2}| + V_{BE(ON)3} + V_{BE(ON)1}$

Analysis

- $(0 < V_{in} < V_{BE(ON)3} + V_{BE(ON)1})$:

  Q_1 is OFF $(V_{in} < V_{BE(ON)1})$

  Q_2 is OFF (V_{BE2} reverse-biased) $\Rightarrow V_{out} = 0$

- $(-|V_{BE(ON)2}| < V_{in} < 0)$:

  Q_1, Q_2 OFF. $V_{out} = 0$

- $(V_{BE(ON)3} + V_{BE(ON)1} < V_{in} < V_{cc})$

  Q_1 ON

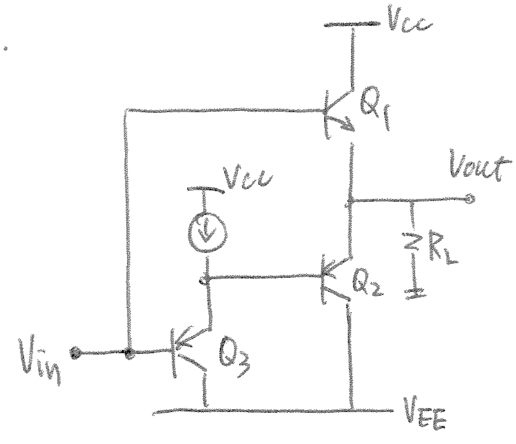
  Q_2 OFF $\quad V_{out} = V_{in} - V_{BE(ON)3} - V_{BE(ON)1}$

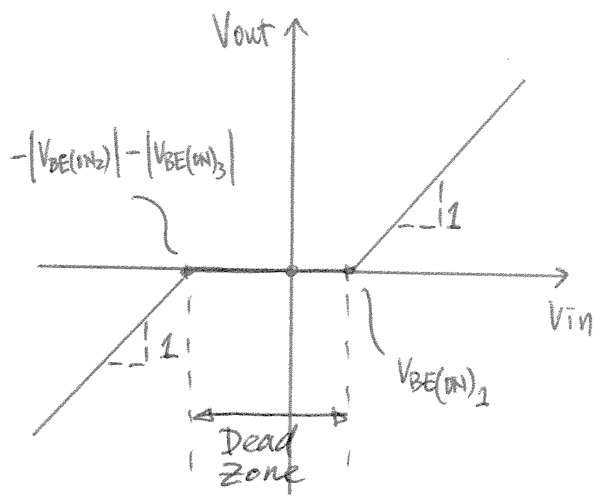
- $(-|V_{EE}| < V_{in} < -|V_{BE(ON)2}|)$

  Q_2 ON

  Q_1 OFF $\quad V_{out} = V_{in} + |V_{BE(ON)2}|$

12.





$-V_{EE} < V_{in} < -\left(\left|V_{BE(ON)_2}\right| + \left|V_{BE(ON)_3}\right|\right)$:

$\Rightarrow$ Q_2, Q_3 ON, Q_1 OFF $\Big\}$ $V_{out} = V_{in} + \left|V_{BE(ON)_3}\right| + \left|V_{BE(ON)_2}\right|$

$-\left(\left|V_{BE(ON)_2}\right| + \left|V_{BE(ON)_3}\right|\right) < V_{in} < V_{BE(ON)_1}$:

$\Rightarrow$ Q_1, Q_2 OFF $\Rightarrow$ $V_{out} \cong 0$

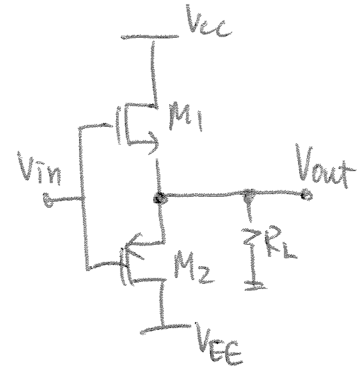
$V_{BE(ON)_1} < V_{in} < V_{cc}$:

$\Rightarrow$ Q_1 ON, Q_2, Q_3 OFF $\Big\}$ $V_{out} = V_{in} - V_{BE(ON)_1}$

Dead Zone $= V_{BE(ON)_1} + \left|V_{BE(ON)_2}\right| + \left|V_{BE(ON)_3}\right|$

13.

(a)



Ignore body effect.

$\underline{-|V_{EE}| < V_{in} < -|V_{t,p}|}$:

$\Rightarrow$ $\left.\begin{array}{l} M_1 \text{ OFF} \\ M_2 \text{ ON} \\ \text{(saturation)} \end{array}\right\}$ $V_{out} = V_{in} + V_{SG,2}$

$\underline{V_{cc} > V_{in} > V_{t,n}}$:

$\Rightarrow$ $\left.\begin{array}{l} M_1 \text{ ON} \\ M_2 \text{ OFF} \end{array}\right\}$ $V_{out} = V_{in} - V_{GS,1}$

$\Rightarrow$ M_1 & M_2 can never on at the same time.

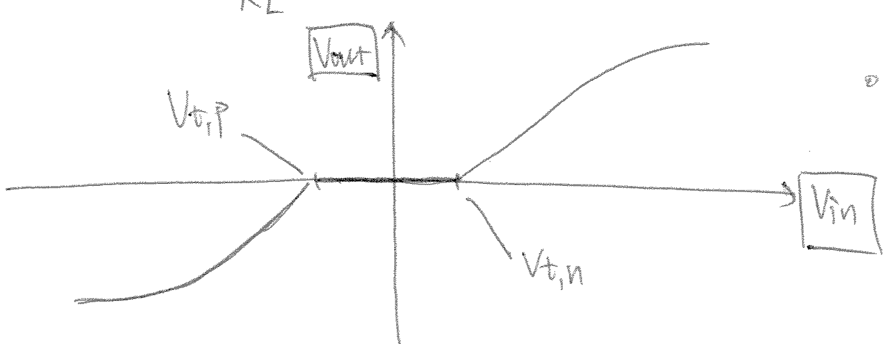
$\underline{-|V_{t,p}| < V_{in} < V_{t,n}}$:

M_1, M_2 OFF $\Rightarrow$ $V_{out} = 0$

For MOS, $I_D = \frac{1}{2}\mu C_{ox}\frac{W}{L}(|V_{GS}| - |V_t|)^2$ — saturation region.

$\Rightarrow$ M_1 ON: $\frac{V_{out}}{R_L} = \frac{1}{2}\mu C_{ox}\frac{W}{L}(V_{in} - V_{out} - V_{t,n})^2$, $V_{out} > 0$

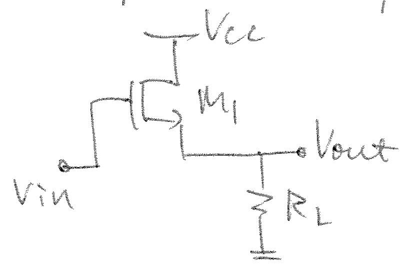
M_2 ON: $-\frac{V_{out}}{R_L} = \frac{1}{2}\mu C_{ox}\frac{W}{L}(V_{out} - V_{in} - V_{t,p})^2$, $V_{out} < 0$



- Solve for V_{out} in both cases.

(b) Outside dead zone
⇒ either M_1 or M_2 is on.

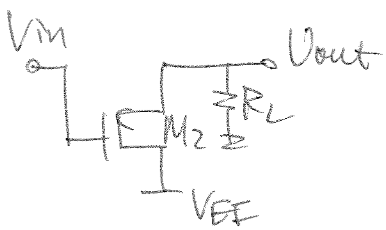
- For positive inputs:



Source follower:

$$\therefore \frac{v_{out}}{v_{in}} = \frac{g_{m1}}{1+g_{m1}R_L}$$

- For negative inputs:

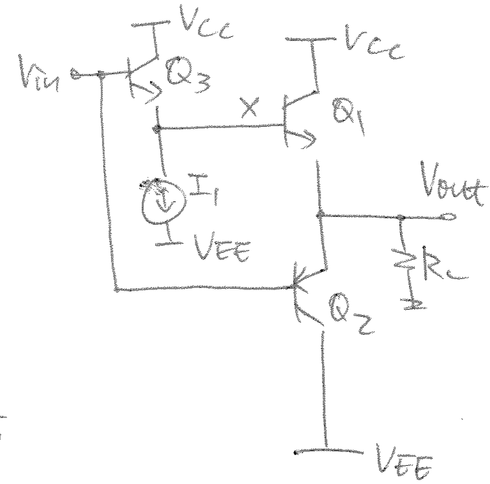


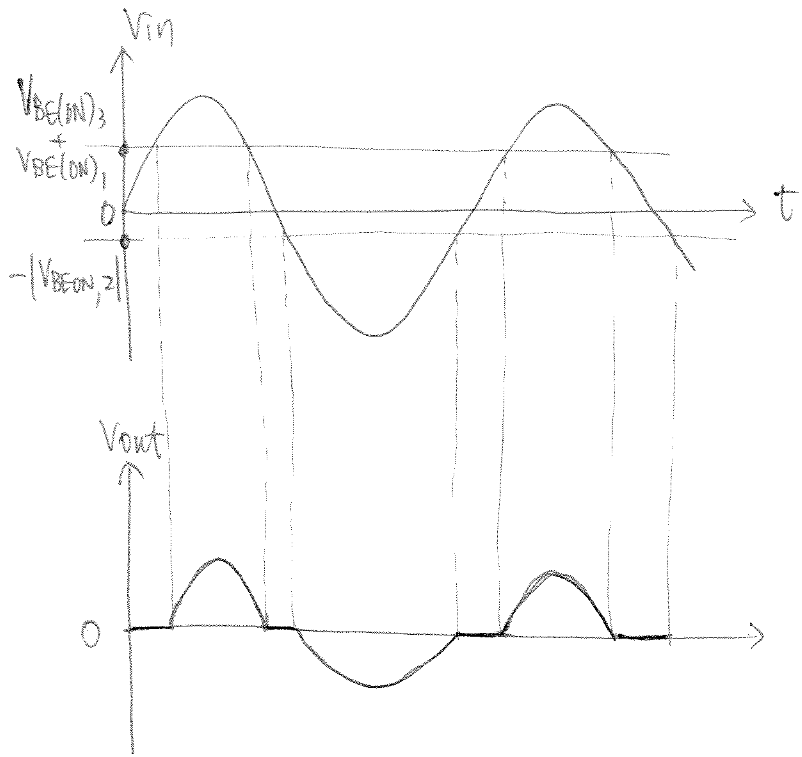
Source follower:

$$\therefore \frac{v_{out}}{v_{in}} = \frac{g_{m2}}{1+g_{m2}R_L}$$

14. Dead zone :

$$V_{out} \in \left[-|V_{BE(ON)_2}| \, , \; V_{BE(ON)_3} + V_{BE(ON)_1} \right]$$

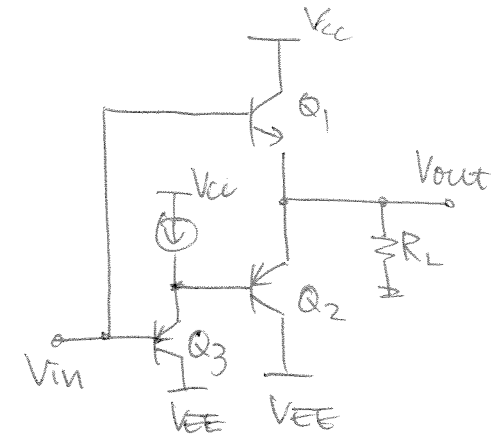


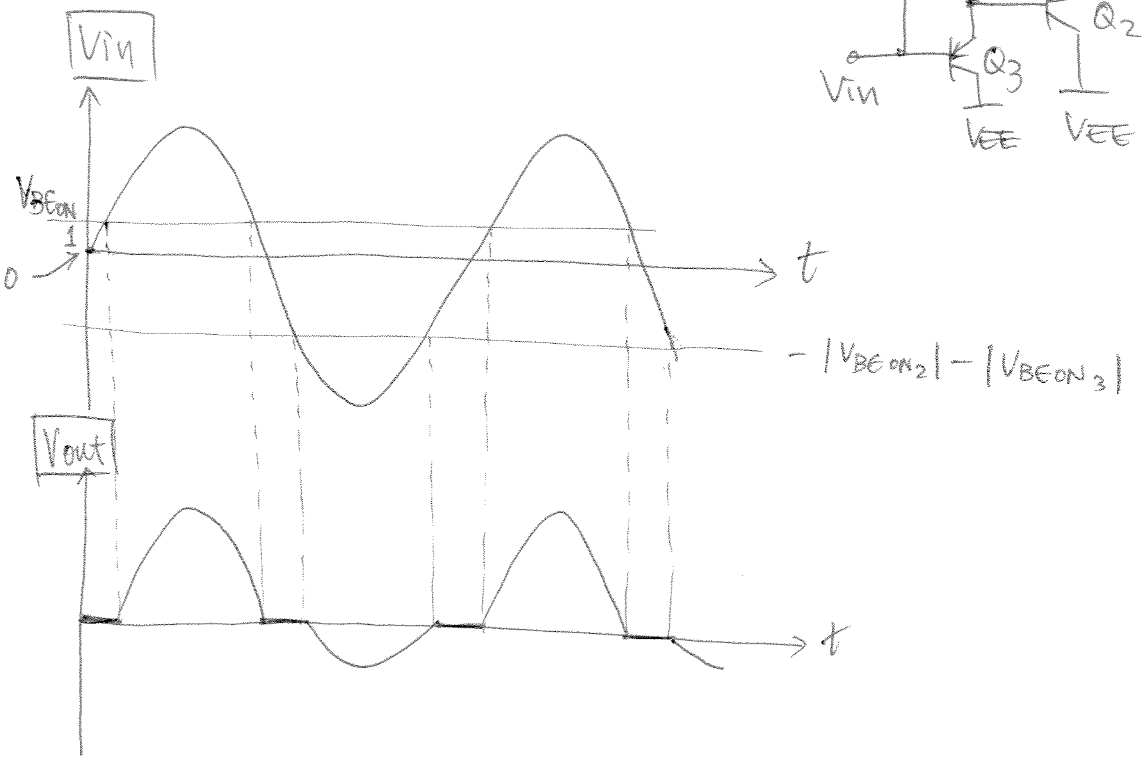


15.

Dead zone:

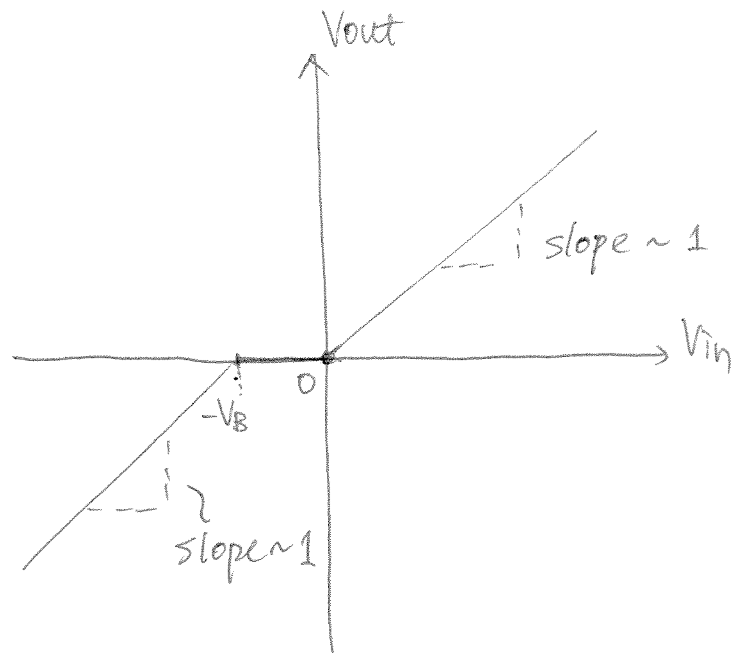
$$V_{out} \in \left[-\left(|V_{BE,ON,2}| + |V_{BE,ON,3}|\right),\ V_{BE,ON,1}\right]$$

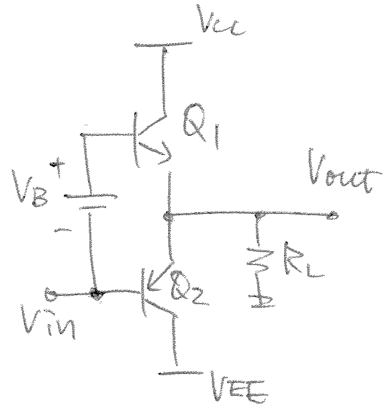




16.

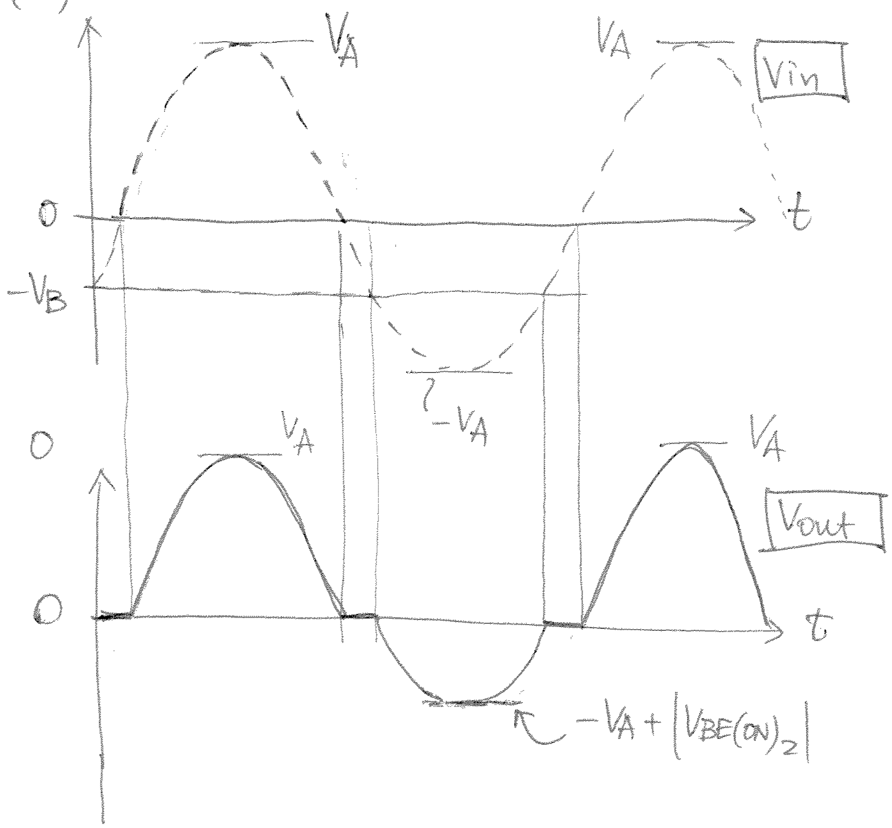
(a)


Vout
Vin
0
-VB
slope ~ 1
slope~1

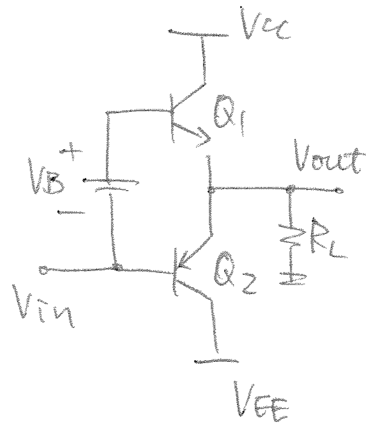

Vcc
Q1
VB
+
-
Vout
RL
Q2
Vin
VEE

($V_B = v_{BE}$)

(b)


VA
VA
Vin
0
t
-VB
-VA
0
VA
VA
Vout
0
t
-VA + |VBE(ON)2|

17.



- $V_{out} = 0$:

$\Rightarrow I_{C_1} = I_{C_2} = I_{BIAS}$

$$\Rightarrow I_{S_1} \exp\left(\frac{V_{in}+V_B-V_{out}}{V_T}\right) = I_{S_2} \exp\left(\frac{|V_{out}-V_{in}|}{V_T}\right)$$

$$\ln\left(\frac{I_{S1}}{I_{S2}}\right) + \frac{V_{in}+V_B-V_{out}}{V_T} = \frac{V_{out}-V_{in}}{V_T}$$

$I_{S_1} = 5 \cdot 10^{-17} A$

$I_{S_2} = 8 \cdot 10^{-17} A$

$I_{BIAS} = 5 mA$
$(V_{out} = 0)$

- For $V_{out} = 0$, $V_T = 0.026 V$:

$$\Rightarrow \ln\left(\frac{5}{8}\right) + \frac{V_{in}+V_B}{0.026} = +\frac{V_{in}}{0.026}$$

- Given $I_{C2} = 5 mA$

$$\Rightarrow I_{S2} \exp\left(\frac{-V_{in}}{0.026}\right) = 5mA \Rightarrow V_{in} = -0.83 V$$

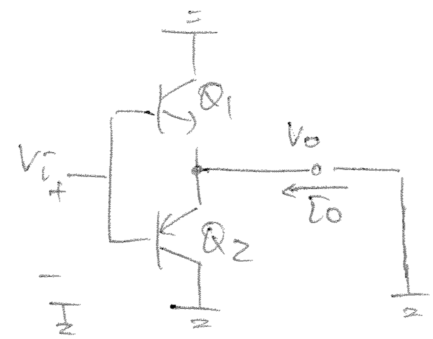
$$I_{C_1} = I_{S_1} \exp\left(\frac{V_{in}+V_B-V_{out}}{V_T}\right) = (5 \cdot 10^{-17} A) \exp \frac{-0.83+V_B}{V_T}$$

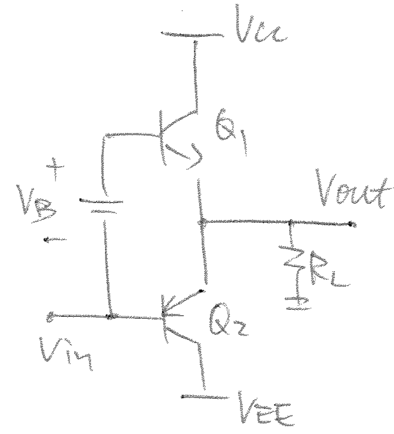
$$\Rightarrow V_B = 0.83 + 0.026 \ln\left(\frac{5mA}{5 \cdot 10^{-17} A}\right)$$

$$\approx 1.67 V$$

18.

(a) Equivalent circuit (small-signal) around $V_{out} = 0$:





$V_{P_{in}} = 2V$

$R_L = 8\Omega$

$$i_o = -g_{m_1} v_i + (-v_i) g_{m_2}$$
$$= -(g_{m_1} + g_{m_2}) v_i$$

$$\therefore G_m = \frac{i_o}{v_i} = -(g_{m_1} + g_{m_2})$$

$$\therefore A_V = \frac{v_o}{v_i} = \frac{i_o \times R_L}{v_i} = -(g_{m_1} + g_{m_2}) R_L$$

(b) $$A_V = -(g_{m_1} + g_{m_2}) R_L = -\left(\frac{I_{C_1}}{V_T} + \frac{I_{C_2}}{V_T}\right) R_L$$
$$= -\left(\frac{5mA}{0.026V} + \frac{5mA}{0.026V}\right)(8\Omega) = -3.08$$

$$\Rightarrow |V_o|_p = |v_i A_V|_p = |(2V)(-3.08)| = 6.16\ V$$

(Assume V_{cc} is large enough)

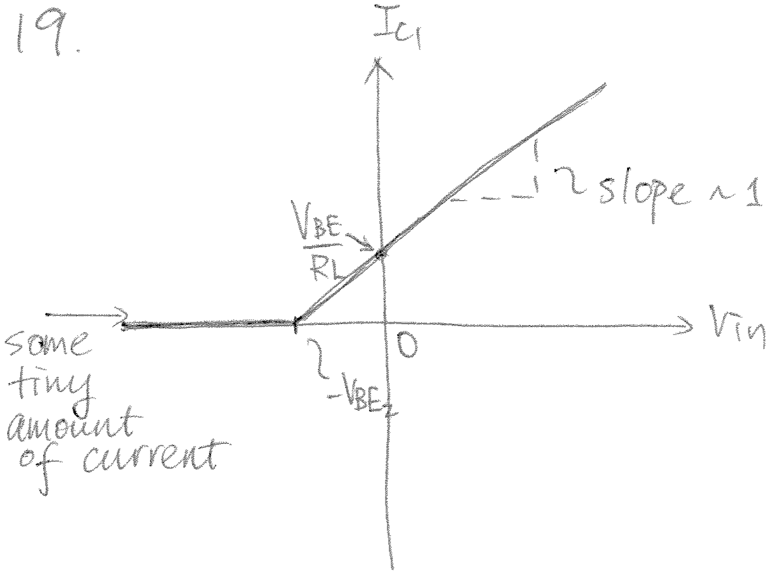
(c) $I_{C1} = I_{C2} + \frac{V_{out}}{R_L}$

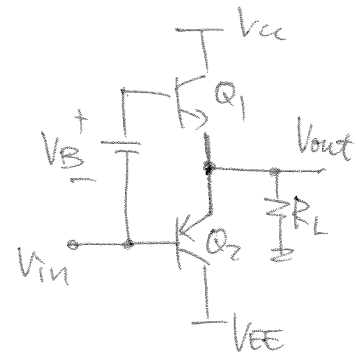
$$I_{C1,peak} = I_{C2} + \frac{V_P}{R_L}$$

$$= 5mA + \frac{6.16V}{8\Omega}$$

$$= 775\,mA$$

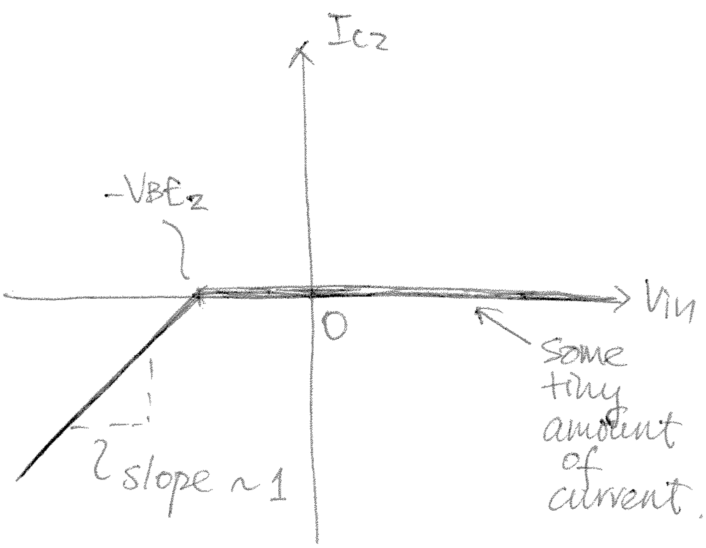
19.



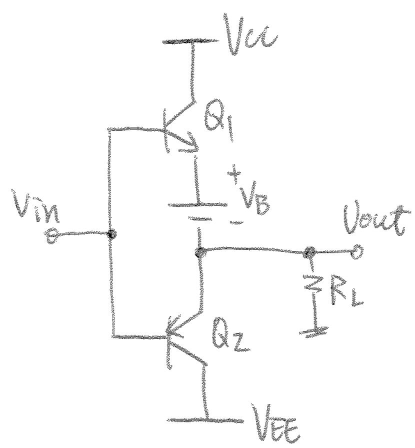


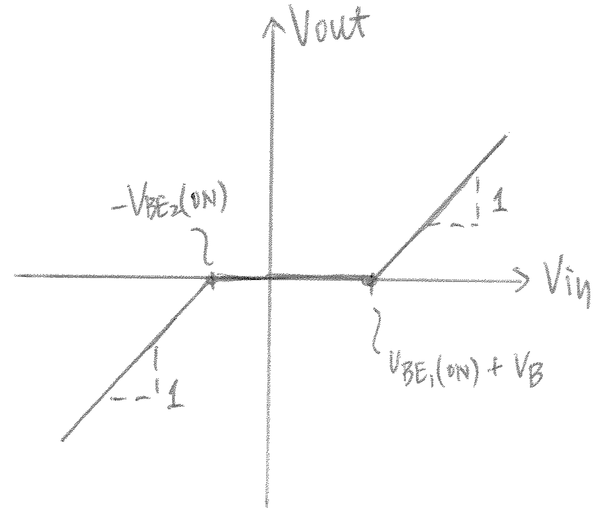
$$V_{out} = V_{in} + |V_{BE_2}|$$

$$\Rightarrow I_{C1} = I_{C2} + \frac{V_{out}}{R_L}$$



20.





- To analyze such circuit, assume $V_{out} = 0$:

$\Rightarrow \quad -V_{BE2(ON)} < V_{in} < V_{BE1(ON)} + V_B$.

$(V_{BE1(ON)} + V_B) < V_{in}$: $\quad V_{out} = V_{in} - V_{BE1(ON)} - V_B$

$V_{in} < -V_{BE2(ON)}$: $\quad V_{out} = V_{in} + |V_{BE2(ON)}|$

21. $V_{BE_1} + |V_{BE_2}| = V_{D_1} + V_{D_2}$

$$\Rightarrow V_T \left[\ln \frac{I_{C_1}}{I_{S_{Q1}}} + \ln \frac{I_{C_2}}{I_{S_{Q2}}} \right] = V_T \left[\ln \frac{I_{D_1}}{I_{S_{D_1}}} + \ln \frac{I_{D_2}}{I_{S_{D_2}}} \right]$$

$$\Rightarrow \quad \frac{I_{C_1} I_{C_2}}{I_{S_{Q_1}} I_{S_{Q_2}}} = \frac{I_{D_1} I_{D_2}}{I_{S_{D_1}} I_{S_{D_2}}}$$

$\therefore$ If $I_{S_{Q_1}} I_{S_{Q_2}} = I_{S_{D_1}} I_{S_{D_2}}$,

then $I_{C_1} I_{C_2} = I_{D_1} I_{D_2}$

22. $V_{BE_1} + |V_{BE_2}| = V_{D_1} + V_{D_2}$

$$\Rightarrow V_T \ln\left(\frac{I_{C_1} I_{C_2}}{I_{S,Q_1} I_{S,Q_2}}\right) = V_T \ln\left(\frac{I_{D_1} I_{D_2}}{I_{S,D_1} I_{S,D_2}}\right)$$

$$\Rightarrow \frac{I_{C_1} I_{C_2}}{I_{S,Q_1} I_{S,Q_2}} = \frac{I_{D_1} I_{D_2}}{I_{S,D_1} I_{S,D_2}} \quad \text{———} \quad ①$$

$I_1 = I_{D_1} = I_{D_2} = 1mA;\ I_{S,Q} = 16 I_{S,D}$

$V_{out} = 0 \Rightarrow I_{C_1} = I_{C_2}$ ——— ②

Substitute all into ①:

$$\frac{I_{C_1} I_{C_1}}{(16 I_{S,D})^2} = \frac{(1mA)^2}{(I_{S,D})^2} \Rightarrow I_{C_1} = I_{C_2} = 16\ mA$$

23. $V_{BE_1} + |V_{BE2}| = V_{D_1} + V_{D2}$

$$\Rightarrow \frac{I_{C_1} I_{C2}}{I_{S,Q_1} I_{S,Q2}} = \frac{I_{D_1} I_{D2}}{I_{S,D_1} I_{S,D2}} \qquad \text{———} \quad 1$$

$$I_{C_1} = I_{C_2} = 5mA \qquad \text{———} \quad 2$$

$$I_{S,Q} = 8 I_{S,D} \qquad \text{———} \quad 3$$

Substitute all into 1:

$$\frac{(5mA)^2}{(8 I_{S,D})^2} = \frac{I_{D_1} I_{D_2}}{(I_{S,D})^2} \Rightarrow I_1 = I_D = 0.625\ mA$$

24. $V_{BE_1} + |V_{BE_2}| = V_{D_1} + V_{D_2}$

$$\Rightarrow \frac{I_{C_1} I_{C_2}}{I_{S,Q_1} I_{S,Q_2}} = \frac{I_{D_1} I_{D_2}}{I_{S,D_1} I_{S,D_2}} \quad \text{———} \quad (1)$$

$I_1 = I_D = 2mA$

$I_{S,Q_1} = 8 I_{S,D_1}$; $I_{S,Q_2} = 16 I_{S,D_2}$

Substitute all into (1):

$$\frac{I_{C_1} I_{C_2}}{(8 I_{S,D_1})(16 I_{S,D_2})} = \frac{(2mA)^2}{I_{S,D_1} I_{S,D_2}}$$

$\Rightarrow I_{C_1} = I_{C_2} \cong 22.6\ mA$

25. $V_{BE_1} + |V_{BE_2}| = V_{D_1} + V_{D_2}$

$$\Rightarrow \frac{kT_Q}{q}\left[\ln\left(\frac{I_{C_1} I_{C_2}}{I_{S,Q_1} I_{S,Q_2}}\right)\right] = \frac{kT_D}{q}\left[\ln\left(\frac{I_{D_1} I_{D_2}}{I_{S,D_1} I_{S,D_2}}\right)\right]$$

Suppose $T_D = (T_Q + \Delta T)$:

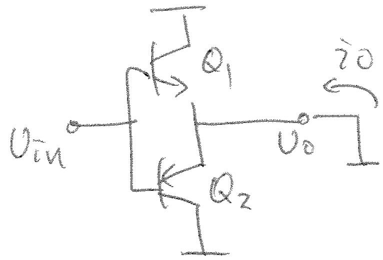
$$\Rightarrow T_Q\left[\ln\frac{I_{C_1} I_{C_2}}{I_{SQ_1} I_{SQ_2}} - \ln\frac{I_{D_1} I_{D_2}}{I_{SD_1} I_{SD_2}}\right] = \Delta T \cdot \ln\frac{I_{D_1} I_{D_2}}{I_{SD_1} I_{SD_2}}$$

$$\Rightarrow I_{C_1} \cdot I_{C_2} = I_{SQ_1} \cdot I_{SQ_2} \cdot \left(\frac{I_{D_1} I_{D_2}}{I_{SD_1} I_{SD_2}}\right)^{1+\frac{\Delta T}{T_Q}}$$

Typically, $\frac{I_{D_1} I_{D_2}}{I_{SD_1} I_{SD_2}} > 1$

$\Rightarrow$ A ΔT introduces a factor $\left(\frac{I_{D_1} I_{D_2}}{I_{SD_1} I_{SD_2}}\right)^{\frac{\Delta T}{T_Q}} < 0$, implying that the $I_{C_1} I_{C_2}$ product drops corresponding to a change (positive) in temperature.

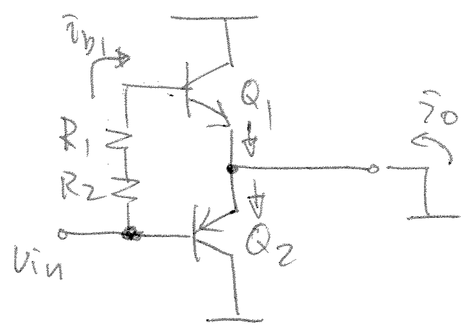
26. Small Signal:



$$G_m = \frac{i_o}{v_{in}} = -(g_{m1} + g_{m2})$$

$$\Rightarrow \frac{v_o}{v_{in}} = \frac{i_o R_L}{v_{in}} = +(g_{m1} + g_{m2}) R_L$$

27. Small-Signal:



$$i_o = -g_{m1} v_{be1} + g_{m2}|v_{be2}| \qquad \left(i_o = i_{c2} - i_{c1}\right)$$

$$|v_{be2}| = v_{in}$$

$$v_{be1} = v_{in} - i_{b1}(R_1+R_2) = v_{in} - \frac{i_{c1}}{\beta_1}(R_1+R_2)$$

$$= v_{in} - \frac{i_{c2} - i_o}{\beta_1}(R_1+R_2)$$

$$= v_{in} + \frac{g_{m2} v_{in} + i_o}{\beta_1}(R_1+R_2)$$

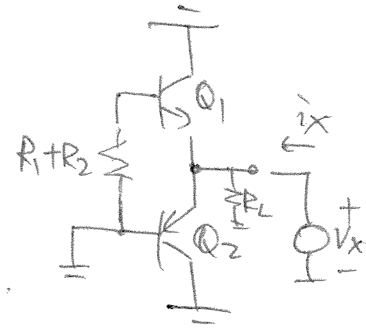
$$\therefore\ g_{m1}\left[v_{in} + \frac{g_{m2} v_{in} + i_o}{\beta_1}(R_1+R_2)\right] + i_o = -g_{m2} v_{in}$$

Solving for $\frac{i_o}{v_{in}}$ gives:

$$G_m = \frac{i_o}{v_{in}} = \frac{-\left[g_{m1} + \frac{g_{m1} g_{m2}}{\beta_1}(R_1+R_2) + g_{m2}\right]}{1 + \frac{g_{m1}(R_1+R_2)}{\beta_1}}$$

Rout:

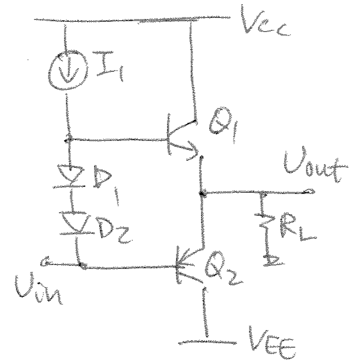
$$\frac{v_x}{i_x} = R_{out} = \left(r_{\pi 2} \parallel \frac{1}{g_{m2}}\right) \parallel \left[\left(r_{\pi 1} + R_1 + R_2\right) \parallel \frac{1}{g_{m1}}\right] \parallel R_L$$



$$\therefore A_v = G_m R_{out}$$

$$= -\left[\frac{g_{m1} + \frac{g_{m1} g_{m2} (R_1 + R_2)}{\beta_1} + g_{m2}}{1 + \frac{g_{m1}(R_1 + R_2)}{\beta_1}}\right] \cdot \left\{\left[r_{\pi 2} \parallel \frac{1}{g_{m2}}\right] \parallel \left[\left(r_{\pi 1} + R_1 + R_2\right) \parallel \frac{1}{g_{m1}}\right] \parallel R_L\right\}$$

28. Small signal gain around $V_{out} = 0$:



$R_L = 8\Omega$

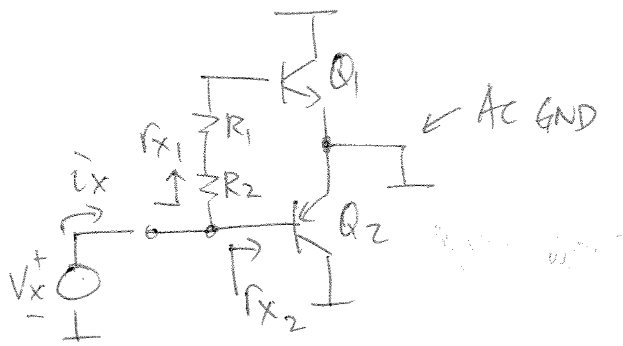
$$A_v = +(g_{m1} + g_{m2}) R_L$$

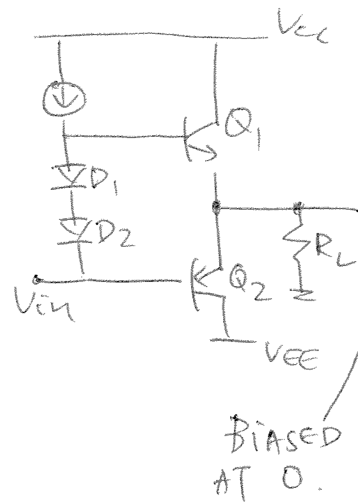
$$0.8 = (I_{C1} + I_{C2}) \frac{R_L}{V_T}$$

If $I_{C1} = I_{C2} = I_{BIAS}$, then

$$I_C = \frac{0.8}{2} \times \frac{V_T}{R_L} = 0.4 \frac{V_T}{R_L} = 0.01 \cdot R_L = 0.08\ A$$

29. Small-signal equivalent:





$$R_{in} = \frac{v_x}{i_x} = r_{x_1} \parallel r_{x_2}$$

$$= (R_1 + R_2 + r_{\pi_1}) \parallel r_{\pi_2}$$

- R_1 & R_2 can be neglected when $r_{\pi_1} \gg (R_1 + R_2)$

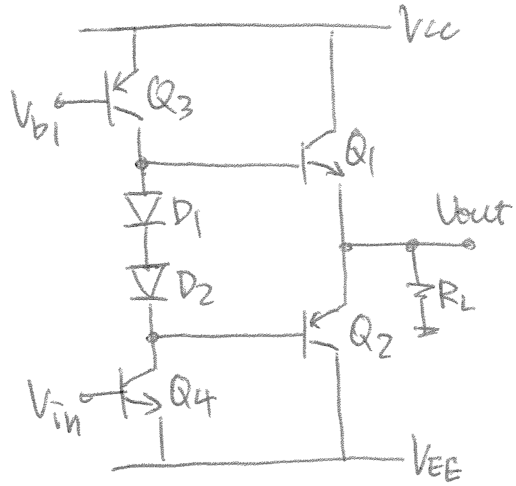
30. $I_{C_1} = I_{C_2} = 10\ mA$

$I_{C_3} = I_{C_4} = 1\ mA$

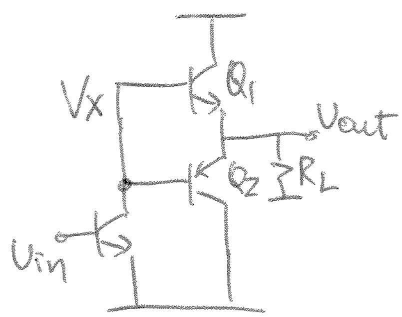
$\beta_1 = 40 \qquad \beta_2 = 20$

$R_L = 8\Omega$

$R_{D_1} = R_{D_2} = 0$



Small-signal



$$A_V = \frac{v_{out}}{v_x} \cdot \frac{v_x}{v_{in}}$$

$$= -g_{m4}\left[(g_{m1}+g_{m2})(r_{\pi_1} \| r_{\pi_2})R_L + (r_{\pi_1} \| r_{\pi_2})\right] \times \frac{R_L}{R_L + \frac{1}{g_{m1}+g_{m2}}}$$

$$= -g_{m4}(r_{\pi_1} \| r_{\pi_2})(g_{m1}+g_{m2})R_L$$

$$\therefore A_V = -\frac{I_{C4}}{V_T}\left(\frac{\beta_1 V_T}{I_{C1}} \| \frac{\beta_2 V_T}{I_{C2}}\right)\left(\frac{I_{C1}}{V_T} + \frac{I_{C2}}{V_T}\right)R_L$$

$$= -\frac{1mA}{0.026}\left[35\right] \cdot \left(2 \times \frac{10mA}{V_T}\right)(8)$$

$$\approx -8.3$$

31.

$$\frac{v_{out}}{v_{in}} = -g_{m4}(r_{\pi_1} \parallel r_{\pi_2})(g_{m1}+g_{m2})R_L \qquad \left(r_\pi = \frac{\beta}{g_m}\right)$$

When $g_{m1} \approx g_{m2}$: $(\Rightarrow r_\pi$

$$\frac{v_{out}}{v_{in}} \approx -g_{m4}R_L(2g_{m1})\left(\frac{\beta_1}{g_{m1}} \parallel \frac{\beta_2}{g_{m1}}\right)$$

$$= -g_{m4}R_L(2g_{m1})\left[\frac{1}{g_{m1}} \cdot \frac{\beta_1\beta_2}{\beta_1+\beta_2}\right]$$

$$= -\frac{2\beta_1\beta_2}{\beta_1+\beta_2} g_{m4}R_L$$

32. From lecture, small-signal gain of the output stage is:

$$\left|\frac{v_{out}}{v_{in}}\right| = +g_{m4}(r_{\pi 1} \parallel r_{\pi 2})(g_{m1}+g_{m2})R_L$$

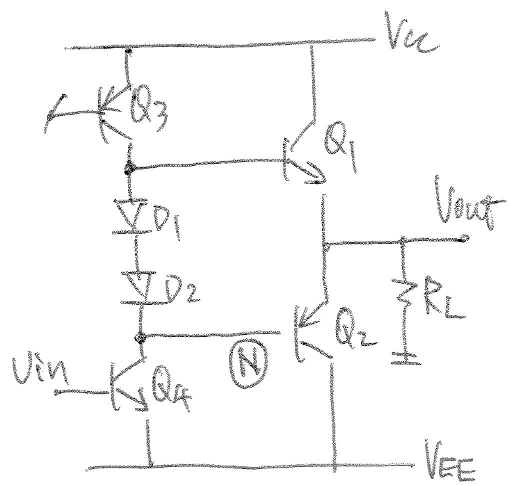
$$\approx +g_{m4}R_L \times \frac{2\beta_1\beta_2}{\beta_1+\beta_2}$$

$$\Rightarrow 4 = +\frac{I_{C4}}{V_T}(8\Omega) \times \frac{2(40)(20)}{40+20}$$

$$\Rightarrow I_{C4} \approx I_{C3}$$

$$= \frac{4V_T}{(8\Omega)} \cdot \frac{40+20}{2(40)(20)}$$

$$= 0.49\ mA$$



$A_V = \frac{v_{out}}{v_{in}} = 4$

$\beta_1 = 40$

$\beta_2 = 20$

$R_L = 8\,\Omega$

33. From lecture,

$$\frac{v_x}{i_x} = \frac{1}{g_{m1} + g_{m2}} + \frac{r_{o3} \parallel r_{o4}}{(g_{m1} + g_{m2})(r_{\pi 1} - r_{\pi 2})}$$

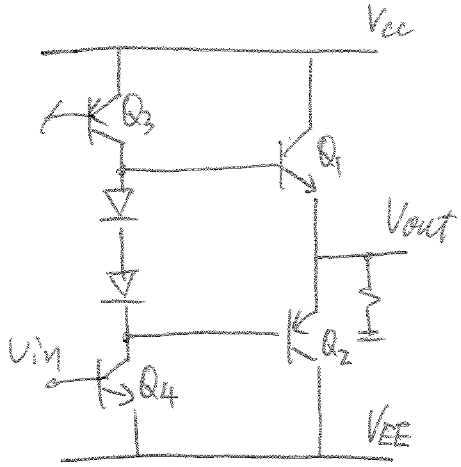
If $g_{m1} \approx g_{m2} = g_m$:

$$\frac{v_x}{i_x} \approx \frac{1}{2g_m} + \frac{r_{o3} \parallel r_{o4}}{2g_m\left(\frac{\beta_1}{g_m} \parallel \frac{\beta_2}{g_m}\right)}$$

$$= \frac{1}{2g_m} + \frac{r_{o3} \parallel r_{o4}}{2g_m\left(\frac{1}{g_m}\ \frac{\beta_1\beta_2}{\beta_1+\beta_2}\right)}$$

$$= \frac{1}{2g_m} + \frac{r_{o3} \parallel r_{o4}}{2\beta_1\beta_2}(\beta_1+\beta_2)$$

34.



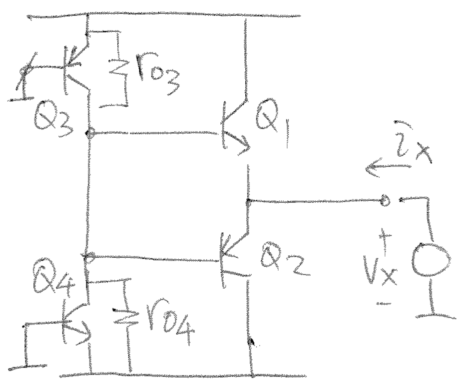
$I_3 = I_4 = 1\,mA$

$I_1 = I_2 = 8\,mA$

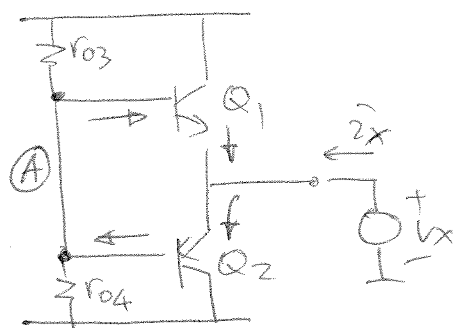
$V_{A3} = 10\,V$

$V_{A4} = 15\,V$

(a) Small-signal equivalent:



$\Rightarrow$



$$v_{eb} = v_x \frac{(r_{\pi_1} \parallel r_{\pi_2})}{(r_{\pi_1} \parallel r_{\pi_2}) + (r_{o3} \parallel r_{o4})}$$

$$v_{be} = v_A - v_x$$

$$i_x + i_{c_1} = i_{c_2} \Rightarrow i_x = i_{c_2} - i_{c_1} = g_{m_2} v_{eb} - g_{m_1} v_{be}$$

$$\therefore i_x = [g_{m_2} + g_{m_1}]\, v_x \frac{(r_{\pi_1} \parallel r_{\pi_2})}{(r_{\pi_1} \parallel r_{\pi_2}) + (r_{o3} \parallel r_{o4})}$$

$$\Rightarrow \frac{v_x}{i_x} = R_{out} = \frac{(r_{\pi_1} \parallel r_{\pi_2}) + (r_{o3} \parallel r_{o4})}{[g_{m_1} + g_{m_2}]\,(r_{\pi_1} \parallel r_{\pi_2})}$$

$$r_{\pi_1} = \frac{\beta_1 V_T}{I_{C1}} = 130\,\Omega \qquad r_{\pi_2} = \frac{\beta_2 V_T}{I_{C2}} = 65\,\Omega$$

$$r_{O3} = \frac{V_{A3}}{I_{C3}} = 10\,k\Omega \qquad r_{O4} = \frac{V_{A4}}{I_{C4}} = 15\,k\Omega$$

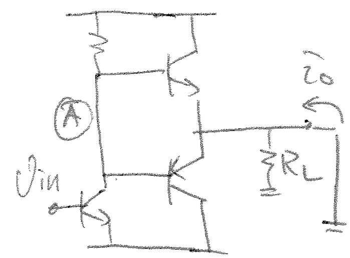
$$g_{m1} = 0.31\,S \qquad g_{m2} = 0.31\,S$$

$$\Rightarrow R_{out} = \frac{43.3 + 6000}{(0.62)(43.3)} \approx 6001\,\Omega$$

(b) Effective $R_{out} = R_{out,a} \parallel 8\,\Omega \approx 8\,\Omega$.

$$G_m = \frac{i_o}{v_A} \cdot \frac{v_A}{v_{in}}$$

$$= -g_{m4}(r_{\pi_1} \parallel r_{\pi_2} \parallel r_{o3}) \cdot (g_{m1} + g_{m2})$$



$$g_{m4} = \frac{I_{C4}}{V_T} = 0.038\,S$$

$$\therefore A_v = G_m R_{out}$$

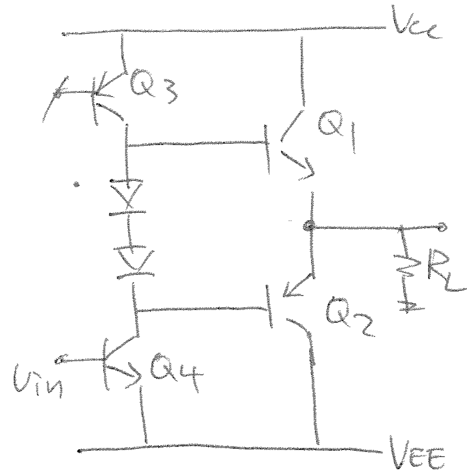
$$= -g_{m4}(r_{\pi_1} \parallel r_{\pi_2} \parallel r_{o3})(g_{m1} + g_{m2}) R_{out}$$

$$= -0.038[130 \parallel 65 \parallel 10k][0.62](8)$$

$$\approx -8.1$$

35. Max current delivered by Q_1 $= I_{C3} \cdot \beta_1 = 1mA \cdot 40$

$= 40\ mA$. (Q_4 off)



$I_{C3} = I_{C4} = 1mA$

$\beta_1 = 40 \quad \beta_2 = 20$

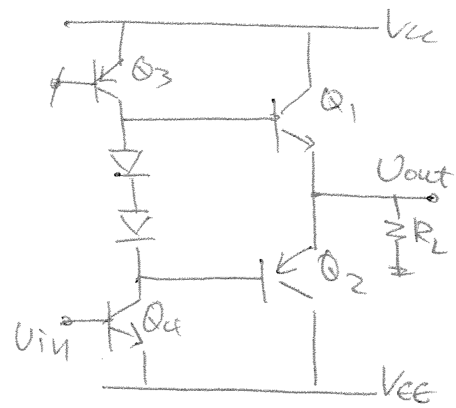
Max current delivered by Q_2 $= I_{C4} \cdot \beta_2$

$= 1mA \cdot 20$

$= 20\ mA$. (Q_3 off)

36. $P = 0.5\,W$ $\quad R_L = 8\,\Omega$

$\beta_1 = 40 \quad \beta_2 = 20.$



$$P_{AVG} = \frac{1}{2}\frac{V_p^2}{R_L} = 0.5$$

$$\Rightarrow V_p^2 = 2(0.5)R_L$$

$$\Rightarrow V_p = \sqrt{R_L} = 2\sqrt{2}$$

At positive V_p, $I_{C_1} = \frac{V_p}{R_L} = \frac{2\sqrt{2}}{8} = 0.35\,A.$

At negative V_p, $I_{C_2} = \frac{V_p}{R_L} \Rightarrow I_{C_2} = 0.35\,A.$

- At $+V_p$, all of I_{C3} supports the base current of Q_1

$$\Rightarrow I_{C3} = I_{B_1} = \frac{I_{C1}}{\beta_1} = \frac{0.35A}{40} = 8.75\,mA$$

- At $-V_p$, all of I_{C4} supports the base current of Q_2

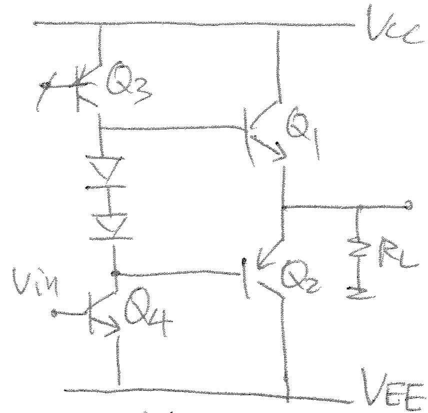
$$\Rightarrow I_{C4} = I_{B_2} = \frac{I_{C2}}{\beta_2} = \frac{0.35A}{20} = 17.5\,mA$$

37. $P_{AVG} = 0.5\,W \qquad R_L = 8\Omega$

$V_{cc} = 5V$

$\Rightarrow 0.5W = \frac{1}{2}\frac{V_p^2}{R_L}$

$\Rightarrow V_p = 2\sqrt{2}\ V$



(Assume negligible currents at Vout=0)

$$P_{Q_1} = \frac{1}{T}\int_0^{T/2} I_{C_1} V_{CE_1}\, dt$$

$$= \frac{1}{T}\int_0^{T/2} \left(\frac{V_p \sin\omega t}{R_L}\right)\cdot(V_{cc} - V_p \sin\omega t)\,dt$$

$$= \frac{1}{T}\int_0^{T/2} \left[\frac{V_{cc}V_p}{R_L}\sin\omega t - \frac{V_p^2}{2R_L}\right] dt$$

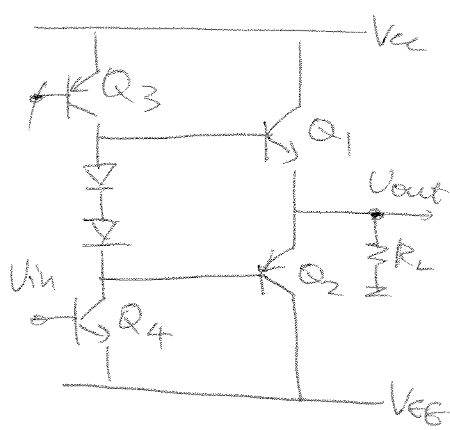
$$= \frac{V_p}{R_L}\left(\frac{V_{cc}}{\pi} - \frac{V_p}{4}\right) = \frac{2\sqrt{2}}{8}\left(\frac{5}{\pi} - \frac{2\sqrt{2}}{4}\right)$$

$$\approx 0.31\ W$$

38. $P_{Q,MAX} = 0.75W$

$R_L = 8\,\Omega$

$V_{CC} = 5V$



- Out of all 4 transistors, Q_1 & Q_2 must sustain the most currents.

$$\underset{(INST)}{P_{Q_1,MAX}} = V_{CE} \times I_{C_1,MAX} = (V_{CC} - V_{out})\, I_{C_1,MAX}$$

$$\Rightarrow \underset{(AVG)}{P_{Q_1,MAX}} = \frac{1}{T}\int_0^{T/2} \frac{V_P \sin\omega t}{R_L}\cdot(V_{CC} - V_P\sin\omega t)\, dt$$

$$= \frac{1}{T}\int_0^{T/2}\left(\frac{V_{CC}V_P}{R_L}\sin\omega t - \frac{V_P^2}{2R_L}\right) dt$$

$$= \frac{V_P}{R_L}\left(\frac{V_{CC}}{\pi} - \frac{V_P}{4}\right)$$

$$\Rightarrow \frac{dP_Q}{dV_P} = \frac{V_{CC}}{\pi R_L} - \frac{V_P}{2R_L}$$

$$= 0 \quad \text{when} \quad V_P = \frac{2V_{CC}}{\pi} = 3.18\,V$$

$$P_Q\Big|_{V_P = \frac{2V_{CC}}{\pi}} = 0.32\,W$$

$$\therefore P_{R_L,MAX} = \frac{1}{2}\frac{V_P^2}{R_L} = 0.63\,W$$

39. $P_{Q_1,MAX} = \left(\frac{V_{CC}}{\pi} - \frac{2V_{CC}}{4\pi} \right) \cdot \frac{2V_{CC}}{\pi R_L} \leq 0.75\ W$

$\Rightarrow V_{CC}|_{MAX} = 7.7\ V$

$\Rightarrow V_{P,MAX} = \frac{2V_{CC\,MAX}}{\pi} = 4.9\ V$

$\Rightarrow P_{R_L,MAX} = \frac{1}{2} \frac{V_{P\,max}^2}{R_L} = 1.5\ W$

40. $I_1 = I_{C1} + I_{E2}$

$$= I_{C1} + \frac{\beta_1+1}{\beta_1} I_{C2}$$

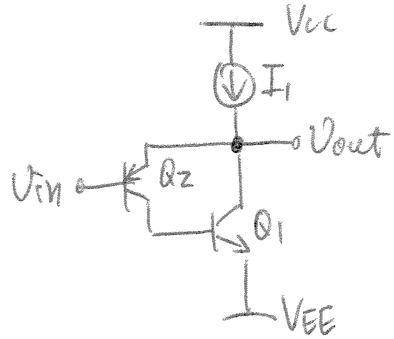
$$= I_{C1} + \frac{\beta_1+1}{\beta_1} I_{B1}$$

$$= \beta_1 I_{B1} + \frac{\beta_1+1}{\beta_1} I_{B1}$$

$$\Rightarrow I_{B1} = \frac{I_1}{\beta_1 + \frac{\beta_1+1}{\beta_1}} = \frac{0.005}{40 + \frac{41}{40}}$$

$$\approx 0.12 \, mA$$

$$\Rightarrow I_{B2} = \frac{I_{C2}}{\beta_2} = \frac{I_{B1}}{\beta_2} = 0.0024 \, mA$$



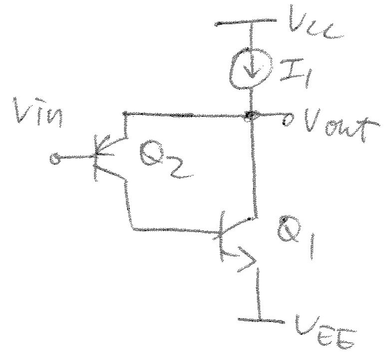
$I_1 = 5 mA$

$\beta_1 = 40$

$\beta_2 = 50.$

41. $V_{in} = 0.5\,V$

$I_{S2} = 6 \cdot 10^{-17}\,A.$



$$I_{B_1} = I_{C2} = 0.12\,mA$$

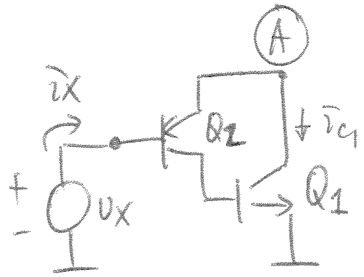
$$\Rightarrow I_{C2} = I_{S2} \cdot \exp\left(\frac{V_{out} - V_{in}}{V_T}\right)$$

$$\therefore V_{out} = V_T \ln\left(\frac{I_{C2}}{I_{S2}}\right) + V_{in}$$

$$= 0.026 \ln\left(\frac{0.12\,mA}{6 \cdot 10^{-17}A}\right) + 0.5$$

$$\approx 1.24\,V$$

42.



$i_{c2} = i_x \beta_2$

$i_{c1} = -g_{m2} v_{eb2} = i_{e2} = i_{c2} + i_{b2}$
$= i_x(\beta_2+1)$

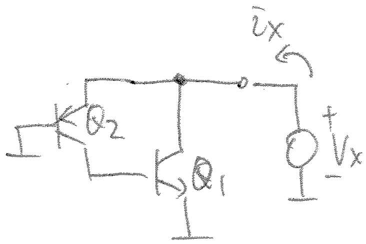
$v_{eb2} = v_A - v_x$

where $v_A = v_x - i_x r_{\pi 2}$

$\therefore\ i_{b1} = -g_{m2}(i_x \cdot r_{\pi 2})$

$i_{c1} = i_{b1} + i_{b1}\cdot\beta_1 = -g_m i_x \cdot r_{\pi 2}(1+\beta_1)$

$\Rightarrow \frac{v_x}{i_x} \to \infty$ (Rin)



$i_x = i_{e2} + i_{c1}$
$= i_{e2} + i_{b1}\beta_1$
$= i_{e2} + i_{c2}\beta_1$
$= i_{c2} + i_{b2} + i_{c2}\beta_1$
$= i_{c2}\left(1+\beta_1+\frac{1}{\beta_1}\right)$
$= v_x g_{m2}\left(1+\beta_1+\frac{1}{\beta_1}\right)$

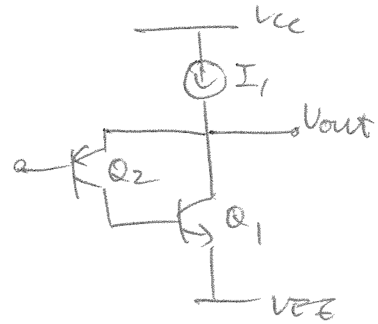
$g_{m2} = \frac{I_{B2}\beta_2}{V_T}$
$= 4.6$ S

$\Rightarrow R_{out} = \frac{v_x}{i_x} = \frac{1}{g_{m2}\left(1+\beta_1+\frac{1}{\beta_1}\right)}$

$= 0.005\ \Omega$.

43. $R_{out} = 1\,\Omega$.

$\beta_1 = 40 \quad \beta_2 = 50$.



$$R_{out} = 1 = \frac{1}{g_{m2}\left(1+\beta_1+\frac{1}{\beta_1}\right)}$$

$$\Rightarrow g_{m2} = 0.024\,S = \frac{I_{B2}\beta_2}{V_T}$$

$$\Rightarrow I_{B_2} = 0.012\ mA.$$

$$I_1 = I_{C_1} + I_{E2} = I_{B_1}\beta_1 + (I_{C2} + I_{B2})$$

$$= I_{C2}\beta_1 + I_{B2}(\beta_2+1)$$

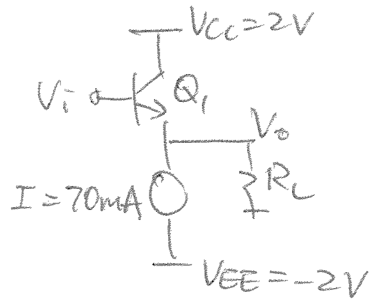
$$= I_{B2}\beta_2\beta_1 + I_{B_2}(\beta_2+1)$$

$$= 0.012\,[50\times40 + 50 + 1]$$

$$= 25.6\ mA$$

44. $V_P = 0.5V$

$R_L = 8\Omega$.



$$P_{R_L} = \frac{V_P^2}{2R_L} = \frac{0.25}{16} = 0.0156\ W$$

$$P_I = -I \times V_{EE} = 0.14\ W$$

$$P_{Q_1} = I_1 \left(V_{CC} - \frac{V_P}{2}\right) = 0.1225\ W$$

$$\therefore \eta = \frac{P_{R_L}}{P_{R_L} + P_I + P_{Q_1}} = \frac{0.0156}{0.2781} = 5.6\%$$

45. $P_{R_L} = \dfrac{V_P^2}{2R_L} = \dfrac{(V_{CC}-V_{BE})^2}{2R_L}$

$P_{Q_1} = I_1\left(V_{CC} - \dfrac{V_{CC}-V_{BE}}{2}\right)$

$P_I = +I_1|V_{EE}|$

◦ Assume $|V_{CC}| = |V_{EE}|$,

$I_1 = V_P/R_L$

$= \dfrac{V_{CC}-V_{BE}}{R_L}$

$$\therefore \eta = \frac{P_{R_L}}{P_{R_L}+P_{Q_1}+P_I} = \frac{\dfrac{(V_{CC}-V_{BE})^2}{2R_L}}{\dfrac{(V_{CC}-V_{BE})^2}{2R_L} + I_1\left[V_{CC} - \dfrac{V_{CC}-V_{BE}}{2} + |V_{EE}|\right]}$$

$$= \frac{\dfrac{1}{2R_L}}{\dfrac{1}{2R_L} + \dfrac{3V_{CC}-V_{BE}}{2R_L(V_{CC}-V_{BE})}}$$

$$= \frac{1}{1 + \dfrac{3V_{CC}-V_{BE}}{V_{CC}-V_{BE}}} \approx \frac{V_{CC}-V_{BE}}{3V_{CC}-V_{BE}}$$

46. $$\eta = \frac{\frac{V_p^2}{2R_L}}{\frac{V_p^2}{2R_L} + \frac{2V_p}{R_L}\left(\frac{V_{cc}}{\pi} - \frac{V_p}{4}\right)}$$

$$= \frac{\pi}{4}\frac{V_p}{V_{cc}}$$

$$\Rightarrow \eta\Big|_{V_p = V_{cc} - V_{BE}} = \frac{\pi}{4} - \frac{\pi}{4}\cdot\frac{V_{BE}}{V_{cc}}$$

47. $$\eta = \frac{\dfrac{(V_p/2)^2}{2R_L}}{\dfrac{(V_p/2)^2}{2R_L} + \dfrac{2(V_p/2)}{R_L}\left(\dfrac{V_{cc}}{\pi} - \dfrac{V_p/2}{4}\right)}$$

$$= \frac{V_p^2/8R_L}{\dfrac{V_p^2}{8R_L} + \dfrac{V_p}{R_L}\left(\dfrac{V_{cc}}{\pi} - \dfrac{V_p}{8}\right)} = \frac{1/8R_L}{1/8R_L + \dfrac{1}{R_L}\left(\dfrac{V_{cc}}{V_p\pi} - \dfrac{1}{8}\right)}$$

$$= \frac{1}{1 + \left(\dfrac{8V_{cc}}{V_p\pi} - 1\right)} = \frac{\pi}{8}\frac{V_p}{V_{cc}} \approx 39\%.$$

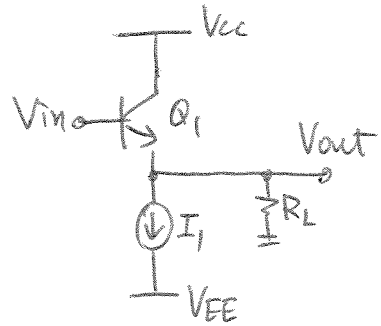
48. $V_{cc} = 3V$ $\quad P_{R_L} = 0.2W$ $\quad R_L = 8\Omega$.

$$P_{R_L} = \frac{1}{2}\frac{V_p^2}{R_L} \Rightarrow V_p = \sqrt{2P_{R_L} \times R_L} = 1.8\ V$$

$$\therefore \eta = \frac{P_{R_L}}{P_{R_L} + \frac{2V_p}{R_L}\left(\frac{V_{cc}}{\pi} - \frac{V_p}{4}\right)} = \frac{0.2}{0.2 + \frac{3.6}{8}\left(\frac{3}{\pi} - \frac{1.8}{4}\right)}$$

$$\approx 18\%.$$

49. Power = 1 W
$R_L = 8\Omega$



$$P_{LOAD} = \frac{1}{2}\frac{V_P^2}{R_L} = 1W$$

$\Rightarrow V_P = 4\,V \Rightarrow I_1 = \frac{V_P}{R_L} = 0.5\text{ mA}$

(Note: the problem does not specify small-signal voltage gain, so choose $V_P = I_1 R_L$)

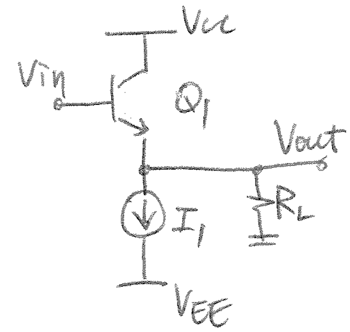
P_{Q_1} (power rating) $= I_1 (V_{CC})$

$= (0.5\text{mA})(5V)$

$= 2.5\text{ mW}$

50. $A_v = 0.8$

$R_L = 4\Omega$



$$A_v = \frac{R_L}{R_L + \frac{1}{g_{m_1}}} = \frac{4}{4 + \frac{0.026}{I_{C_1}}} = 0.8$$

$\Rightarrow I_{C_1} = 26 \text{ mA}$

$\therefore I_1 = I_{C_1} = 26\text{mA}$ (Vout biased at 0 V.)

Max Output Swing $= I_1 R_L$

$\approx (26\text{mA})(8\Omega)$

$= 0.208\text{V}$

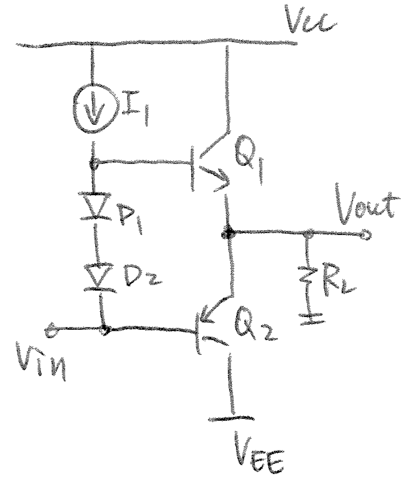
P_{Q_1} (power rating) $= I_1 V_{cc}$ ($V_p = 0$)

$= (26 \text{ mA})(5\text{V}) = 130 \text{ mW}$

51. $A_v = 0.6$

$R_L = 8\ \Omega$

$r_{D_1} = r_{D_2} = 0$



$$A_v = \frac{R_L}{R_L + \frac{1}{g_{m_1}}} = \frac{(8\Omega)}{(8\Omega) + \frac{0.026\,V}{I_{Q_1}}}$$

$$= 0.6$$

$$\Rightarrow I_{Q_1} = I_{Q_2} = 4.8\ mA$$

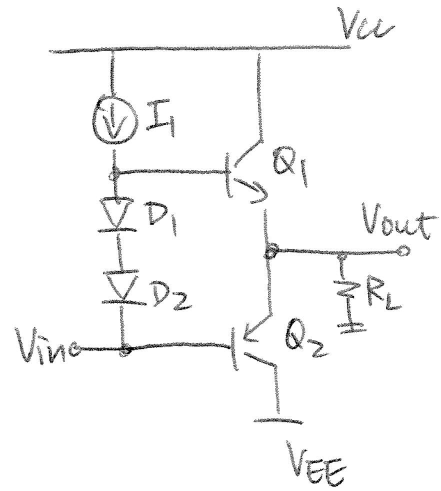
(V_{out} biased at 0 V.)

52. Power = 1 W (to load)

$R_L = 8\Omega$

$|V_{BE}| \approx 0.8$ V

$\beta_1 = 40$



$$P_L = \frac{1}{2}\frac{V_p^2}{R_L} = 1\text{ W}$$

$$\Rightarrow V_p = 4\text{ V}$$

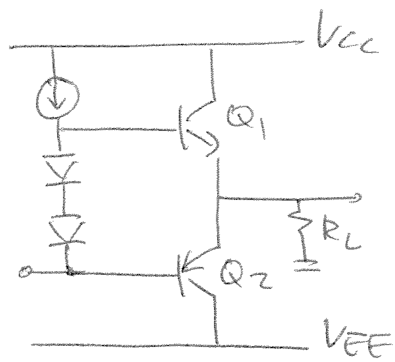
$\therefore$ Min allowable supply voltage $= V_p + |V_{BE}| = 4.8$ V

- At $+V_p$, all of I_1 goes to base of Q_1

$$\Rightarrow I_1 = I_{B_1} = \frac{I_{C_1}}{\beta_1} = \frac{V_p}{R_L}\cdot\frac{1}{\beta_1} \quad (Q_2 \text{ off})$$

$$= \frac{4}{8}\cdot\frac{1}{40} = \frac{1}{80} = 12.5\text{ mA}.$$

53. $P_{Q,MAX} = 2W$
$R_L = 8\Omega$.



For this circuit,

$$P_{AVG,MAX} = \frac{V_{CC}^2}{\pi^2 R_L} \quad \left(V_p = \frac{2V_{CC}}{\pi}\right)$$
$$= 2W$$

$\Rightarrow V_{CC}\big|_{MAX} = 12.6\ V \quad \Rightarrow V_p\big|_{max} = \frac{25.2}{\pi} = 8.02\ V$

$\therefore P_{R_L\ MAX} = \frac{V_{Pmax}^2}{2R_L} = \frac{(8.02)^2}{2 \cdot 8} = 4.02\ W$

54. For this circuit,

$P_{Q,MAX} = 2W$

$R_L = 4\Omega$

$$P_{AVG,MAX} = \frac{V_{CC}^2}{\pi^2 R_L} \qquad \left(V_P = \frac{2V_{CC}}{\pi}\right)$$

$$\Rightarrow V_{CC\,max} = \sqrt{\frac{\pi^2 R_L P_{Q\,MAX}}{1}} = 8.9\ V$$

$$\Rightarrow V_{P_{max}} = \frac{2V_{CC\,max}}{\pi} = 5.6\ V$$

$$\therefore P_{R_L,MAX} = \frac{V_{P,MAX}^2}{2R_L} = \frac{32}{2(4)} = 4W$$

55. $A_v = 4$ $R_L = 8\Omega$ $I_{C1} \approx I_{C2}$

$\beta_1 = 40$ $\beta_2 = 20$

Suppose we want 1st-stage (CE amplifier) to have gain $= 5 \Rightarrow$ 2nd stage gain $= 0.8$.

$$\Rightarrow 0.8 = \frac{R_L}{R_L + \frac{1}{g_{m1} + g_{m2}}}$$

$$0.8 = \frac{8}{8 + \frac{1}{2g_{m1}}} \Rightarrow g_{m1} = .5 \Rightarrow I_{C1} = I_{C2} = 6.5\,mA$$

$$r_{\pi 1} \| r_{\pi 2} = \frac{\beta_1 V_T}{I_{C1}} \| \frac{\beta_2 V_T}{I_{C2}} = \frac{40(0.026)}{6.5\,mA} \| \frac{20(0.026)}{6.5\,mA} \approx 133\Omega$$

- $A_v = 4 = g_{m4}(r_{\pi 1} \| r_{\pi 2})(g_{m1} + g_{m2}) R_L$

$$= \frac{I_{C4}}{V_T}(133)(0.5)\,8$$

$$\Rightarrow I_{C4} = I_{C3} = \frac{4 V_T}{8(133)(0.5)} = 0.195\,mA$$

Max I_{Q1} when all of I_{C3}/I_{C4} supports base current of Q_1

$\Rightarrow I_{Q1,\,MAX} = I_{C4} = 0.195\,mA$

56. $A_v = 4 \qquad R_L = 4\,\Omega \qquad I_{C_1} \approx I_{C_2}$
$\beta_1 = 40 \qquad \beta_2 = 20$

1st stage gain $= 5$ (CE amplifier)
2nd " " $= 0.8$

- $$0.8 = \frac{R_L}{R_L + \frac{1}{g_{m_1} + g_{m_2}}} = \frac{4}{4 + \frac{1}{2g_{m_1}}}$$

$\Rightarrow g_{m_1} = 0.5\,S^{1} \Rightarrow I_{C_1} = I_{C_2} = 13\,mA$

$$r_{\pi_1} \| r_{\pi_2} = \frac{\beta_1 V_T}{I_{C_1}} \| \frac{\beta_2 V_T}{I_{C_2}} = 80 \| 40 = 26.7\,\Omega$$

- $$A_v = 4 = g_{m_4}(r_{\pi_1} \| r_{\pi_2})(g_{m_1} + g_{m_2}) R_L$$
$$= \frac{I_{C4}}{V_T}(26.7)(1)(4)$$

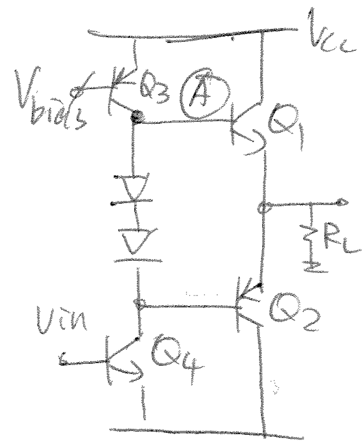
$$\Rightarrow I_{C4} = I_{C3} = \frac{4 V_T}{(26.7)(1)(4)} = 0.974\ mA.$$

- Max I_{Q_1} ($I_{Q_1 MAX}$) when $I_{C4} = I_{Q MAX} = 0.974 mA$.
- For a reduction of 2x the R_L, we have to provide ~5x current to base of Q_1. $\left(\frac{0.974}{0.195} \approx 5\right)$

57. $P_{R_L} = 2W$ $\beta_1 = 40$

$R_L = 8\,\Omega$ $\beta_2 = 20$

$|V_{BE}| = 0.8V$



(a) $P_{R_L} = \frac{1}{2}\frac{V_p^2}{R_L} \Rightarrow V_p \approx 5.6\ V$

- At $+V_p$, $V_A = V_p + |V_{BE}|$.
- For Q_3 in active region, $V_A \le V_{bias}$

$\Rightarrow V_{CC} \ge V_{bias} + |V_{BE}| = V_p + 2|V_{BE}|$

$\ge 5.6 + 1.6 = 7.2\ V.$

(b) $I_p = \frac{V_p}{R_L} = 0.7\ A.\ (\approx I_{E_1}),\ (\approx I_{E_2})$

$\Rightarrow I_{B_1} = \frac{I_{E_1}}{1+\beta_1} = 17\ mA.$

$\therefore$ We bias Q_3 & Q_4 with $I_C = 17\ mA$.

(c) $P_{AV} = \frac{V_P}{R_L}\left(\frac{V_{CC}}{\pi} - \frac{V_P}{4}\right)$

$= \frac{5.6}{8}\left(\frac{5}{\pi} - \frac{5.6}{4}\right) = 3.66\ W$

(d) $P_{IQ3} = 2V_{CC} \times I_{Q3} = 10 \times 17mA = 170\,mW$

$P_{AV,Q_1} = \frac{V_P}{R_L}\left(\frac{V_{CC}}{\pi} - \frac{V_P}{4}\right) = 3.66\ W$

$P_{R_L} = 2W$

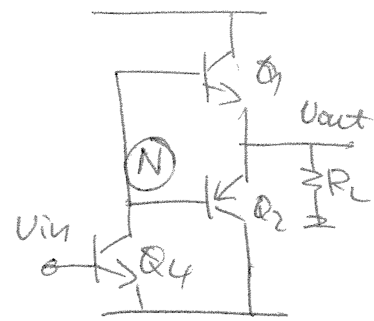
$\Rightarrow \eta = \frac{P_{R_L}}{P_{IQ3} + 2 \cdot P_{AV,Q_1} + P_{R_L}}$

$= \frac{2}{170m + 3.66 \times 2 + 2} = 0.21 = 21\%$

58.

(a) $A_v = 5 \quad R_L = 4\Omega \quad \beta_1 = 40 \quad \beta_2 = 20.$

Assume $I_{C1} \approx I_{C2}$.



$$\frac{v_{out}}{v_N} = \frac{R_L}{(g_{m1}+g_{m2})^{-1} + R_L} = 0.8$$

$$\Rightarrow 2g_{m1}^{-1} = 1 \Rightarrow I_{C1} = 2V_T$$

$$= 0.052\ A.$$

$$\Rightarrow \frac{v_{out}}{v_{in}} = +g_{m4}(r_{\pi1} \| r_{\pi2})(g_{m1}+g_{m2})R_L = 5$$

Assume $g_{m1} \approx g_{m2}$:

$$\Rightarrow I_{C4} = V_T \frac{5}{(r_{\pi1} \| r_{\pi2})(g_{m1}+g_{m2})R_L}$$

$$= V_T \times \frac{5}{(r_{\pi1} \| r_{\pi2})(g_{m1} \times 2)R_L}$$

$$= 0.026 \frac{5}{(6.7\Omega)(2\times 2)(4)}$$

$$\approx 1.2\ mA.$$

$\Rightarrow$ Max I by $Q_1 = \beta_1 \times I_{C4} = 48\ mA$

$\Rightarrow P_{R_L} = \frac{1}{2} I^2 R_L = 24 \times 4\ mW = 96\ mW$, BELOW requirement!

(b) $P = 5\,W = \frac{1}{2}\frac{V_p^2}{R_L} \Rightarrow V_p = 6.3\,V$

$\Rightarrow I_p = \frac{V_p}{R_L} = 1.6\,A$

$\Rightarrow I_{B2,MAX} = \frac{I_p}{\beta_2} = \frac{1.6}{20} = 79\ mA$

$\Rightarrow$ I_{C4} must equal 79 mA to allow max output swing V_p

$\Rightarrow g_{m4} = \frac{I_{C4}}{V_T} = 3.04\ \mho$

Suppose 2nd stage gain $= 0.8$ $(I_{C1} \simeq I_{C2})$

$\Rightarrow \frac{v_{out}}{v_N} = \frac{R_L}{R_L + \frac{1}{g_{m1}+g_{m2}}} = 0.8$ $\Rightarrow g_{m1} = 0.5\ \mho$

$\Rightarrow I_{C1} = I_{C2} = 13\ mA.$

$r_{\pi 1} \| r_{\pi 2} = \frac{\beta_1 V_T}{I_{C1}} \| \frac{\beta_2 V_T}{I_{C2}} = 26.7\,\Omega.$

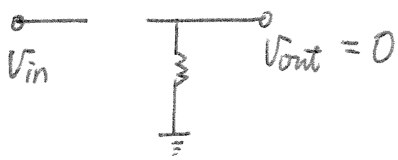
$\therefore \frac{v_{out}}{v_{in}} = -(3.04)(26.7\,\Omega)(0.5+0.5)4$

$= -324$!! (Huge! Impractical)

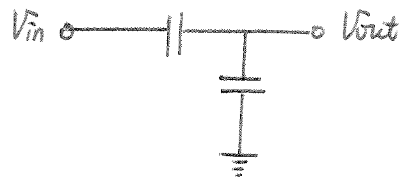
- Even when the 2nd stage gets close to 1, we still need huge gain from first stage.

Ch. 14

1. (a).



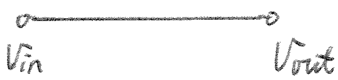
(Low freq.)



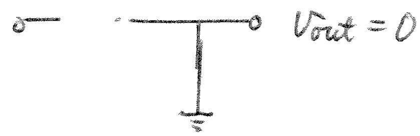
(High freq.)

This is a high pass filter.

(b).



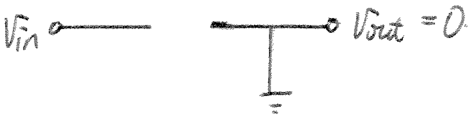
(Low freq.)



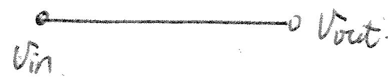
(High freq.)

This is a low pass filter.

(c).



(Low freq)

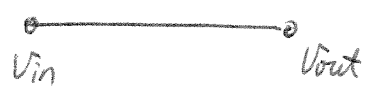


(High freq.).

This is a high pass filter.

(d)

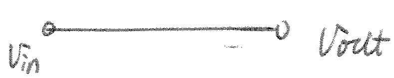
(Low freq.)



(High freq.)

This is a high pass filter.

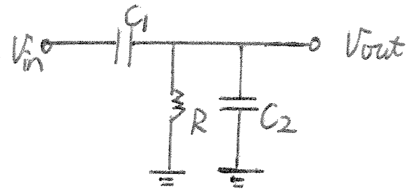
(e)



(Low freq.).

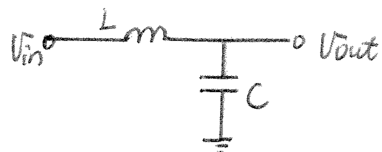
(High freq.)

This is a low pass filter.

2. (a). 

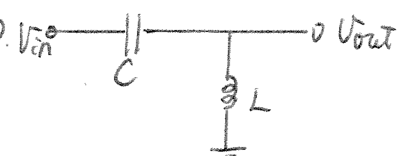
$$V_{out} = \frac{R // \frac{1}{sC_2}}{\frac{1}{sC_1} + R // \frac{1}{sC_2}} V_{in}.$$

$$\frac{V_{out}}{V_{in}} = \frac{\frac{C_1}{C_1+C_2} s}{s + \frac{1}{R(C_1+C_2)}}, \quad zero = 0; \quad pole = -\frac{1}{R(C_1+C_2)}$$

(b) 

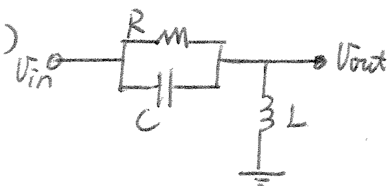
$$V_{out} = \frac{\frac{1}{sC}}{sL + \frac{1}{sC}} V_{in}$$

$$\frac{V_{out}}{V_{in}} = \frac{1/(LC)}{s^2 + \frac{1}{LC}}$$ zero: No finite zero; poles $= \pm i \frac{1}{\sqrt{LC}}$

(c) 

$$V_{out} = \frac{sL}{sL + \frac{1}{sC}} V_{in}$$

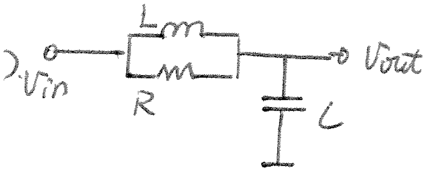
$$\frac{V_{out}}{V_{in}} = \frac{s^2 \cancel{LC}}{s^2 + \frac{1}{LC}}$$ zeros: Two zeros at 0; poles $= \pm i \frac{1}{\sqrt{LC}}$

(d) 

$$V_{out} = \frac{sL}{sL + R // \frac{1}{sC}} V_{in}$$

$$\frac{V_{out}}{V_{in}} = \frac{s(s + \frac{1}{RC})(RC)^2}{s^2 + \frac{s}{RC} + \frac{1}{LC}}.$$ zeros: $0, -\frac{1}{RC}$

poles $= \dfrac{-\frac{1}{RC} \pm \sqrt{(\frac{1}{RC})^2 - \frac{4}{LC}}}{2}$

(e). 

$$V_{out} = \frac{\frac{1}{sC}}{\frac{1}{sC} + sL // R} V_{in}$$

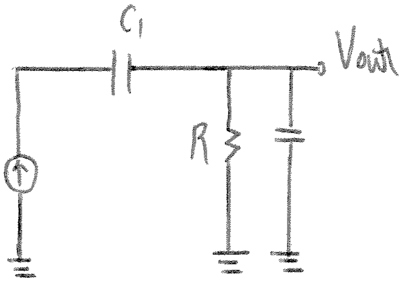
$$\frac{V_{out}}{V_{in}} = \frac{\frac{1}{RC}(s + \frac{R}{L})}{s^2 + \frac{s}{RC} + \frac{1}{LC}},$$ zero $= -\frac{R}{L}$; poles $= \dfrac{-\frac{1}{RC} \pm \sqrt{(\frac{1}{RC})^2 - \frac{4}{LC}}}{2}$.

3.

Since $\frac{V_{out}}{V_{in}} = \frac{1}{(s+a)(s+b)}$, where a and b are real and positive, the transfer function contains no finite zero and two real poles on the left hand plane. But, after reviewing Problem #2 we discover that NONE of the networks yield this case.

4.

a)

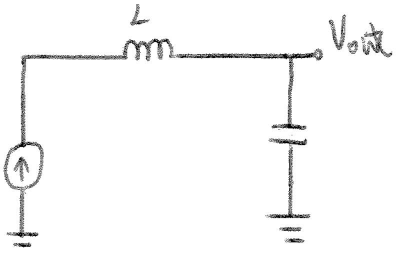


$$\frac{V_{out}}{I_{in}} = \frac{1/C_2}{s + 1/(RC_2)}$$

Zero: No finite zero

Pole: $-1/(RC_2)$

b)

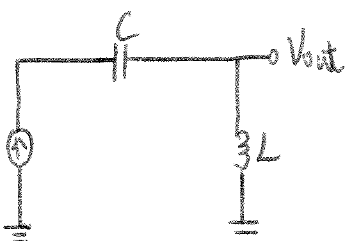


$$\frac{V_{out}}{I_{in}} = \frac{1}{sC}$$

Zero: No finite zero

Pole: 0

c)

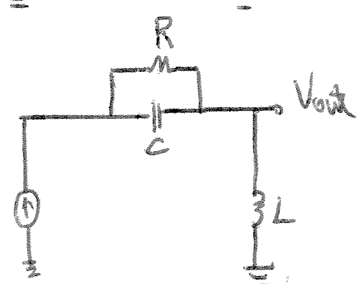


$$\frac{V_{out}}{I_{in}} = sL$$

Zero: 0

Pole: No finite pole

d)

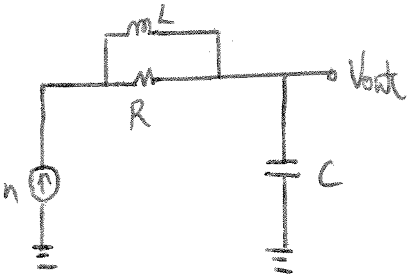


$$\frac{V_{out}}{I_{in}} = sL$$

Zero: 0

Pole: No finite pole

e)

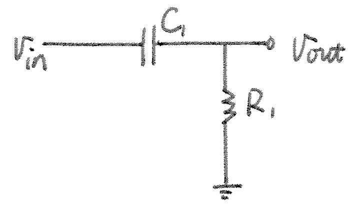


$$\frac{V_{out}}{I_{in}} = \frac{1}{sC}$$

Zero: No finite zero

Pole: 0

5).



$$\frac{V_{out}}{V_{in}} = \frac{R_1}{R_1 + \frac{1}{C_1 S}} = \frac{S}{S + \frac{1}{R_1 C_1}}$$

$$\text{zero} = 0; \quad \text{pole} = -\frac{1}{R_1 C_1}.$$

$$\frac{dP}{dC_1} = \frac{1}{(R_1 C_1)^2} \cdot R_1 = \frac{1}{R_1 C_1^2}.$$

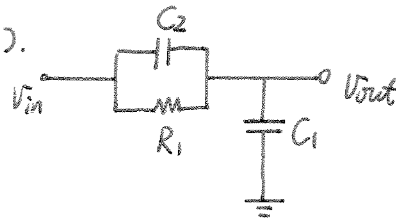
$$S_{C_1}^{P} = \frac{\frac{dP}{P}}{\frac{dC_1}{C_1}} = \frac{dP}{dC_1} \cdot \frac{C_1}{P} = -\frac{1}{R_1 C_1^2} \cdot C_1 \cdot (R_1 C_1) = -1.$$

Similarly

$$S_{R_1}^{P} = -1.$$

As for the sensitivity of zero, since the zero is at 0, which is indepedant of R_1 and C_1, $S_{R_1}^{z} = S_{C_1}^{z} = 0$.

6).



$$\frac{V_{out}}{V_{in}} = \frac{\frac{1}{C_1 S}}{\frac{1}{C_1 S} + R_1 // \frac{1}{SC_2}}.$$

$$= \frac{1}{R_1 C_2} \cdot \frac{S + \frac{1}{R_1 C_2}}{R_1(C_1+C_2)\left[S + \frac{1}{R_1(C_1+C_2)}\right]}$$

$$\text{zero} = -\frac{1}{R_1 C_2}, \qquad \text{pole} = -\frac{1}{R_1(C_1+C_2)}$$

$$\frac{dP}{dR_1} = [R_1(C_1+C_2)]^{-2} \cdot (C_1+C_2) = -\frac{C_1+C_2}{R_1(C_1+C_2)} \cdot P. = -\frac{P}{R_1}$$

$$S_{R_1}^{P} = \frac{\frac{dP}{P}}{\frac{dR_1}{R_1}} = \frac{dP}{dR_1} \cdot \frac{R_1}{P} = -1.$$

$$\frac{dP}{dC_1} = [R_1(C_1+C_2)]^{-2} \cdot R_1 = -\frac{P}{C_1+C_2}$$

$$S_{C_1}^{P} = \frac{\frac{dP}{P}}{\frac{dC_1}{C_1}} = \frac{dP}{dC_1} \cdot \frac{C_1}{P} = -\frac{C_1}{C_1+C_2}$$

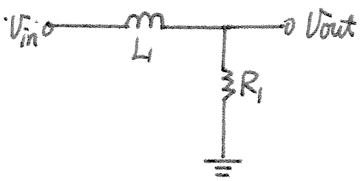
Conversely,

$$S_{C_2}^{P} = -\frac{C_2}{C_1+C_2}.$$

From Problem 5).

$$S_{R_1}^{z} = S_{C_2}^{z} = -1.$$

7)



$$\frac{V_{out}}{V_{in}} = \frac{R_1}{R_1 + L_1 S} = \frac{R_1/L_1}{S + R_1/L_1}$$

$$\text{pole} = -\frac{R_1}{L_1}$$

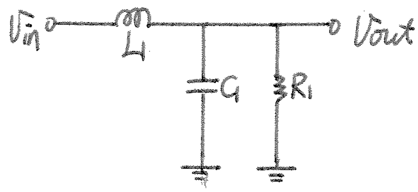
$$dp = \frac{\partial p}{\partial R_1} \cdot dR_1 + \frac{\partial p}{\partial L_1} dL_1 = -\frac{1}{L_1} dR_1 + \frac{R}{L^2} dL$$

$$\Rightarrow \frac{dp}{p} = \frac{dR_1}{R_1} - \frac{dL_1}{L_1}$$

$$\left|\frac{dp}{p}\right| \le 5\%, \quad \text{and} \quad \left|\frac{dR_1}{R_1}\right| \le 3\%$$

$$\Rightarrow \left|\frac{dL_1}{L_1}\right| \le 2\%$$

8).



a). $V_{out} = V_{in} \cdot \dfrac{R_1 \parallel \frac{1}{C_1 S}}{R_1 \parallel \frac{1}{C_1 S} + SL_1}$

$= V_{in} \cdot \dfrac{\frac{R_1}{R_1 C_1 S + 1}}{\frac{R_1}{R_1 C_1 S + 1} + SL_1}$

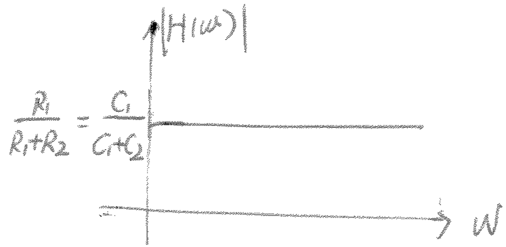
$$\frac{V_{out}}{V_{in}} = \frac{R_1}{R_1 C_1 L_1 S^2 + L_1 S + R_1} = \frac{1}{L_1 C_1} \cdot \frac{1}{S^2 + \frac{1}{R_1 C_1} S + \frac{1}{L_1 C_1}}$$

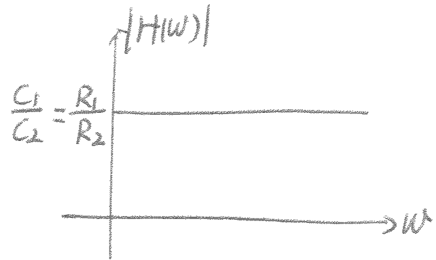
b). $\text{poles} = \dfrac{-\frac{1}{R_1 C_1} \pm \sqrt{\left(\frac{1}{R_1 C_1}\right)^2 - \frac{4}{L_1 C_1}}}{2}.$

For them to be real $\Rightarrow \left(\frac{1}{R_1 C_1}\right)^2 - \frac{4}{L_1 C_1} \geqslant 0$

$\Rightarrow \quad \dfrac{1}{R_1 C_1} \geqslant \dfrac{2}{\sqrt{L_1 C_1}}$

9). If the zero and pole coincide, they will neutralize each other, and also render the transfer function flat.





10).

$$H(s) = \frac{\alpha s^2 + \beta s + \gamma}{s^2 + \frac{w_n}{Q} s + w_n^2}$$

$$P_{1,2} = -\frac{w_n}{2Q} \pm j w_n \sqrt{1 - \frac{1}{4Q^2}}$$

If $Q = \frac{1}{2}$, then $P_{1,2} = -\frac{w_n}{2Q}$.

11.

$$|H(j\omega)|^2 = \frac{r^2}{(\omega_n^2-\omega^2)^2+\left(\frac{\omega_n}{Q}\omega\right)^2}$$

No peaking means no local minimum for $(\omega_n^2-\omega^2)^2+\left(\frac{\omega_n}{Q}\omega\right)^2$, which is also known as $D(\omega)$.
A local min exists if $\frac{\partial D(\omega)}{\partial \omega} = 0$.

$$\frac{\partial D(\omega)}{\partial \omega} = \left(\frac{\partial D(\omega)}{\partial \omega^2}\right)\left(\frac{\partial \omega^2}{\partial \omega}\right), \quad \frac{\partial D(\omega)}{\partial \omega^2} = -2(\omega_n^2-\omega^2)+\left(\frac{\omega_n}{Q}\right)^2$$

$$\frac{\partial \omega^2}{\partial \omega} = 2\omega, \text{ so } \frac{\partial D(\omega)}{\partial \omega} = 2\omega\left[2(\omega^2-\omega_n^2)+\left(\frac{\omega_n}{Q}\right)^2\right]=0$$

Solving for ω, we have $\omega = 0, \pm\sqrt{\omega_n^2-\frac{1}{2}\left(\frac{\omega_n}{Q}\right)^2}$

Will bring $D(\omega)$ to its min value.
At $\omega=0$, we have the DC value of the transfer function.
However if $Q^2<\frac{1}{2}$ or $Q<\frac{1}{\sqrt{2}}$, $\omega_n^2-\frac{1}{2}\left(\frac{\omega_n}{Q}\right)^2$ becomes negative, which is not physical. Therefore, there is no peaking for $Q<\frac{1}{\sqrt{2}}$. And at $Q=\frac{1}{\sqrt{2}}$, we have $\omega=\pm 0$, which corresponding to the DC value of the transfer function, not peaking. Therefore, the only option left is for $Q>\frac{1}{\sqrt{2}}$, and that is the condition for peaking.

12). $|H(j\omega)|^2 = \dfrac{\gamma^2}{(\omega_n^2-\omega^2)^2 + (\frac{\omega_n}{Q}\omega)^2}$

If $Q > \sqrt{2}/2$, it will peak at $\omega_0 = \omega_n\sqrt{1-1/(2Q)^2}$

$$H(j\omega) = \frac{\gamma}{\sqrt{(\omega_n^2-\omega^2)^2 + (\frac{\omega_n}{Q}\omega)^2}}$$

$$\Rightarrow H(j\omega_0) = \frac{\gamma}{\sqrt{\left[\omega_n^2 - (\omega_n\sqrt{1-\frac{1}{2Q^2}})^2\right]^2 + (\frac{\omega_n}{Q}\omega_n\sqrt{1-\frac{1}{2Q^2}})^2}}$$

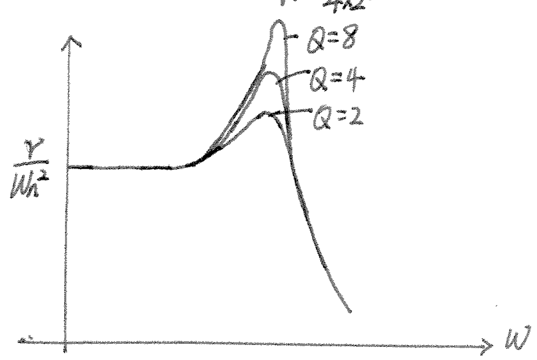
$$= \frac{\gamma}{\sqrt{(\omega_n^2\cdot\frac{1}{2Q^2})^2 + \frac{\omega_n^4}{Q^2}(1-\frac{1}{2Q^2})}}$$

$$= \frac{\gamma}{\sqrt{\frac{\omega_n^4}{4Q^4} + \frac{\omega_n^4}{Q^2} - \frac{\omega_n^4}{2Q^4}}}$$

$$= \frac{Q\gamma}{\omega^2\sqrt{1-\frac{1}{4Q^2}}}$$

Normalize to passband $\Rightarrow \dfrac{Q}{\sqrt{1-\frac{1}{4Q^2}}}$

$Q=2$, peak $= \dfrac{2}{\sqrt{1-\frac{1}{4\times2^2}}} = 2.07$; $Q=4$, peak $=4.03$; $Q=8$, peak $=8.02$



13).

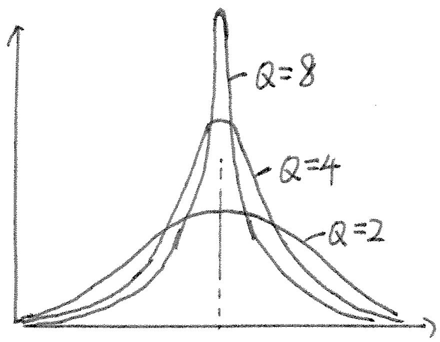
$$H(s) = \frac{\beta s}{s^2 + \frac{\omega_n}{Q} s + \omega_n^2}, \qquad H(j\omega) = \frac{j\beta\omega}{\omega_n^2 + j\frac{\omega_n}{Q}\omega - \omega^2}$$

$$|H(j\omega)| = \frac{\beta\omega}{\sqrt{(\omega_n^2 - \omega^2)^2 + (\frac{\omega_n}{Q}\omega)^2}}$$

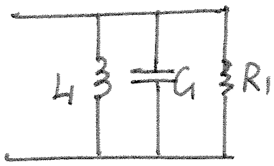
At $\omega = \omega_n$,

$$|H(j\omega_n)| = \frac{\beta\omega_n}{\frac{\omega_n}{Q}\omega} = \frac{Q}{\omega_n}\beta.$$

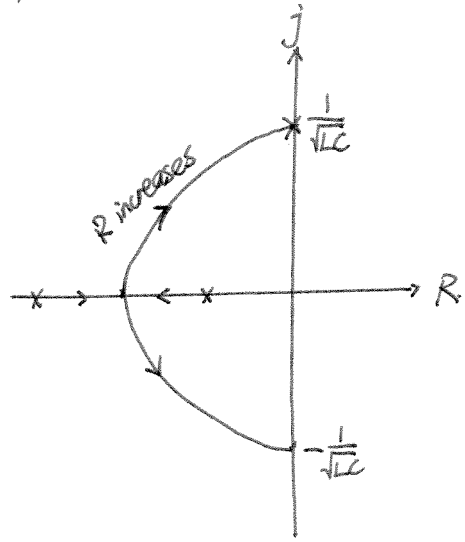
So if we normalize to β, we get $\frac{Q}{\omega_n}$.



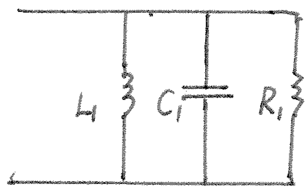
14).

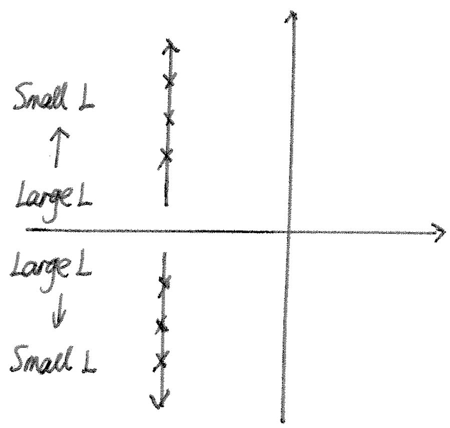


Assume R is never negative



15).





16.

1 dB peaking $\Rightarrow \dfrac{Q^2}{\left(1-\frac{1}{4Q^2}\right)} = (1.1)^2 = 1.21$

$Q^2 = 1.21\left(1-\frac{1}{4Q^2}\right) \Rightarrow 4Q^4 - 4(1.1)^2Q^2 + (1.1)^2 = 0$

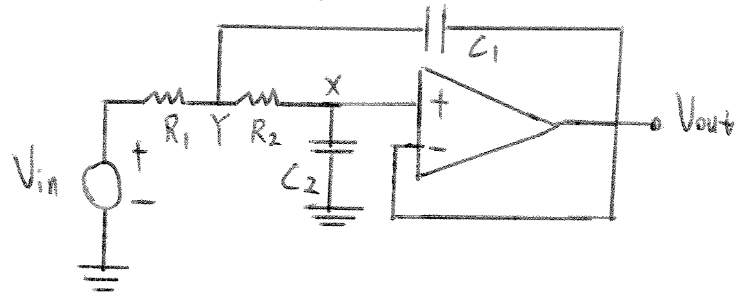
$Q^2 = 0.85704,\ 0.35296,\ Q = 0.925765,\ 0.59410$

$Q = 0.925765$, since $Q > \frac{1}{\sqrt{2}}$ for peaking

$Q = \dfrac{\omega_n}{\beta} = \dfrac{RC}{\sqrt{LC}} = R\sqrt{\dfrac{C}{L}} = 0.925765$

17.

Sallen and Key filter



$$H(s) = \frac{1}{R_1 R_2 C_1 C_2 s^2 + (R_1+R_2) C_2 s + 1} \ , \quad Q = \frac{1}{R_1+R_2}\sqrt{R_1 R_2 \frac{C_1}{C_2}} \ ,$$

$$\omega_n = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}} \ ,$$

$$H(s) = \left(s^2 + \frac{(R_1+R_2)C_2}{R_1 R_2 C_1 C_2} s + \frac{1}{R_1 R_2 C_1 C_2} \right)^{-1}$$

$$P_{1,2} = -\frac{(R_1+R_2)}{R_1 R_2 C_1} \pm \sqrt{\left(\frac{R_1+R_2}{R_1 R_2 C_1}\right)^2 - \frac{4}{R_1 R_2 C_1 C_2}}$$

$$P_{1,2} = -\frac{1}{2(R_1 /\!/ R_2) C_1} \pm \sqrt{\left(\frac{1}{(R_1 /\!/ R_2) C_1}\right)^2 - \frac{4}{R_1 C_1 R_2 C_2}}$$

Assuming $\frac{4}{R_1 C_1 R_2 C_2} > \frac{1}{[(R_1 /\!/ R_2) C_1]^2}$

$$P_{1,2} = -\frac{1}{2(R_1 /\!/ R_2) C_1} \pm j2\sqrt{\frac{1}{R_1 C_1 R_2 C_2} - \frac{1}{4[(R_1 /\!/ R_2) C_1]^2}}$$

17.

a) $R_1: 0 \to \infty$

When $R_1 = 0$, poles are at $\pm\infty$, so no finite poles. As $R_1 \uparrow$, $\frac{1}{R_1 C_1 R_2 C_2}$ approaches 0, and

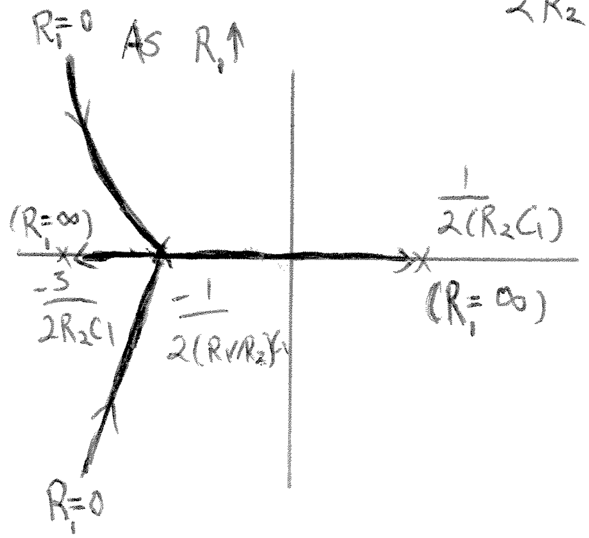
$\frac{1}{4[(R_1 // R_2)C_1]^2}$ approaches $\frac{1}{4[R_2 C_1]^2}$. There exists

a R_1 such that $\frac{1}{R_1 C_1 R_2 C_2} = \frac{1}{4[(R_1 // R_2)C_1]^2} \Rightarrow$

$$P_{1,2} = -\frac{1}{2(R_1 // R_2)C_1}$$

As $R_1 \to \infty$, $P_{1,2} = -\frac{1}{2R_2C_1} \pm \frac{1}{R_2C_1} = -\frac{3}{2R_2C_1}, \frac{1}{2R_2C_1}$

As $R_1 \uparrow$



17. b)

R_2 from $0 \to \infty$

when $R_2 = 0$, $P_{1,2}$ are at $\pm\infty$

As $R_2 \uparrow$, $\dfrac{-1}{2(R_1 /\!/ R_2)C_1} \to -\dfrac{1}{2R_1C_1}$

$\dfrac{1}{R_1C_1R_2C_2} \to 0$, and $\dfrac{1}{4[(R_1/\!/R_2)C_1]^2} \to \dfrac{1}{4[R_1C_1]^2}$

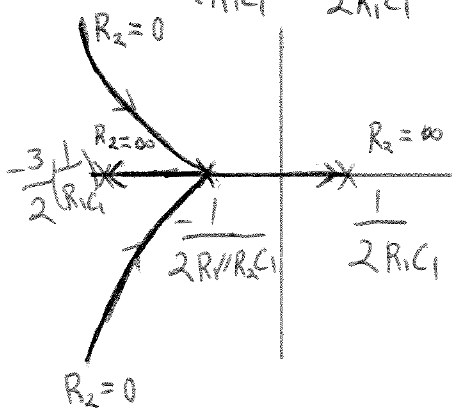
For a certain R_2, $\dfrac{1}{R_1C_1R_2C_2} = \dfrac{1}{4[(R_1/\!/R_2)C_1]^2}$

and $P_{1,2} = \dfrac{-1}{2(R_1/\!/R_2)C_1}$

Finally, when $R_2 = \infty$,

$$P_{1,2} = -\frac{1}{2R_1C_1} \pm 2\sqrt{\frac{1}{4[R_1C_1]^2}} = -\frac{1}{2R_1C_1} \pm \frac{1}{R_1C_1}$$

$$P_{1,2} = -\frac{3}{2}\frac{1}{R_1C_1}, \quad \frac{1}{2R_1C_1}$$

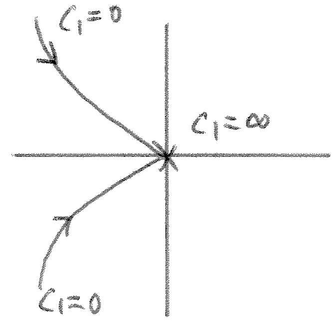


17. c)

As $C_1 : 0 \to \infty$

When $C_1 = 0$, Poles are at $\pm\infty$

As $C_1 \uparrow$, Poles approach 0.

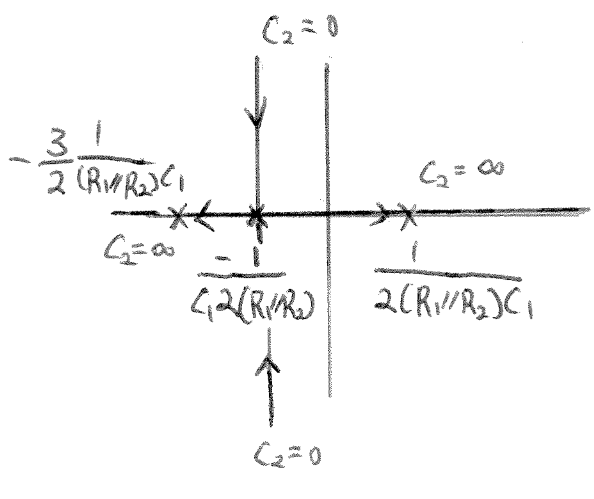


d) As $C_2 : 0 \to \infty$

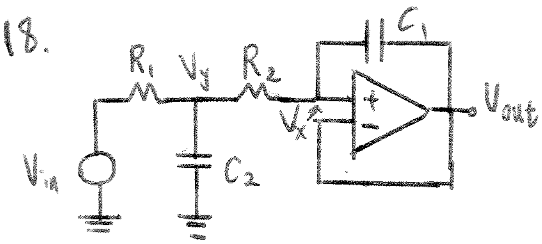
When $C_2 = 0$, Poles are at $\pm\infty$

When $C_2 = \infty$, poles: $-\dfrac{1}{2(R_1//R_2)C_1} \pm \dfrac{1}{R_1//R_2C_1} = -\dfrac{3}{2}\left(\dfrac{1}{R_1//R_2C_1}\right)$

$+ \dfrac{1}{2R_1//R_2C_1}$ (note, real part doesn't depend on C_2).



18.

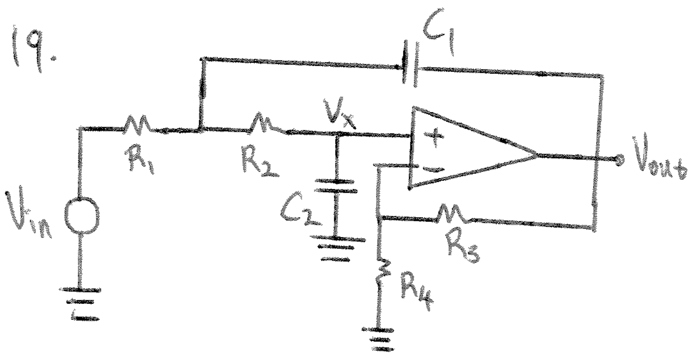


Assuming an ideal Op-amp, $V_x = V_{out}$. Therefore, no current will flow thru C_1. Moreover, since the input impedance of an Op-amp (ideal) is infinite, no current will flow thru R_2 as well, which means $V_y = V_x = V_{out}$.

$$\Rightarrow V_y = V_{out} = \frac{1/C_2 s}{R_1 + 1/C_2 s}$$

Not very useful since it's only a simple single pole lowpass filter. We can implement it with passive components, instead of op-amp.

19.



$K=4, \quad C_1=C_2$

$Q=4$

$$K = 1 + \frac{R_3}{R_4} = 4 \Rightarrow \frac{R_3}{R_4} = 3, \quad \frac{C_1}{C_2} = 1$$

$$\frac{1}{Q} = \sqrt{\frac{R_1 C_2}{R_2 C_1}} + \sqrt{\frac{R_2}{R_1}\frac{C_2}{C_1}} - \sqrt{\frac{R_1 C_1}{R_2 C_2}}\,\frac{R_3}{R_4}$$

$$\frac{1}{Q} = \sqrt{\frac{R_1}{R_2}} + \sqrt{\frac{R_2}{R_1}} - 3\sqrt{\frac{R_1}{R_2}} \Rightarrow \sqrt{\frac{R_2}{R_1}} - 2\sqrt{\frac{R_1}{R_2}} = \frac{1}{Q}$$

$$\frac{1}{Q} = \left(\frac{R_1}{R_2}\right)^{\frac{1}{2}} - 2\left(\frac{R_1}{R_2}\right)^{\frac{1}{2}} \Rightarrow \text{squaring both sides} \Rightarrow$$

$$\frac{1}{Q^2} = 4\left(\frac{R_1}{R_2}\right) - 4 + \left(\frac{R_1}{R_2}\right)^{-1} \Rightarrow \frac{1}{16} = 4\left(\frac{R_1}{R_2}\right) - 4 + \left(\frac{R_2}{R_1}\right)$$

$$\left(\frac{1}{16}+4\right)\frac{R_1}{R_2} = \frac{R_1}{R_2}\left(4\frac{R_1}{R_2} + \frac{R_2}{R_1}\right) \Rightarrow 4.0625\frac{R_1}{R_2} = 4\left(\frac{R_1}{R_2}\right)^2 + 1$$

$$4\left(\frac{R_1}{R_2}\right)^2 - 4.0625\left(\frac{R_1}{R_2}\right) + 1 = 0, \quad \frac{R_1}{R_2} = 0.41908,\ 0.59\cancel{6}55$$

This leads to a negative Q.

19.

$$S_{R_1}^{Q} = -\frac{1}{2}\left[\sqrt{\frac{R_1 C_2}{R_2 C_1}} - \sqrt{\frac{R_2 C_2}{R_1 C_1}} - (K-1)\sqrt{\frac{R_1 C_1}{R_2 C_2}}\right] Q$$

$$S_{R_1}^{Q} = -\frac{1}{2}\left[\sqrt{0.41908} - \sqrt{1/0.41908} - 3\sqrt{0.41908}\right] 4$$

$$S_{R_1}^{Q} = 5.68$$

20.

$$H(s) = \frac{1}{R_1R_2C_1C_2s^2 + (R_1+R_2)C_2s + 1}$$

$$Q = \frac{1}{R_1+R_2}\sqrt{R_1R_2\frac{C_1}{C_2}}\ , \qquad \omega_n = \frac{1}{\sqrt{R_1R_2C_1C_2}}$$

$$\frac{Q}{\sqrt{1-\frac{1}{4Q^2}}} = 1.1 \Rightarrow Q^2 = (1.1)^2\left(1-\frac{1}{4Q^2}\right)$$

$$\Rightarrow 3.3058\,Q^4 - 4Q^2 + 1 = 0$$

$Q^2 = 0.85704,\ 0.35296,$

$Q = \pm 0.925765,\ \pm 0.5941$

In order to peak, $Q > \frac{1}{\sqrt{2}} \Rightarrow Q = 0.925765$

$$\Rightarrow \frac{1}{R_1+R_2}\sqrt{R_1R_2\frac{C_1}{C_2}} = 0.925765$$

let $\frac{C_1}{C_2} = 1 \Rightarrow \frac{1}{R_1+R_2}\sqrt{R_1R_2} = 0.925765$

$$\Rightarrow R_1R_2 = (0.925765)^2(R_1+R_2)^2$$

$$\frac{R_1R_2}{R_1+R_2} = R_1 /\!/ R_2 = 0.85704\,(R_1+R_2)$$

$R_1 /\!/ R_2 = 0.85704\,(R_1+R_2)$

only if $\frac{C_1}{C_2} = 1$

21.

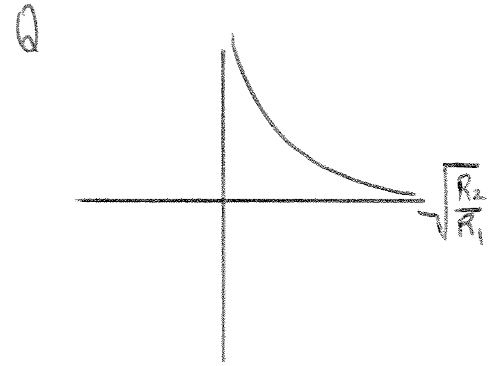
$S^Q_{R_1} = 2, \quad C_2 = C_1, \quad Q = f\left(\sqrt{\frac{R_2}{R_1}}\right)$

Range of Q and $\sqrt{R_2/R_1}$

$$S^Q_{R_1} = -\frac{1}{2}\left[\sqrt{\frac{R_1 C_2}{R_2 C_1}} - \sqrt{\frac{R_2 C_2}{R_1 C_1}} - (K-1)\sqrt{\frac{R_1 C_1}{R_2 C_2}}\right] Q$$

$$S^Q_{R_1} = -\frac{1}{2} + Q\sqrt{\frac{R_2 C_2}{R_1 C_1}} \implies 2 = -\frac{1}{2} + Q\sqrt{\frac{R_2}{R_1}}$$

$$\implies 2.5 = Q\sqrt{\frac{R_2}{R_1}}, \quad Q = \frac{2.5}{\sqrt{R_2/R_1}}$$



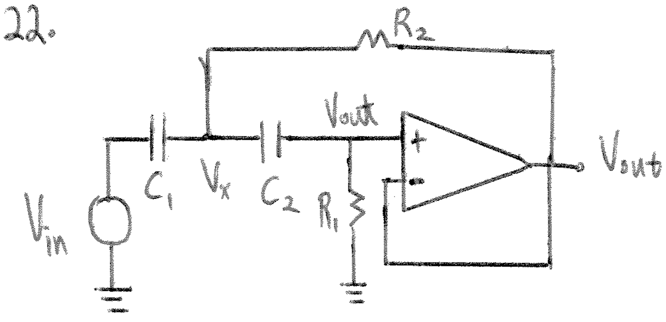
($\frac{R_2}{R_1}$ can't be negative)

Range: $0 < Q < \infty$

$0 < \sqrt{\frac{R_2}{R_1}} < \infty$

If the transfer function does not want to experience peaking, then $0 < Q \leq \frac{1}{\sqrt{2}} \implies 3.5356 \leq \sqrt{\frac{R_2}{R_1}} < \infty$

22.



Assuming an ideal Op amp.

1) $(V_x - V_{in})C_1 s + (V_x - V_{out})\left(C_2 s + \dfrac{1}{R_2}\right) = 0$, nodal equation at V_x.

2) $(V_x - V_{out})C_2 s - \dfrac{V_{out}}{R_1} = 0$, nodal equation at V_{out}.

2) $\Rightarrow$ $V_x = V_{out}\left[\dfrac{C_2 s + 1/R_1}{C_2 s}\right]$ (A) The stuff in the bracket becomes "A"

1) $\Rightarrow$ $(A V_{out} - V_{in})C_1 s + (A V_{out} - V_{out})\left[C_2 s + \dfrac{1}{R_2}\right] = 0$

$\Rightarrow A V_{out} C_1 s + V_{out}(A-1)(C_2 s + 1/R_2) = V_{in} C_1 s$

$A - 1 = \dfrac{1}{R_1 C_2 s}$, $A = \dfrac{C_2 s + 1/R_1}{C_2 s}$

Subsitute (A-1) and A into 1) $\Rightarrow$

$$\left(\frac{C_2 s + 1/R_1}{C_2 s}\right) C_1 s V_{out} + \left(C_2 s + \frac{1}{R_2}\right)\frac{1}{R_1 C_2 s} V_{out} = V_{in} C_1 s$$

22.

$$\frac{V_{out}}{V_{in}} = \frac{C_1 s}{\frac{C_1}{C_2}\left(C_2 s + \frac{1}{R_1}\right) + \frac{1}{R_1 C_2 s}\left(C_2 s + \frac{1}{R_2}\right)}$$

Rearranging

$$H(s) = \frac{V_{out}}{V_{in}} = \frac{s^2}{s^2 + \left(\frac{C_1 + C_2}{C_2 R_1 C_1}\right)s + \frac{1}{R_2 C_2 R_1 C_1}}$$

$$\omega_n^2 = \frac{1}{R_2 C_2 R_1 C_1}, \quad \frac{\omega_n}{Q} = \frac{C_1 + C_2}{C_2 R_1 C_1}$$

$$\omega_n = \frac{1}{\sqrt{R_2 C_2 R_1 C_1}}, \quad Q = \sqrt{\frac{C_2 C_1 R_1}{R_2}}\left(\frac{1}{C_1 + C_2}\right)$$

23.

$$Q = \frac{1}{C_1+C_2}\sqrt{\frac{C_2C_1R_1}{R_2}} \Rightarrow \frac{1}{Q} = (C_1+C_2)\sqrt{\frac{R_2}{C_2C_1R_1}}$$

1) $$\frac{d}{dQ}\left[\frac{1}{Q}\right] = -\frac{1}{Q^2} \Rightarrow d\left[\frac{1}{Q}\right] = -\frac{1}{Q^2}\,dQ$$

2) $$\frac{d\left[\frac{1}{Q}\right]}{dR_2} = \frac{1}{2}\frac{C_1+C_2}{\sqrt{C_2C_1R_1R_2}} \Rightarrow d\left[\frac{1}{Q}\right] = \frac{1}{2}\frac{C_1+C_2}{\sqrt{C_2C_1R_1R_2}}\,dR_2$$

Equating 1) and 2) and multiple 2) by $\frac{R_2}{R_2}$

$$-\frac{dQ}{Q^2} = \frac{1}{2}\frac{(C_1+C_2)R_2}{\sqrt{C_2C_1R_1R_2}}\;\frac{dR_2}{R_2}$$

$$\frac{dQ}{Q}\Big/\frac{dR_2}{R_2} = -\frac{Q}{2}(C_1+C_2)\sqrt{\frac{R_2}{C_1C_2R_1}}$$

$$S^Q_{R_2} = -\frac{Q}{2}(C_1+C_2)\sqrt{\frac{R_2}{C_1C_2R_1}} = -\frac{1}{2}$$

$$\frac{1}{Q} = (C_1+C_2)\sqrt{\frac{R_2}{C_2C_1R_1}} = C_1\sqrt{\frac{R_2}{C_2C_1R_1}} + C_2\sqrt{\frac{R_2}{C_2C_1R_1}}$$

$$\frac{\partial\left(\frac{1}{Q}\right)}{\partial C_1} = \frac{1}{2}\sqrt{\frac{R_2}{C_2R_1C_1}} - \frac{C_2}{2C_1}\sqrt{\frac{R_2}{C_2R_1C_1}}, \quad \frac{d\left(\frac{1}{Q}\right)}{dQ} = -\frac{1}{Q^2}$$

Rearranging $\Rightarrow$ $$-\frac{\partial Q}{Q^2} = \frac{\partial C_1}{C_1}\left(\frac{C_1-C_2}{2}\sqrt{\frac{R_2}{C_2R_1C_1}}\right)$$

$$\frac{\partial Q}{Q}\Big/\frac{\partial C_1}{C_1} = S^Q_{C_1} = -Q\left(\frac{C_1-C_2}{2}\sqrt{\frac{R_2}{C_2R_1C_1}}\right)$$

23. Similarly:

$$S_{C_2}^{Q} = -Q\left(\frac{C_2 - C_1}{2} \sqrt{\frac{R_2}{C_2 R_1 C_1}} \right)$$

$$S_{R_1}^{Q} = Q\left(\frac{C_1 + C_2}{2} \sqrt{\frac{R_2}{C_2 C_1 R_1}} \right) = \frac{1}{2}$$

24.

$$\frac{V_{out}}{V_{in}}(s) = \frac{\alpha s^2}{s^2 + \frac{\omega_n}{Q} s + \omega_n^2}$$

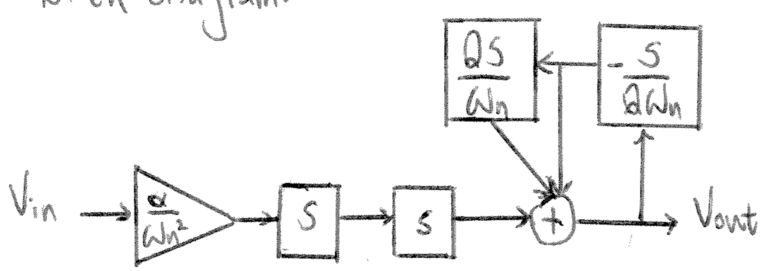
Cross-multiply.

$$V_{out} s^2 + V_{out} \frac{\omega_n}{Q} s + V_{out}\,\omega_n^2 = V_{in}\,\alpha s^2$$

Rearranging

$$V_{out} = V_{in} \frac{\alpha}{\omega_n^2} s^2 - V_{out} \frac{s^2}{\omega_n^2} - V_{out} \frac{s}{Q\omega_n}$$

Block diagram:



25.

$Q = 2, \quad \omega_n = (2\pi)(2\times10^6)$

$R_6 = R_3, \quad R_1 = R_2, \quad C_1 = C_2$

$10\,pF < \text{Total } C < 1\,nF, \quad 1K\Omega < \text{Total } R < 50\,K\Omega$

$$\frac{\omega_n}{Q} = \frac{R_4}{R_4+R_5}\left(\frac{1}{R_1C_1}\right), \quad \omega_n^2 = \frac{R_6}{R_3}\left(\frac{1}{R_1R_2C_1C_2}\right)$$

Since $R_6 = R_3 \Rightarrow \omega_n^2 = \left(\frac{1}{R_1C_1}\right)^2 = (2\pi\times2\times10^6)^2$

$$\frac{1}{R_1C_1} = 2\pi\times2\times10^6 = \omega_n$$

$$Q = \frac{R_4+R_5}{R_4} = 2 \Rightarrow R_5 = R_4$$

Let $C_1 = C_2 = 100\,pf$, $R_1 = \frac{1}{(2\pi)(2\times10^6)(100\,pf)} = 795.77\Omega$

So $R_1 = R_2 = 795.77\,\Omega$, $C_1 = C_2 = 100\,pf$.

Since R_3, R_4, R_5, R_6 don't affect Q and ω_n, let them be $500\,\Omega$ each.

Total R: $(4)(500) + (2)(795.77) = 3.6\,K\Omega$

Total C: $100\,pf + 100\,pf = 200\,pf$.

26.

$$\frac{\omega_n}{Q} = \frac{R_4}{R_4+R_5}\,\frac{1}{R_1C_1}, \quad \omega_n^2 = \frac{R_6}{R_3}\cdot\frac{1}{R_1R_2C_1C_2}$$

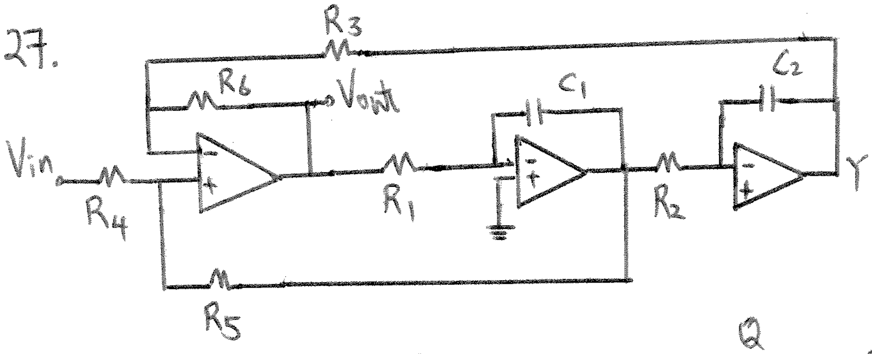
$$Q = \omega_n\left(\frac{R_4+R_5}{R_4}\right)R_1C_1, \quad \omega_n = \sqrt{\frac{R_6}{R_3}\left(\frac{1}{R_1R_2C_1C_2}\right)}$$

$$Q = \sqrt{\frac{R_6}{R_3}\left(\frac{1}{R_1R_2C_1C_2}\right)}\left(\frac{R_4+R_5}{R_4}\right)R_1C_1$$

$$Q = \sqrt{\frac{R_6}{R_3}}\sqrt{\frac{R_1C_1}{R_2C_2}}\left(\frac{R_4+R_5}{R_4}\right)$$

If $R_6 = R_3$, Q doesn't depend on R_6 and R_3, hence zero sensitivity.

27.



Low pass, low freq gain of 2. $S^{Q}_{R_3,R_6} = 0$

$$\frac{V_Y}{V_{in}} = \left(\frac{\alpha s^2}{s^2 + \frac{\omega_n}{Q}s + \omega_n^2}\right)\left(\frac{1}{R_1R_2C_1C_2 s^2}\right)$$ Low pass transfer function

Low freq gain: $\dfrac{\alpha}{\omega_n^2 R_1R_2C_1C_2}$, where $\alpha = \dfrac{R_5}{R_4+R_5}\left(1+\dfrac{R_6}{R_3}\right)$

and $\omega_n^2 = \dfrac{R_6}{R_3}\left(\dfrac{1}{R_1R_2C_1C_2}\right)$

Therefore, Low freq gain: $\dfrac{\alpha}{\frac{R_6}{R_3}} = \dfrac{R_5}{R_4+R_5}\left(1+\dfrac{R_6}{R_3}\right)\left(\dfrac{R_3}{R_6}\right)$

$$S^{Q}_{R_3,R_6} = \frac{Q}{2}\,\frac{|R_3-R_6|}{1+R_5/R_4}\sqrt{\frac{R_2C_2}{R_3R_6R_1C_1}}$$

To obtain $S^{Q}_{R_3,R_6} = 0$, $R_3 = R_6$, however this makes the low freq gain: $2\left(\dfrac{R_5}{R_4+R_5}\right) \neq 2$.

Therefore, it's impossible a low freq gain of 2 if $S^{Q}_{R_3,R_6} = 0$.

28.

Peaking : 1 dB, $R_3 = R_6$

Normalized Peak Value: $\dfrac{Q}{\sqrt{1-(4Q^2)^{-1}}} = 1.1$

solving for Q^2: 0.8570, $\cancel{0.3830}$ (not possible for peaking)

$$\frac{\omega_n}{Q} = \frac{R_4}{R_4+R_5}\left(\frac{1}{R_1 C_1}\right), \quad \left(\frac{\omega_n}{Q}\right)^2 = \left(\frac{R_4}{R_4+R_5}\right)^2\left(\frac{1}{R_1 C_1}\right)^2$$

$$\omega_n^2 = \frac{R_6}{R_3}\left(\frac{1}{R_1 R_2 C_1 C_2}\right) = \frac{1}{R_1 R_2 C_1 C_2} \quad (\text{since } R_6 = R_3)$$

Therefore, $Q^2 = \left(\dfrac{R_1 C_1}{R_2 C_2}\right)\left(\dfrac{R_4+R_5}{R_4}\right)^2 = 0.8570$

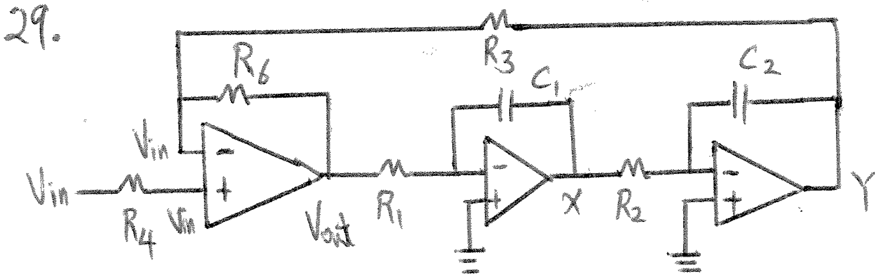
Low pass gain: $2\dfrac{R_5}{R_4+R_5} = \alpha$ (Since $R_6 = R_3$)

So $\dfrac{\alpha}{2} = \dfrac{R_5}{R_4+R_5}$, $\dfrac{R_4}{R_4+R_5} = 1-\dfrac{\alpha}{2} \Rightarrow \dfrac{R_4+R_5}{R_4} = \left(1-\dfrac{\alpha}{2}\right)^{-1}$

So $\left(\dfrac{R_1 C_1}{R_2 C_2}\right)\left(1-\dfrac{\alpha}{2}\right)^{-2} = 0.8570$, if $\alpha = 1 \Rightarrow \dfrac{R_1 C_1}{R_2 C_2} = 0.214$.

However, can't go down any further without knowing more information

29.



$$V_x = -\frac{V_{out}}{R_1}\left(\frac{1}{C_1 S}\right), \quad V_Y = -\frac{V_x}{R_2}\left(\frac{1}{C_2 S}\right), \quad V_{out} = V_{in} - \frac{(V_Y - V_{in}) R_6}{R_3}$$

Subsituting V_x into $V_y \Rightarrow V_y = \frac{V_{out}}{R_1}\left(\frac{1}{C_1 S}\right)\left(\frac{1}{R_2 C_2 S}\right)$

Subsituting V_y into V_{out} and rearranging:

$$\frac{V_{out}}{V_{in}} = \frac{(R_1C_1)(R_2C_2)S^2\left(1+\frac{R_6}{R_3}\right)}{(R_1C_1)(R_2C_2)S^2 + \frac{R_6}{R_3}}$$

simplifying

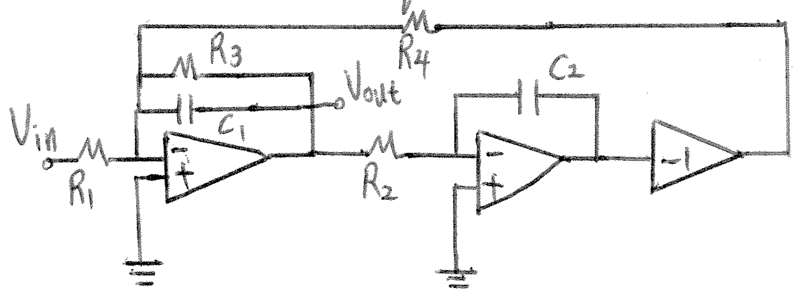
$$\frac{V_{out}}{V_{in}} = \frac{S^2\left(1+\frac{R_6}{R_3}\right)}{S^2 + \frac{R_6}{R_3}\left(\frac{1}{R_1C_1R_2C_2}\right)}$$

$$\omega_n^2 = \frac{R_6}{R_3}\left(\frac{1}{R_1C_1R_2C_2}\right), \quad Q = \infty$$

$$\alpha = \left(1 + \frac{R_6}{R_3}\right)$$

30.

Tow-Thomas Biquad:



$$\omega_n = \frac{1}{\sqrt{R_2 R_4 C_1 C_2}} \quad , \quad Q^{-1} = \frac{1}{R_3}\sqrt{\frac{R_2 R_4 C_2}{C_1}}$$

$$\frac{\partial \omega_n}{\partial R_2} = -\frac{1}{2}\,\frac{1}{R_2\sqrt{R_2 R_4 C_1 C_2}} = -\frac{1}{2}\,\frac{\omega_n}{R_2}$$

$$\frac{\partial \omega_n}{\omega_n} \Big/ \frac{\partial R_2}{R_2} = S^{\omega_n}_{R_2} = -\frac{1}{2}$$

Since R_2, R_4, C_1, C_2 are equivalent in ω_n's definition, all of their sensitivities $= -\frac{1}{2}$

Sensitivities of Q:

$$\frac{\partial Q}{\partial R_3} = \sqrt{\frac{C_1}{R_2 R_4 C_2}}\left(\frac{R_3}{R_3}\right) \Rightarrow \frac{\partial Q}{Q} = \frac{\partial R_3}{R_3} \Rightarrow S^{Q}_{R_3} = 1$$

$$\frac{\partial Q}{\partial C_1} = \frac{1}{2} R_3 \left(\frac{C_1}{R_2 R_4 C_2}\right)^{-\frac{1}{2}}\left(\frac{1}{R_2 R_4 C_2}\right)\frac{C_1}{C_1} \Rightarrow \frac{\partial Q}{Q} = \frac{1}{2}\,\frac{\partial C_1}{C_1} \Rightarrow S^{Q}_{C_1} = \frac{1}{2}$$

$$\frac{\partial Q}{\partial R_2} = -\frac{1}{2} R_3 \left(\frac{C_1}{R_2 R_4 C_2}\right)^{-\frac{1}{2}}\frac{C_1}{R_4 C_2}\left(\frac{1}{R_2^2}\right) \Rightarrow \frac{\partial Q}{Q} = -\frac{1}{2}\,\frac{\partial R_2}{R_2} \Rightarrow S^{Q}_{R_2} = -\frac{1}{2}$$

30.

Since R_2, R_4 and C_2 are equivalent in the expression

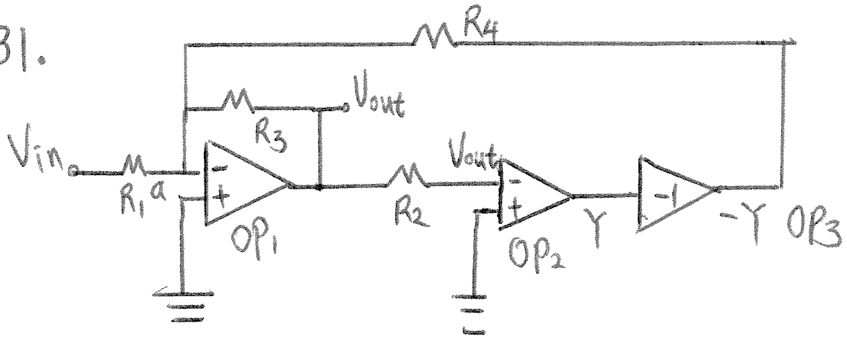
$$S^{Q}_{R_2,R_4,C_2} = -\frac{1}{2}$$

So, $S^{\omega_n}_{R_2,R_4,C_1,C_2} = -\frac{1}{2}$, $S^{\omega_n}_{R_1,R_3} = 0$

$$S^{Q}_{R_2,R_4,C_2} = -\frac{1}{2}, \quad S^{Q}_{C_1} = \frac{1}{2}, \quad S^{Q}_{R_3} = 1$$

$$S^{Q}_{R_1} = 0$$

31.



(Low freq)

V_{out} equals zero because of OP_2's negative feedback.
Likewise, V_a equals to zero as well.
So, summing all the currents thru R_3, we have

$$-\left(\frac{0-V_Y}{R_4}+\frac{V_{in}}{R_1}\right)R_3 = V_{out} = 0$$

$$\Rightarrow \frac{V_{in}}{R_1} = \frac{V_Y}{R_4} \Rightarrow \frac{V_Y}{V_{in}} = \frac{R_4}{R_1}$$

32.

$$\frac{V_Y}{V_{in}} = \frac{R_3 R_4}{R_1}\left(\frac{1}{R_2R_3R_4C_1C_2S^2 + R_2R_4C_2S + R_3}\right)$$

$\omega_n = (2\pi)(10\ MHz)$, $R_3 = 1K$, $R_2 = R_4$, $C_1 = C_2$

$$\omega_n = \frac{1}{\sqrt{R_2R_4C_1C_2}}, \quad Q^{-1} = \frac{1}{R_3}\sqrt{\frac{R_2R_4C_2}{C_1}}$$

Peaking: 1dB

$$\frac{Q}{\sqrt{1-(4Q^2)^{-1}}} = 1.1, \quad Q^2 = 0.8570$$

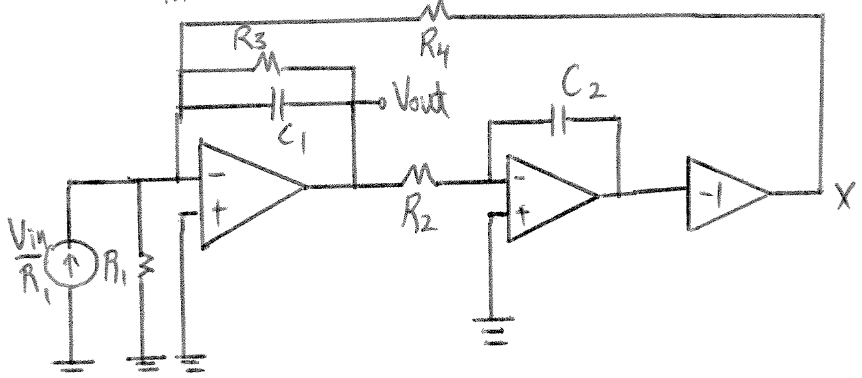
$$\omega_n = \frac{1}{\sqrt{(R_2C_1)^2}} = (2\pi)(10\times10^6) \Rightarrow \frac{1}{R_2C_1} = (2\pi)(10\times10^6)$$

$$\frac{1}{Q} = \frac{1}{1000}\sqrt{R_2^2} = \frac{1}{Q} = \frac{R_2}{1000} \Rightarrow \begin{array}{l} R_2 = 1166.86\,Ohm \\ R_2 = 1.2\,K\Omega \end{array}$$

Solving for C_1 we have: $C_1 = 13.64\,pf$.

33.

$$\frac{V_Y}{V_{in}} = \frac{R_3 R_4}{R_1}\left(\frac{1}{R_2 R_3 R_4 C_1 C_2 s^2 + R_2 R_4 C_2 s + R_3}\right)$$



When R_1 and V_{in} are replaced with its Norton equivalent, we see that the "upper" terminal of R_1 is at virtual ground. Since R_1's two terminals are at the same potential, no current will flow throw it, therefore it can be seen as an open. So R_1 is not in the signal path, and therefore will not affect the frequency response. However, since its magnitude is embedded in the Norton current source, it will affect the DC gain.

34.

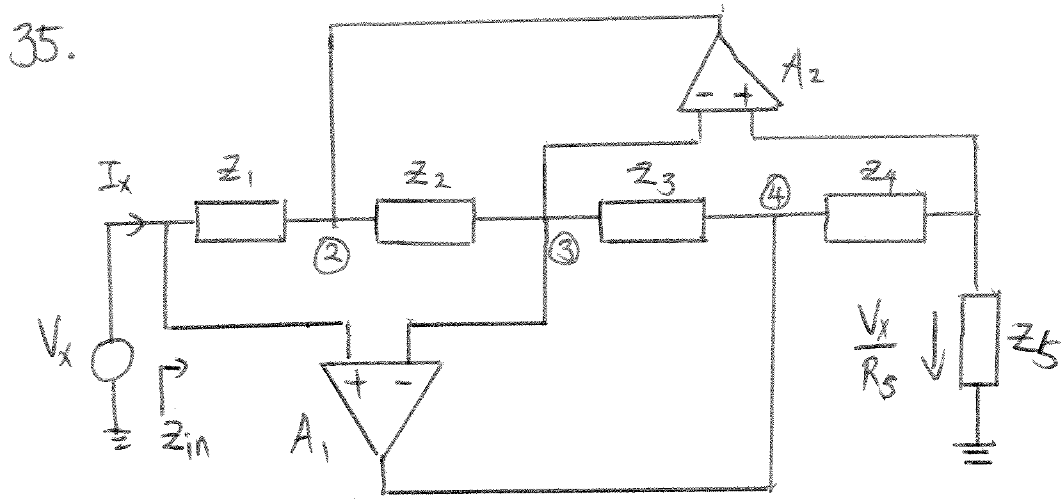
$$Z_{in} = \frac{Z_1 Z_3 Z_5}{Z_2 Z_4}$$ (for circuit diagram, please refer to Problem # 35)

For Z_{in} to be inductive, the following combinations will work.

1	2	3
$Z_5 = R$	$Z_5 = R$	$Z_5 = R$
$Z_4 = R$	$Z_4 = C$	$Z_4 = C$
$Z_3 = R$	$Z_3 = R$	$Z_3 = R$
$Z_2 = C$	$Z_2 = C$	$Z_2 = R$
$Z_1 = R$	$Z_1 = C$	$Z_1 = R$

Any other combination will result in DC path blockage at a node. Moreover, in #2 it's assumed that the input can provide a DC bias.

35.



$$Z_{in} = \frac{Z_1 Z_3 Z_5}{Z_2 Z_4}$$

For Z_{in} to be capacitive the following combinations can be used.

1	2	3	4	5
$Z_5 = C$	$Z_5 = C$	$Z_5 = R$	$Z_5 = R$	$Z_5 = R$
$Z_4 = R$	$Z_4 = R$	$Z_4 = R$	$Z_4 = R$	$Z_4 = C$
$Z_3 = R$	$Z_3 = R$	$Z_3 = R$	$Z_3 = C$	$Z_3 = C$
$Z_2 = R$	$Z_2 = C$	$Z_2 = R$	$Z_2 = R$	$Z_2 = R$
$Z_1 = R$	$Z_1 = C$	$Z_1 = C$	$Z_1 = R$	$Z_1 = C$

Any other combination results in a DC path blockage at a node. Moreover, in # 2, 3, 5, it is assumed that the input node will produce a DC bias.

36.

$$Z_{in} = \frac{Z_1 Z_3 Z_5}{Z_2 Z_4}$$

$$Z_5 = R_x + \frac{1}{CS}, \quad Z_4 = R_x, \quad Z_3 = R_x, \quad Z_2 = R_x,$$

$$Z_1 = \frac{1}{CS}$$

$$Z_{in} = \frac{\frac{R_x}{CS}\left(R_x + \frac{1}{CS}\right)}{R_x^2} = \frac{1}{CSR_x}\left(R_x + \frac{1}{CS}\right)$$

$$Z_{in} = \frac{1}{CS} + \frac{1}{C^2 S^2 R_x}$$

$$V_{out} = \frac{V_{in}\left[S^2[C^2 R_x] + CS\right]}{\left[S^2[C^2 R_x] + SC + R_1[S^3 C^3 R_x]\right]}$$

$$\frac{V_{out}}{V_{in}} = \frac{SCR_x + 1}{S^2 R_1 R_x C^2 + SCR_x + 1}$$

37. $Z_{in} = \frac{Z_1 Z_3 Z_5}{Z_2 Z_4}$ (for circuit diagram, please refer to problem # 35)

Let Z_5 be a capacitor, Z_2 and Z_4 be large resistors and Z_1 and Z_3 be small resistors compared to Z_2 and Z_4.

For example, let Z_1 and Z_3 equal 50Ω and Z_2 and Z_4 equal $5K\Omega$. Then there's a $(100)^2 = 10000$ multiplication factor onto C_5.

38.

Butterworth filter: roll-off of 1dB @ $\omega = 0.9\,\omega_0$

$$|H(j\omega)| = \frac{1}{\sqrt{1+(0.9)^{2n}}} = 0.9 \Rightarrow 2n = \frac{\log(0.2345679)}{\log(0.9)}$$

$n = 6.88$

So we need a 7th order.

39.

$$|H(j\omega)| = \frac{1}{\sqrt{1+(0.9)^{2n}}}$$

$n=0 \Rightarrow -3\,dB$

$n=1 \Rightarrow -2.577\,dB$

$n=3 \Rightarrow -1.851\,dB$

$n=5 \Rightarrow -1.299\,dB$

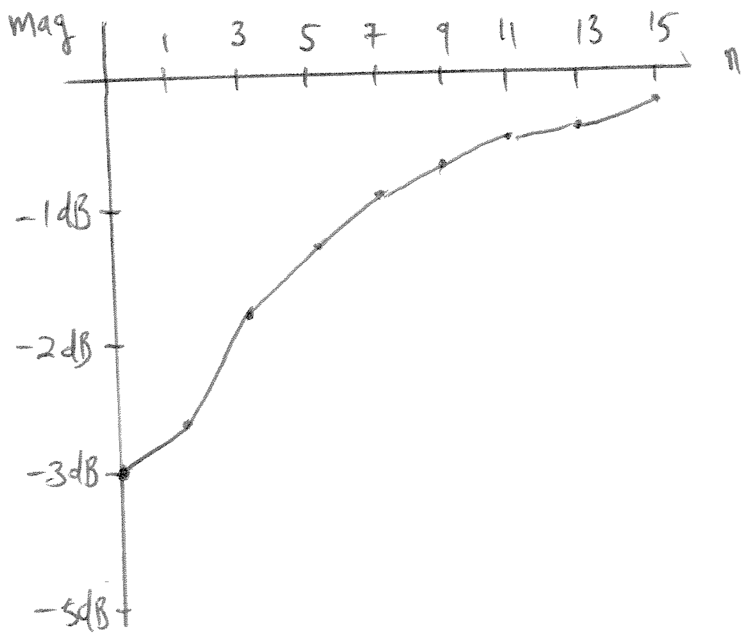
$n=7 \Rightarrow -0.895\,dB$

$n=9 \Rightarrow -0.607\,dB$

$n=11 \Rightarrow -0.408\,dB$

$n=13 \Rightarrow -0.272\,dB$

$n=15 \Rightarrow -0.180\,dB$



40.

$1.1\omega_0$

$$|H(j\omega)| = \frac{1}{\sqrt{1+(1.1)^{2n}}} = 0.1 \Rightarrow 2n = \frac{\log(99)}{\log(1.1)}$$

$n = 24.106$ so needs $n = 25$.

$n=0 \Rightarrow -3$ dB

$n=1 \Rightarrow -3.4439$ dB

$n=3 \Rightarrow -4.427$ dB

$n=5 \Rightarrow -5.555$ dB

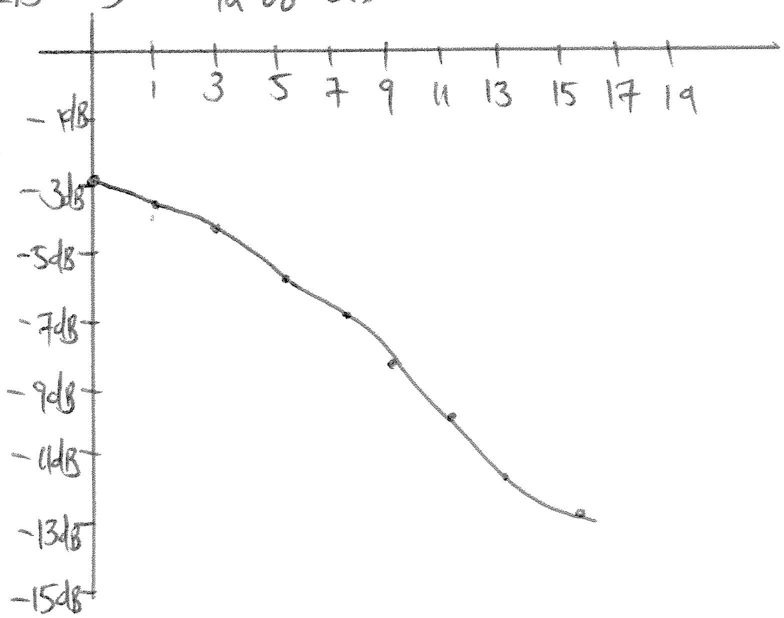
$n=7 \Rightarrow -6.810$ dB

$n=9 \Rightarrow -8.169$ dB

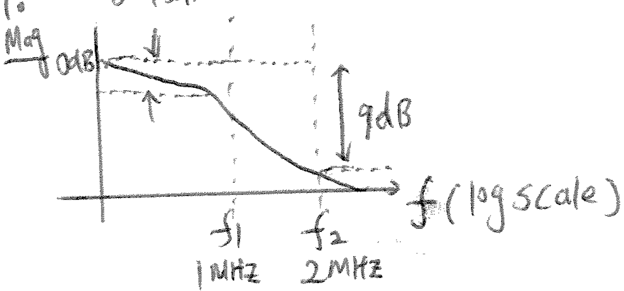
$n=11 \Rightarrow -9.61$ dB

$n=13 \Rightarrow -11.112$ dB

$n=15 \Rightarrow -12.66$ dB



4.



$$|H(f)| = \frac{1}{\left(1 + \left(\frac{2\pi f}{\omega_0}\right)^6\right)^{\frac{1}{2}}}$$

$$|H(5\text{MHz})| = 0.02438$$

Suppression: $20 \log (0.02438) = -32.26$ dB

42.

Low-pass Butterworth: Passband flatness of 0.5 dB
$f_1 = 1\text{ MHz}$, $f_2 = 2\text{ MHz}$, order < 5

$-0.5\text{dB} = 20\log(x) \Rightarrow x = 0.944$

$$|H(j\omega)| = \frac{1}{\sqrt{1+\left(\frac{\omega}{\omega_0}\right)^{2n}}} = 0.944 \Rightarrow \frac{1}{\left(1+\left(\frac{1}{f_0}\right)^{2n}\right)} = (0.944)^2$$
@ $f = 1\text{MHz}$

$$\Rightarrow 1 + \frac{1}{(f_0)^{2n}} = \frac{1}{(0.944)^2} \Rightarrow f_0 = 10^{\frac{0.91306}{2n}}$$

for $n=1$, $f_0 \approx 2.86\text{ MHz}$

for $n=5$, $f_0 \approx 1.234\text{ MHz}$
Therefore, for greatest attenuation $n=5$

so $$H(2\text{MHz}) = \frac{1}{\sqrt{1+\left(\frac{2}{1.234}\right)^{10}}} = 0.089$$

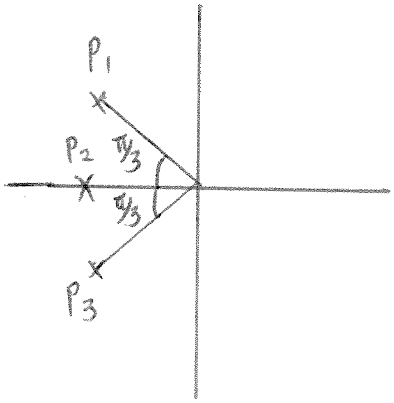
$20\log(0.089) = -21.0\text{ dB}$ at $n=5$

43.

$$P_K = \omega_0 \exp\left(\frac{j\pi}{2}\right) \exp\left(j\,\frac{2K-1}{2n}\pi\right), \quad K = 1, 2, \cdots n$$

The poles lie on a circle because all of their magnitude, which is the distance from the origin to the poles, are the same (ω_0) with each K; only the phase, which is the angle the poles make with the positive real axis, differ. Therefore, a circle is formed.

44.



$$P_1 = 2\pi(1.45\,MHz)\left[\cos\left(\frac{2\pi}{3}\right) + j\sin\left(\frac{2\pi}{3}\right)\right]$$

$$P_2 = (2\pi)(1.45\,MHz)$$

$$P_3 = (2\pi)(1.45\,MHz)\left[\cos\left(\frac{2\pi}{3}\right) - j\sin\left(\frac{2\pi}{3}\right)\right]$$

$$H(s) = \frac{(-P_1)(-P_3)}{(s-P_1)(s-P_3)} = \frac{[2\pi(1.45\,MHz)]^2}{s^2 - [4\pi(1.45\,MHz)\cos(2\pi/3)]s + [2\pi(1.45\,MHz)]^2}$$

KHN Low pass Transfer function:

$$\frac{\alpha s^2}{s^2 + \frac{\omega_n}{Q}s + \omega_n^2} \cdot \frac{1}{R_1R_2C_1C_2s^2} = \frac{\alpha/(R_1R_2C_1C_2)}{s^2 + \frac{\omega_n}{Q}s + \omega_n^2}$$

$$\frac{\alpha}{R_1R_2C_1C_2} = (2\pi \times 1.45 \times 10^6)^2, \quad \omega_n^2 = \frac{R_6}{R_3}\left(\frac{1}{R_1R_2C_1C_2}\right) = [2\pi \times 1.45 \times 10^6]^2$$

$$\frac{\omega_n}{Q} = \frac{R_4}{R_4+R_5}\left(\frac{1}{R_1C_1}\right) = -(4\pi \times 1.45 \times 10^6 \times \cos(2\pi/3))$$

$$\frac{\alpha}{R_1R_2C_1C_2} = \frac{R_5}{R_4+R_5}\left(1 + \frac{R_6}{R_3}\right)\left(\frac{1}{R_1R_2C_1C_2}\right)$$

Let $R_6 = R_3$, $R_2 = 4R_1$, $C_1 = C_2$

$$\omega_n^2 = \left(\frac{1}{4R_1C_1}\right)^2 = (2\pi \times 1.45 \times 10^6)^2 \Rightarrow \frac{1}{2R_1C_1} = 2\pi \times 1.45 \times 10^6$$

44.

Let $R_1 = 5K \Rightarrow C_1 = 10.98pf$, $R_2 = 20K$, $C_2 = 10.98pf$

$$\frac{\omega_n}{Q} = \frac{R_4}{R_4 + R_5}\left(\frac{1}{R_1 C_1}\right) = 9110618.7$$

$$\Rightarrow \frac{R_4}{R_4 + R_5} = \frac{1}{2}$$

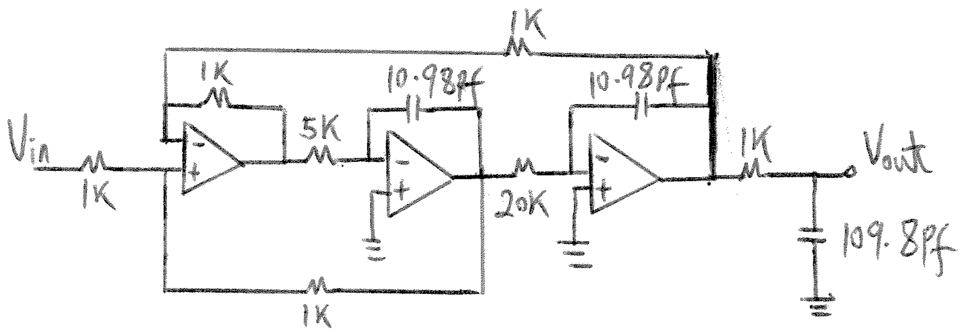
$$\frac{R_5}{R_4 + R_5}\left(1 + \frac{R_6}{R_3}\right)\left(\frac{1}{R_1 R_2 C_1 C_2}\right) = \left(\frac{1}{2}\right)(2)\left(2\pi \times 1.45 \times 10^6\right)^2 = \left(2\pi \times 1.45 \times 10^6\right)^2$$

let R_5 and R_4 be 1K apiece.

So $R_5 = R_4 = R_6 = R_3 = 1K$

$R_1 = 5K$, $R_2 = 20K$

$C_1 = C_2 = 10.98pf$



45.

Tow-Thomas Biquad

$$\frac{V_Y}{V_{in}} = \frac{R_3R_4}{R_1}\left(\frac{1}{R_2R_3R_4C_1C_2s^2 + R_2R_4C_2s + R_3}\right)$$

$$\frac{V_Y}{V_{in}} = \frac{1/(R_1R_2C_1C_2)}{s^2 + 1/(R_3C_1)s + 1/(R_2R_4C_1C_2)}$$

$$\frac{V_Y}{V_{in}} = \frac{(2\pi \times 1.45\times10^6)^2}{s^2 - (4\pi\times1.45\times10^6\times\cos(2\pi/3))s + (2\pi\times1.45\times10^6)^2}$$

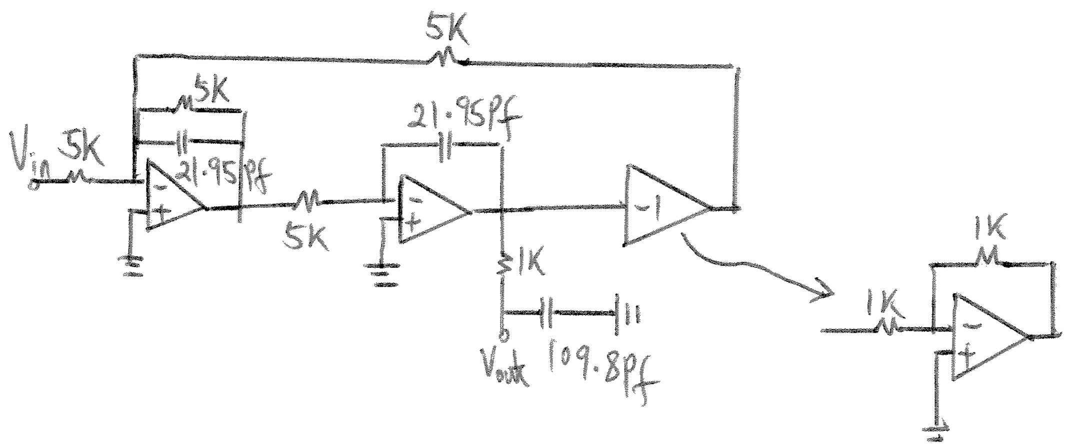
$$\frac{1}{R_1R_2C_1C_2} = (2\pi\times1.45\times10^6)^2, \quad \frac{1}{R_2R_4C_1C_2} = (2\pi\times1.45\times10^6)^2$$

$$\frac{1}{R_3C_1} = 2\pi\times1.45\times10^6$$

Let $R_1 = R_2 = R_3 = R_4$, $C_1 = C_2$

let $R_3 = 5K \Rightarrow C_1 = 21.95\,pf$

so $R_1 = R_2 = R_3 = R_4 = 5K$, and $C_1 = C_2 = 21.95\,pf$



46.

$$|H(j\omega)| = \frac{1}{\sqrt{1+\epsilon^2 C_n^2\left(\frac{\omega}{\omega_0}\right)}} \qquad \begin{array}{l} n=4 \\ \epsilon = 0.2 \end{array}$$

$$C_n\left(\frac{\omega}{\omega_0}\right) = \cos\left(n\cos^{-1}\frac{\omega}{\omega_0}\right) = \cos\left(4\cos^{-1}\frac{\omega}{\omega_0}\right)$$

$$C_n^2\left(\frac{\omega}{\omega_0}\right) = \cos^2\left(n\cos^{-1}\frac{\omega}{\omega_0}\right) = \frac{1}{2}\left(1+\cos\left(2n\cos^{-1}\frac{\omega}{\omega_0}\right)\right)$$

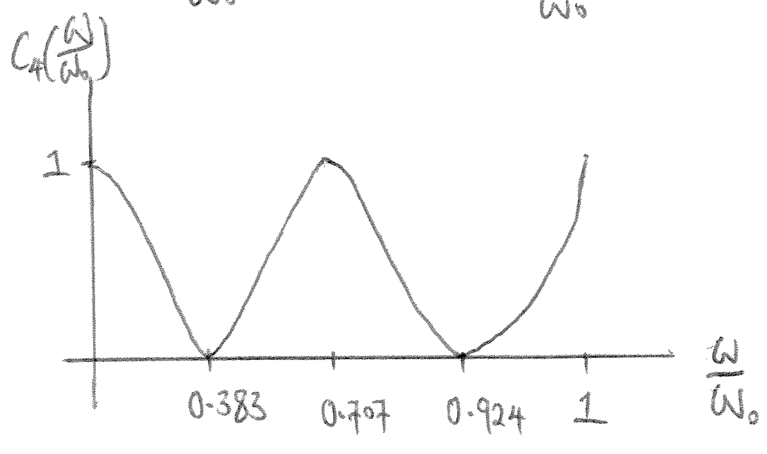
$$2n\cos^{-1}\frac{\omega}{\omega_0} = \pi, \quad n=4 \Rightarrow \frac{\omega}{\omega_0} = 0.924$$

$$2n\cos^{-1}\frac{\omega}{\omega_0} = 3\pi, \quad n=4 \Rightarrow \frac{\omega}{\omega_0} = 0.383$$

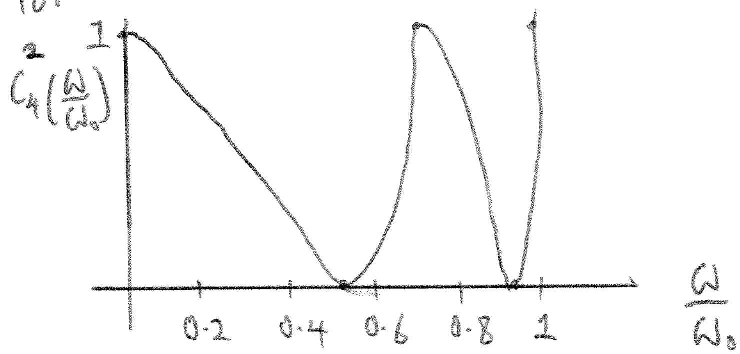
$$2n\cos^{-1}\frac{\omega}{\omega_0} = 0, \quad n=4 \Rightarrow \frac{\omega}{\omega_0} = 1$$

$$2n\cos^{-1}\frac{\omega}{\omega_0} = 2\pi, \quad n=4 \Rightarrow \frac{\omega}{\omega_0} = 0.707$$

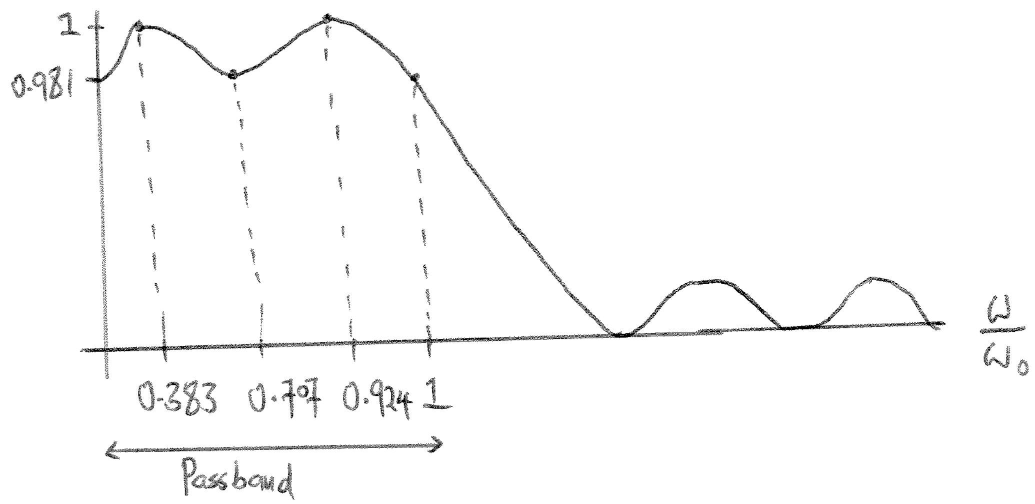
$$2n\cos^{-1}\frac{\omega}{\omega_0} = 4\pi, \quad n=4 \Rightarrow \frac{\omega}{\omega_0} = 0$$



46.



$$H(j\omega) = \frac{1}{\sqrt{1 + (0.2)^2\left[C_4^2\left(\frac{\omega}{\omega_0}\right)\right]}}$$



47. Chebyshev: 25 dB at 5 MHz.

$n = 5, \quad W_0 = 2\text{MHz}, \quad \frac{W}{W_0} = \frac{5}{2}.$

$$\frac{1}{\sqrt{1+\epsilon^2 \cosh^2(n \cosh^{-1}\frac{5}{2})}} = -25\,dB = 0.056234$$

$$\Rightarrow \frac{1}{1+\epsilon^2(1.5939\times10^6)} = 0.0031622771$$

$$\Rightarrow \epsilon^2 = 1.9777\times10^{-4}$$

$\Rightarrow$ Minimum Ripple

$$\frac{1}{\sqrt{1+(1.9777\times10^{-4})}} = 0.99990 = -8.6\times10^{-4}\,dB.$$

48. $n = 6$

$$\cosh^2\left(6\cos^{-1}\left(\frac{5}{2}\right)\right) = 36590401$$

$$\frac{1}{\sqrt{1+\epsilon^2(36590401)}} = 0.056234$$

$$\epsilon^2 = 8.615 \times 10^{-6}$$

$$\text{Minimum Ripple} = \frac{1}{\sqrt{1+8.615\times10^{-6}}} = -3.74\times10^{-5}\ dB$$

Smaller than when $n=5$.

49. $\epsilon = 0.509, \quad n = 4$

$$P_{1,4} = -0.140w_0 \pm 0.983jw_0$$

$$P_{2,3} = -0.337w_0 \pm 0.407jw_0$$

$$H_{1,4}(w) = \frac{0.986\,w_0^2}{s^2 + 0.28w_0 s + 0.986\,w_0^2} = \frac{\alpha/(R_1R_2C_1C_2)}{s^2 + \frac{w_n}{Q}s + w_n^2}$$

$$Q = 3.55$$

$$w_n = (2\pi)(4.965\,MHz)$$

$$w_n^2 = [(2\pi)(4.965\times10^6)]^2 = \frac{R_6}{R_3}\left(\frac{1}{R_1R_2C_1C_2}\right)$$

$$\frac{w_n}{Q} = (0.28)(5MHz)(2\pi) = (1.4MHz)(2\pi) = \frac{R_4}{R_4+R_5}\cdot\left(\frac{1}{R_1C_1}\right)$$

$$\frac{R_5}{R_4+R_5}\left(1+\frac{R_6}{R_3}\right)\left(\frac{1}{R_1R_2C_1C_2}\right) = \frac{R_6}{R_3}\left(\frac{1}{R_1R_2C_1C_2}\right)$$

$$\Rightarrow \frac{R_5}{R_4+R_5}\left(1+\frac{R_6}{R_3}\right) = \frac{R_6}{R_3}$$

$$\Rightarrow \frac{R_5}{R_4+R_5} = \frac{\frac{R_6}{R_3}}{1+\frac{R_6}{R_3}}$$

$$\Rightarrow 1-\alpha = \frac{\frac{R_6}{R_3}}{1+\frac{R_6}{R_3}}, \quad \alpha = \frac{R_4}{R_4+R_5}$$

$$\Rightarrow \frac{R_6}{R_3} = \frac{1-\alpha}{\alpha}$$

Let $\alpha = 0.5 \Rightarrow \frac{R_6}{R_3} = 1$

49.

$$\Rightarrow W_n^2 = \frac{R_6}{R_3}\left(\frac{1}{R_1 C_1 R_2 C_2}\right) = \frac{1}{R_1 C_1 R_2 C_2} \qquad (*)$$

Since $\frac{W_n}{Q} = \alpha\left(\frac{1}{R_1 C_1}\right) = 1.4\times10^6 \times 2\pi$, $\alpha = 0.5$

$$\Rightarrow \frac{1}{R_1 C_1} = 1.76\times10^7 \qquad (1)$$

Consider $(*)$

$$\Rightarrow \frac{1}{R_2 C_2} = W_n^2 \cdot R_1 C_2 = 5.53\times10^7 \qquad (2)$$

$R_6 = R_3 = R_5 = R_4 = 1K$. According to (1).(2), choose

$R_1 = 5K$, $C_1 = 11.368p$

$R_2 = 5K$, $C_2 = 3.62p$

For $H_{2,3}(S) = \dfrac{0.279 W_0^2}{S^2 + 0.674 W_0 S + 0.279 W_0^2}$

$W_n = (2\pi)(2.64\times10^6)$

$\frac{W_n}{Q} = (2\pi)(0.674\times5\times10^6) = (2\pi)(3.37\times10^6)$

Let $\alpha = 0.5$

49. $\frac{W_n}{Q} = (\alpha)\left(\frac{1}{R_1C_1}\right)$

$$\Rightarrow \frac{1}{R_1C_1} = \frac{W_n}{Q}\cdot\frac{1}{\alpha} = 4.23\times10^7 \qquad ③$$

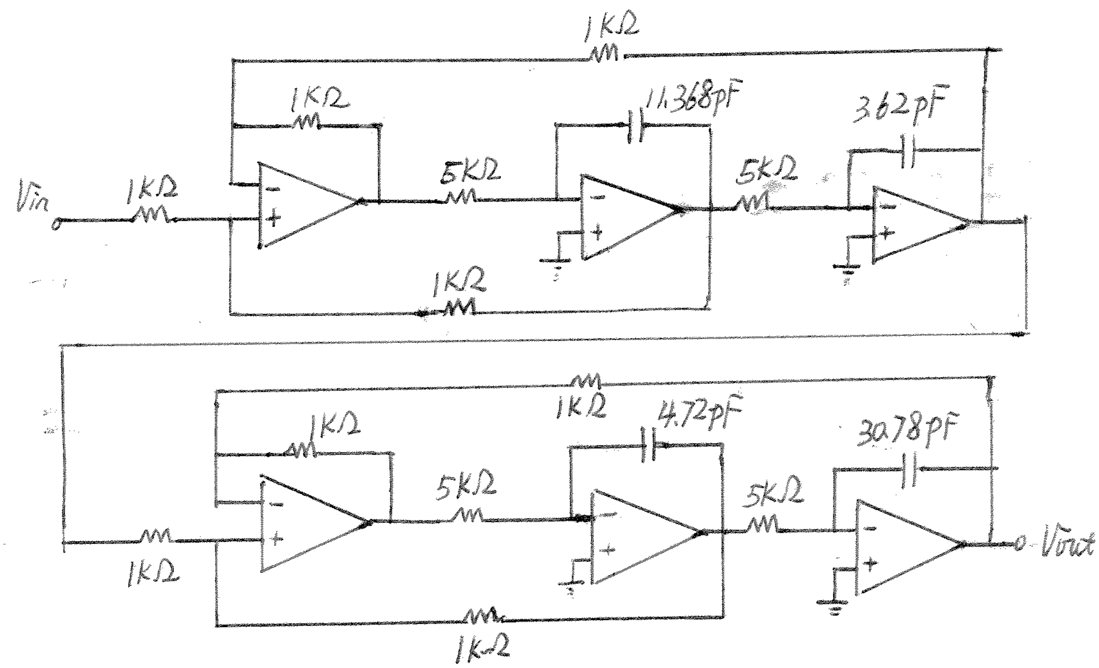
$$\frac{R_6}{R_3} = 1, \quad W_n^2 = \frac{R_6}{R_3}\left(\frac{1}{R_1C_1R_2C_2}\right) = \frac{1}{R_1C_1R_2C_2}$$

$$\Rightarrow \frac{1}{R_2C_2} = W_n^2\cdot R_1C_1 = 6.50\times10^6 \qquad ④$$

Consider ③ ④ , choose

$R_1 = 5K$, $C_1 = 4.72p$; $R_2 = 5K$, $C_2 = 30.78p$

$R_6 = R_3 = R_5 = R_4 = 1K$.



50. Tow Thomas

Low Pass Transfer Function

$$P_{1,4} = \frac{V_4}{V_{in}} = \frac{\frac{1}{R_1R_2C_1C_2}}{s^2 + \frac{1}{R_3C_1}s + \frac{1}{R_2R_4C_1C_2}} = \frac{0.986\,w_o^2}{s^2 + 0.28\,w_o s + 0.986\,w_o^2}$$

$$\frac{1}{R_1R_2C_1C_2} = \frac{1}{R_2R_4C_1C_2} = [(2\pi)(4.965\times10^6)]^2 \quad (*)$$

$$\frac{1}{R_3C_1} = 2\pi\times1.4\times10^6$$

Let $R_3 = 5K$, $C_1 = 22.736P$

Let $C_1 = C_2$, $C_2 = 22.736P$

$$\left.\begin{array}{l} R_1 = R_2 \overset{(*)}{\Rightarrow} R_1 = R_2 = R_4 = 1.4K \\ R_3 = 5K, \quad C_1 = C_2 = 22.736P \end{array}\right\} \text{for } P_{1,4}$$

$$P_{2,3} = \frac{0.279\,w_o^2}{s^2 + 0.674w_o s + 0.279w_o^2} = \frac{\frac{1}{R_1R_2C_1C_2}}{s^2 + \frac{1}{R_3C_1}s + \frac{1}{R_2R_4C_1C_2}}$$

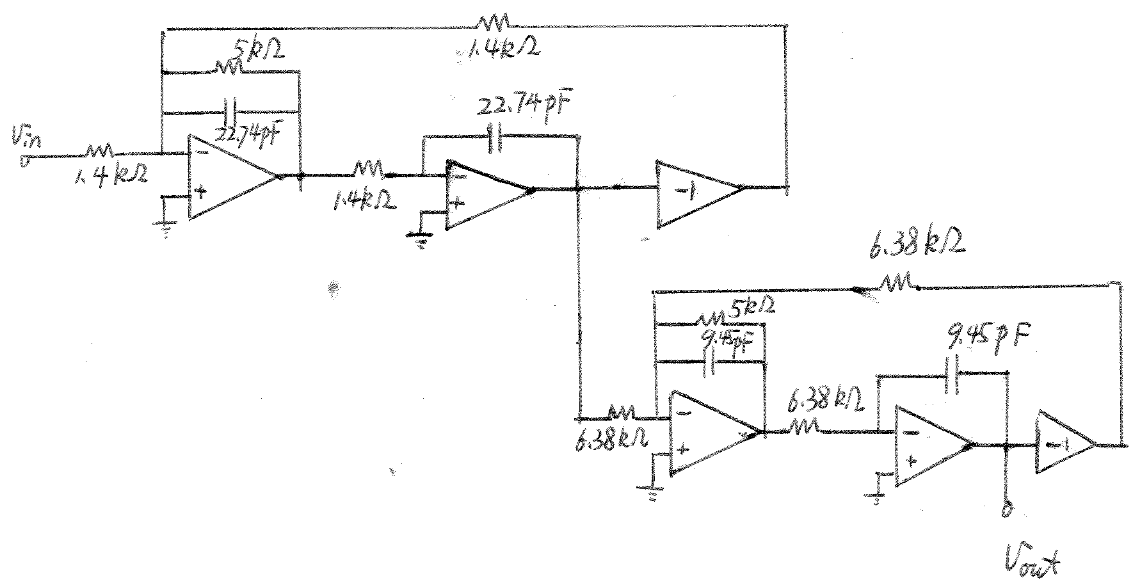
$$\frac{1}{R_1R_2C_1C_2} = \frac{1}{R_2R_4C_1C_2} = [(2\pi)(2.64\times10^6)]^2 \quad (*)$$

$$\frac{1}{R_3C_1} = (2\pi)(3.37\times10^6)$$

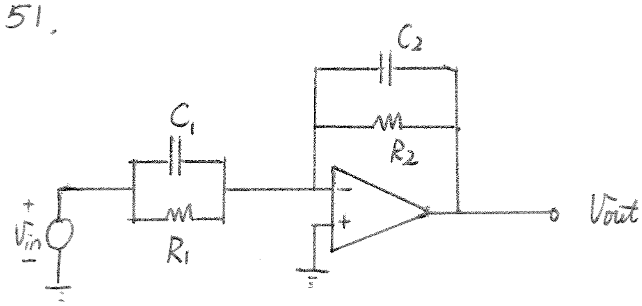
50. Let $R_3 = 5K$, $C_1 = 9.45p$

Let $C_1 = C_2 = 9.45p$, $R_1 = R_2 \xRightarrow{(*)'} R_1 = R_2 = R_4 = 6.38K$.

For $P_{1,4}$. $\begin{cases} R_1 = R_2 = R_4 = 6.38K, \quad R_3 = 5K \\ C_1 = C_2 = 9.45\,p. \end{cases}$

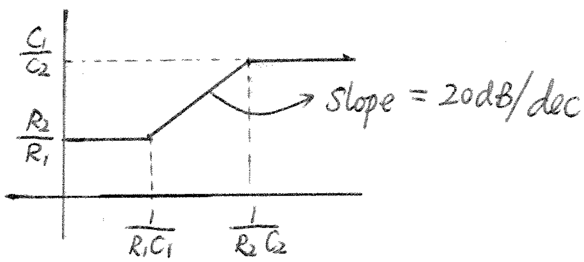


51.



High pass, 1MHz ⇒ 10dB atten

$f > 5\,MHz$, gain = 1



$$\frac{1}{R_2C_2} = (5\,MHz)(2\pi)$$

Let $\frac{C_1}{C_2} = 1$, $\frac{R_2}{R_1} = -10\,dB = 0.316$

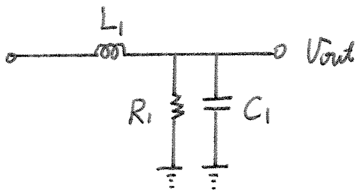
So $\frac{1}{0.316} = 3.1623$ $\frac{5\,MHz}{3.1623} = 1.58\,MHz$

$$\Rightarrow \frac{1}{R_1C_1} = (1.58\,MHz)(2\pi)$$

Choose $C_2 = 31.83\,pF \Rightarrow R_2 = 1K\Omega$

$C_1 = 31.83\,pF \Rightarrow R_1 = 3.16\,k\Omega$

52.



Peaking : 1dB

bandwidth : 100MHz

$L_1 < 100nH$

$$\frac{V_{out}}{V_{in}} = \frac{\frac{1}{L_1C_1}}{S^2+\frac{1}{R_1C_1}S+\frac{1}{L_1C_1}} = \frac{\gamma}{S^2+\frac{W_n}{Q}S+W_n^2}\Bigg|_{S=jw} = \frac{\gamma}{(jw)^2+\frac{W_n}{Q}(jw)+W_n^2}$$

$$H(jw) = \frac{\gamma}{(W_n^2-W^2)+\frac{W_n}{Q}Wj}$$

$$|H(jw)| = \frac{\gamma}{\sqrt{(W_n^2-W^2)^2+(\frac{W_n}{Q}W)^2}}$$

At W_1, $$|H(jW_1)| = \frac{\gamma}{\sqrt{(W_n^2-W_1^2)^2+(\frac{W_n}{Q}W_1)^2}} = \frac{\gamma}{W_n^2\sqrt{2}}$$

$$\Rightarrow \sqrt{\frac{(W_n^2-W_1)^2+(\frac{W_n}{Q}W_1)^2}{W_n^4}} = \sqrt{2}.$$

$$\Rightarrow (W_n^2-W_1)^2+(\frac{W_n}{Q}W_1)^2 = 2W_n^4 \qquad (*).$$

$$\frac{Q}{\sqrt{1-(4Q^2)^{-1}}} = 1.1 \Rightarrow Q = 0.9258,\ 0.5941 \left(<\frac{1}{\sqrt{2}},\ \text{can't produce peaking}\right)$$

So $Q = 0.9258$.

Solve (*) gives $W = \sqrt{1.5}\,W_n$.

$$W = \underbrace{\sqrt{1.5}\frac{1}{\sqrt{L_1C_1}} = (2\pi)(100\times10^6)}_{} \qquad (1)$$

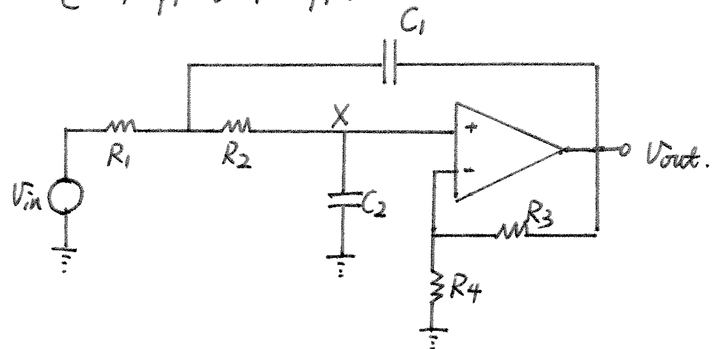
52.

$$\frac{\omega_n}{Q} = \frac{1}{R_1 C_1} \Rightarrow Q = R_1 C_1 \frac{1}{\sqrt{L_1 C_1}} = R_1 \sqrt{\frac{C_1}{L_1}} = 0.9258 \quad ②$$

$$\text{Let } L_1 = 90nH \xRightarrow{①} C_1 = 42.22pF \xRightarrow{②} R_1 = 42.74\Omega.$$

53. $W_n = (2\pi)(50\,MHz)$, $Q = 1.5$, Low fequency gain $= 2$.

$C = 10pF$ to $100pF$.



$$\frac{V_{out}}{V_{in}} = \frac{1+\frac{R_3}{R_4}}{R_1R_2C_1C_2S^2 + (R_1C_2 + R_2C_2 - R_1\frac{R_3}{R_4}C_1)S + 1}$$

$$= \frac{(1+\frac{R_3}{R_4})/(R_1R_2C_1C_2)}{S^2 + \frac{(R_1C_2 + R_2C_2 - \frac{R_1R_3}{R_4}C_1)S}{R_1R_2C_1C_2} + \frac{1}{R_1R_2C_1C_2}}$$

$$W_n = \frac{1}{\sqrt{R_1R_2C_1C_2}} \quad , \quad \frac{W_n}{Q} = \frac{R_1C_2 + R_2C_2 - R_1\frac{R_3}{R_4}C_1}{R_1R_2C_1C_2.}$$

Low fequency gain $(1+\frac{R_3}{R_4}) = 2$, Let $\frac{R_3}{R_4} = 1$.

$$\frac{1}{\sqrt{R_1R_2C_1C_2}} = W_n = (2\pi)(50\times10^6) \quad ①$$

$$R_1C_2 + R_2C_2 - R_1C_1 = \frac{W_n}{Q}(R_1R_2C_1C_2) = \frac{1}{(1.5)(2\pi)(50\times10^6)} \quad ②$$

Let $C_1 = C_2 = 10pF$

② $\Rightarrow R_2 = \frac{1}{(1.5)(2\pi)(50\times10^6)} \cdot \frac{1}{C_2} = 212.2\,(\Omega)$

① $\Rightarrow R_1 = 1/[(2\pi)^2(50\times10^6)^2 \cdot R_2C_1C_2] = 477.5\,(\Omega)$

Let $R_3 = R_4 = 1k\Omega$.

54. $W_{3dB} = (30\times10^6)(2\pi)$, gain $= 2$, sensitivities no greater than 1.

$$H(s) = \frac{K W_n^2}{s^2 + \frac{W_n}{Q}s + W_n^2}, \quad s = jw \Rightarrow$$

$$H(jw) = \frac{K W_n^2}{W_n^2 - W^2 + \frac{W_n}{Q}wj}$$

$$|H(jw)| = \frac{K W_n^2}{\sqrt{(W_n^2 - W^2)^2 + (\frac{W_n}{Q}W)^2}}$$

$$|H(jw)| = \frac{K}{\sqrt{2}} \Rightarrow \frac{W_n^2}{\sqrt{(W_n^2-W^2)^2 + (\frac{W_n}{Q}W)^2}} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow (W_n^2 - W^2)^2 + (\frac{W_n}{Q}W)^2 = 2W_n^4$$

$$\Rightarrow W_n^4 (1 - \frac{W^2}{W_n^2})^2 + W_n^4 [(\frac{1}{Q})^2 \cdot (\frac{W}{W_n})^2] = 2W_n^4$$

$$\Rightarrow [1 - (\frac{W}{W_n})^2]^2 + (\frac{1}{Q})^2 (\frac{W}{W_n})^2 = 2$$

$$\Rightarrow (\frac{W}{W_n})^4 + [(\frac{1}{Q})^2 - 2](\frac{W}{W_n})^2 - 1 = 0$$

$S^{W_n}_{R_2, C_1, C_2, R_1} = -\frac{1}{2}$ (sensitivities of W_n all < 1)

$$S^Q_{R_1} = -S^Q_{R_2} = -\frac{1}{2} + Q\sqrt{\frac{R_2C_2}{R_1C_1}}$$

$$S^Q_{C_1} = -S^Q_{C_2} = -\frac{1}{2} + Q\left(\sqrt{\frac{R_1C_2}{R_2C_1}} + \sqrt{\frac{R_2C_2}{R_1C_1}}\right) = \frac{1}{2} + Q\sqrt{\frac{R_1C_1}{R_2C_2}}$$

$$S^Q_K = QK\sqrt{\frac{R_1C_1}{R_2C_2}} = 2Q\sqrt{\frac{R_1C_1}{R_2C_2}}$$

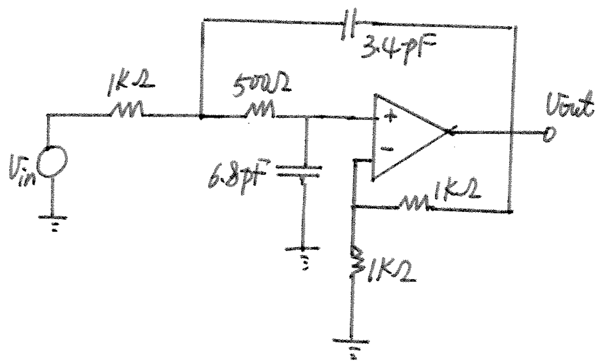
Let $\sqrt{\dfrac{R_1 C_2}{R_2 C_2}} = 1$, and $Q = \dfrac{1}{2}$,

$$S_K^Q = 2 \cdot \left(\frac{1}{2}\right) = 1, \qquad S_{C_1}^Q = \frac{1}{2} + \frac{1}{2} = 1$$

$$S_{C_2}^Q = -1, \qquad S_{R_1}^Q = -\frac{1}{2} + \frac{1}{2} = 0, \qquad S_{R_2}^Q = 0$$

Since $Q = \frac{1}{2}$.,

$$\left(\frac{w}{w_n}\right)^4 + 2\left(\frac{w}{w_n}\right)^2 - 1 = 0$$



$$\Rightarrow \left(\frac{w}{w_n}\right)^2 = 0.4142$$

$$\Rightarrow w = \sqrt{0.4142}\, w_n.$$

Since $R_1 C_1 = R_2 C_2$,

$$w_n = \frac{1}{\sqrt{(R_1 C_1)^2}} = \frac{1}{R_1 C_1}$$

$$\Rightarrow \sqrt{0.4142}\, w_n = \frac{\sqrt{0.4142}}{R_1 C_1} = (2\pi)(30 \times 10^6) \quad ①$$

Also $\dfrac{1}{Q w_n} = R_1 C_2 + R_2 C_2 - R_1 C_1 = R_1 C_2$ ②

$$\Rightarrow \frac{R_1 C_1}{R_1 C_2} = \frac{\frac{1}{w_n}}{\frac{1}{Q w_n}} = Q = \frac{1}{2}.$$

$$\Rightarrow \frac{C_1}{C_2} = \frac{1}{2}.$$

Let $R_1 = 1K\Omega \overset{①}{\Longrightarrow} C_1 = \dfrac{\sqrt{0.4142}}{(2\pi)(30 \times 10^6) \cdot R_1} = 3.4pF.$

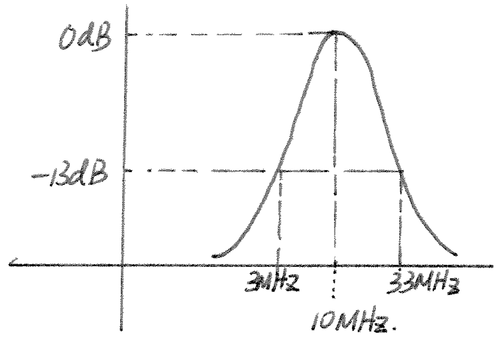
$$\Rightarrow C_2 = 2C_1 = 6.8pF.$$

$$\Rightarrow R_2 = R_1 \frac{C_1}{C_2} = 500\Omega.$$

And as before, $R_3 = R_4 = 1K\Omega$.

55. 10MHz, Gain $=1$ (peak), $R_6 = R_3$, $-13\,dB$ @ 3MHz, 33MHz.

$$\frac{V_x}{V_{in}} = \frac{\alpha S^2}{S^2 + \frac{W_n}{Q}S + W_n^2} \cdot \frac{-1}{R_1C_1S}$$



$$1 = \left(\frac{\alpha}{R_1C_1}\right)\cdot\frac{Q}{W_n}, \qquad \alpha = \frac{R_5}{R_4+R_5}\left(1+\frac{R_6}{R_3}\right), \quad \frac{W_n}{Q} = \frac{R_4}{R_4+R_5}\cdot\frac{1}{R_1C_1}.$$

since $R_6 = R_3$, $\alpha = 2\frac{R_5}{R_4+R_5} \Rightarrow \frac{\alpha}{2} = \frac{R_5}{R_4+R_5}$

$$\Rightarrow \frac{R_4}{R_4+R_5} = 1-\frac{\alpha}{2} \Rightarrow \frac{Q}{W_n} = \frac{R_1C_1}{1-\frac{\alpha}{2}}.$$

$$\Rightarrow \left(\frac{\alpha}{R_1C_1}\right)\cdot\left(\frac{R_1C_1}{1-\frac{\alpha}{2}}\right) = 1 \Rightarrow \alpha = 1-\frac{\alpha}{2}.$$

$$\Rightarrow \alpha = \frac{2}{3}, \quad \frac{R_5}{R_4+R_5} = \frac{1}{3}$$

$$\Rightarrow R_5 = \frac{1}{2}R_4.$$

$$\left.\begin{aligned} \frac{W_n}{Q} &= \left(\frac{2}{3}\right)\left(\frac{1}{R_1C_1}\right) \\ W_n &= \frac{1}{\sqrt{R_1R_2C_1C_2}} \end{aligned}\right\} \Rightarrow \frac{3}{2}\frac{R_1C_1}{\sqrt{R_1R_2C_1C_2}} = Q$$

$$\Rightarrow \frac{3}{2}\sqrt{\frac{R_1C_1}{R_2C_2}} = Q$$

Let $R_1C_1 = R_2C_2 \Rightarrow Q = \frac{3}{2}$, $W_n = \frac{1}{R_1C_1}$.

$$H(jw) = \frac{\frac{2}{3}w^2}{\frac{w}{w_n}\sqrt{(w_n^2-w^2)^2+(\frac{2}{3}w_n w)^2}} = \frac{\frac{2}{3}w^2}{\frac{w}{w_n}\sqrt{w_n^4-\frac{14}{9}(w_n w)^2+w^4}}$$

$$H(jw) = 1$$

$$\Rightarrow \frac{4}{9}w^2 = w_n^2 - \frac{14}{9}w^2 + \frac{w^4}{w_n^2}$$

$$\Rightarrow w_n^4 - 2w^2 w_n^2 + w^4 = 0$$

$$\Rightarrow w_n^2 = w^2 = [(2\pi)(10MHz)]^2$$

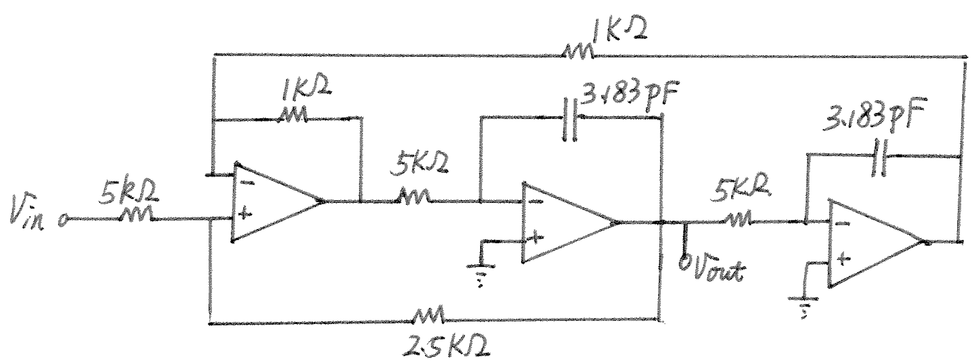
As derived, $w_n = \frac{1}{R_1 C_1}$

Let $R_1 = 5K\Omega \Rightarrow C_1 = 3.183pF$.

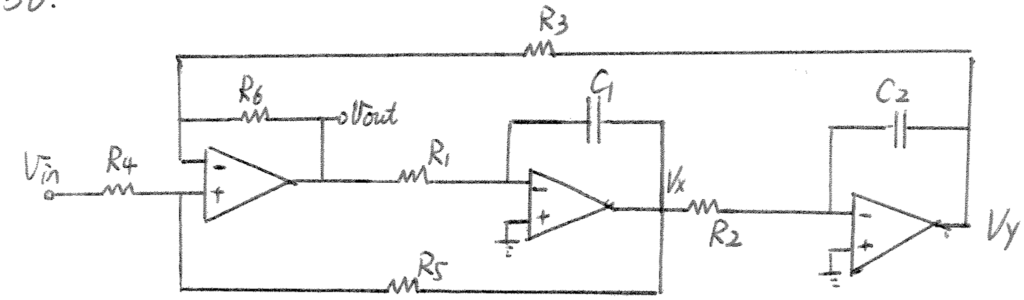
Let $R_2 = R_1 = 5k\Omega \Rightarrow C_2 = C_1 = 3.183pF$

Let $R_4 = 5K\Omega \Rightarrow R_5 = \frac{1}{2}R_4 = 2.5k\Omega$.

Let $R_3 = R_6 = 1K\Omega$.



56.



Low pass, $$\frac{V_y}{V_{in}} = \frac{\alpha s^2}{s^2+\frac{W_n}{Q}s+W_n^2}\cdot\frac{1}{R_1R_2C_1C_2s^2}$$

$$= \frac{\alpha}{(s^2+\frac{W_n}{Q}s+W_n^2)(R_1R_2C_1C_2)}$$

$$H(s) = \frac{\alpha/(R_1R_2C_1C_2)}{s^2+\frac{W_n}{Q}s+W_n^2}$$

$$H(jw) = \frac{\alpha/(R_1R_2C_1C_2)}{(W_n^2-W^2)+j\frac{W_n}{Q}W}$$

$$|H(jw)| = \frac{\alpha/(R_1R_2C_1C_2)}{\sqrt{(W_n^2-W^2)^2+(\frac{W_nW}{Q})^2}}$$

$$|H(jW_{3dB})| = \frac{\alpha/(R_1R_2C_1C_2)}{\sqrt{(W_n^2-W_{3dB}^2)^2+(\frac{W_nW_{3dB}}{Q})^2}} = \frac{\alpha}{W_n^2(R_1R_2C_1C_2)\sqrt{2}}$$

$$\Rightarrow \frac{\sqrt{(W_n^2-W_{3dB}^2)^2+(\frac{W_nW_{3dB}}{Q})^2}}{W_n^2} = \sqrt{2}$$

$$\Rightarrow 1-2\left(\frac{W_{3dB}}{W_n}\right)^2+\left(\frac{W_{3dB}}{W_n}\right)^4+\frac{1}{Q^2}\left(\frac{W_{3dB}}{W_n}\right)^2 = 2$$

$$\Rightarrow \left(\frac{W_{3dB}}{W_n}\right)^4 + \left(\frac{1}{Q^2} - 2\right)\left(\frac{W_{3dB}}{W_n}\right)^2 - 1 = 0$$

$$Q = 1.5 \Rightarrow \left(\frac{W_{3dB}}{W_n}\right)^4 - 1.556\left(\frac{W_{3dB}}{W_n}\right)^2 - 1 = 0$$

$$\Rightarrow \left(\frac{W_{3dB}}{W_n}\right)^2 = 2.0446, \quad -0.4891 \text{ (impossible)}$$

$$\Rightarrow W_{3dB} = 1.43\, W_n$$

$\Rightarrow$ Low pass corner $= 14.3\,MHz$.

High pass:

$$\frac{V_{out}}{V_{in}}(S) = \frac{\alpha S^2}{S^2 + \frac{W_n}{Q} S + W_n^2}$$

$$|H(jw)| = \frac{\alpha w^2}{\sqrt{(W_n^2 - W^2)^2 + \left(\frac{W W_n}{Q}\right)^2}}$$

$$|H(jW_{3dB})| = \frac{\alpha W_{3dB}^2}{\sqrt{(W_n^2 - W_{3dB}^2)^2 + \left(\frac{W_{3dB} W_n}{Q}\right)^2}} = \frac{\alpha}{\sqrt{2}}$$

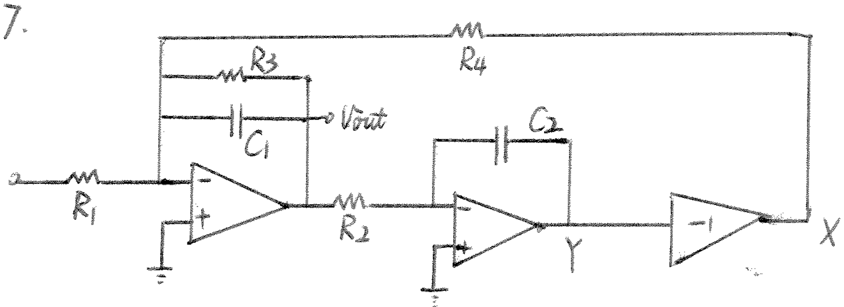
$$\Rightarrow \left(\frac{W_n}{W_{3dB}}\right)^4 + \left(\frac{1}{Q^2} - 2\right)\left(\frac{W_n}{W_{3dB}}\right)^2 - 1 = 0$$

Since $Q = 1.5$

$$\Rightarrow \left(\frac{W_n}{W_{3dB}}\right)^2 = 2.0446 \Rightarrow W_{3dB} = \frac{W_n}{1.43}$$

$\Rightarrow W_{3dB} = 7\,MHz$. (high pass corner)

57.



$W_n = 10\,MHz,$

$-13dB = 3MHz,$

$33MHz.$

$$\frac{V_{out}}{V_{in}} = -\frac{R_2R_3R_4}{R_1}\left(\frac{C_2S}{R_2R_3R_4C_1C_2S^2+R_2R_4C_2S+R_3}\right)$$

Same as in #55, $Q = \frac{10}{6.684} = 1.5.$

$$\frac{V_{out}}{V_{in}} = -\frac{\frac{1}{R_1C_1}S}{S^2+\frac{1}{R_3C_1}S+\frac{1}{R_2R_4C_1C_2}}$$

$$\frac{W_n}{Q} = \frac{1}{R_3C_1}, \quad W_n = \frac{1}{\sqrt{R_2R_4C_1C_2}}.$$

$$Q = \frac{R_3C_1}{\sqrt{R_2R_4C_1C_2}} = R_3\sqrt{\frac{C_1}{R_2R_4C_2}}$$

$$\frac{V_{out}}{V_{in}} = \frac{-\beta S}{S^2+\frac{W_n}{Q}s+W_n^2};$$

At $W = W_n \Rightarrow |H(jW_n)| = 1 = \frac{\beta Q}{W_n}.$

$$\frac{\beta Q}{W_n} = \left(\frac{1}{R_1C_1}\right)(R_3C_1) = \frac{R_3}{R_1} = 1$$

$$Q = R_3\sqrt{\frac{C_1}{R_2R_4C_2}} = 1.5, \qquad W_n = \frac{1}{\sqrt{R_2R_4C_2C_1}} = (10\times10^6)(2\pi)$$

Let $R_2 = R_4 = 1K\Omega$.

$$\frac{1}{\sqrt{10^6\times C_1C_2}} = (10\times10^6)(2\pi). \Rightarrow C_1C_2 = 2.533\times10^{-22}$$

Let $C_1 = C_2 = 15.9pF$

$$R_3\sqrt{\frac{1}{1000\times1000}} = 1.5$$

$$\Rightarrow R_3 = 1.5\,k\Omega = R_1$$

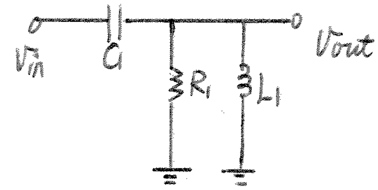
So : $R_1 = 1.5\,K\Omega$, $R_2 = 1K\Omega$, $R_3 = 1.5K\Omega$, $R_4 = 1K\Omega$.

$C_1 = C_2 = 15.9pF$.

58. Peaking : 1dB @ 7MHz.

Corner : 3.69 MHz

−13.6dB @ 2MHz.



$$\frac{V_{out}}{V_{in}} = \frac{s^2}{s^2 + \frac{1}{R_1C_1}s + \frac{1}{L_1C_1}}$$

Peaking 1dB ⇒ $Q = 0.926$.

$$\frac{\omega_n}{\sqrt{1 - 1/(2Q^2)}} = (2\pi)(7MHz) \Rightarrow \omega_n = (2\pi)(4.52MHz)$$

$$\frac{\omega_n}{Q} = \frac{(2\pi)(4.52MHz)}{0.926} = (2\pi)(4.88MHz) = \frac{1}{R_1C_1}$$

$$\omega_n^2 = [(2\pi)(4.52MHz)]^2 = \frac{1}{L_1C_1}$$

Let $C_1 = 100pF$, $L_1 = 12.4\mu H$, $R_1 = 326.1\Omega$.

With simulated inductor

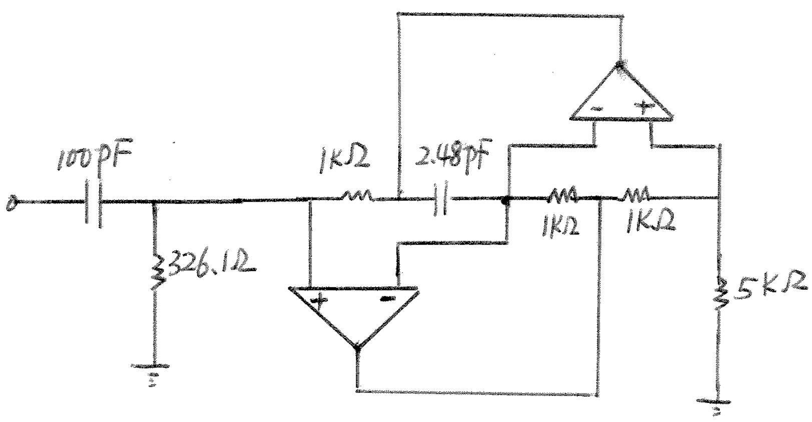
$$Z_{in} = \left(\frac{Z_1 Z_3}{Z_2 Z_4}\right) Z_5 = R_Y R_X C_s$$

Let $Z_1 = Z_3 = Z_4 = R_Y$, $Z_5 = R_X$, $Z_2 = C_s^{-1}$

Let $R_Y = 1K\Omega$, $R_X = 5K\Omega$.

$12.4 \times 10^{-6} = (1000)(5000)\,C$.

⇒ $C = 2.48pF$ to simulate an L of $12.4\mu H$.


100pF
326.1Ω
1KΩ
2.48pF
1KΩ
1KΩ
5KΩ

59. Corner @ 16.38 MHz, Peaking 0.5 dB @ 8 MHz.

5.9 dB ≈ 6 dB attenuation @ 20 MHz.

$$\frac{V_{out}}{V_{in}} = \frac{1}{R_1 R_x C^2 S^2 + R_1 C S + 1} = \frac{1/(R_1 R_x C^2)}{S^2 + \frac{S}{R_x C} + \frac{1}{R_1 R_x C^2}}$$

$0.5\,dB \Leftrightarrow 1.05292.$

$$\frac{Q}{\sqrt{1-\frac{1}{4Q^2}}} = 1.05292 \Rightarrow Q = 0.8636$$

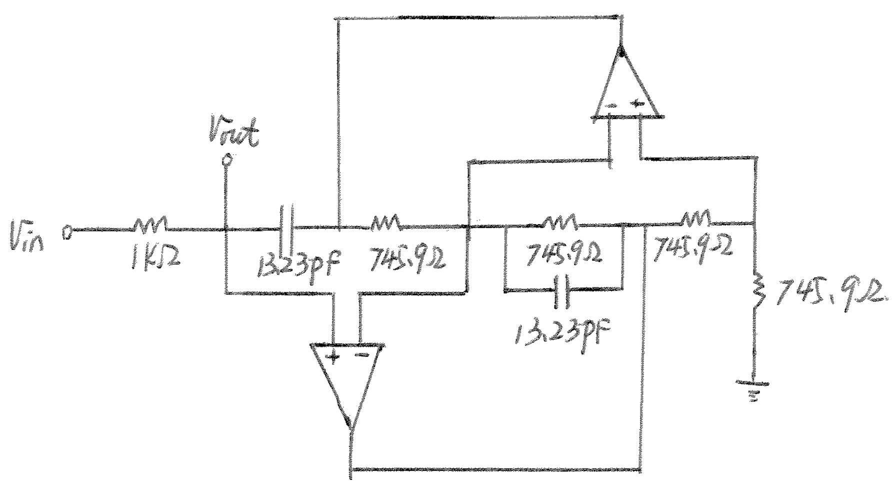
$$W_n \sqrt{1-\frac{1}{2Q^2}} = (2\pi)(8\times10^6)$$

$$\Rightarrow W_n = (2\pi)(13.934\times10^6).$$

$$\frac{1}{R_1 R_x C^2} = W_n^2, \quad \frac{1}{R_x C} = \frac{W_n}{Q} = (2\pi)(16.134\times10^6).$$

$$\Rightarrow \frac{1}{R_1 C} = 7.56\times10^7$$

Let $R_1 = 1k\Omega$, $C = 13.23\,pF$, $R_x = 745.9\,\Omega$.



60. Butterworth

a) Passband 0.5dB @ 1MHz , $-0.5dB \Leftrightarrow 0.944$

Attenuation 12dB @ 2.5MHz , $-12dB \Leftrightarrow 0.2512$

$$|H(j\omega)|^2_{1MHz} = \frac{1}{1+\left[\frac{(2\pi)(10^6)}{W_0}\right]^{2n}} = 0.944^2 \qquad ①$$

$$|H(j\omega)|^2_{2.5MHz} = \frac{1}{1+\left(\frac{2\pi\times 2.5\times 10^6}{W_0}\right)^{2n}} = 0.2512^2 \qquad ②$$

$$① \Rightarrow 1 = (0.944)^2\left[\left(\frac{2\pi\times 10^6}{W_0}\right)^{2n}+1\right]$$

$$\Rightarrow W_0^{2n} = 8.186\times(2\pi\times 10^6)^{2n} \qquad ③$$

$$② \Rightarrow 1 = (0.2512)^2\left[\frac{(2\pi\times 2.5\times 10^6)^{2n}}{8.186\times(2\pi\times 10^6)^{2n}}+1\right]$$

$$\Rightarrow n = 2.62$$

So choose $n=3$. $\overset{③}{\Rightarrow}$ $W_0 = 2\pi\times 1.42\text{MHz}$.

$$|H(j\omega)| = \frac{1}{\sqrt{1+\left(\frac{W}{2\pi\times 1.42\times 10^6}\right)^6}}$$

b). Passband : 0.1dB @ 1MHz

$$\frac{1}{1+\left(\frac{2\pi\times10^6}{W_0}\right)^{2n}} = (0.98855)^2 \qquad ①$$

Stopband attenuation : 12dB @ 2.5MHz

$$\frac{1}{1+\left(\frac{2\pi\times2.5\times10^6}{W_0}\right)^{2n}} = (0.2512)^2 \qquad ②$$

$$① \Rightarrow W_0^{2n} = 42.931\times(2\pi\times10^6)^{2n}$$

$$② \Rightarrow n = 3.52$$

$$\text{Choose } n=4 \xrightarrow{③} W_0 = 2\pi\times1.6\text{MHz}.$$

$$|H(jw)| = \frac{1}{\sqrt{1+\left(\frac{w}{2\pi\times1.6\times10^6}\right)^8}}$$

c). Passband 1dB @ 1MHz $\Rightarrow \frac{1}{1+\left(\frac{2\pi\times10^6}{W_0}\right)^{2n}} = (0.90)^2$ ④

Attenuation 18dB @ 2.5MHz $\Rightarrow \frac{1}{1+\left(\frac{2\pi\times2.5\times10^6}{W_0}\right)^{2n}} = (0.1259)^2$ ⑤

$$④ \Rightarrow W_0^{2n} = 4.263\times(2\pi\times10^6)^{2n} \qquad ⑥$$

$$⑤ \Rightarrow n = 3.0$$

$$\text{Choose } n=3 \xrightarrow{⑥} W_0 = 2\pi\times1.27\text{MHz}$$

$$|H(jw)| = \frac{1}{\sqrt{1+\left(\frac{w}{2\pi\times1.27\times10^6}\right)^6}}$$

d) Passband: 0.5dB @ 1MHz $\Rightarrow \dfrac{1}{1+\left(\frac{2\pi\times10^6}{W_0}\right)^{2n}} = 0.944^2$ ①

Attenuation: 18dB @ 2.5MHz $\Rightarrow \dfrac{1}{1+\left(\frac{2\pi\times2.5\times10^6}{W_0}\right)^{2n}} = 0.1259^2$ ②

① $\Rightarrow W_0^{2n} = 8.186\times(2\pi\times10^6)^{2n}$ ③

② $\Rightarrow n = 3.4$

Choose $n=4 \overset{③}{\Longrightarrow} W_0 = 2\pi\times1.3MHz$

$$|H(jw)| = \frac{1}{\sqrt{1+\left(\frac{w}{2\pi\times1.3\times10^6}\right)^8}}$$

<u>Chebyshev</u>

a) Passband 0.5dB @ 1MHz $\Rightarrow 0.5 = 20\log(\sqrt{1+\epsilon^2})$

$\Rightarrow \epsilon = 0.3493$, $W_0 = 1MHz$.

Attenuation 12dB @ 2.5MHz

$\Rightarrow \dfrac{1}{\sqrt{1+\epsilon^2\cosh^2[n\cosh^{-1}(\frac{w}{W_0})]}} = 0.2512$, when $w = 2.5\times10^6\times2\pi$.

Since W, W_0, ϵ, known

$\Rightarrow n = 1.9733$

Choose $n=2$, $|H(jw)| = \dfrac{1}{\sqrt{1+0.3493^2\, C_2^2(\frac{w}{W_0})}}$, $W_0 = 2\pi\times1MHz$

b). Passband 0.1 dB @ 1MHz , $w_0 = 1MHz$

$\Rightarrow 0.1 = 20\log(\sqrt{1+\epsilon^2}) \Rightarrow \epsilon = 0.1526$.

Attenuation 12dB @ 2.5MHz

$$\Rightarrow \frac{1}{1+0.1526^2\cosh^2[n\cosh^{-1}(2.5)]} = 0.2512^2$$

$\Rightarrow n = 2.5$

Choose $n=3$, $|H(jw)| = \dfrac{1}{\sqrt{1+0.1526^2 C_3^2(\frac{w}{w_0})}}$, $w_0 = (2\pi)(1MHz)$

c). Passband 1dB @ 1MHz, $w_0 = 1MHz$

$\Rightarrow 1 = 20\log\sqrt{1+\epsilon^2} \Rightarrow \epsilon = 0.5089$.

Attenuation 18dB @ 2.5MHz

$$\Rightarrow \frac{1}{1+0.5089^2\cosh^2[n\cosh^{-1}(2.5)]} = 0.1259^2 \Rightarrow n = 2.19$$

Choose $n=3$, $|H(jw)| = \dfrac{1}{\sqrt{1+0.5089^2 C_3^2(\frac{w}{w_0})}}$, $w_0 = (2\pi)(1MHz)$

d). Passband 0.5dB @ 1MHz $\Rightarrow \epsilon = 0.3493$

Attenuation 18dB @ 2.5MHz

$$\Rightarrow \frac{1}{1+0.3493^2\cosh^2[n\cosh^{-1}(2.5)]} = 0.1259^2 \Rightarrow n = 2.43$$

Choose $n=3$, $|H(jw)| = \dfrac{1}{\sqrt{1+0.3493^2 C_3^2(\frac{w}{w_0})}}$, $w_0 = (2\pi)(1MHz)$

61. a) Butterworth in Sallen and Key

$n=3,\ W_0 = (2\pi)(1.42MHz)$

$P_K = W_0 \cdot \exp(\frac{j\pi}{2})\ \exp(j\frac{2K-1}{2n}\pi),\ K=1,2,3.$

$P_1 = W_0 \exp(j\frac{2\pi}{3}) = (2\pi)(1.42MHz) \times (\cos\frac{2\pi}{3} + j\sin\frac{2\pi}{3})$

$P_2 = W_0 \exp(j\pi) = -(2\pi)(1.42MHz)$

$P_3 = W_0 \exp(j\frac{4\pi}{3}) = (2\pi)(1.42MHz) \times (\cos\frac{2\pi}{3} - j\sin\frac{2\pi}{3})$

$$H_{P_{1,3}}(S) = \frac{(-P_1)(-P_3)}{(S-P_1)(S-P_3)}$$

$$= \frac{[2\pi \times (1.42MHz)]^2}{S^2 - [4\pi \times (1.42MHz)\cos\frac{2\pi}{3}]S + [2\pi \times (1.42MHz)]^2}$$

$W_n = 2\pi \times 1.42MHz \left(= \frac{1}{\sqrt{R_1R_2C_1C_2}}\right)$

$\frac{W_n}{Q} = 2\pi \times 1.42MHz \cdot \cos(\frac{2\pi}{3}) \Rightarrow Q = \frac{-1}{2\cos(\frac{2\pi}{3})} \left(= \frac{1}{R_1+R_2}\sqrt{R_1R_2\frac{C_1}{C_2}}\right)$

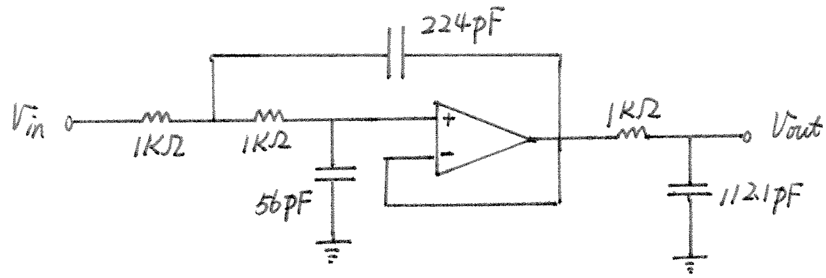
Let $C_1 = 4C_2$, $R_1 = R_2$, so that it satisfies $Q = \frac{-1}{2\cos(\frac{2\pi}{3})} = 1$

Also $\frac{1}{\sqrt{R_1R_2C_1C_2}} = 2\pi \times 1.42MHz$

Let $R_1 = R_2 = 1K\Omega \Rightarrow C_1 = 224pF,\ C_2 = 56pF$

$P_2 = -W_0, \quad \frac{1}{R_3C_3} = (2\pi)(1.42\,MHz)$

Let $R_3 = 1K\Omega \Rightarrow C_3 = 1121\,pF$.



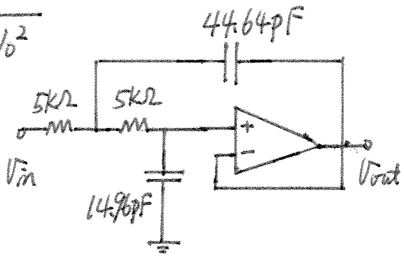
Chebyshev in Sallen and Key

$$P_k = -W_0 \sin\frac{(2K-1)\pi}{2n} \sinh\left(\frac{1}{n}\sinh^{-1}\frac{1}{\epsilon}\right) + jW_0 \cos\frac{(2K-1)\pi}{2n} \cosh\left(\frac{1}{n}\sinh^{-1}\frac{1}{\epsilon}\right)$$

$n=2, \quad W_0 = (2\pi)(1\,MHz), \quad \epsilon = 0.3493, \quad K=1,2.$

$P_{1,2} = -0.7128W_0 \pm j1.0041\,W_0$

$$H_{SK}(s) = \frac{(-P_1)(-P_2)}{(S-P_1)(S-P_2)} = \frac{(1.2314)^2 W_0^2}{S^2 + 1.4256W_0S + (1.2314)^2W_0^2}$$



$W_n = 1.2314W_0 = (2\pi)(1.2314\,MHz)$

$\frac{W_n}{Q} = 1.4256W_0 \Rightarrow Q = \frac{1.2314}{1.4256} = 0.8638$.

$W_n = \frac{1}{\sqrt{R_1R_2C_1C_2}}, \quad Q = \frac{1}{R_1+R_2}\sqrt{R_1R_2\frac{C_1}{C_2}}$

Let $R_1 = R_2 \quad Q = \frac{1}{2}\sqrt{\frac{C_1}{C_2}} \Rightarrow \frac{C_1}{C_2} = 4Q^2 = 2.9844$

$\frac{1}{\sqrt{R_1R_2C_1C_2}} = W_n = 2\pi(1.2314\,MHz), \Rightarrow \frac{1}{R_1C_2\cdot\sqrt{2.9844}} = 2\pi(1.2314\,MHz)$

Let $R_1 = R_2 = 5K\Omega \Rightarrow C_2 = 14.96\,pF, \; C_1 = 44.64\,pF$

b). Butterworth with SK

$n=4,\quad w_0=(2\pi)(1.6\,MHz)$

$P_K = w_0 \exp(j\frac{\pi}{2})\exp(j\frac{2K-1}{2n}\pi),\quad K=1,2,3,4.$

$P_1 = w_0\exp(j\frac{5\pi}{8}),\ P_2 = w_0\exp(j\frac{7\pi}{8}),\ P_3 = w_0\exp(-j\frac{5\pi}{8}),\ P_4 = w_0\exp(-j\frac{7\pi}{8})$

$$H_{SK1,4}(S) = \frac{(-P_1)(-P_4)}{(S-P_1)(S-P_4)} = \frac{[(2\pi)(1.6\times10^6)]^2}{S^2-[4\pi\times(1.6\times10^6)\cos(\frac{5\pi}{8})]S+[2\pi\times(1.6\times10^6)]^2}$$

$W_n = 2\pi\times1.6\times10^6$

$\frac{W_n}{Q} = (4\pi)(1.6\times10^6)\cos(\frac{5\pi}{8}) \Rightarrow Q=1.31.$

$W_n = \frac{1}{\sqrt{R_1R_2C_1C_2}},\quad Q=\frac{1}{R_1+R_2}\sqrt{R_1R_2\frac{C_1}{C_2}}.$

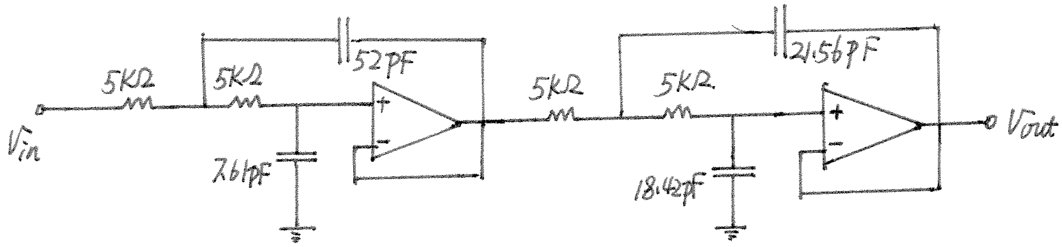
Let $R_1=R_2$, $Q=\frac{1}{2}\sqrt{\frac{C_1}{C_2}} \Rightarrow \frac{C_1}{C_2}=4Q^2=6.83.$

$W_n = \frac{1}{\sqrt{6.83}\,R_1C_2} = 2\pi\times1.6\times10^6$

Let $R_1=R_2=5K\Omega \Rightarrow C_2=7.61pF,\ C_1=52pF.$

Similarly, $H_{SK2,3}(S) = \frac{(-P_2)(-P_3)}{(S-P_2)(S-P_3)}$, it can be derived for $H_{SK2,3}$.

$R_1=R_2=5K\Omega,\ C_2=18.42pF,\ C_1=21.55pF.$



b). Chebyshev in SK.

$n=3$, $W_0 = (2\pi)(1\times10^6)$, $\epsilon = 0.1526$

$P_1 = -W_0(0.9694)\sin(\frac{1}{6}\pi) + jW_0(1.3927)\cos(\frac{\pi}{6}) = -0.4847W_0 + j1.2061W_0$

$P_2 = -W_0(0.9694)\sin(\frac{3}{6}\pi) + jW_0(1.3927)\cos(\frac{3\pi}{6}) = -0.9496W_0$

$P_3 = -W_0(0.9694)\sin(\frac{5}{6}\pi) + jW_0(1.3927)\sin(\frac{5\pi}{6}) = -0.4847W_0 - j1.2061W_0$

$$H_{SK}(s) = \frac{(-P_1)(-P_3)}{(s-P_1)(s-P_3)} = \frac{1.3^2 W_0^2}{s^2 + 0.9694W_0 s + (1.3)^2 W_0^2}$$

$W_n = 1.3W_0$

$\frac{W_n}{Q} = 0.9694W_0 \Rightarrow Q = 1.3410.$

$W_n = \frac{1}{\sqrt{R_1R_2C_1C_2}}$, $\quad Q = \frac{1}{R_1+R_2}\sqrt{R_1R_2\frac{C_1}{C_2}}$

Let $R_1 = R_2 \Rightarrow Q = \frac{1}{2}\sqrt{\frac{C_1}{C_2}} \Rightarrow \frac{C_1}{C_2} = (2Q)^2 = 7.1931.$

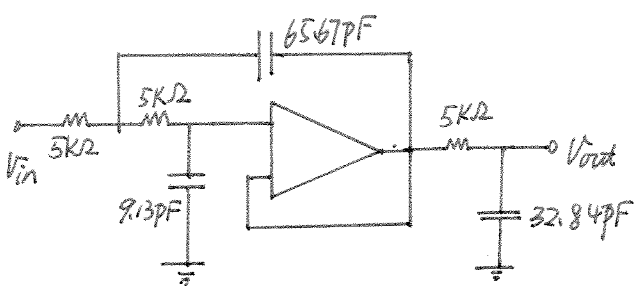
$W_n = \frac{1}{\sqrt{7.1931}\,R_1C_2} = (1.3)(2\pi)(1\times10^6)$

Let $R_1 = R_2 = 5K\Omega \Rightarrow C_2 = 9.13\,pF \Rightarrow C_1 = 65.67\,pF$

$P_2 = (2\pi)(0.9694\times10^6)$, and

$\frac{1}{R_3C_3} = P_2$

Let $R_3 = 5K\Omega \Rightarrow C_3 = 32.84\,pF$



c). Butterworth in SK.

$n=3$, $w_0 = (2\pi)(1.27\times10^6)$

$P_K = w_0 \exp(j\frac{\pi}{2}) \exp(j\frac{2K-1}{2n}\pi)$, $K=1,2,3.$

$$H_{P_{1,3}}(s) = \frac{(-P_1)(-P_3)}{(s-P_1)(s-P_3)} = \frac{[(2\pi)(1.27\times10^6)]^2}{s^2-[4\pi\times(1.27\times10^6)\cos(\frac{2\pi}{3})]s+[2\pi(1.27\times10^6)]^2}$$

$w_n = (2\pi)(1.27\times10^6)$

$\frac{w_n}{Q} = (4\pi)(1.27\times10^6)\cos(\frac{2\pi}{3}) \Rightarrow Q=1.$

$w_n = \frac{1}{\sqrt{R_1R_2C_1C_2}}$, $Q = \frac{1}{R_1+R_2}\sqrt{R_1R_2\frac{C_1}{C_2}}$

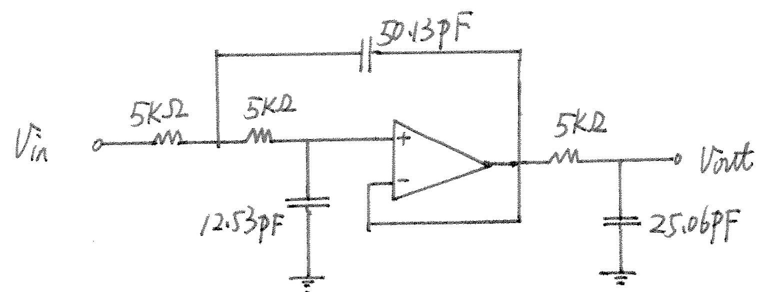
Let $C_1 = 4C_2$, $R_1 = R_2$

$w_n = \frac{1}{2R_1C_2} = (2\pi)(1.27\times10^6)$

Let $R_1 = R_2 = 5K\Omega \Rightarrow C_2 = 12.53pF$, $C_1 = 50.13pF$.

$P_2 = -w_0 = (2\pi)(1.27\times10^6) = \frac{1}{R_3C_3}$

Let $R_3 = 5K\Omega \Rightarrow C_3 = 25.06pF$



c). Chebyshev in SK.

$n=3,\ \epsilon \doteq 0.5089,\ \omega_0=(2\pi)(10^6).$

$P_1 = -0.2470\,\omega_0 + j\,0.9660\,\omega_0$

$P_2 = -0.4941\,\omega_0$

$P_3 = -0.2470\,\omega_0 - j\,0.9660\,\omega_0$

$$H_{P_{1,3}}(S) = \frac{(-P_1)(-P_3)}{(S-P_1)(S-P_3)} \doteq \frac{[(2\pi)(0.9971\times10^6)]^2}{S^2+(0.4940)(2\pi)(10^6)S+(2\pi\times0.9971\times10^6)^2}$$

$\omega_n = (2\pi)(0.9971\times10^6)$

$\frac{\omega_n}{Q} = (0.4940)(2\pi\times10^6) \Rightarrow Q = 2.02.$

$\omega_n = \frac{1}{\sqrt{R_1R_2C_1C_2}},\quad Q = \frac{1}{R_1+R_2}\sqrt{R_1R_2\frac{C_1}{C_2}}.$

Let $R_1=R_2 \Rightarrow Q = \frac{1}{2}\sqrt{\frac{C_1}{C_2}} \Rightarrow \frac{C_1}{C_2} = 4Q^2 = 16.296.$

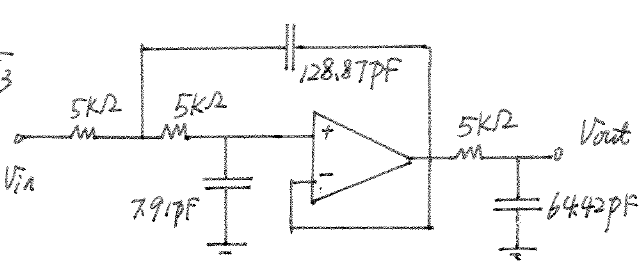
$\omega_n = \frac{1}{\sqrt{16.296}\,R_1C_2} = (2\pi)(0.9971\times10^6)$

Let $R_1=R_2=5\,k\Omega \Rightarrow C_2 = 7.91\,pF \Rightarrow C_1 = 128.87\,pF$

$P_2 = 2\pi\times0.4941\times10^6 = \frac{1}{R_3C_3}$

Let $R_3 = 5\,k\Omega$

$\Rightarrow C_3 = 64.42\,pF$



d). Butterworth in SK.

$n=4, \quad w_0=(2\pi)(1.3\times10^6)$

$P_K = w_0 \exp(j\frac{\pi}{2}) \exp(j\frac{2K-1}{2n}\pi), \quad K=1,2,3,4.$

$H_{SK_{1,4}}(S) = \dfrac{(-P_1)(-P_4)}{(S-P_1)(S-P_4)} = \dfrac{w_0^2}{S^2-[2w_0\cos(\frac{5\pi}{8})]S+w_0^2}$

$W_n = W_0 = (2\pi)(1.3\times10^6)$

$\dfrac{W_n}{Q} = 4\pi(1.3\times10^6)\cos(\frac{5\pi}{8}) \Rightarrow Q = 1.31$

$W_n = \dfrac{1}{\sqrt{R_1R_2C_1C_2}}, \quad Q = \dfrac{1}{R_1+R_2}\sqrt{R_1R_2\frac{C_1}{C_2}}.$

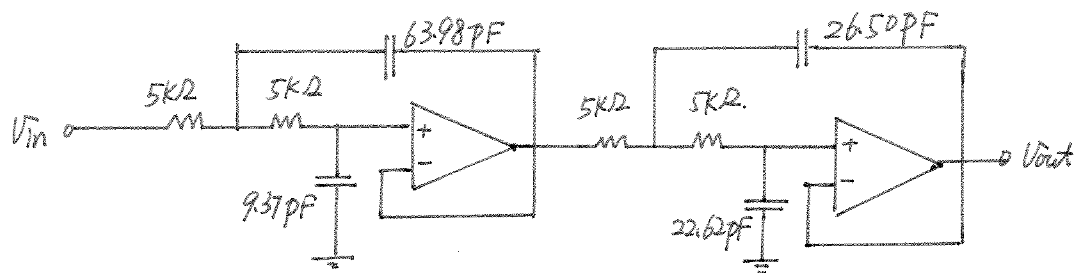
Let $R_1=R_2 \Rightarrow Q=\frac{1}{2}\sqrt{\frac{C_1}{C_2}} \Rightarrow \frac{C_1}{C_2}=4Q^2=6.828$

$W_n = \dfrac{1}{\sqrt{6.828}\,R_1C_2} = (2\pi)(1.3\times10^6)$

Let $R_1=R_2=5K\Omega \Rightarrow C_2=9.37pF \Rightarrow C_1=63.98\,pF$

Similarly, $H_{SK_{2,3}}(S) = \dfrac{(-P_2)(-P_3)}{(S-P_2)(S-P_3)}$. It can be derived that

$R_1=R_2=5K\Omega, \; C_2=22.62pF, \; C_1=26.50pF.$



d). Chebyshev in SK.

$n=3,\ \epsilon=0.3493,\ W_0=(2\pi)(1\times10^6)$

$P_1 = -W_0\,0.6265\sin(\frac{1}{6}\pi) + jW_0(1.11800)\cos(\frac{1}{6}\pi)$

$P_2 = -W_0\,0.6265$

$P_3 = -W_0\,0.6265\sin(\frac{5}{6}\pi) + jW_0(1.1800)\cos(\frac{5}{6}\pi)$

$$H_{P_{1,3}} = \frac{(-P_1)(-P_3)}{(S-P_1)(S-P_3)} = \frac{[2\pi\times1.069\times10^6]^2}{S^2+(0.6265)(2\pi\times10^6)S+(2\pi\times1.069\times10^6)^2}$$

$W_n=(2\pi)(1.069\times10^6)$

$\frac{W_n}{Q}=(0.6265)(2\pi\times10^6) \Rightarrow Q=1.7063$

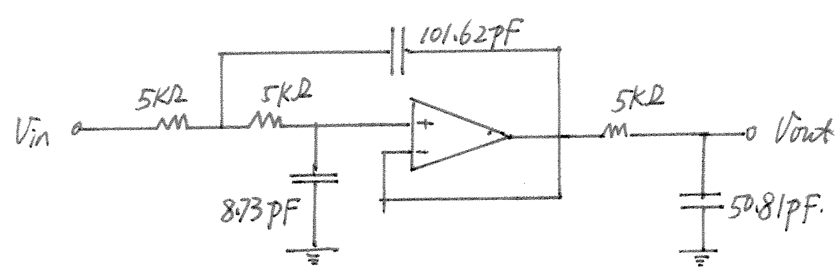
Let $R_1=R_2 \Rightarrow Q=\frac{1}{R_1+R_2}\sqrt{R_1R_2\frac{C_1}{C_2}}=\frac{1}{2}\sqrt{\frac{C_1}{C_2}} \Rightarrow \frac{C_1}{C_2}=4Q^2=11.6459.$

$$W_n=\frac{1}{\sqrt{R_1R_2C_1C_2}}=\frac{1}{\sqrt{11.6459}\,R_1C_2}=(2\pi)(1.069\times10^6)$$

Let $R_1=R_2=5K\Omega \Rightarrow C_2=8.73pF \Rightarrow C_1=101.62pF.$

$-P_2=(0.6265)(2\pi\times10^6)=\frac{1}{R_3C_3}$

Let $R_3=5K\Omega \Rightarrow C_3=50.81pF.$



62) a). Butterworth TT

$n=3$, $W_0 = (2\pi)(1.42\times10^6)$

$P_1 = W_0 \exp(j\frac{2\pi}{3})$, $P_2 = -W_0$, $P_3 = W_0 \exp(-j\frac{2\pi}{3})$.

$$H_{P_{1,3}} = \frac{[2\pi\times(1.42\times10^6)]^2}{s^2 - [4\pi\times(1.42\times10^6)\cos(\frac{2\pi}{3})]s + [2\pi\times1.42\times10^6)^2}$$

$W_n = (2\pi)(1.42\times10^6)$.

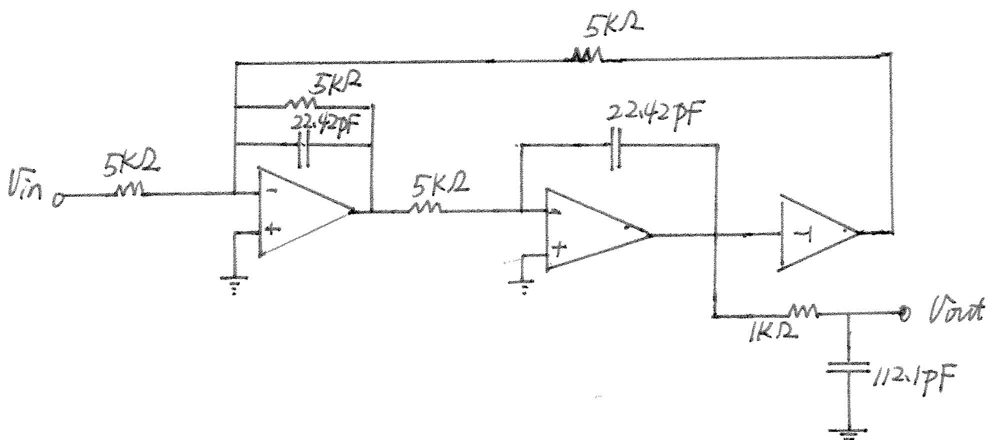
$$Q = \frac{-1}{2\cos(\frac{2\pi}{3})} = 1$$

$W_n = \frac{1}{\sqrt{R_2R_4C_1C_2}}$, $Q = R_3\sqrt{\frac{C_1}{C_2R_2R_4}}$.

Let $R_2 = R_4 = R_3 = 5K\Omega$, $C_1 = C_2 = 22.42pF$

$-P_2 = +(2\pi)(1.42\times10^6) = \frac{1}{R_3C_3}$.

Let $R_3 = 1K\Omega \Rightarrow C_3 = 112.1\,pF$



$R_1 = R_2 = R_4$, to match low frequency gain requirement.

a) Chebyshev II

$n=2,\ w_0=(2\pi)(1MHz),\ \epsilon=0.3493$

$$H_{p_{1,2}} = \frac{(1.2314)^2 w_0^2}{s^2+1.4256 w_0 s+(1.2314)^2 w_0^2}$$

$w_n=(2\pi)(1.2314\times10^6)$

$Q=\frac{1.2314}{1.4256}=0.8638$

$Q=R_3\sqrt{\frac{C_1}{R_2R_4C_2}},\quad w_n=\frac{1}{\sqrt{R_2R_4C_1C_2}}$

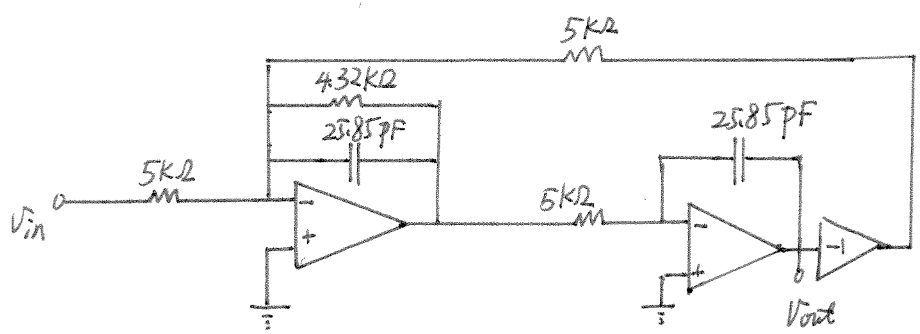
Let $C_1=C_2,\ R_2=R_4$

$w_n=\frac{1}{R_2C_1}=(2\pi)(1.2314\times10^6)$

Let $R_2=R_4=5K\Omega,\ C_1=C_2=25.85pF$

$R_1=R_2=R_4=5K\Omega$, to match low frequency gain of unity.

$Q=\frac{R_3}{R_2} \Rightarrow R_3=4.32K\Omega.$



b). Butterworth with TT

$n=4,\ w_0=(2\pi)(1.6\times10^6)$

$P_1 = w_0 \exp(j\frac{5\pi}{8})$, $P_4 = w_0 \exp(-j\frac{5\pi}{8})$

$P_2 = w_0 \exp(j\frac{7\pi}{8})$, $P_3 = w_0 \exp(-j\frac{7\pi}{8})$

$$H_{P_{1,4}} = \frac{w_0^2}{s^2 - [2w_0 \cos(\frac{5\pi}{8})]s + w_0^2}$$

$W_n = W_0 = (2\pi)(1.6\times10^6)$

$\frac{W_n}{Q} = (4\pi)(1.6\times10^6)\cos(\frac{5\pi}{8}) \Rightarrow Q = 1.31$

$W_n = \frac{1}{\sqrt{R_2R_4C_1C_2}}$, $Q = R_3\sqrt{\frac{C_1}{R_2R_4C_2}}$

Let $R_2 = R_4,\ C_1 = C_2 \Rightarrow W_n = \frac{1}{R_2C_1} = (2\pi)(1.6\times10^6)$

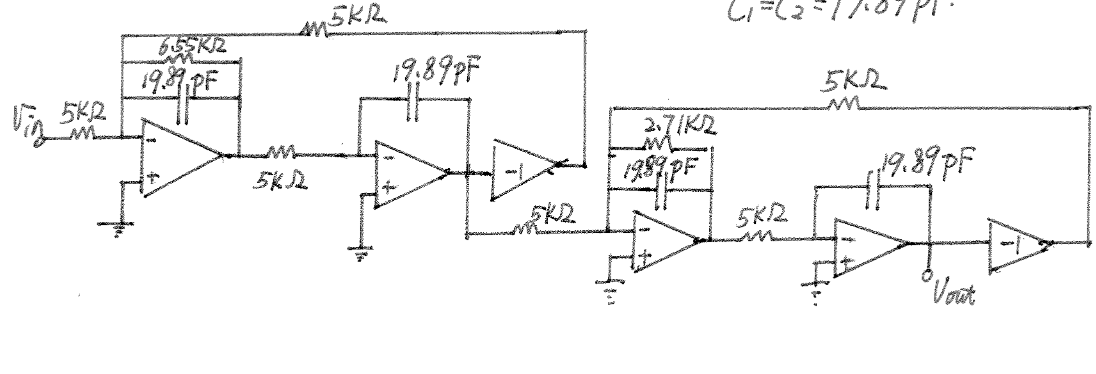
Let $R_2 = R_4 = 5K\Omega \Rightarrow C_1 = C_2 = 19.89pF$.

$R_1 = R_2 = R_4 = 5K\Omega$, to obtain a low-frequency gain of unity.

$Q = \frac{R_3}{R_2} = 1.31 \Rightarrow R_3 = 1.31R_2 = 6.55K\Omega$.

Similarly for $H_{P_{2,3}}$, it can be derived that $R_1=R_2=R_4=5K\Omega, R_3 = 2.7K\Omega$

$C_1=C_2=19.89pF$.



b). Chebyshev with TT

$n=3, \quad \epsilon = 0.1526, \quad W_0 = (2\pi)(1\times10^6)$

$P_{1,3} = 0.4847 W_0 \pm j\,1.2061 W_0$

$P_2 = -0.9694 W_0$

$$H_{P_{1,3}}(S) = \frac{(1.3)^2 W_0^2}{S^2 + 0.9694 W_0 S + (1.3)^2 W_0^2}$$

$W_n = 1.3 W_0, \quad \frac{W_n}{Q} = 0.9694 W_0$

$$Q = \frac{1.3}{0.9694} = 1.3410$$

$$Q = R_3 \sqrt{\frac{C_1}{R_2 R_4 C_2}}, \quad W_n = \frac{1}{\sqrt{R_2 R_4 C_1 C_2}}$$

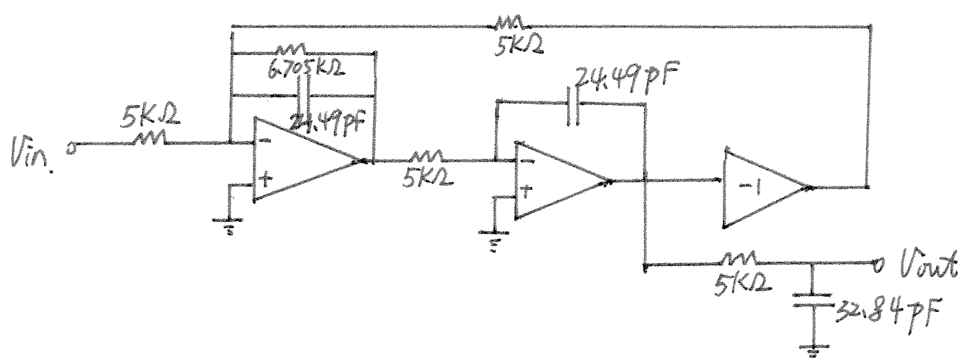
Let $R_2 = R_4,\ C_1 = C_2,\ \frac{1}{R_2 C_1} = (1.3)(2\pi)(10^6)$

Let $R_2 = R_4 = 5K\Omega, \Rightarrow C_1 = C_2 = 24.49\,pF$.

$R_1 = R_2 = R_4 = 5K\Omega$, to obtain Low-frequency gain of unity.

$R_3 = Q R_2 = 6.705K\Omega$.

$-P_2 = (2\pi)(0.9694\times10^6) = \frac{1}{R_5 C_5}$, Let $R_5 = 5K\Omega \Rightarrow C_5 = 32.84\,pF$.



c) Butterworth with TT

$n=3,\ w_0=(2\pi)(1.27\times10^6)$

$P_1 = w_0 \exp(j\frac{2\pi}{3}),\quad P_3 = w_0 \exp(-j\frac{2\pi}{3}),\quad P_2 = -w_0.$

$$H_{P_{1,3}}(s) = \frac{[(2\pi)(1.27\times10^6)]^2}{s^2 - [(4\pi)(1.27\times10^6)\cos(\frac{2\pi}{3})]s + [(2\pi)(1.27\times10^6)]^2}$$

$W_n = (2\pi)(1.27\times10^6),\quad \frac{W_n}{Q} = (4\pi)(1.27\times10^6)\cos(\frac{2\pi}{3})$

$Q = -\frac{1}{2\cos(\frac{2\pi}{3})} = 1.$

$Q = R_3\sqrt{\frac{C_1}{R_2R_4C_2}},\quad W_n = \frac{1}{\sqrt{R_2R_4C_1C_2}}$

Let $R_2=R_4,\ C_1=C_2,\ \frac{1}{R_2C_1} = (2\pi)(1.27\times10^6)$

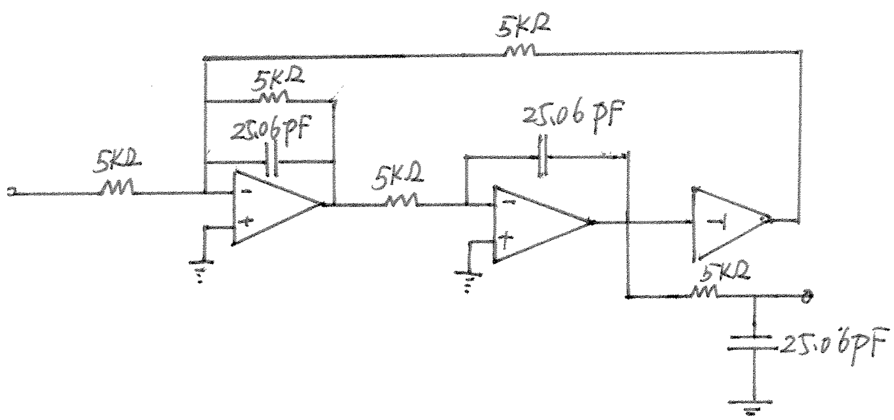
Let $R_2=R_4=5K\Omega \Rightarrow C_1=C_2=25.06\ pF.$

$R_1=R_2=R_4=5K\Omega$, to obtain Low-fequency gain of unity.

$R_3 = QR_2 = 5K\Omega.$

$-P_2 = W_0 \Rightarrow \frac{1}{R_5C_5} = (2\pi)(1.27\times10^6)$

Let $R_5=5K\Omega \Rightarrow C_5 = 25.06pF$



c) Chebyshev TT

$n=3, \quad \epsilon = 0.5089, \quad w_0 = (2\pi)(1\times10^6)$

$P_{1,3} = -0.2470\, w_0 \pm j\, 0.9660\, w_0 \ , \quad P_2 = -0.4941\, w_0$

$$H_{P_{1,3}}(s) = \frac{[(2\pi)(0.9971\times10^6)]^2}{s^2 + (0.4940)(2\pi\times10^6)s + [2\pi\times0.9971\times10^6]^2}$$

$W_n = (2\pi)(0.9971\times10^6)$

$\frac{W_n}{Q} = (0.4940)(2\pi\times10^6) \Rightarrow Q = 2.02$

$Q = R_3\sqrt{\frac{C_1}{R_2R_4C_2}} \ , \qquad W_n = \frac{1}{\sqrt{R_2R_4C_1C_2}}$

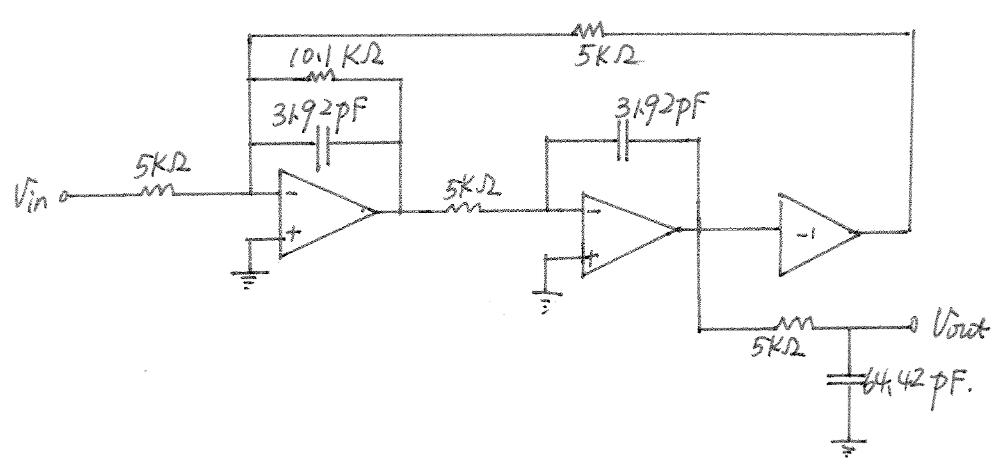
Let $R_2 = R_4$, $C_1 = C_2 \Rightarrow \frac{1}{R_2C_1} = (2\pi)(0.9971\times10^6)$

Let $R_2 = R_4 = 5K\Omega \Rightarrow C_1 = C_2 = 31.92pF$

$R_1 = R_2 = R_4 = 5K\Omega$, to obtain Low-frequency gain of unity.

$R_3 = QR_2 = 10.1\ K\Omega$.

$-P_2 = (2\pi)(0.4941\times10^6) = \frac{1}{R_5C_5}$. Let $R_5 = 5K\Omega \Rightarrow C_5 = 64.42pF$.



d). Butterworth in TT

$n=4,\ W_0=(2\pi)(1.3\times10^6)$

$P_{1,4}=W_0\exp(\pm j\frac{5\pi}{8}),\quad P_{2,3}=W_0\exp(\pm j\frac{7\pi}{8})$

$$H_{P_{1,4}}(S)=\frac{W_0^2}{S^2-[2W_0\cos(\frac{5\pi}{8})]S+W_0^2}$$

$W_n=(2\pi)(1.3\times10^6)$

$\frac{W_n}{Q}=(4\pi)(1.3\times10^6)\cos(\frac{5\pi}{8})\Rightarrow Q=-\frac{1}{2\cos(\frac{5\pi}{8})}=1.31.$

$W_n=\frac{1}{\sqrt{R_2R_4C_1C_2}},\quad Q=R_3\sqrt{\frac{C_1}{R_2R_4C_2}}$

Let $R_2=R_4,\ C_1=C_2,\ W_n=\frac{1}{\sqrt{R_2^2C_1^2}}=\frac{1}{R_2C_1}=(2\pi)(1.3\times10^6)$

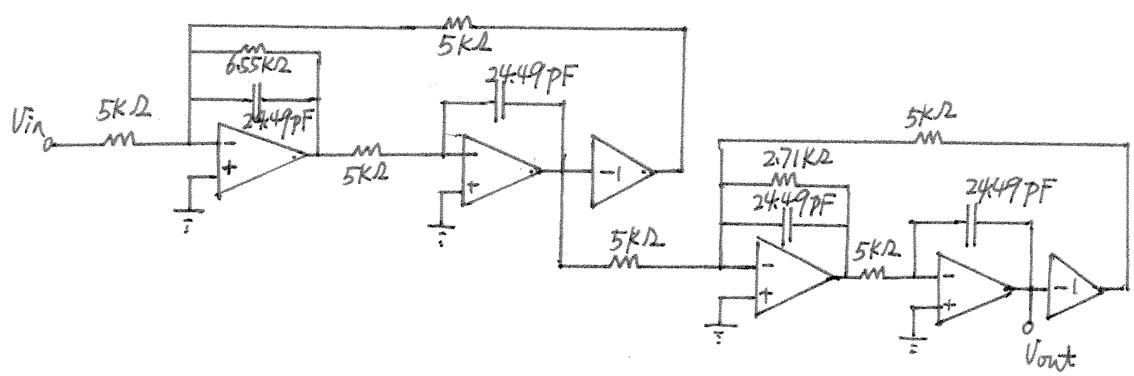
Let $R_2=R_4=5K\Omega\Rightarrow C_1=C_2=24.49pF.$

$R_1=R_2=R_4=5K\Omega$, to obtain a low-frequency gain of unity.

$R_3=QR_2=6.55K\Omega.$

Similarly, $H_{P_{2,3}}(S)=\frac{W_0^2}{S^2-[2W_0\cos(\frac{7\pi}{8})]S+W_0^2}$

It can be obtained that $R_1=R_2=R_4=5K\Omega$, $R_3=2.71K\Omega$, $C_1=C_2=24.49pF$.



d). Chebyshev II

$n=3,\quad \epsilon = 0.3493,\quad \omega_0 = (2\pi)(1\times10^6)$

$P_{1,3} = -0.3133\,\omega_0 \pm j1.022\,\omega_0.,\quad P_2 = -0.6265\,\omega_0.$

$$H_{P_{1,3}} = \frac{[2\pi\times1.069\times10^6]^2}{s^2+(0.6265)(2\pi\times10^6)s+(2\pi\times1.069\times10^6)^2}$$

$\omega_n = (2\pi)(1.069\times10^6)$

$\frac{\omega_n}{Q} = (0.6265)(2\pi\times10^6) \Rightarrow Q = \frac{1.069}{0.6265} = 1.7063$

$\omega_n = \frac{1}{\sqrt{R_2R_4C_1C_2}}\ ,\quad Q = R_3\sqrt{\frac{C_1}{R_2R_4C_2}}$

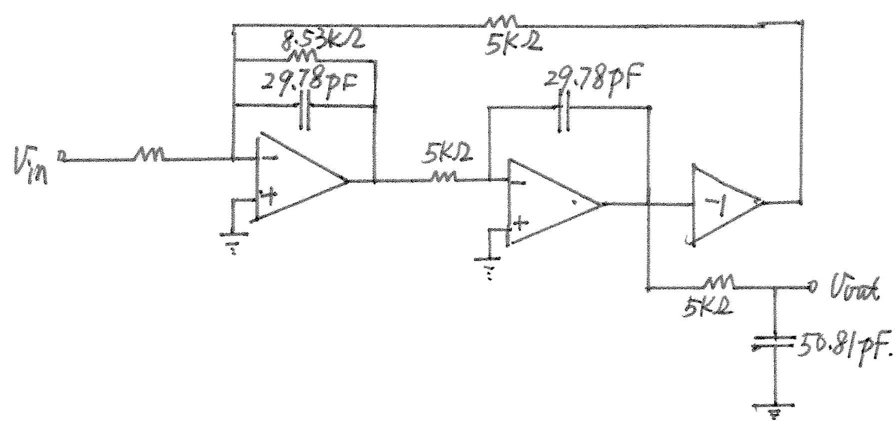
Let $R_2=R_4,\ C_1=C_2,\ \frac{1}{R_2C_1} = (2\pi)(1.069\times10^6)$

Let $R_2=R_4=5K\Omega \Rightarrow C_1=C_2=29.78\,pF.$

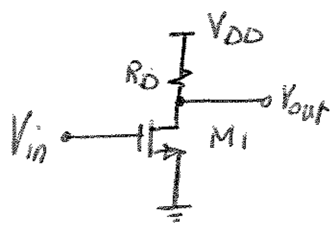
$R_1=R_2=R_4=5K\Omega$, to obtain a low-frequency gain of unity.

$R_3 = QR_2 = 8.53\,K\Omega.$

$-P_2 = 0.6265\times(2\pi\times10^6) = \frac{1}{R_5C_5}$, Let $R_5 = 5K\Omega \Rightarrow C_5 = 50.81\,pF$



1.



M_1 operates in the triode region

$$I_D = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left[2(V_{DD}-V_{TH})V_{out,min} - V_{out,min}^2\right]$$

$R_D = 10K$

$$V_{out} = V_{DD} - R_D I_D$$

$\left(W/L\right)_1 = 3/0.18$

$V_{out,min} = ?$ when $V_{in} = V_{DD}$

$$V_{out,min} = V_{DD} - R_D I_{D,max}$$

$$= V_{DD} - \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left[2(V_{DD}-V_{TH})V_{out,min} - V_{out,min}^2\right] \times R_D$$

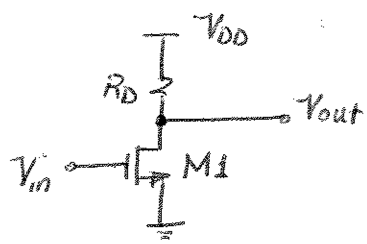
If the second term in the square brackets is neglected, then

$$V_{out,min} \simeq \frac{V_{DD}}{1+\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{DD}-V_{TH}) \times R_D}$$

$$= \frac{1.8}{1+100\times10^{-6}\times 3/0.18 \times (1.8-0.4)\times10^5}$$

$$\boxed{V_{out,min} \simeq 74\text{ mV}}$$

2.



$V_{out,min} \leq 100\,mV$

$R_D = 5K\Omega$

$(W/L)_{1,min} = ?$

Output low level establishes for $V_{in} = V_{DD}$, driving M_1 into the triode region.

$$I_{D\,max} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1\left[2(V_{DD}-V_{TH})V_{out,min} - V_{out,min}^2\right]$$

$$V_{out,min} = V_{DD} - R_D \times I_{D,max}$$

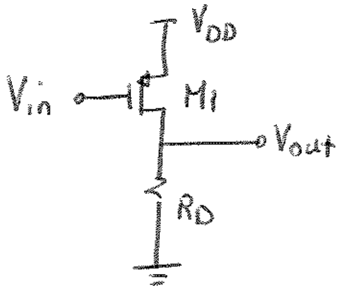
$$= V_{DD} - \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1\left[2(V_{DD}-V_{TH})V_{out,min} - V_{out,min}^2\right] \times R_D$$

$$\left(\frac{W}{L}\right)_1 = \frac{V_{DD} - V_{out,min}}{\frac{1}{2}\mu_n C_{ox}\left[2(V_{DD}-V_{TH})V_{out,min} - V_{out,min}^2\right] \times R_D}$$

$$\left(\frac{W}{L}\right)_{1,min} = \frac{1.8 - 100\times10^{-3}}{\frac{1}{2}\times100\times10^{-6}\left[2(1.8-0.4)100\times10^{-3} - (100\times10^{-3})^2\right]\times5\times10^3}$$

$$\boxed{\left(\frac{W}{L}\right)_{1,min} = 25}$$

3.



$\left(\frac{W}{L}\right)_1 = 20/0.18$, $R_D = 5K$

V_{OL} , $V_{OH} = ?$

(1) $V_{in} = V_{DD} \longrightarrow M_1$ off $\longrightarrow I_D = 0 \longrightarrow \boxed{V_{out} = V_{OL} = 0}$

(2) $V_{in} = 0 \longrightarrow M_1$ operates in the triode region

$$I_D = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_1\left[2(V_{SG} - |V_{THP}|)V_{SD} - V_{SD}^2\right]$$

$$I_{D,max} = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_1\left[2(V_{DD} - |V_{THP}|)(V_{DD} - V_{out}) - (V_{DD} - V_{out})^2\right] \quad (1)$$

$$I_{D,max} = \frac{V_{out}}{R_D} \quad (2)$$

Equating (1) and (2) and neglecting the second order term in the brackets

$$\frac{V_{out}}{R_D} = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_1 \times 2(V_{DD} - |V_{THP}|)(-V_{out} + V_{DD})$$

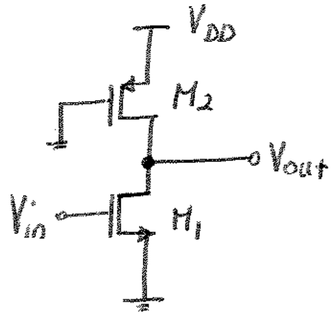
$$V_{out}\left[\frac{1}{R_D} + \mu_p C_{ox}\left(\frac{W}{L}\right)_1 (V_{DD} - |V_{THP}|)\right] = \mu_p C_{ox}\left(\frac{W}{L}\right)_1 (V_{DD} - |V_{THP}|) \times V_{DD}$$

$$V_{out} = \frac{R_D}{R_D + \dfrac{1}{\mu_p C_{ox}\left(\frac{W}{L}\right)_1 (V_{DD} - |V_{THP}|)}} V_{DD}$$

$$V_{out} = \frac{5000}{5000 + \frac{1}{50\times10^{-6} \times \left(\frac{20}{0.18}\right) \times (1.8 - 0.5)}} \times 1.8$$

$$\boxed{V_{out} = V_{OH} = 1.75\ V}$$

4.



$\left(\frac{W}{L}\right)_1 = 3/0.18 \quad \left(W/L\right)_2 = 2/0.18$

(a) if $V_{in} = V_{DD}$, M_2 Saturated $\rightarrow V_{OL} = ?$

(b) if $V_{in} = V_{out} \longrightarrow V_{in} = ?$

(a) $I_{D2} = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 \left(V_{SG} - |V_{THP}|\right)^2$

$I_{D2} = \frac{1}{2} \times 50 \times 10^{-6} \times \left(\frac{2}{0.18}\right)\left(1.8 - 0.5\right)^2$, Note that $V_{SG} = V_{DD}$

$I_{D2} = 4.7 \times 10^{-4}\ A$

$$I_{D1} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left[2\left(V_{GS} - V_{THN}\right)V_{DS} - V_{DS}^2\right]$$

$$= \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left[2\left(V_{DD} - V_{THN}\right)V_{OL} - V_{OL}^2\right]$$

However $I_{D1} = I_{D2}$

$$4.7 \times 10^{-4} = \frac{1}{2} \times 100 \times 10^{-6} \times \left(\frac{3}{0.18}\right)\left[2(1.8 - 0.4)V_{OL} - V_{OL}^2\right]$$

Neglecting the second-order term yields:

$$\boxed{V_{OL} = 0.2\ V}$$

As $\left(V_{in} - V_{THN}\right) = \left(V_{DD} - V_{THN}\right) = (1.8 - 0.4) = 1.4 > V_{DS_1} = V_{OL} = 0.2\ V$

the assumption of M_1 being in Triode region is correct

We define , $V_X = V_{in} - V_{TH,N} \rightarrow V_{in} = V_X + V_{TH,N}$

$$\frac{\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1}{\frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2} V_X^2 = 2(V_{DD} - |V_{TH,P}|)(V_{DD} - V_{TH,N} - V_X) - (V_{DD} - V_{TH,N} - V_X)^2$$

$$\frac{100}{50} \times \frac{3/0.18}{2/0.18} V_X^2 = 2(1.8 - 0.5)(1.8 - 0.4 - V_X) - (1.8 - 0.4 - V_X)^2$$

$$3V_X^2 = 2.6(1.4 - V_X) - (1.4 - V_X)^2$$

$$3V_X^2 = 3.64 - 2.6V_X - 1.96 + 2.8V_X - V_X^2$$

$$4V_X^2 - 0.2V_X - 1.68 = 0$$

$$V_X = \frac{0.2 \pm \sqrt{0.2^2 + 4 \times 4 \times 1.68}}{8} \longrightarrow \boxed{V_X = 0.67\ V}$$

$$V_{in} = V_X + V_{TH,N} = 0.67 + 0.4 \rightarrow \boxed{V_{in} = V_{out} = 1\ V}$$

This value of V_{out} guarantees that M_2 operates in the triode region.

Now, let's investigate the region of operation of M_2

$V_{SD2} = V_{DD} - V_{out}$

$= 1.8 - 0.2$

$V_{SD2} = 1.6\ V$

$V_{SG2} - |V_{THP}| = V_{DD} - |V_{THP}|$

$= 1.8 - 0.5$

$V_{SG2} - |V_{THP}| = 1.3$

As $V_{SD2} > V_{SG2} - |V_{THP}|$, M_2 operates in the saturation region and the initial assumption is valid.

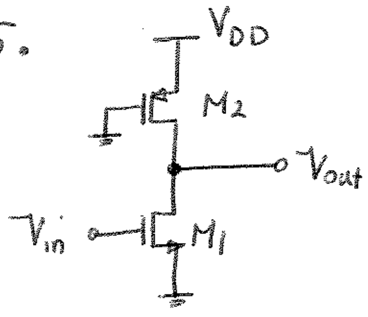
(b) As $V_{in} = V_{out} \longrightarrow M_1$ is saturated.

We assume that M_2 is in the triode region and check the validity of this assumption

$I_{D1} = I_{D2}$

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{in} - V_{THN})^2 = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 \Big[2(V_{DD} - |V_{THP}|) \times (V_{DD} - V_{in}) - (V_{DD} - V_{in})^2\Big]$$

5.



$V_{OL} \leqslant 100\ mV$

$(W/L)_2 = 3/0.18$

$\left(\frac{W}{L}\right)_{1,min} = ?$

$V_{in} = V_{DD} \rightarrow$ M_1 operates in the triode region and M_2 in the saturation.

$$I_{D2} = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 (V_{SG} - |V_{TH,p}|)^2$$

$$= \frac{1}{2} \times 50\times10^{-6} \times \left(\frac{3}{0.18}\right) \times (1.8 - 0.5)^2$$

$$I_{D2} = 7.041 \times 10^{-4} A$$

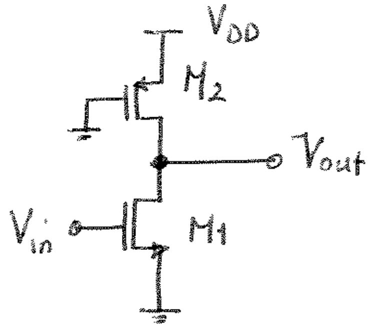
$$I_{D1} = I_{D2} = 7.041\times10^{-4} A$$

$$I_{D1} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left[2(V_{GS} - V_{TH,N})V_{DS} - V_{DS}^2\right]$$

$$7.041\times10^{-4} = \frac{1}{2}\times 100\times10^{-6}\left(\frac{W}{L}\right)_1 \left[2(1.8-0.4)0.1 - (0.1)^2\right]$$

$$\boxed{\left(\frac{W}{L}\right)_{1,min} = 52.16}$$

6.



$V_{OL} \leqslant 80\ mV$

$\left(\frac{W}{L}\right)_1 = 2/0.18$

$\left(\frac{W}{L}\right)_{2,max}$

$V_{in} = V_{DD} \longrightarrow M_1$ operates in the triode region and M_2 in the saturation

$$I_{D1} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1\left[2(V_{GS} - V_{TH,N})V_{DS} - V_{DS}^2\right]$$

$$I_{D1} = \frac{1}{2} \times 100\times10^{-6} \times \left(\frac{2}{0.18}\right) \times \left[2(1.8-0.4)0.08 - 0.08^2\right]$$

$$I_{D1} = 1.2\times10^{-4}\ A$$

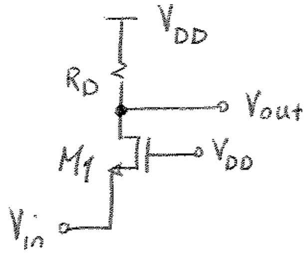
$$I_{D2} = I_{D1} = 1.2\times10^{-4}\ A$$

$$I_{D2} = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 (V_{SG} - |V_{TH,P}|)^2$$

$$1.2\times10^{-4} = \frac{1}{2}\times 50\times10^{-6}\left(\frac{W}{L}\right)_2 (1.8-0.5)^2$$

$$\boxed{\left(\frac{W}{L}\right)_{2,max} = 2.86}$$

7.



(a) If $V_{in} = 0$, V_{DD} , $V_{out} = ?$

If $V_{in} = 0 \longrightarrow M_1$ operates in the triode region.

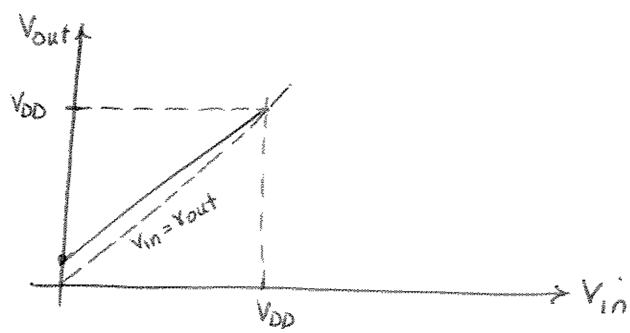
$$R_{on1} = \frac{1}{\mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{DD} - V_{TH,N})}$$

$$V_{out} \cong \frac{R_{on1}}{R_{on1} + R_D} \times V_{DD} \longrightarrow V_{out} \cong \frac{1}{1 + \mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{DD} - V_{TH,N}) R_D} \times V_{DD}$$

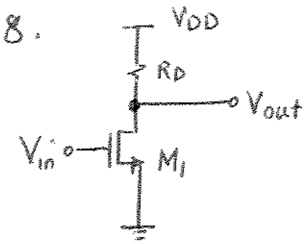
If $V_{in} = V_{DD} \longrightarrow V_{out} = V_{DD}$

No, this circuit does not invert.

(b) A trip point cannot be found for this circuit because $V_{out} = V_{in}$ line does not intersect the transfer characteristic of this buffer.



8.



$$\left(\frac{W}{L}\right)_1 = 5/0.18$$

$$R_D = 2K\Omega$$

$$NM_L\ ,\ NM_H = ?$$

Small signal gain of the circuit is equal to $-g_m R_D$

and $g_m = \mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{GS} - V_{TH,N})$

$$\mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{GS} - V_{TH,N}) R_D = 1 \quad , \quad V_{GS} = V_{IL}$$

$$\mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{IL} - V_{TH,N}) R_D = 1$$

$$V_{IL} = \frac{1}{\mu_n C_{ox} \left(\frac{W}{L}\right)_1 R_D} + V_{TH} = \frac{1}{100 \times 10^{-6} \times \frac{5}{0.18} \times 2000} + 0.4$$

$$\boxed{V_{IL} = 0.58\ V}$$

To determine NM_H, we note that V_{in} drives M_1 into the triode region

$$V_{out} = V_{DD} - R_D I_D$$

$$= V_{DD} - \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 \left[2(V_{in} - V_{TH,N}) V_{out} - V_{out}^2\right] R_D \quad (1)$$

$$\frac{\partial V_{out}}{\partial V_{in}} = -\frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 \left[2(V_{in} - V_{TH,N}) \frac{\partial V_{out}}{\partial V_{in}} - 2 V_{out} \frac{\partial V_{out}}{\partial V_{in}} + 2 V_{out}\right] R_D$$

$$\frac{\partial V_{out}}{\partial V_{in}} = -1 \ @\ V_{IH}$$

$$-1 = -\frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 \left[-2(V_{in} - V_{TH,N}) + 4 V_{out}\right] R_D$$

$$1 = \mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left[-V_{in} + V_{TH,N} + 2V_{out}\right] R_D$$

$$\frac{1}{\mu_n C_{ox}\left(\frac{W}{L}\right)_1 R_D} = -(V_{in} - V_{TH,N}) + 2V_{out}$$

$$V_{out} = \frac{1}{2\mu_n C_{ox}\left(\frac{W}{L}\right)_1 R_D} + \frac{V_{in} - V_{TH,N}}{2} \longrightarrow V_{out} = 0.5V_{in} - 0.11$$

Substituting This in (1) yields:

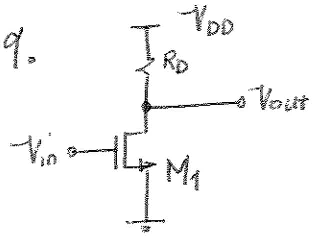
$$0.5V_{in} - 0.11 = 1.8 - \frac{1}{2} \times 100 \times 10^{-6} \times \frac{5}{0.18} \times 2000 \left[2(V_{in} - 0.4)(0.5V_{in} - 0.11) - (0.5V_{in} - 0.11)^2\right]$$

$$0.75V_{in}^2 - 0.33V_{in} - 0.6117 = \emptyset$$

$$V_{in} = V_{IH} = 1.15$$

$$NM_H = V_{DD} - V_{IH} = 1.8 - 1.15 \longrightarrow \boxed{NM_H = 0.65\,V}$$

9.



Small signal gain of the inverter is equal to $-g_m R_D$

and $g_m = \mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{GS} - V_{TH,N})$

$\mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{GS} - V_{TH,N}) R_D = 1$, $V_{GS} = V_{IL}$

$\mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{IL} - V_{TH,N}) R_D = 1 \longrightarrow V_{IL} = \dfrac{1}{\mu_n C_{ox} \left(\frac{W}{L}\right)_1 R_D} + V_{TH,N}$

If we double the value of $\left(\frac{W}{L}\right)_1$ or R_D

$$V_{IL} = \frac{1}{100\times10^{-6}\times\frac{5}{0.18}\times2000\times2} + 0.4 \longrightarrow \boxed{V_{IL} = 0.49}$$

To determin NM_H, we note that V_{in} drives M_1 into the triode region

$V_{out} = V_{DD} - R_D I_D$

$$= V_{DD} - \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left[2(V_{in} - V_{TH})V_{out} - V_{out}^2\right] R_D. \quad (1)$$

$$\frac{\partial V_{out}}{\partial V_{in}} = -\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left[2V_{out} + 2(V_{in} - V_{TH})\frac{\partial V_{out}}{\partial V_{in}} - 2V_{out}\frac{\partial V_{out}}{\partial V_{in}}\right]$$

$\frac{\partial V_{out}}{\partial V_{in}} = -1$ @ V_{IH}

$$V_{out} = \frac{1}{2\mu_n C_{ox}\left(\frac{W}{L}\right)_1 R_D} + \frac{V_{in} - V_{TH,N}}{2}$$

Doubling $\left(\frac{W}{L}\right)_1$ or R_D leads to $\boxed{V_{out} = 0.5V_{in} - 0.155}$

Substituting in (1) yields:

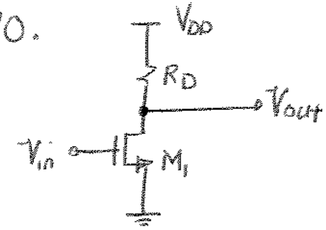
$$0.5V_{in} - 0.155 = 1.8 - \frac{1}{2} \times 100 \times 10^{-6} \times \frac{5}{0.18} \times 2000 \times 2\left[2(V_{in} - 0.4)(0.5V_{in} - 0.155) - (0.5V_{in} - 0.155)^2\right]$$

$$0.75V_{in}^2 - 0.465V_{in} - 0.251925 = 0$$

$$V_{in} = 0.967\,V \longrightarrow NM_H = 1.8 - 0.967$$

$$\boxed{NM_H = 0.833\,V}$$

10.



$$\left(\frac{W}{L}\right)_1 = \frac{5}{0.18}$$

$R_D = 2K$

NM_L and $NM_H = ?$ if $\frac{\partial V_{out}}{\partial V_{in}} = -0.5$ instead of -1

Small signal gain of the inverter is equal to "$-g_m R_D$"

and $g_m = \mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{GS} - V_{TH,N})$

$$\mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{IL} - V_{TH,N}) R_D = 0.5$$

$$V_{IL} = \frac{1}{2\mu_n C_{ox} \left(\frac{W}{L}\right)_1 R_D} + V_{TH,N} = \frac{1}{2 \times 100 \times 10^{-6} \times \frac{5}{0.18} \times 2000} + 0.4$$

$\boxed{V_{IL} = 0.49}$ which is less than 0.58 obtained in problem 8.

To determine NM_H, note that M_1 operates in the triode region

$$V_{out} = V_{DD} - R_D I_D$$

$$= V_{DD} - \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 \left[2(V_{in} - V_{TH}) V_{out} - V_{out}^2\right] R_D \quad (1)$$

$$\frac{\partial V_{out}}{\partial V_{in}} = -\frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 \left[2V_{out} + 2(V_{in} - V_{TH}) \frac{\partial V_{out}}{\partial V_{in}} - 2V_{out} \frac{\partial V_{out}}{\partial V_{in}}\right] R_D$$

$\frac{\partial V_{out}}{\partial V_{in}} = -0.5$ @ V_{IH}

$$-0.5 = -\frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 \left[-(V_{in} - V_{TH,N}) + 3V_{out}\right] R_D$$

$$V_{out} = \frac{1}{3\mu_n C_{ox} \left(\frac{W}{L}\right)_1 R_D} + \frac{V_{in} - V_{THN}}{3} \longrightarrow V_{out} = -73.33 \times 10^{-3} + 0.33 V_{in}$$

or $V_{out} = -\frac{0.22}{3} + \frac{V_{in}}{3}$

Substituting in (1) yields:

$$-\frac{0.22}{3} + \frac{V_{in}}{3} = 1.8 - \frac{1}{2} \times 100 \times 10^{-6} \times \frac{5}{0.18} \times 2000 \left[2 \left(V_{in} - 0.4 \right) \left(-\frac{0.22}{3} + \frac{V_{in}}{3} \right) - \left(-\frac{0.22}{3} + \frac{V_{in}}{3} \right)^2 \right]$$

$$5V_{in}^2 - 2.2V_{in} - 5.59 = 0$$

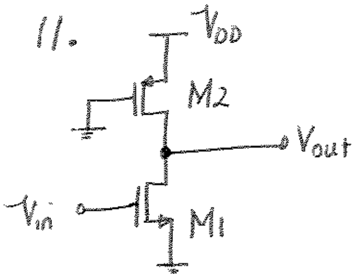
$$V_{in} = V_{IH} = 1.3$$

$$NM_H = 1.8 - 1.3$$

$$\boxed{NM_H = 0.5\ V}$$

less than 0.65 V obtained in problem 8 because V_{IH} is now further pushed up toward V_{DD}.

11.



$$\left(\frac{W}{L}\right)_1 = \frac{4}{0.18}$$

$$\left(\frac{W}{L}\right)_2 = \frac{9}{0.18}$$

To calculate V_{IL}, we assume that M_1 and M_2 operate in saturation and triode region respectively.

$$I_{D1} = I_{D2} \qquad (1)$$

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{in} - V_{TH,N})^2 = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 \left[2(V_{DD} - |V_{TH,P}|)(V_{DD} - V_{out}) - (V_{DD} - V_{out})^2\right]$$

$$\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{in} - V_{THN}) = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 \left[2(V_{DD} - |V_{TH,P}|)\left(-\frac{\partial V_{out}}{\partial V_{in}}\right) - 2(V_{DD} - V_{out})\left(-\frac{\partial V_{out}}{\partial V_{in}}\right)\right]$$

By substituting $\frac{\partial V_{out}}{\partial V_{in}}$ with "-1" in the above relationship:

$$\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{in} - V_{TH,N}) = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 \left[2(V_{DD} - |V_{TH,P}|) - 2(V_{DD} - V_{out})\right]$$

$$V_{out} = \frac{\mu_n C_{ox}\left(\frac{W}{L}\right)_1}{\mu_p C_{ox}\left(\frac{W}{L}\right)_2}(V_{in} - V_{THN}) + |V_{THP}| = \frac{100\times10^{-6}\times 4/0.18}{50\times10^{-6}\times 9/0.18}(V_{in} - 0.4) + 0.5$$

$$V_{out} = 0.144 + 0.88\,V_{in} \quad \text{or} \quad \boxed{V_{out} = \frac{8}{9}V_{in} + \frac{1.3}{9}}$$

Substituting V_{out} in (1) by the derivation versus V_{in} gives:

$$136\,V_{in}^2 - 108.8\,V_{in} - 115.13 = 0$$

$$\boxed{V_{in} = V_{IL} = NM_L = 1.4\,V}$$

To Calculate V_{IH}, we assume that M_1 and M_2 operate in the triode and saturation region respectively.

$I_{D1} = I_{D2}$

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1\left[2(V_{in}-V_{THN})V_{out} - V_{out}^2\right] = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2\left(V_{DD}-|V_{THP}|\right)^2 \quad (2)$$

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1\left[2V_{out} + 2(V_{in}-V_{THN})\frac{\partial V_{out}}{\partial V_{in}} - 2V_{out}\frac{\partial V_{out}}{\partial V_{in}}\right] = \emptyset$$

$$\frac{\partial V_{out}}{\partial V_{in}} = -1$$

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1\left[2V_{out} - 2(V_{in}-V_{THN}) + 2V_{out}\right] = \emptyset$$

$V_{out} = \dfrac{V_{in}-V_{THN}}{2}$ supstituted in (2) yields:

$$V_{in} = \sqrt{\frac{3}{2}}\left(V_{DD}-|V_{THP}|\right) + V_{THN}$$

$V_{in} = 2 \longrightarrow V_{out} = 0.8$ This value of V_{out} puts M_2 into the triode region so our initial assumption is not correct

Now we assume that both M_1 and M_2 operate in the triode region.

$I_{D1} = I_{D2}$

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1\left[2(V_{in}-V_{THN})V_{out} - V_{out}^2\right] = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2\left[2\left(V_{DD}-|V_{THP}|\right)\left(V_{DD}-V_{out}\right) - \left(V_{DD}-V_{out}\right)^2\right] \quad (3)$$

$$\mu_n C_{ox}\left(\frac{W}{L}\right)_1\left[2V_{out}+2(V_{in}-V_{THN})\frac{\partial V_{out}}{\partial V_{in}}-2V_{out}\frac{\partial V_{out}}{\partial V_{in}}\right]=\mu_p C_{ox}\left(\frac{W}{L}\right)_2 \times$$

$$\left[2(V_{DD}-|V_{THP}|)\left(-\frac{\partial V_{out}}{\partial V_{in}}\right)-2(V_{DD}-V_{out})\left(-\frac{\partial V_{out}}{\partial V_{in}}\right)\right]$$

$$\mu_n C_{ox}\left(\frac{W}{L}\right)_1\left[2V_{out}-2(V_{in}-V_{THN})+2V_{out}\right]=\mu_p C_{ox}\left(\frac{W}{L}\right)_2 \times$$

$$\left[2(V_{DD}-|V_{THP}|)-2(V_{DD}-V_{out})\right]$$

$$V_{out}=\frac{\frac{\mu_n C_{ox}(\frac{W}{L})_1}{\mu_p C_{ox}(\frac{W}{L})_2}(V_{in}-V_{THN})-|V_{THP}|}{2\frac{\mu_n C_{ox}(\frac{W}{L})_1}{\mu_p C_{ox}(\frac{W}{L})_2}-1}$$

$$V_{out}=\frac{8}{7}V_{in}-1.1$$

After substituting in (3) it leads to :

$$2.1769\,V_{in}^2-4.19\,V_{in}+0.576=\phi$$

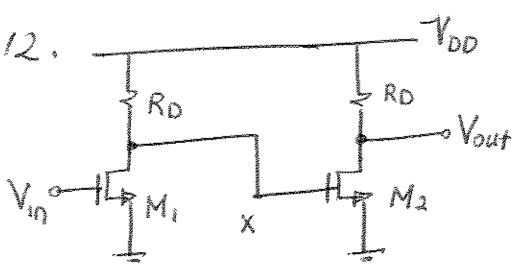
$$\boxed{V_{in}=1.77\,V}$$

$V_{out}=0.93\,V \longrightarrow$ The assumption is correct

$V_{IH}=1.77\,V \longrightarrow NM_H=1.8-1.77$

$$\boxed{NM_H=0.03\,V}$$

12.



The small signal gain of the circuit is equal to $-g_m R_D$ and since

$$g_m = \mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{GS} - V_{THN})$$

$$\mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{IL} - V_{THN}) R_D = 1$$

$$V_{IL} = \frac{1}{\mu_n C_{ox} \left(\frac{W}{L}\right)_1 R_D} + V_{THN} = \frac{2}{S} + 0.4 \;\; ; \;\; \left(\frac{W}{L}\right)_{1,2} = S$$

Now we calculate the output of M_1 for $V_{in} = V_{DD}$:

$$V_{DD} - R_D I_D = V_{out}$$

$$V_{DD} - \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left[2(V_{DD} - V_{THN}) V_{out} - V_{out}^2\right] R_D = V_{out} \;\; ; \;\; \left(\frac{W}{L}\right)_{1,2} = S$$

$$1.8 - \frac{1}{2} \times 100 \times 10^{-6} \times S \left[2(1.8 - 0.4)\left(\frac{2}{S} + 0.4\right) - \left(\frac{2}{S} + 0.4\right)^2\right] \times 5000 = \left(\frac{2}{S} + 0.4\right)$$

$$1.8 - 0.25 \times \left[2.8(2 + 0.4S) - S\left(\frac{2}{S} + 0.4\right)^2\right] = \frac{2}{S} + 0.4$$

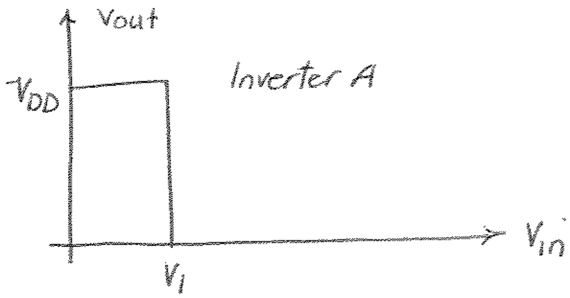
$$1.8S - 0.25 \times \left[2.8(2S + 0.4S^2) - S^2\left(\frac{2}{S} + 0.4\right)^2\right] = 2 + 0.4S$$

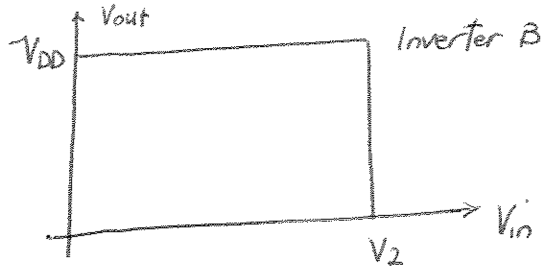
$$1.8S - 0.25 \times \left[5.6S + 1.12S^2 - 4 - 1.6S - 0.16S^2\right] = 2 + 0.4S$$

$$0.24S^2 - 0.4S + 1 = 0$$

$$\Delta < 0 \,!$$

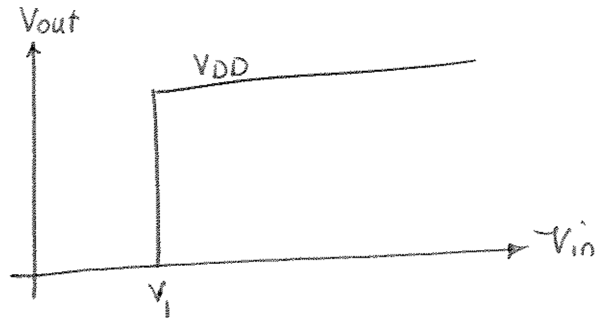
13.


Vout
VDD
Inverter A
V1
Vin

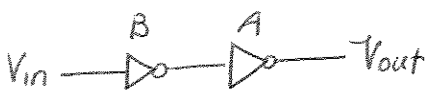

Vout
VDD
Inverter B
V2
Vin

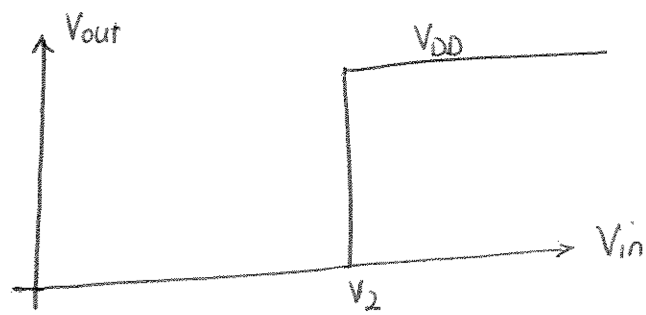
(a)


Vin
A
B
Vout

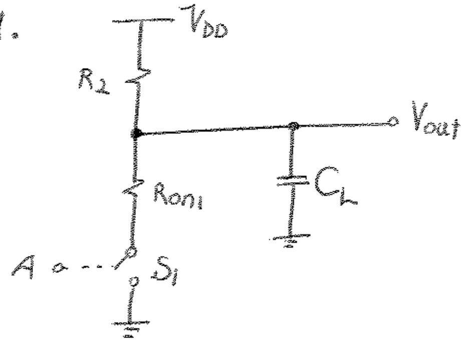

Vout
VDD
Vin
V1

(b)


Vin
B
A
Vout


Vout
VDD
Vin
V2

14.



$R_{on1} << R_2 \longrightarrow V_{out,min} \approx 0$

(a) $V_{out}(t) = V_{out}(0^-) + \left[V_{DD} - V_{out}(0^-)\right]\left(1 - \exp\frac{-t}{R_2C_L}\right) \quad t > 0$

Note that $V_{out}(0^-) = 0$, $V_{out}(\infty) = V_{DD}$

$V_{out}(t) = V_{DD} \times \left(1 - \exp\frac{-t}{R_2C_L}\right) \quad t > 0$

$0.95\, V_{DD} = V_{DD} \times \left(1 - \exp\frac{-T_{95\%}}{R_2C_L}\right)$

$\boxed{T_{95\%} = 3R_2C_L}$

(b) $V_{out}(t) = V_{out}(0^-) + \left[V_{out}(\infty) - V_{out}(0^-)\right] \times \left(1 - \exp\frac{-t}{R_2C_L}\right)$

$V_{out}(t) = V_{DD} + \left[0 - V_{DD}\right] \times \left(1 - \exp\frac{-t}{R_2C_L}\right)$

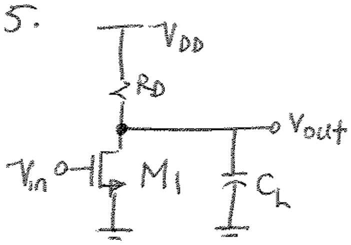
$V_{out}(t) = V_{DD}\exp\frac{-t}{R_2C_L}$

$0.05\, V_{DD} = V_{DD}\exp\frac{-T_{0.05}}{R_2C_L}$

$\boxed{T_{5\%} = 3R_2C_L}$

If $R_{on1} << R_2$, inverter exhibits equal rise and fall time (or low-to-high and high-to-low delay) at the output.

15.



$C_L = 50\,fF$

$T_R = 100\,pS$

$T_R = 3\,\tau_{out}$

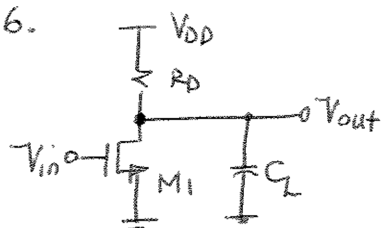
$R_{D,max} = ?$

$T_R = 3 R_D C_L = 100\,pS$

$$R_D \leqslant \frac{100\,pS}{3 \times 50\,fF}$$

$$\boxed{R_D \leqslant 666.67\,\Omega}$$

16.



$C_L = 100\,fF$

$V_{out,min} = 50\,mV$

$T_R = 200\,pS$

$R_D\,,\ \left(\frac{W}{L}\right)_1 = ?$

$T_R = 3\tau_{out}$

$T_R = 3R_DC_L$

$200\times10^{-12} = 3\times R_D\times 100\times10^{-15}$

$$\boxed{R_D = 666.667\ \Omega}$$

$V_{in} = V_{DD}$ places M_1 in the triode region

$$V_{out,min} = V_{DD} - R_D I_{D,max}$$

$$= V_{DD} - \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 R_D\left[2(V_{DD}-V_{THN})V_{out,min} - V_{out,min}^2\right]$$

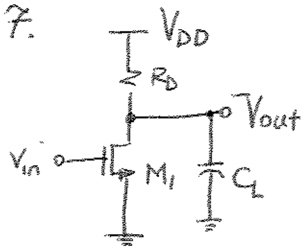
Neglecting the 2^{nd}-order term in the square brackets yields:

$$V_{out,min} = \frac{V_{DD}}{1+\mu_n C_{ox}\left(\frac{W}{L}\right)_1 R_D(V_{DD}-V_{TH})}$$

$$50\times10^{-3} = \frac{1.8}{1+100\times10^{-6}\times\left(\frac{W}{L}\right)_1\times 666.7\times(1.8-0.4)}$$

$$\boxed{\left(\frac{W}{L}\right)_1 = 375}$$

17.



$C_L = 100\,fF$

$I_{D_{max}} \leq 1mA$

$T_{R,min} = ?$

$V_{out,min} \approx 0$

$$I_{D,max} = \frac{V_{DD} - V_{out,min}}{R_D}$$

$$10^{-3} = \frac{1.8 - 0}{R_D}$$

$$\boxed{R_D = 1.8\,K\Omega}$$

$$V_{out}(t) = V_{out}(0^-) + \left[V_{out}(\infty) - V_{out}(0^-)\right]\left(1 - \exp\frac{-t}{R_D C_L}\right) \quad t > 0$$

$$V_{out}(t) = V_{out,min} + \left[V_{DD} - V_{out,min}\right] \times \left(1 - \exp\frac{-t}{R_D C_L}\right) \quad t > 0$$

$$V_{out}(t) = V_{DD}\left(1 - \exp\frac{-t}{R_D C_L}\right) \quad t > 0$$

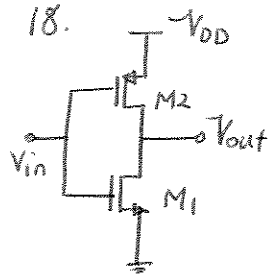
$$0.1 V_{DD} = V_{DD}\left(1 - \exp\frac{-T_{10\%}}{R_D C_L}\right) \longrightarrow T_{10\%} = 0.105\, R_D C_L$$

$$0.9 V_{DD} = V_{DD}\left(1 - \exp\frac{-T_{90\%}}{R_D C_L}\right) \longrightarrow T_{90\%} = 2.3\, R_D C_L$$

$$T_R = T_{90\%} - T_{10\%} = 2.197\, R_D C_L = 2.197 \times 1.8 \times 10^{3} \times 100 \times 10^{-15}$$

$$\boxed{T_R = 395.5\,pS}$$

18.



$$\left(\frac{W}{L}\right)_1 = \frac{2}{0.18}$$

$$\left(\frac{W}{L}\right)_2 = \frac{3}{0.18}$$

$I_{D1} = I_{D2}$

At the trip point $V_{in} = V_{out}$; therefore, both M_1 and M_2 operate in the Saturation region.

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{in}^{o} - V_{THN})^2 = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 (V_{DD} - V_{in}^{o} - |V_{THP}|)^2$$

$$V_{in}^{o} = \frac{V_{DD} - |V_{THP}| + \sqrt{\frac{\mu_n C_{ox} (W/L)_1}{\mu_p C_{ox} (W/L)_2}} \times V_{THN}}{1 + \sqrt{\frac{\mu_n C_{ox} (W/L)_1}{\mu_p C_{ox} (W/L)_2}}}$$

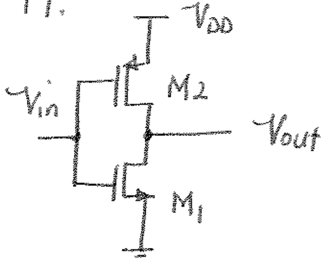
$$V_{in}^{o} = \frac{1.8 - 0.5 + \left(\frac{100 \times 2}{50 \times 3}\right)^{1/2} \times 0.4}{1 + \left(\frac{100 \times 2}{50 \times 3}\right)^{1/2}}$$

$$\boxed{V_{in} = V_{out} = 0.82\,V}$$

$$I_{D1} = I_{D2} = \frac{1}{2} \times 100 \times 10^{-6} \times \left(\frac{2}{0.18}\right) (0.82 - 0.4)^2$$

$$\boxed{I_{D1} = I_{D2} = 97\,\mu A}$$

19.



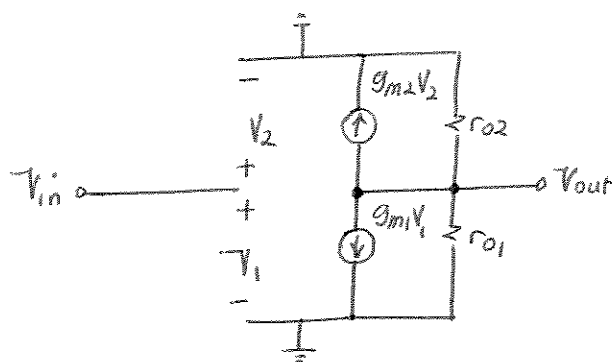
$$\left(\frac{W}{L}\right)_1 = \frac{2}{0.18}$$

$$\left(\frac{W}{L}\right)_2 = \frac{3}{0.18}$$

$$\lambda_N = 0.1\,V^{-1}$$

$$\lambda_P = 0.2\,V^{-1}$$

Replacing M_1 and M_2 with their small-signal model in the saturation region yields:



$$V_{out} = (-g_{m1}V_1 - g_{m2}V_2)(r_{o1} \| r_{o2})$$

$$V_1 = V_2 = V_{in}$$

$$V_{out} = -(g_{m1} + g_{m2})(r_{o1} \| r_{o2})V_{in}$$

$$\frac{V_{out}}{V_{in}} = -(g_{m1} + g_{m2})(r_{o1} \| r_{o2})$$

$$I_{D1} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{in} - V_{THN})^2$$

$$g_{m1} = \frac{\partial I_{D1}}{\partial V_{in}} = \mu_n C_{ox}\left(\frac{W}{L_1}\right)_1 (V_{in} - V_{THN}) = \frac{2I_{D1}}{V_{in} - V_{THN}}$$

$$g_{m1} = \frac{2 \times 9.7 \times 10^{-5}}{(0.817 - 0.4)} \rightarrow \boxed{g_{m1} = 4.641 \times 10^{-4}\ \mho}$$

$$g_{m2} = \frac{2 I_{D2}}{(V_{SG} - |V_{THP}|)} = \frac{2 \times 9.7 \times 10^{-5}}{(1.8 - 0.817 - 0.5)} \rightarrow \boxed{g_{m2} = 4.02 \times 10^{-4}\ \mho}$$

$$I_D = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right) (V_{GS} - V_{TH})^2 (1 + \lambda V_{DS})$$

$$g_o = \frac{\partial I_D}{\partial V_{DS}} = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right) (V_{GS} - V_{TH})^2 \lambda \simeq \lambda I_D$$

$$r_o \cong \frac{1}{\lambda I_D}$$

$$r_{oN} \simeq \frac{1}{0.1 \times 9.7 \times 10^{-5}} = 103.17\,K\Omega$$

$$r_{oP} \simeq \frac{1}{0.2 \times 9.7 \times 10^{-5}} = 51.58\,K\Omega$$

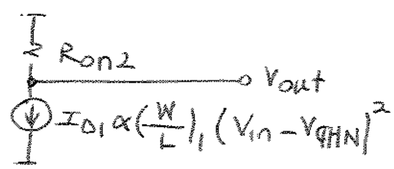
$$\text{Gain} = \frac{V_{out}}{V_{in}} = -(4.641 \times 10^{-4} + 4.02 \times 10^{-4})(51.58K \| 103.17K)$$

$$\boxed{\text{Gain} = -29.8}$$

20.

(a) Length of M_1 is increased

Let's assume that $V_{in} < V_{TH1}$, as a result M_1 is off and M_2 is on operating in the triode region. As V_{in} increases beyond V_{TH1}, M_1 starts pulling current (Conducting) in the saturation region while M_2 is still in the triode region. operating as a resistor; therefore, CMOS inverter Can be modelled as follows:

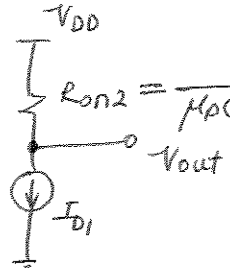


By increasing L_1, I_{D1} is weakened due to the inverse proportionality; as a result, an excess V_{in} is required to drop V_{out} to the point where $V_{out} = V_{in} + |V_{TH2}|$ and M_2 is placed at the edge of Saturation. Therefore Characteristic is shifted to the right and it will be steeper at the gain region where both M_1 and M_2 are in Saturation region.

(b) Length of M_2 is increased

Again if we assume that $V_{in} < V_{TH1}$, M_1 is off and M_2 is operating in the triode region with no current. By increasing

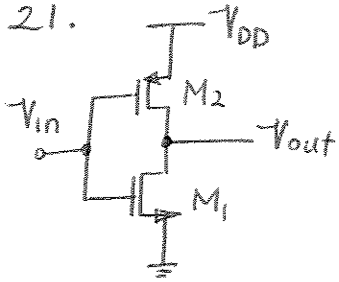
V_{in} above V_{TH1}, M_1 conducts in the saturation region while M_2 is operating in the triode region. Using the same model, as used in part (a) yields:



$$R_{on2} = \frac{1}{\mu_p C_{ox} \left(\frac{W}{L}\right)_2 \left(V_{DD} - V_{in} - |V_{THP}|\right)}$$

By increasing L_2, R_{on2} becomes larger; as a result, lower value of I_{D1} causes comparable voltage drop at the output. This will drive M_2 into the saturation with lower current (I_{D1}) and; hence, lower value of V_{in}. Therefore, characteristic is shifted to the left and small signal gain will be higher.

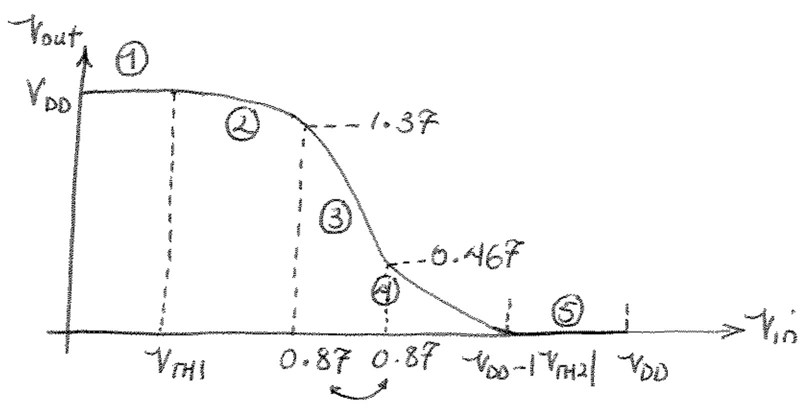
21.



$$\left(\frac{W}{L}\right)_1 = \frac{3}{0.18}$$

$$\left(\frac{W}{L}\right)_2 = \frac{7}{0.18}$$

VTC looks like the following figure



1 M_1 off, M_2 in triode region

$$I_{D1} = \emptyset$$

$$I_{D2} = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 \left[2\left(V_{DD} - V_{in} - |V_{TH2}|\right)V_{SD} - V_{SD}^2\right] = \emptyset$$

$$V_{SD} = \emptyset \rightarrow V_{out} = V_{DD} \quad (1)$$

2 M_1 in saturation, M_2 in triode region

$$I_{D1} = I_{D2}$$

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{in} - V_{TH1})^2 = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 \left[2\left(V_{DD} - V_{in} - |V_{THP}|\right)(V_{DD} - V_{out}) - (V_{DD} - V_{out})^2\right]$$

$$\frac{1}{2}\times 100\times 10^{-6}\times\frac{3}{0.18}\times(V_{in}-0.4)^2=\frac{1}{2}\times 50\times 10^{-6}\times\frac{7}{0.18}\times\left[2(1.8-V_{in}-0.5)\times(1.8-V_{out})-(1.8-V_{out})^2\right]$$

$$6(V_{in}-0.4)^2=7\left[2(1.3-V_{in})(1.8-V_{out})-(1.8-V_{out})^2\right]\quad(2)$$

If V_{out} falls significantly, M_2 enters saturation. That is $V_{out}=V_{in}+|V_{TH2}|$.
then M_2 is about to exit the triode region.

Replacing V_{out} by $V_{in}+|V_{TH2}|$ in (2) leads to:

$$6(V_{in}-0.4)^2=7\left[2(1.3-V_{in})(1.8-V_{in}-0.5)-(1.8-V_{in}-0.5)^2\right]$$

$$6(V_{in}-0.4)^2=7(1.3-V_{in})^2\rightarrow\sqrt{\frac{6}{7}}(V_{in}-0.4)=(1.3-V_{in})$$

$V_{in}=0.867\,V$, $V_{out}=1.37$

③ $$\frac{1}{2}\mu_nC_{ox}\left(\frac{W}{L}\right)_1(V_{in}-V_{TH1})^2(1+\lambda_1V_{out})=\frac{1}{2}\mu_pC_{ox}\left(\frac{W}{L}\right)_2(V_{DD}-V_{in}-|V_{TH2}|)^2\times\left[1+\lambda_2(V_{DD}-V_{out})\right]$$

$$V_{out}=\frac{\mu_pC_{ox}\left(\frac{W}{L}\right)_2(V_{DD}-V_{in}-|V_{TH2}|)^2-\mu_nC_{ox}\left(\frac{W}{L}\right)_1(V_{in}-V_{TH1})^2}{\lambda_2\mu_pC_{ox}\left(\frac{W}{L}\right)_2(V_{DD}-V_{in}-|V_{TH2}|)^2+\lambda_1\mu_nC_{ox}\left(\frac{W}{L}\right)_1(V_{in}-V_{TH1})^2}$$

$$V_{out}=\frac{\cancel{3}\,7(1.3-V_{in})^2-6(V_{in}-0.4)^2}{7\lambda_2(1.3-V_{in})^2+6\lambda_1(V_{in}-0.4)^2}\quad(3)$$

in region ③ M_1 and M_2 are both in saturation.

④ M_1 in triode region, M_2 in saturation

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1\left[2(V_{in}-V_{TH1})V_{out}-V_{out}^2\right]=\frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 \times (V_{DD}-V_{in}-|V_{TH2}|)^2 \quad (A)$$

$$6\left[2(V_{in}-0.4)V_{out}-V_{out}^2\right]=7(1.3-V_{in})^2$$

If V_{out} falls sufficiently, M_1 enters the triode region. That is, if $V_{in}=V_{out}+V_{TH1}$. Then M_1 is about to enter the triode region.

By substituting V_{in} with $V_{out}+0.4$ in (A), we have:

$$6\left[2V_{out}^2-V_{out}^2\right]=7(0.9-V_{out})^2$$

$V_{out}=0.467$, $V_{in}=0.867$

As channel length modulation has been neglected in this calculation tha value of input voltage that makes CMOS inverter transition from region ② to ③ is the same as that which makes inverter transition from region ③ to ④.

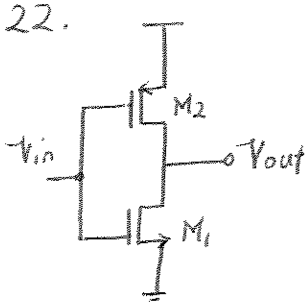
The slope in region ③ is infinit; however, we assume a finite slope in that region to emphasize the behavior of inverter as to producing a high gain.

⑤ M_1 in triode region, M_2 off

$I_{D2}=\emptyset$; $I_{D1}=\emptyset$

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1\left[2(V_{in}-V_{TH1})V_{out}-V_{out}^2\right]=\emptyset \longrightarrow V_{out}=\emptyset$$

22.



$V_{in} = V_{out} = 0.5\,V$

M_1 and M_2 are both in saturation region

$I_{D1} = I_{D2}$

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{in} - V_{TH1})^2 = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 (V_{DD} - V_{in} - |V_{TH2}|)^2$$

$$\frac{1}{2} \times 100 \times 10^{-6} \times \left(\frac{W}{L}\right)_1 (0.5 - 0.4)^2 = \frac{1}{2} \times 50 \times 10^{-6} \times \left(\frac{W}{L}\right)_2 (1.8 - 0.5 - 0.5)^2$$

$$\left(\frac{W}{L}\right)_1 \Big/ \left(\frac{W}{L}\right)_2 = 32$$

23. The value of the trip point has to be larger than the threshold voltage of NMOS transistor, 0.4 V. Therefore, 0.3 V Cannot be the trip point of such an inverter.

24.

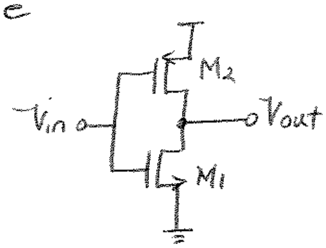
(a) If the inverter exhibits a very high voltage gain around the trip point, the range of input voltage values which guarantees that M_1 and M_2 are in saturation region is very narrow. Therefore this range can be fairly approximated with only one value of input voltage.

(b) $(W/L)_1 = 3/0.18$ and $(W/L)_2 = 7/0.18$

To calculate the minimum input voltage at which both transistors operate in saturation we assume

M_1 saturation

M_2 triode



$I_{D1} = I_{D2}$

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{in} - V_{TH1})^2 = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 \left[2(V_{DD} - V_{in} - |V_{TH2}|)(V_{out} - V_{DD}) - (V_{out} - V_{DD})^2\right]$$

$V_{out} = V_{in} + |V_{TH2}|$ places M_2 at the edge of saturation

$$2 \times 3 \times (V_{in} - 0.4)^2 = 7 \times \left[2(1.8 - V_{in} - 0.5)(1.8 - V_{in} - 0.5) - (1.8 - V_{in} - 0.5)^2\right]$$

$$\boxed{V_{in_{min}} = 0.867}$$

To calculate $V_{in,max}$, we assume that M_1 and M_2 are in triode and saturation region respectively

$I_{D1} = I_{D2}$

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left[2(V_{in} - V_{TH1})V_{out} - V_{out}^2\right] = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 (V_{DD} - V_{in} - |V_{TH2}|)^2$$

when M_1 is just going to leave the saturation and enters the triode region

$$V_{in} = V_{out} + 0.4^{(V_{TH1})}$$

$$\frac{1}{2} \times 100 \times 10^{-6} \times \frac{3}{0.18} \times \left[2(V_{out} + V_{TH1} - V_{TH1})V_{out} - V_{out}^2\right] = \frac{1}{2} \times 50 \times 10^{-6} \times \frac{7}{0.18} \times (V_{DD} - V_{in} - |V_{TH2}|)^2$$

$$\frac{6}{7} V_{out}^2 = (V_{DD} - V_{out} - V_{TH1} - |V_{TH2}|)^2$$

$$\frac{6}{7} V_{out}^2 = (0.9 - V_{out})^2$$

$$V_{out} = 0.467\,V, \boxed{V_{in_{max}} = 0.867}$$

To find the trip point, M_1 and M_2 are assumed to be in saturation.

$$I_{D1} = I_{D2}$$

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{in} - V_{TH1})^2 = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 (V_{DD} - V_{in} - |V_{TH2}|)^2$$

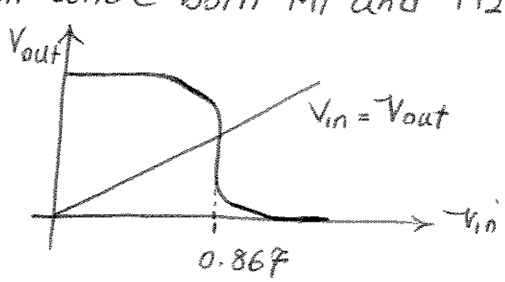
$$2 \times 3 \times (V_{in} - 0.4)^2 = 7 \times (1.8 - V_{in} - 0.5)^2$$

$$\boxed{V_{in}^{o} = 0.867 \text{ @ trip point}}$$

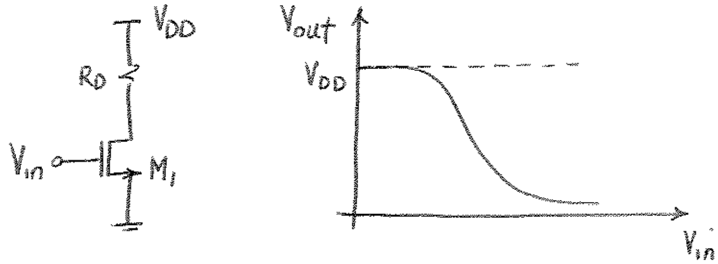
$$V_{in,trip} - V_{in,min} = \emptyset$$

$$V_{in,max} - V_{in,trip} = \emptyset$$

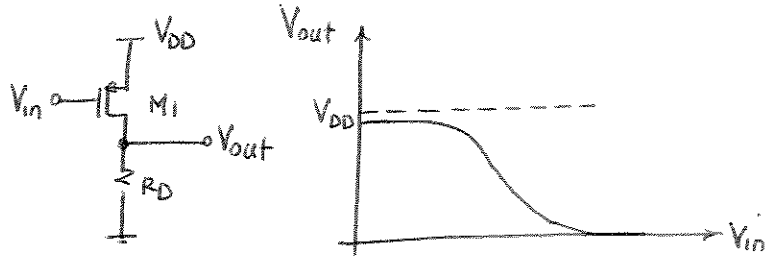
This result is not surprising because VTC of inverter has infinite slope at the region where both M_1 and M_2 are in saturation region



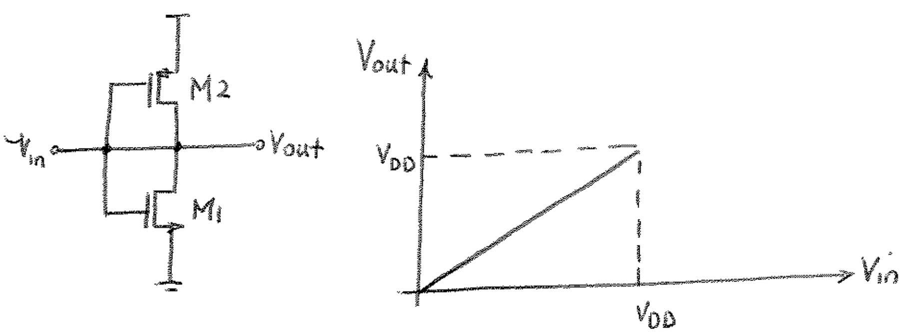
25.



V_{DD}
R_D
V_{in}
M_1
V_{out}
V_{DD}
V_{in}

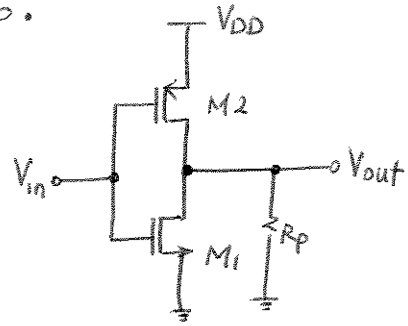


V_{DD}
V_{in}
M_1
V_{out}
R_D
V_{out}
V_{DD}
V_{in}



M_2
V_{in}
V_{out}
M_1
V_{out}
V_{DD}
V_{in}
V_{DD}

26.



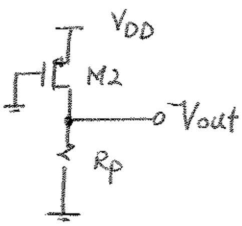
$R_p = 2K$

$\left(\frac{W}{L}\right)_1 = 3/0.18$

$\left(\frac{W}{L}\right)_2 = 5/0.18$

$V_{OL}, V_{OH}, V_{in,trip} = ?$

To calculate V_{OH}, V_{in} is assumed to be $0V$



$$I_{D2} = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2\left[2(V_{DD} - |V_{TH2}|)(V_{DD} - V_{out}) - (V_{DD} - V_{out})^2\right]$$

$$I_{D2} = \frac{V_{out}}{R_p}$$

$$\frac{V_{out}}{R_p} = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2\left[2(V_{DD} - |V_{TH2}|)(V_{DD} - V_{out}) - (V_{DD} - V_{out})^2\right]$$

$$V_{out}^2 - 0.28V_{out} - 1.44 = 0$$

$$\boxed{V_{out} = V_{OH} = 1.348\,V}$$

$\boxed{V_{OL} = 0}$ because M_2 is off for $V_{in} = V_{DD}$

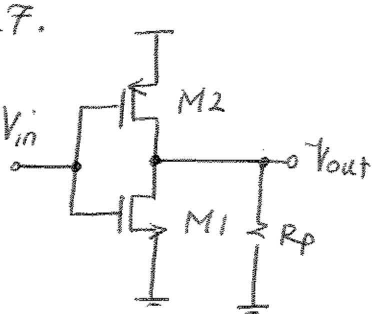
(a) trip point $V_{in} = V_{out}$

$$I_{D2} = I_{D1} + \frac{V_{out}}{R_p}$$

$$\frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2\left(V_{DD} - V_{in} - |V_{TH2}|\right)^2 = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1\left(V_{in} - V_{TH1}\right)^2 + \frac{V_{out}}{2000}$$

$$0.05V_{out}^2 + 0.59V_{out} - 0.3745 = 0 \longrightarrow \boxed{V_{in} = V_{out} = 0.6\,V}$$

27.



$R_P = 2K$

$\left(\frac{W}{L}\right)_1 = 3/0.18$

$\left(\frac{W}{L}\right)_2 = 5/0.18$

$V_{in} = V_{out} = 0.6V$ @ trip point

With R_P

$$I_{D2} = I_{D1} + \frac{V_{out}}{R_P}$$

$$\frac{1}{2}\mu_P C_{ox}\left(\frac{W}{L}\right)_2 \left(V_{DD} - V_{in} - |V_{TH2}|\right)^2 = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left(V_{in} - V_{TH1}\right)^2 + \frac{V_{out}}{2000}$$

$$0.05\,V_{out}^2 + 0.59\,V_{out} - 0.3745 = 0$$

$$V_{in} = V_{out} = 0.6\,V$$

$$I_{D1} = \frac{1}{2} \times 100\times10^{-6} \times \frac{3}{0.18} \times (0.6 - 0.4)^2$$

$$\boxed{I_{D1} = 3.33\times10^{-5}\,A}$$

$$I_{D2} = I_{D1} + \frac{V_{out}}{R_P} = 3.33\times10^{-5} + \frac{0.6}{2000}$$

$$\boxed{I_{D2} = 3.35\times10^{-4}\,A}$$

$$g_{m1} = \frac{2I_{D1}}{V_{eff1}} = \frac{2\times3.33\times10^{-5}}{(0.6-0.4)} \rightarrow \boxed{g_{m1} = 3.3\times10^{-3}\,\mho}$$

$$g_{m2} = \frac{2I_{D2}}{V_{eff2}} = \frac{2\times3.35\times10^{-4}}{(1.8-0.6-0.5)} \rightarrow \boxed{g_{m2} = 9.4\times10^{-3}}$$

$$A_V = -(g_{m1} + g_{m2}) \times R_P$$

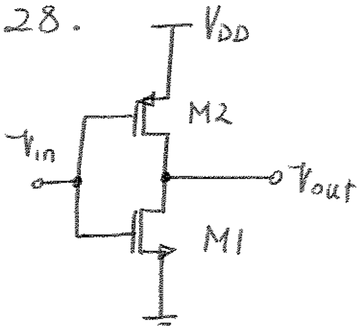
$$A_V = -(3.3 \times 10^{-3} + 9.4 \times 10^{-3}) \times 2000$$

$$\boxed{A_V = -25.4}$$

Without R_P

$$A_V \Rightarrow -\infty$$

28.



$(W/L)_1 = 5/0.18$

$(W/L)_2 = 11/0.18$

NM_L and $NM_H = ?$

To calculate NM_L, M_1 and M_2 are assumed to operate in the saturation and triode region respectively.

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{in}-V_{TH1})^2 = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2\left[2(V_{DD}-V_{in}-|V_{TH2}|)(V_{DD}-V_{out}) - (V_{DD}-V_{out})^2\right] \quad (1)$$

$$\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{in}-V_{TH1}) = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2\left[-2(V_{DD}-V_{out}) - 2(V_{DD}-V_{in}-|V_{TH2}|)\times\frac{\partial V_{out}}{\partial V_{in}} + 2(V_{DD}-V_{out})\frac{\partial V_{out}}{\partial V_{in}}\right]$$

$\frac{\partial V_{out}}{\partial V_{in}} = -1$, $V_{in} = V_{IL}$

$$\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{IL}-V_{TH1}) = \mu_p C_{ox}\left(\frac{W}{L}\right)_2\left[2V_{OH} - V_{IL} - |V_{TH2}| - V_{DD}\right] \quad (2)$$

Obtaining V_{OH} from (2) and substituting in (1) yields:

$$V_{IL} = \frac{2\sqrt{a}\,(V_{DD}-V_{TH1}-|V_{TH2}|)}{(a-1)\sqrt{a+3}} - \frac{V_{DD}-aV_{TH1}-|V_{TH2}|}{a-1}$$

$$a = \frac{\mu_n C_{ox}\left(\frac{W}{L}\right)_1}{\mu_p C_{ox}\left(\frac{W}{L}\right)_2} = \frac{100}{50}\times\frac{5}{11} = \frac{10}{11}$$

$$V_{IL} = \frac{2\sqrt{10/11}\,(1.8-0.4-0.5)}{(10/11-1)\sqrt{10/11+3}} - \frac{1.8-(10/11)\times 0.4-0.5}{10/11-1}$$

$$\boxed{V_{IL} = 0.7516\ V}$$

To determine NM_H, M_1 and M_2 are assumed to operate in the triode and saturation region respectively.

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1\left[2(V_{in}-V_{TH1})V_{out}-V_{out}^2\right]=\frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2\left(V_{DD}-|V_{TH2}|-V_{in}\right)^2 \quad (3)$$

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1\left[2V_{out}+2(V_{in}-V_{TH1})\frac{\partial V_{out}}{\partial V_{in}}-2V_{out}\frac{\partial V_{out}}{\partial V_{in}}\right]=-\mu_p C_{ox}\left(\frac{W}{L}\right)_2 \times \left(V_{DD}-V_{in}-|V_{TH2}|\right)$$

$\frac{\partial V_{out}}{\partial V_{in}}=-1$ yields

$$\mu_n C_{ox}\left(\frac{W}{L}\right)_1\left[V_{out}-V_{in}+V_{TH1}+V_{out}\right]=-\mu_p C_{ox}\left(\frac{W}{L}\right)_2\left(V_{DD}-V_{in}-|V_{TH2}|\right)$$

$$100\times 5\times\left[2V_{out}-(V_{in}-0.4)\right]=-50\times 11\times(1.8-0.5-V_{in})$$

$$V_{out}=1.05V_{in}-0.915 \quad (4)$$

Substituting (4) in (3) yields an equation versus V_{in} as follows:

$$10\times\left[2(V_{in}-0.4)(1.05V_{in}-0.915)-(1.05V_{in}-0.915)^2\right]=11(1.3-V_{in})^2$$

$$1.025V_{in}^2-21.115V_{in}+19.64225=0$$

$$V_{in}=V_{IH}=0.9765\ V$$

$$NM_H=V_{DD}-V_{IH}$$

$$\boxed{NM_H=0.823\ V}$$

29. $NM_L = 0.6\,V$

$$(W/L)_1 / (W/L)_2 = ?$$

$$V_{IL} = \frac{2\sqrt{a}\,(V_{DD} - V_{TH1} - |V_{TH2}|)}{(a-1)\sqrt{a+3}} - \frac{V_{DD} - aV_{TH1} - |V_{TH2}|}{a-1}$$

$$a = \frac{\mu_n C_{ox} \left(\frac{W}{L}\right)_1}{\mu_p C_{ox} \left(\frac{W}{L}\right)_2}$$

$$0.6 = \frac{2\sqrt{a}\,(1.8 - 0.4 - 0.5)}{(a-1)\sqrt{a+3}} - \frac{1.8 - a\,0.4 - 0.5}{a-1}$$

$$a = 3\sqrt{\frac{a}{a+3}} - \frac{1.3 - 0.4a}{0.6} + 1$$

$$a = \frac{a+3}{9} \times \left[a - 1 + \frac{1.3 - 0.4a}{0.6}\right]^2$$

$$\boxed{a = 1}$$

$$(W/L)_1 / (W/L)_2 = \frac{\mu_p C_{ox}}{\mu_n C_{ox}} = \frac{1}{2}$$

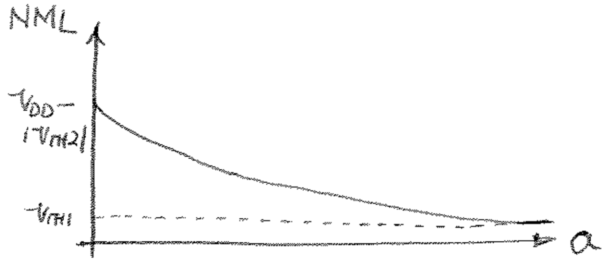
$$\boxed{(W/L)_1 / (W/L)_2 = \frac{1}{2}}$$

30. $V_{IL} = \dfrac{2\sqrt{a}\,(V_{DD} - V_{TH1} - |V_{TH2}|)}{(a-1)\sqrt{a+3}} - \dfrac{V_{DD} - aV_{TH1} - |V_{TH2}|}{a-1}$

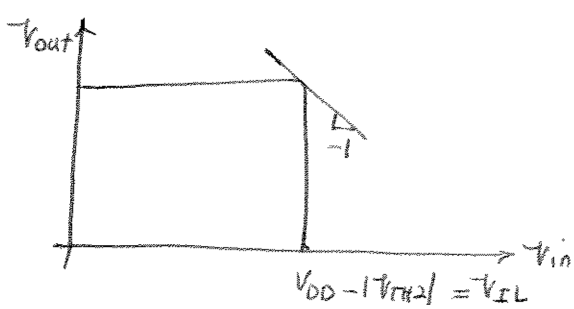
$$a = \frac{\mu_n \left(\frac{W}{L}\right)_1}{\mu_p \left(\frac{W}{L}\right)_2}$$

$a \to 0 \quad V_{IL} = V_{DD} - |V_{TH2}|$

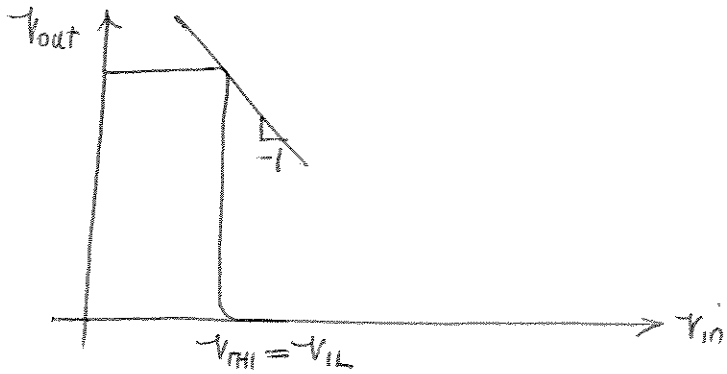
$a \to \infty \quad V_{IL} = V_{TH1}$



If $a = \dfrac{\mu_n}{\mu_p} \times \dfrac{(W/L)_1}{(W/L)_2} \to 0$, it implies that PMOS transistor is extremely stronger than NMOS. Therefor, as V_{in} increases from 0V, the output of inverter stays at V_{DD} until input reaches $V_{DD} - |V_{TH2}|$. At that point, PMOS is cut off and V_{out} sharply drops to 0V.



When $a \to \infty$, NMOS is prevailing and once input voltage hits the threshold voltage of NMOS, output voltage falls sharply to 0V.



Vout
-1
Vin
VTH1 = VIL

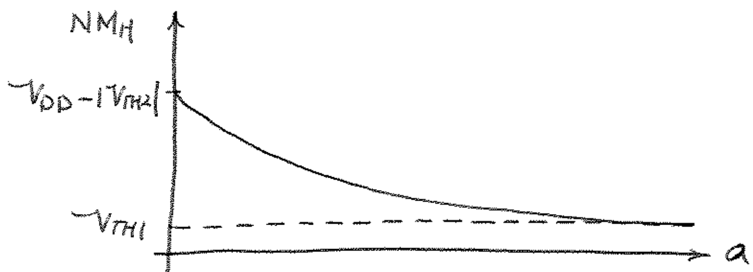
31.

$$NM_H = V_{DD} - \frac{2a\left(V_{DD} - V_{TH1} - |V_{TH2}|\right)}{(a-1)\sqrt{1+3a}} + \frac{V_{DD} - aV_{TH1} - |V_{TH2}|}{a-1}$$

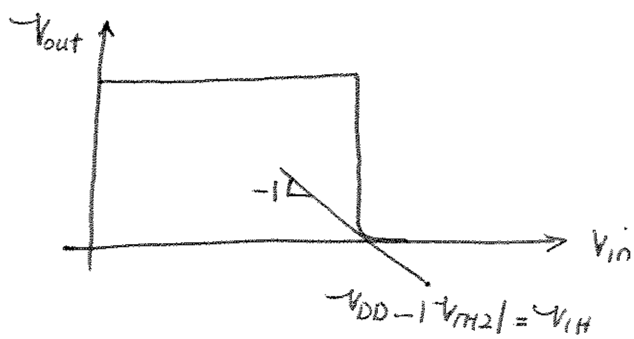
$$a = \frac{\mu_n \left(\frac{W}{L}\right)_1}{\mu_p \left(\frac{W}{L}\right)_2}$$

$a \to 0 \quad NM_H = |V_{TH2}|\ , \quad V_{IH} = V_{DD} - |V_{TH2}|$

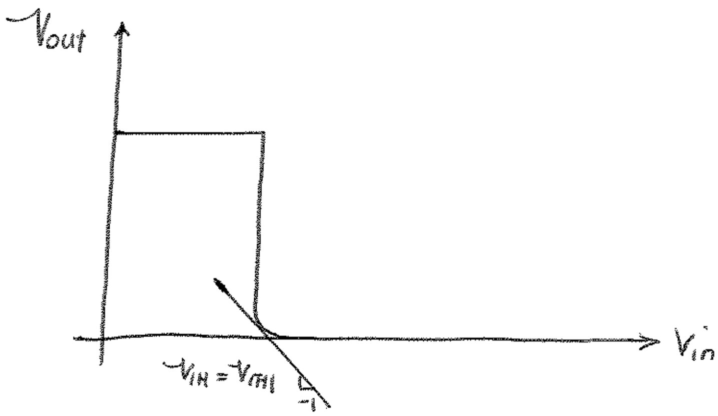
$a \to \infty \quad NM_H = V_{DD} - V_{TH1}\ , \quad V_{IH} = V_{TH1}$



If $a = \frac{\mu_n}{\mu_p} \times \frac{(W/L)_1}{(W/L)_2} \to 0$, it implies that PMOS transistor is much stronger than NMOS. Therefor, as V_{in} increases from 0 V, the output of inverter remains at V_{DD} until input reaches $V_{DD} - |V_{TH2}|$. At that point, PMOS is cutoff and V_{out} sharply drops to 0 V.

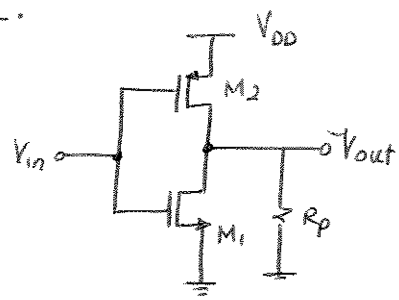


When "a" approaches infinity, NMOS is prevailing and once input voltage hits the threshold voltage of NMOS, output voltage falls sharply to 0V.



Note that the separation between V_{IH} and V_{IL} depends on the slope of VTC in the transition region. If "a" approaches either "0" or infinity, VTC exhibits infinite gain in its transition region. Therefore V_{IL} and V_{IH} Coincide.

32.



$R_p = 2K$

$\left(W/L\right)_1 = 3/0.18$

$\left(W/L\right)_2 = 5/0.18$

$NM_L,\ NM_H = ?$

To calculate NM_L, M_1 and M_2 are assumed to be in the saturation and triode region respectively.

$$I_{D2} = I_{D1} + \frac{V_{out}}{R_p} \quad (V_{in} = V_{IL})$$

$$\frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2\left[2\left(V_{DD}-V_{in}-|V_{TH2}|\right)\left(V_{DD}-V_{out}\right)-\left(V_{DD}-V_{out}\right)^2\right] =$$

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1\left(V_{in}-V_{TH1}\right)^2 + \frac{V_{out}}{R_p} \qquad (1)$$

$$\frac{\partial V_{out}}{\partial V_{in}} = -1, \quad V_{in} = V_{IL}$$

$$\frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2\left[-2\left(V_{DD}-V_{out}\right)-2\left(V_{DD}-V_{in}-|V_{TH2}|\right)\frac{\partial V_{out}}{\partial V_{in}}+2\left(V_{DD}-V_{out}\right)\frac{\partial V_{out}}{\partial V_{in}}\right] =$$

$$\mu_n C_{ox}\left(\frac{W}{L}\right)_1\left(V_{in}-V_{TH1}\right) + \frac{1}{R_p}\frac{\partial V_{out}}{\partial V_{in}}$$

$$\mu_n C_{ox}\left(\frac{W}{L}\right)_1\left(V_{IL}-V_{TH1}\right) - \frac{1}{R_p} = \mu_p C_{ox}\left(\frac{W}{L}\right)_2\left[2V_{OH}-V_{IL}-|V_{TH2}|-V_{DD}\right] \qquad (2)$$

$$\boxed{V_{OH} = 1.01\,V_{IL} + 0.73}$$

Replacing V_{out} in (1) with its equivalent versus V_{IL} obtained from (2) yields:

$$\frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2\left[2(V_{DD}-V_{IL}-|V_{TH2}|)(V_{DD}-1.1V_{IL}-0.73)-(V_{DD}-1.1V_{IL}-0.73)^2\right]=$$

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1(V_{IL}-V_{TH1})^2+\frac{1.1V_{IL}+0.73}{R_p}$$

$$\frac{1}{2}\times 50\times 10^{-6}\times\frac{5}{0.18}\left[2(1.8-V_{IL}-0.5)(1.8-1.1V_{IL}-0.73)-(1.8-1.1V_{IL}-0.73)^2\right]=$$

$$\frac{1}{2}\times 100\times 10^{-6}\times\frac{3}{0.18}(V_{IL}-0.4)^2+\frac{1.1V_{IL}+0.73}{2000}$$

$$-52.5\times 10^{-3}V_{IL}-0.6195V_{IL}+0.229875=0$$

$\boxed{V_{IL}=NM_L=0.36\,V}$ $< V_{TH1}$ Not Acceptable !

This is less than threshold voltage of M_1; Therefore, this answer is not acceptable. It means that M_1 is off and should be left out in this calculation.

$I_{D1}=0$, $I_{D2}=\frac{V_{out}}{R_p}$

$$\mu_p C_{ox}\left(\frac{W}{L}\right)_2\left[2V_{OH}-V_{IL}-|V_{TH2}|-V_{DD}\right]=-\frac{1}{R_p}$$

$$50\times 10^{-6}\times\frac{5}{0.18}\times\left[2V_{OH}-V_{IL}-0.5-1.8\right]=-\frac{1}{2000}$$

$\boxed{V_{OH}=V_{out}=0.5V_{IL}+0.97}$ (3)

$$\frac{1}{2}\times 50\times 10^{-6}\times\frac{5}{0.18}\times\left[2(1.8-V_{IL}-0.5)(1.8-0.5V_{IL}-0.97)-(1.8-0.5V_{IL}-0.97)^2\right]=$$

$$\frac{0.5V_{IL}+0.97}{2000}$$

$$0.1875V_{IL}^2-0.6225V_{IL}+0.192675=0$$

$\boxed{V_{IL}=NM_L=0.345\,V}$

To determine NM_H, M_1 and M_2 are assumed to operate in the triode and saturation region respectively.

$$I_{D2} = I_{D1} + \frac{V_{out}}{R_p} \quad (V_{in} = V_{IH})$$

$$\frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 \left(V_{DD} - V_{in} - |V_{TH2}|\right)^2 = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left[2\left(V_{in} - V_{TH1}\right)V_{out} - V_{out}^2\right] + \frac{V_{out}}{R_p} \quad (4)$$

$$-\mu_p C_{ox}\left(\frac{W}{L}\right)_2 \left(V_{DD} - V_{in} - |V_{TH2}|\right) = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left[2V_{out} + 2\left(V_{in} - V_{TH1}\right)\frac{\partial V_{out}}{\partial V_{in}} - 2V_{out}\frac{\partial V_{out}}{\partial V_{in}}\right] + \frac{\partial V_{out}}{\partial V_{in}}\frac{1}{R_p}$$

$$-\mu_p C_{ox}\left(\frac{W}{L}\right)_2 \left(V_{DD} - V_{in} - |V_{TH2}|\right) = \mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left[V_{out} - V_{in} + V_{TH1} + V_{out}\right] - \frac{1}{R_p}$$

$$-50\times10^{-6}\times\frac{5}{0.18}\times\left(1.8 - V_{in} - 0.5\right) = 100\times10^{-6}\times\frac{3}{0.18}\times\left(2V_{out} - V_{in} + 0.4\right) - \frac{1}{2000}$$

$$\boxed{V_{out} = \frac{0.1V_{in} + 0.59}{1.2}} \quad (5)$$

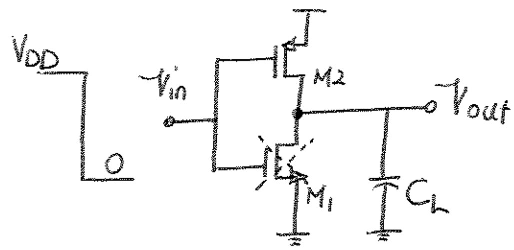
Combining equs (4) and (5) yields:

$$\frac{1}{2}\times50\times10^{-6}\times\frac{5}{0.18}\left(1.8 - V_{in} - 0.5\right)^2 = \frac{1}{2}\times100\times10^{-6}\times\frac{3}{0.18}\left[2\left(V_{in} - 0.4\right)\frac{0.1V_{in}+0.59}{1.2} - \frac{\left(0.1V_{in}+0.59\right)^2}{1.2^2}\right] + \frac{0.1V_{in}+0.59}{1.2\times2000}$$

$$-0.291V_{in}^2 + 1.3182V_{in} - 0.75531 = 0$$

$$V_{in} = V_{IH} = 0.673\,V \longrightarrow \boxed{NM_H = V_{DD} - V_{IH} = 1.127\,V}$$

33.



$V_{out}(t=0)=0$

$(W/L)_2 = 6/0.18$

$C_L = 50fF$

$0 < V_{out} < |V_{TH2}|$; M_2 in the saturation

$$C_L \frac{dV_{out}}{dt} = I_{D2} = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 \left(V_{DD}-V_{in}-|V_{TH2}|\right)^2 = \frac{1}{2}\times 50\times 10^{-6}\times \frac{6}{0.18}(1.8-0.5)^2$$

$$= 1.4\times 10^{-3} A$$

$$V_{out}(t) = \frac{I_{D2}}{C_L}\times t$$

$$|V_{TH2}| = \frac{I_{D2}}{C_L}\cdot T_1 \longrightarrow T_1 = \frac{C_L \times |V_{TH2}|}{I_{D2}} = 50\times 10^{-15}\times(1.4\times 10^{-3})^{-1}\times 0.5$$

$$\boxed{T_1 = 17.75\ pS}$$

$|V_{TH2}| < V_{out} < V_{DD}/2$; M_2 in triode

$$C_L \frac{dV_{out}}{dt} = I_{D2} = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2\left[2(V_{DD}-|V_{TH2}|)(-V_{out}+V_{DD}) - (V_{DD}-V_{out})^2\right]$$

$$\frac{dV_{out}}{2(V_{DD}-|V_{TH2}|)(V_{DD}-V_{out})-(V_{DD}-V_{out})^2} = \frac{1}{2}\mu_p \frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_2 dt$$

$$\frac{1}{(V_{DD}-V_{out})\left[2(V_{DD}-|V_{TH2}|)-(V_{DD}-V_{out})\right]} = \frac{1}{2(V_{DD}-|V_{TH2}|)}\left[\frac{1}{2(V_{DD}-|V_{TH2}|)-(V_{DD}-V_{out})} + \frac{1}{V_{DD}-V_{out}}\right]$$

$$\frac{1}{\cancel{2}(V_{DD}-|V_{TH2}|)}\left[\frac{dV_{out}}{2(V_{DD}-|V_{TH2}|)-(V_{DD}-V_{out})}+\frac{dV_{out}}{V_{DD}-V_{out}}\right]=\frac{1}{\cancel{2}}\mu_p\frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_2 dt$$

$$Ln\,\frac{2(V_{DD}-|V_{TH2}|)-(V_{DD}-V_{out})}{V_{DD}-V_{out}}=\mu_p\frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_2(V_{DD}-|V_{TH2}|)t+C$$

$$\frac{2(V_{DD}-|V_{TH2}|)-(V_{DD}-V_{out})}{V_{DD}-V_{out}}=K.\exp\left[\mu_p\frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_2(V_{DD}-|V_{TH2}|)t\right]$$

Time origin is assumed to be at $t=T_1=17.75\,pS$

$V_{out}(t=0)=|V_{TH2}| \longrightarrow K=1$

$$\frac{2(V_{DD}-|V_{TH2}|)-(V_{DD}-V_{out})}{V_{DD}-V_{out}}=e^{\mu_p\frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_2(V_{DD}-|V_{TH2}|)t}$$

ⓐ $V_{out}=V_{DD}/2$ $\quad T_2=\dfrac{Ln(3-4|V_{TH2}|/V_{DD})}{\mu_p\frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_2(V_{DD}-|V_{TH2}|)}$

$$=\frac{Ln(3-4\times0.5/1.8)}{50\times10^{-6}\times\frac{1}{50\times10^{-15}}\times\frac{6}{0.18}\times(1.8-0.5)}$$

$$\boxed{T_2=1.467\times10^{-11}}$$

$T_{0\to V_{DD}/2}=T_1+T_2=17.75+14.67$

$$\boxed{T_{0\to V_{DD}/2}=32.43\,pS}$$

34. $|V_{TH2}| < V_{out} < 0.95 V_{DD}$ M_2 in Triode

$$\frac{2(V_{DD}-|V_{TH2}|)-(V_{DD}-V_{out})}{V_{DD}-V_{out}} = e^{\mu_p \frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_2 (V_{DD}-|V_{TH2}|)t}$$

(a) $V_{out} = 0.95 V_{DD}$, $T_2 = \dfrac{\ln(39 - 40|V_{TH2}|/V_{DD})}{\mu_p \frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_2 (V_{DD}-|V_{TH2}|)}$

$$= \frac{\ln(39 - 40 \times 0.5/1.8)}{50\times10^{-6} \times \frac{1}{50\times10^{-15}} \times \frac{6}{0.18} \times (1.8-0.5)}$$

$$\boxed{T_2 = 7.68 \times 10^{-11}}$$

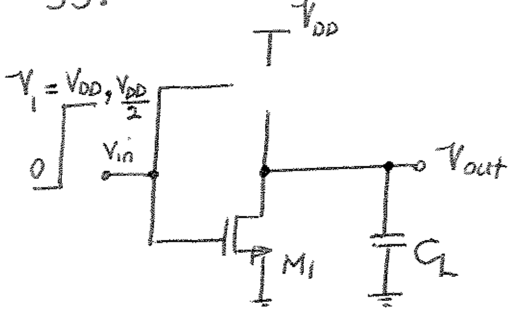
$T_1 = 17.75$ ps from previous problem

$T_{0 \to 0.95 V_{DD}} = T_1 + T_2 = 17.75 + 76.8$

$$\boxed{T_{0 \to 0.95 V_{DD}} = 94.55 \text{ ps}}$$

$$\boxed{(T_{0 \to 0.95 V_{DD}})/(T_{0 \to V_{DD}/2}) \approx 3}$$

35.



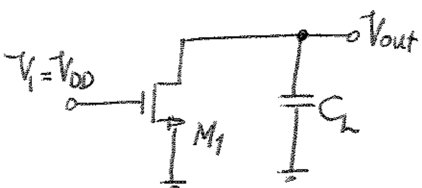
$V_{out}(t=0) = V_{DD}$ $\quad C_L = 30fF$

$(W/L)_1 = 1/0.18$ $\quad T_{V_{DD} \to V_{DD}/2} = ?$

$(W/L)_2 =$ Not necessary $\quad V_1 = V_{DD}$ & $V_{DD}/2$

(a) $V_1 = V_{DD}$

$V_{DD} - V_{TH1} \leqslant V_{out} \leqslant V_{DD}$ $\quad M_1$ saturation



$\frac{V_{DD}}{2} \leqslant V_{out} \leqslant V_{DD} - V_{TH1}$ $\quad M_1$ Triode

$$C_L \frac{dV_{out}}{dt} = -I_{D1} = -\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{DD} - V_{TH1})^2 = -\frac{1}{2} \times 100 \times 10^{-6} \times \frac{1}{0.18}(1.8 - 0.4)^2 = 5.44 \times 10^{-4} A$$

$$dV_{out} = -\frac{I_{D1}}{C_L} \cdot dt$$

$$V_{out}(t) - V_{DD} = -\frac{I_{D1}}{C_L} t \longrightarrow V_{out}(t) = V_{DD} - \frac{I_{D1}}{C_L} t$$

$$T_{V_{DD} \to V_{DD} - V_{TH1}} = \frac{V_{TH1} \times C_L}{I_{D1}} = \frac{0.4 \times 30 \times 10^{-15}}{5.44 \times 10^{-4}} = 2.2 \times 10^{-11} S$$

$$C_L \frac{dV_{out}}{dt} = -I_{D1} = -\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left[2(V_{DD} - V_{TH1})V_{out} - V_{out}^2\right]$$

$$\frac{dV_{out}}{2(V_{DD} - V_{TH1})V_{out} - V_{out}^2} = -\frac{1}{2}\mu_n \frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_1 dt$$

$$\frac{1}{\left[2(V_{DD} - V_{TH1}) - V_{out}\right] V_{out}} = \frac{1}{2(V_{DD} - V_{TH1})}\left[\frac{1}{2(V_{DD} - V_{TH1}) - V_{out}} + \frac{1}{V_{out}}\right]$$

$$\frac{1}{\cancel{2}(V_{DD}-V_{TH1})}\left[\frac{dV_{out}}{2(V_{DD}-V_{TH1})-V_{out}}+\frac{dV_{out}}{V_{out}}\right]=-\frac{1}{\cancel{2}}\mu_n\frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_1 dt$$

$$-Ln\left[2(V_{DD}-V_{TH1})-V_{out}\right]+Ln\,V_{out}=-\mu_n\frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_1(V_{DD}-V_{TH1})t+C$$

$$\frac{V_{out}}{2(V_{DD}-V_{TH1})-V_{out}}=K\cdot\exp\left[-\mu_n\frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_1(V_{DD}-V_{TH1})t\right]$$

$V_{out}(t=0)=V_{DD}-V_{TH1}$ Note that time origin is assumed to be 2.2×10^{-11}

$K=1 \longrightarrow$

$$\frac{V_{out}}{2(V_{DD}-V_{TH1})-V_{out}}=e^{-\mu_n\frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_1(V_{DD}-V_{TH1})t}$$

$$V_{out}=V_{DD}/2\longrightarrow\frac{V_{DD}/2}{2(V_{DD}-V_{TH1})-V_{DD}/2}=e^{-\mu_n\frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_1(V_{DD}-V_{TH1})T_{(V_{DD}-V_{TH1})\to V_{DD}/2}}$$

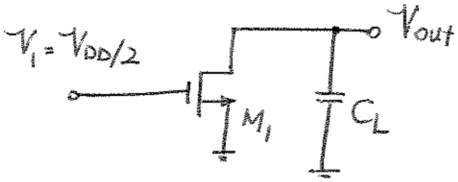
$$T_{(V_{DD}-V_{TH1})\to V_{DD}/2}=\frac{Ln\left(3-\frac{4V_{TH1}}{V_{DD}}\right)}{\mu_n\frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_1(V_{DD}-V_{TH1})}$$

$$=\frac{Ln\,(3-4\times0.4/1.8)}{100\times10^{-6}\times\frac{1}{30\times10^{-15}}\times\frac{1}{0.18}\times(1.8-0.4)}=2.88\times10^{-11}\,S$$

$$T_{V_{DD}\to V_{DD}/2}=T_{V_{DD}\to V_{DD}-V_{TH1}}+T_{(V_{DD}-V_{TH1})\to V_{DD}/2}$$

$$\boxed{T_{V_{DD}\to V_{DD}/2}=5\times10^{-11}=50.86\,pS}$$

(b) $V_1 = V_{DD}/2$



$V_{DD}/2 - V_{TH1} < V_{out} < V_{DD}$ M_1 in Saturation

$V_{DD}/2 < V_{out} < V_{DD}$ M_1 in Saturation

$$C_L \frac{dV_{out}}{dt} = -I_{D1} = -\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left(V_{DD}/2 - V_{TH1}\right)^2 = -\frac{1}{2} \times 100 \times 10^{-6} \times \frac{1}{0.18}(0.9-0.4)^2$$

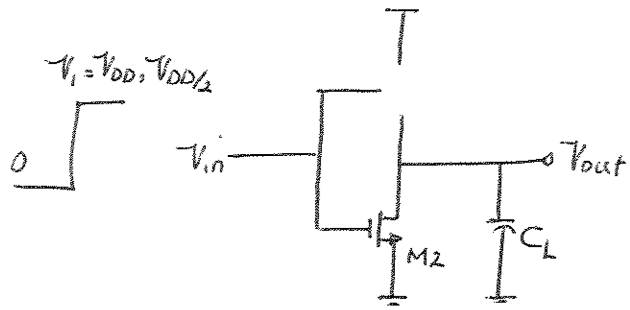
$$= 6.944 \times 10^{-5}$$

$$V_{out}(t) = V_{DD} - \frac{I_{D1}}{C_L} \times t$$

$$V_{DD}/2 = V_{DD} - \frac{I_{D1}}{C_L} \times T_{(V_{DD} \to V_{DD}/2)} \rightarrow T_{(V_{DD} \to V_{DD}/2)} = \frac{(V_{DD}/2) \times C_L}{I_{D1}}$$

$$\boxed{T_{(V_{DD} \to V_{DD}/2)} = 3.888 \times 10^{-10}}$$

36.



$V_{out}(0) = V_{DD}$

$(W/L)_1 = 1/0.18$

$C_L = 30fF$

$T_{V_{DD} \to 0.05 V_{DD}} = ?$

(a) $V_1 = V_{DD}$ $\quad V_{DD} - V_{TH1} < V_{out} < V_{DD}$ M_1 in saturation

$0.05 V_{DD} < V_{out} < V_{DD} - V_{TH1}$ M_1 in Triode

$$C_L \frac{dV_{out}}{dt} = -I_{D1} = -\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{DD} - V_{TH1})^2 = -\frac{1}{2} \times 100 \times 10^{-6} \times \frac{1}{0.18}(1.8 - 0.4)^2$$
$$= 5.44 \times 10^{-4} A$$

$$V_{out}(t) = V_{DD} - \frac{I_{D1}}{C_L} \times t$$

$$T_{V_{DD} \to V_{DD} - V_{TH1}} = \frac{V_{TH1} \times C_L}{I_{D1}} = \frac{0.4 \times 30 \times 10^{-15}}{5.44 \times 10^{-4}} = 2.2 \times 10^{-11} S$$

$$C_L \frac{dV_{out}}{dt} = -I_{D1} = -\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left[2(V_{DD} - V_{TH1})V_{out} - V_{out}^2\right]$$

$$\frac{1}{2(V_{DD} - V_{TH1})}\left[\frac{dV_{out}}{2(V_{DD} - V_{TH1}) - V_{out}} + \frac{dV_{out}}{V_{out}}\right] = -\frac{1}{2}\mu_n \frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_1 dt$$

$V_{out}(t = 0) = V_{DD} - V_{TH1}$ Note that time origin is assumed to be $2.2 \times 10^{-11} S$

$$\frac{V_{out}}{2(V_{DD} - V_{TH1}) - V_{out}} = e^{-\mu_n \frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_1 (V_{DD} - V_{TH1}) t}$$

$V_{out} = 0.05 V_{DD}$

$$\frac{0.05 V_{DD}}{2(V_{DD} - V_{TH1}) - 0.05 V_{DD}} = e^{-\mu_n \frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_1 (V_{DD} - V_{TH1}) T_{(V_{DD}-V_{TH1}) \to 0.05 V_{DD}}}$$

$$T_{(V_{DD}-V_{TH1}) \to 0.05 V_{DD}} = \frac{\ln(39 - 40 V_{TH1}/V_{DD})}{\mu_n \frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_1 (V_{DD} - V_{TH1})}$$

$$= \frac{\ln(39 - 40 \times 0.4/1.8)}{100\times10^{-6} \times \frac{1}{30\times10^{-15}} \times \frac{1}{0.18}(1.8-0.4)}$$

$$T(V_{DD} - V_{TH1}) \to 0.05 V_{DD} = 131.33\ pS$$

$$T_{(V_{DD} \to 0.05 V_{DD})} = T_{(V_{DD} \to V_{DD} - V_{TH1})} - T_{(V_{DD} - V_{TH1} \to 0.05 V_{DD})}$$

$$= 2.2\times10^{-11} + 1.3133\times10^{-10}$$

$$\boxed{T_{(V_{DD} \to 0.05 V_{DD})} = 153.33\ pS}$$

(b) $V_1 = V_{DD}/2$ $\quad V_{DD}/2 - V_{TH1} < V_{out} < V_{DD}$ $\quad M_1$ in Saturation

$0.05 V_{DD} < V_{out} < V_{DD}/2 - V_{TH1}$ $\quad M_1$ in Triode

$$C_L \frac{dV_{out}}{dt} = -I_{D1} = -\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left(V_{DD}/2 - V_{TH1}\right)^2 = -\frac{1}{2}\times 100\times10^{-6} \times \frac{1}{0.18}(0.9-0.4)^2$$

$$= 6.944\times10^{-5} A$$

$$V_{out}(t) = V_{DD} - \frac{I_{D1}}{C_L}\times t$$

$$V_{DD}/2 - V_{TH1} = V_{DD} - \frac{I_{D1}}{C_L}\, T_{(V_{DD} \to V_{DD}/2 - V_{TH1})}$$

$$T_{(V_{DD} \to V_{DD}/2 - V_{TH1})} = \frac{(V_{DD}/2 + V_{TH1}) \times C_L}{I_{D1}}$$

$$T_{(V_{DD} \to V_{DD}/2 - V_{TH1})} = 5.616 \times 10^{-10}$$

<u>for $0.05V_{DD} < V_{out} < V_{DD}/2 - V_{TH1}$</u>

$$C_L \frac{dV_{out}}{dt} = -I_{D1} = -\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left[2(V_{DD}/2 - V_{TH1})V_{out} - V_{out}^2\right]$$

$$\frac{V_{out}}{2(V_{DD}/2 - V_{TH1}) - V_{out}} = e^{-\mu_n \frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_1 (V_{DD}/2 - V_{TH1})\, t}$$

$$V_{out} = 0.05V_{DD} \to \frac{0.05V_{DD}}{2(V_{DD}/2 - V_{TH1}) - 0.05V_{DD}} = e^{-\mu_n \frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_1 (V_{DD}/2 - V_{TH1}) \times T}$$

$$T_{(V_{DD}/2 - V_{TH1} \to 0.05V_{DD})} = \frac{Ln(19 - 40V_{TH1}/V_{DD})}{\mu_n \frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_1 (V_{DD}/2 - V_{TH1})}$$

$$= \frac{Ln(19 - 40 \times 0.4/1.8)}{100 \times 10^{-6} \times \frac{1}{30 \times 10^{-15}} \times \frac{1}{0.18}(0.9 - 0.4)}$$

$$= 2.5 \times 10^{-10}$$

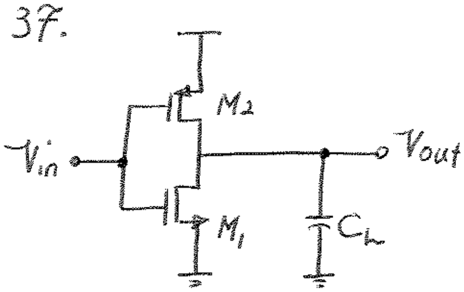
$$T_{(V_{DD} \to 0.05V_{DD})} = T_{(V_{DD} \to V_{DD}/2 - V_{TH1})} + T_{(V_{DD}/2 - V_{TH1} \to 0.05V_{DD})}$$

$$\boxed{T_{(V_{DD} \to 0.05V_{DD})} = 5.616 \times 10^{-10} + 2.5 \times 10^{-10} = 811.5\,PS}$$

By decreasing V_{in} from V_{DD} to $V_{DD}/2$, the time it takes the output to reach $0.05V_{DD}$ will be 5.3 time larger!

$$\frac{T(V_{DD} \rightarrow 0.05V_{DD})(V_{in} = V_{DD})}{T(V_{DD} \longrightarrow 0.05V_{DD})(V_{in} = V_{DD}/2)} = \frac{811.5p}{153.33p} \approx 5.3$$

37.



$\left(\frac{W}{L}\right)_1 = 1/0.18$

$\left(\frac{W}{L}\right)_2 = 3/0.18$

$C_L = 80fF$

$T_{PHL}, T_{PLH} = ?$

To calculate T_{PLH} $\quad 0 < V_{out} < |V_{TH2}| \quad M_2$ in Saturation

$|V_{TH2}| < V_{out} < V_{DD}/2 \quad M_2$ in Triode

$$|I_{D2}| = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 (V_{DD} - |V_{TH2}|)^2$$

$$V_{out}(t) = \frac{|I_{D2}|}{C_L} t$$

$$= \frac{1}{2}\mu_p \frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_2 (V_{DD} - |V_{TH2}|)^2 t.$$

$$V_{out}(T_{PLH1}) = |V_{TH2}|$$

$$T_{PLH1} = \frac{|V_{TH2}| \times C_L}{\frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 (V_{DD} - |V_{TH2}|)^2}.$$

for M_2 operating in Triode region

$$C_L \frac{dV_{out}}{dt} = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 \left[2(V_{DD} - |V_{TH2}|)(V_{DD} - V_{out}) - (V_{DD} - V_{out})^2\right]$$

$$\frac{dV_{out}}{2(V_{DD} - |V_{TH2}|)(V_{DD} - V_{out}) - (V_{DD} - V_{out})^2} = \frac{1}{2}\mu_p \frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_2 dt.$$

Defining $V_{DD} - V_{out} = u$ and noting that $\int \frac{du}{au - u^2} = \frac{1}{a} \ln \frac{u}{a-u}$,

$$\left. \frac{-1}{2(V_{DD} - |V_{TH2}|)} \ln \frac{V_{DD} - V_{out}}{V_{DD} - 2|V_{TH2}| + V_{out}} \right|_{V_{out} = |V_{TH2}|}^{V_{out} = V_{DD}/2} = \frac{1}{2} \mu_p \frac{C_{ox}}{C_L} \left(\frac{W}{L}\right)_2 T_{PLH2}$$

$$T_{PLH2} = \frac{C_L}{\mu_p C_{ox} \left(\frac{W}{L}\right)_2 (V_{DD} - |V_{TH2}|)} \ln \left(3 - 4 \frac{|V_{TH2}|}{V_{DD}}\right)$$

$$T_{PLH} = T_{PLH1} + T_{PLH2} = \frac{C_L}{\mu_p C_{ox} \left(\frac{W}{L}\right)_2 (V_{DD} - |V_{TH2}|)} \left[\frac{2|V_{TH2}|}{V_{DD} - |V_{TH2}|} + \ln \left(3 - 4 \frac{|V_{TH2}|}{V_{DD}}\right)\right]$$

$$T_{PLH} = \frac{80 \times 10^{-15}}{50 \times 10^{-6} \times \frac{3}{0.18} (1.8 - 0.5)} \left[\frac{2 \times 0.5}{1.8 - 0.5} + \ln \left(3 - 4 \frac{0.5}{1.8}\right)\right]$$

$$\boxed{T_{PLH} = 1.0377 \times 10^{-10}}$$

To calculate T_{PHL} $\quad V_{DD} - V_{TH1} < V_{out} < V_{DD}$ $\quad M_1$ in Saturation

$V_{DD}/2 < V_{out} < V_{DD} - V_{TH1}$ $\quad M_1$ in Triode

$$T_{PHL1} = \frac{-\Delta V_{out} \times C_L}{I_{D1}} = \frac{V_{TH1} \times C_L}{\frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{DD} - V_{TH1})^2}$$

after this point in time.

$$C_L \frac{dV_{out}}{dt} = -\frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 \left[2(V_{DD} - V_{TH1}) V_{out} - V_{out}^2\right]$$

$$V_{out}(t=0) = V_{DD} - V_{TH1}$$

$$\frac{1}{2(V_{DD}-V_{TH1})} Ln \frac{V_{out}}{2(V_{DD}-V_{TH1})-V_{out}} \Bigg|_{V_{out}=V_{DD}-V_{TH1}}^{V_{out}=V_{DD}/2} = -\frac{1}{2}\mu_n \frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_1 T_{HL2}$$

$$T_{PHL2} = \frac{C_L}{\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{DD}-V_{TH1})} \times Ln\left(3-4\frac{V_{TH1}}{V_{DD}}\right)$$

$$T_{PHL} = T_{PHL1} + T_{PHL2} = \frac{C_L}{\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{DD}-V_{TH1})} \times \left[\frac{2V_{TH1}}{V_{DD}-V_{TH1}} + Ln\left(3-4\frac{V_{TH1}}{V_{DD}}\right)\right]$$

$$T_{PHL} = \frac{80\times10^{-15}}{100\times10^{-6}\times\frac{1}{0.18}(1.8-0.4)} \times \left[\frac{2\times0.4}{1.8-0.4} + Ln\left(3-4\frac{0.4}{1.8}\right)\right]$$

$$\boxed{T_{PHL} = 1.3563\times10^{-10}}$$

38. $V_{DD} = 1.8 + 1.8 \times 0.1 = 1.98$

$$T_{PLH} = \frac{C_L}{\mu_p C_{ox}\left(\frac{W}{L}\right)_2 (V_{DD} - |V_{TH2}|)} \left[\frac{2|V_{TH2}|}{V_{DD} - |V_{TH2}|} + \ln\left(3 - 4\frac{|V_{TH2}|}{V_{DD}}\right)\right]$$

$$= \frac{80 \times 10^{-15}}{50 \times 10^{-6} \times \frac{3}{0.18} \times (1.98 - 0.5)} \times \left[\frac{2 \times 0.5}{1.98 - 0.5} + \ln\left(3 - 4 \times \frac{0.5}{1.98}\right)\right]$$

$$\boxed{T_{PLH} = 8.846 \times 10^{-11}}$$

$$\text{Decrease in } T_{PLH} = \left|\frac{8.846 \times 10^{-11} - 1.0377 \times 10^{-10}}{1.0377 \times 10^{-10}}\right| \times 100$$

$$= 14.75\%$$

$$T_{PHL} = \frac{C_L}{\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{DD} - V_{TH1})} \times \left[\frac{2V_{TH1}}{V_{DD} - V_{TH1}} + \ln\left(3 - 4\frac{V_{TH1}}{V_{DD}}\right)\right]$$

$$= \frac{80 \times 10^{-15}}{100 \times 10^{-6} \times \frac{1}{0.18} \times (1.98 - 0.4)} \left[\frac{2 \times 0.4}{1.98 - 0.4} + \ln\left(3 - 4\frac{0.4}{1.98}\right)\right]$$

$$\boxed{T_{PHL} = 1.1767 \times 10^{-10}}$$

$$\text{Decrase in } T_{PHL} = \left|\frac{1.1767 \times 10^{-10} - 1.3563 \times 10^{-10}}{1.3563 \times 10^{-10}}\right| \times 100$$

$$= 13.24\%$$

39. $V_{DD} = 0.9\,V$

$$C_L \frac{dV_{out}}{dt} = I_{D2} = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 (V_{DD} - |V_{TH2}|)^2$$

$$T_{PLH} = \frac{\Delta V_{out} \times C_L}{I_{D2}} = \frac{(V_{DD}/2) \times C_L}{\frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 (V_{DD} - |V_{TH2}|)^2}$$

$$= \frac{0.45 \times 80 \times 10^{-15}}{\frac{1}{2} \times 50 \times 10^{-6} \times \frac{3}{0.18} \times (0.9 - 0.5)^2}$$

$$\boxed{T_{PLH} = 5.4 \times 10^{-10} = 540\ pS}$$

$$T_{PHL} = \frac{C_L}{\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{DD} - V_{TH1})}\left[\frac{2V_{TH1}}{V_{DD} - V_{TH1}} + Ln\left(3 - 4\frac{V_{TH1}}{V_{DD}}\right)\right]$$

$$= \frac{80 \times 10^{-15}}{100 \times 10^{-6} \times \frac{1}{0.18} \times (0.9 - 0.4)} \times \left[\frac{2 \times 0.4}{0.9 - 0.4} + Ln\left(3 - 4\frac{0.4}{0.9}\right)\right]$$

$$\boxed{T_{PHL} = 5.186 \times 10^{-10} = 518.6\ pS}$$

$$\text{Increase in } T_{PLH} = \left|\frac{5.4 \times 10^{-10} - 1.0377 \times 10^{-10}}{1.0377 \times 10^{-10}}\right| \times 100$$

$$= 420.38\,\%$$

$$\text{Increase in } T_{PHL} = \left|\frac{5.186 \times 10^{-10} - 1.3563 \times 10^{-10}}{1.3563 \times 10^{-10}}\right| \times 100$$

$$= 282.36\,\%$$

40. $T_{PLH} = T_{PHL} = 80\,pS$

$C_L = 50\,fF$

$\left(W/L\right)_1 \,,\, \left(W/L\right)_2 = ?$

$$T_{PLH} = \frac{C_L}{\mu_p C_{ox}\left(\frac{W}{L}\right)_2 \left(V_{DD} - |V_{TH2}|\right)}\left[\frac{2|V_{TH2}|}{V_{DD} - |V_{TH2}|} + \ln\left(3 - 4\frac{|V_{TH2}|}{V_{DD}}\right)\right]$$

$$80\times10^{-12} = \frac{50\times10^{-15}}{50\times10^{-6}\times(1.8-0.5)\times\left(\frac{W}{L}\right)_2}\times\left[\frac{2\times0.5}{1.8-0.5} + \ln\left(3-4\times\frac{0.5}{1.8}\right)\right]$$

$$\boxed{\left(\frac{W}{L}\right)_2 = \frac{2.4}{0.18}}$$

$$T_{PHL} = \frac{C_L}{\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left(V_{DD} - V_{TH1}\right)}\times\left[\frac{2V_{TH1}}{V_{DD} - V_{TH1}} + \ln\left(3 - 4\frac{V_{TH1}}{V_{DD}}\right)\right]$$

$$80\times10^{-12} = \frac{50\times10^{-15}}{100\times10^{-6}\times(1.8-0.4)\times\left(\frac{W}{L}\right)_1}\times\left[\frac{2\times0.4}{1.8-0.4} + \ln\left(3-4\times\frac{0.4}{1.8}\right)\right]$$

$$\boxed{\left(\frac{W}{L}\right)_1 = \frac{1}{0.18}}$$

41.

$$T_{PHL} = \frac{C_L}{\mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{DD} - V_{TH1})} \left[\frac{2V_{TH1}}{V_{DD} - V_{TH1}} + \ln\left(3 - 4\frac{V_{TH1}}{V_{DD}}\right) \right]$$

$V_{TH1} = 0.4$

$$\frac{2V_{TH1}}{V_{DD} - V_{TH1}} = \ln\left(3 - 4\frac{V_{TH1}}{V_{DD}}\right) \longrightarrow V_{DD} = V_{TH1}\left[1 + \frac{2}{\ln\left(3 - 4\frac{V_{TH1}}{V_{DD}}\right)}\right]$$

$V_{TH1} = 0.4 \longrightarrow \boxed{V_{DD} = 1.57}$

$$\frac{2V_{TH1}}{V_{DD} - V_{TH1}} = 0.1 \times \ln\left(3 - 4\frac{V_{TH1}}{V_{DD}}\right) \longrightarrow V_{DD} = V_{TH1}\left[1 + \frac{20}{\ln\left(3 - 4\frac{V_{TH1}}{V_{DD}}\right)}\right]$$

$V_{TH1} = 0.4 \longrightarrow \boxed{V_{DD} = 8.16}$

42. $\left(\frac{W}{L}\right)_1 = 1/0.18$

$$T_{PHL} = \frac{C_L}{\mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{DD} - V_{TH1})} \left[\frac{2V_{TH1}}{V_{DD} - V_{TH1}} + \ln\left(3 - 4\frac{V_{TH1}}{V_{DD}}\right) \right]$$

$T_{PHL} = 100\,pS$

$C_L = 80\,fF$

$V_{DD} = ?$

$$100\times10^{-12} = \frac{80\times10^{-15}}{100\times10^{-6} \times \frac{1}{0.18} \times (V_{DD} - 0.4)} \times \left[\frac{2\times0.4}{V_{DD} - 0.4} + \ln\left(3 - 4\frac{0.4}{V_{DD}}\right) \right]$$

$$V_{DD} = 0.4 + 1.44 \left[\frac{0.8}{V_{DD} - 0.4} + \ln\left(3 - \frac{1.6}{V_{DD}}\right) \right]$$

$$\boxed{V_{DD} = 2.22}$$

43. $T_{PHL} = 120\ ps$ $\quad (W/L)_1 = ?$

$C_L = 90\ fF$ $\quad V_{TH1} = ?$

$V_{DD} = 1.8$

$T_{PHL} = 160\ ps$

$$T_{PHL} = \frac{C_L}{\mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{DD} - V_{TH1})} \left[\frac{2V_{TH1}}{V_{DD} - V_{TH1}} + Ln\left(3 - 4\frac{V_{TH1}}{V_{DD}}\right)\right]$$

$V_{DD} = 1.5\ V$

$C_L = 90 fF$

$$120 \times 10^{-12} = \frac{90 \times 10^{-15}}{100 \times 10^{-6} \left(\frac{W}{L}\right)_1 (1.8 - V_{TH1})} \times \left[\frac{2V_{TH1}}{1.8 - V_{TH1}} + Ln\left(3 - 4\frac{V_{TH1}}{1.8}\right)\right] \quad (1)$$

$$160 \times 10^{-12} = \frac{90 \times 10^{-15}}{100 \times 10^{-6} \left(\frac{W}{L}\right)_1 (1.5 - V_{TH1})} \times \left[\frac{2V_{TH1}}{1.5 - V_{TH1}} + Ln\left(3 - 4\frac{V_{TH1}}{1.5}\right)\right] \quad (2)$$

Dividing Equations (1) and (2) yields:

$$0.75 = \frac{1.5 - V_{TH1}}{1.8 - V_{TH1}} \times \frac{\frac{2V_{TH1}}{1.8 - V_{TH1}} + Ln\left(3 - 4\frac{V_{TH1}}{1.8}\right)}{\frac{2V_{TH1}}{1.5 - V_{TH1}} + Ln\left(3 - 4\frac{V_{TH1}}{1.5}\right)}$$

$$V_{TH1} = 1.8 - \left(\frac{1.5 - V_{TH1}}{0.75}\right) \times \frac{\frac{2V_{TH1}}{1.8 - V_{TH1}} + Ln\left(3 - 4\frac{V_{TH1}}{1.8}\right)}{\frac{2V_{TH1}}{1.5 - V_{TH1}} + Ln\left(3 - 4\frac{V_{TH1}}{1.5}\right)}$$

This equation does not lead to a real Value for V_{TH1} so we use another derivation

$$V_{TH1} = 0.45 \times \left\{3 - e^{\left[0.75 \frac{1.8 - V_{TH1}}{1.5 - V_{TH1}} \times \left[\frac{2V_{TH1}}{1.5 - V_{TH1}} + Ln\left(3 - 4\frac{V_{TH1}}{1.5}\right)\right] - \frac{2V_{TH1}}{1.8 - V_{TH1}}\right]}\right\}$$

$$\boxed{V_{TH1} = 0.39}$$

$$\boxed{\left(\frac{W}{L}\right)_1 = \frac{1.26}{0.18}}$$

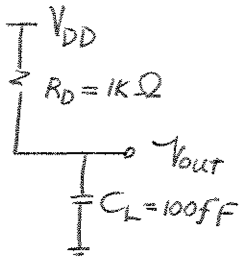
44.

$$T_{PHL} = \frac{C_L}{\mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{DD} - V_{TH1})} \left[\frac{2V_{TH1}}{V_{DD} - V_{TH1}} + Ln\left(3 - 4\frac{V_{TH1}}{V_{DD}}\right) \right]$$

$Ln\left(3 - 4\frac{V_{TH1}}{V_{DD}}\right)$ is meaningless if $V_{DD} < 4V_{TH1}/3$.

Let's consider the case where $V_{DD} = \frac{4}{3}V_{TH1}$; then, T_{PHL} is the time it takes for the output to drop from $V_{DD} = \frac{4}{3}V_{TH1}$ to $\frac{V_{DD}}{2} = \frac{2}{3}V_{TH1}$. However, $\left(V_{in}^{o} = V_{DD} = \frac{4}{3}V_{TH1}\right) - \left(V_{out} = \frac{2}{3}V_{TH1}\right) = \frac{2}{3}V_{TH1} < V_{TH1}$. In other words, M_1 never enters the triode region in the region where T_{PHL} is calculated. The logrithmic term is derived from equation in which M_1 was assumed to be in Triode region. Therefore the logarithmic term is meaningless for $V_{DD} < \frac{4}{3}V_{TH1}$.

45.



$$V_{R_D} = (V_{DD} - V_{out})$$

$$I_{R_D} = C_L \frac{dV_{out}}{dt}$$

$$P_{R_D}(t) = V_{R_D} \cdot I_{R_D} = C_L (V_{DD} - V_{out}) \frac{dV_{out}}{dt}$$

$$E_{R_D} = \int_{t=0}^{\infty} P_{R_D}(t)\, dt = \int_{V_{out}=0}^{V_{DD}} (V_{DD} - V_{out})\, dV_{out} = \frac{1}{2} C_L V_{DD}^2$$

$$= \frac{1}{2} \times 100 \times 10^{-15} \times (1.8)^2$$

$$\boxed{E_{R_D} = 0.162\ pJ}$$

46. 10^6 Gates

$f = 2\,GHz$

20% of gates switch in every clock cycle

$C_L = 20fF$ for each gate

$P_{av} = ?$

$$P_{av,\,gate} = f_{in} C_L V_{DD}^2$$

$$P_{av,\,total} = 0.2 \times 10^6 \times f_{in} C_L V_{DD}^2$$

$$= 0.2 \times 10^6 \times 2 \times 10^9 \times 20 \times 10^{-15} \times (1.8)^2$$

$$\boxed{P_{av,\,total} = 25.92\ W}$$

47. $f = 2\,GHz$

5×10^{6} Transistors with $W = 1\,\mu m$, $L = 0.18\,\mu m$, $C_{ox} = 10\,fF/\mu m^{2}$

$$C_{gate} = WLC_{ox}$$

$$C_{Load} = 5\times10^{6}\,C_{gate}$$

$$= 5\times10^{6}\times WLC_{ox}$$

$$= 5\times10^{6}\times 1\,\mu m\times 0.18\,\mu m\times 10\,fF/\mu m^{2}$$

$$\boxed{C_{Load} = 9\,pF}$$

$$P_{av} = f_{in}\,C_L\,V_{DD}^{2}$$

$$= 2\times10^{9}\times 9\times10^{-9}\times(1.8)^{2}$$

$$\boxed{P_{av} = 58.32\,W}$$

48.

$$V_{DD} = V_{DD} + 0.1\,V_{DD} = 1.98$$

$$\left(\frac{W}{L}\right)_1 = 2/0.18$$

$$\left(\frac{W}{L}\right)_2 = 4/0.18$$

$$I_{Peak}\Big|_{V_{DD}=1.8} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1\left(\frac{V_{DD}}{2} - V_{TH1}\right)^2\left(1 + \lambda_1\frac{V_{DD}}{2}\right)$$

$$= \frac{1}{2} \times 100 \times 10^{-6} \times \left(\frac{2}{0.18}\right)(0.9 - 0.4)^2$$

$$I_{Peak}\Big|_{V_{DD}=1.8} = 1.388 \times 10^{-4}$$

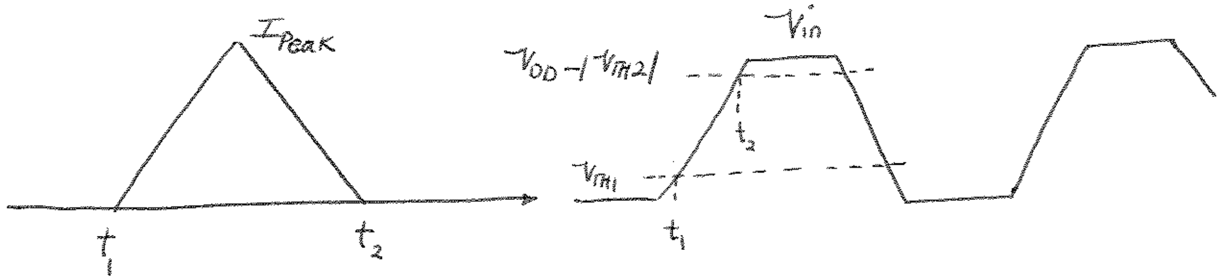
$$I_{Peak}\Big|_{V_{DD}=1.98} = \frac{1}{2} \times 100 \times 10^{-6} \times \left(\frac{2}{0.18}\right)(0.99 - 0.4)^2$$

$$I_{Peak}\Big|_{V_{DD}=1.98} = 1.9338 \times 10^{-4}$$

$$\text{Change in Crowbar Current} = \frac{1.9338 \times 10^{-4} - 1.388 \times 10^{-4}}{1.388 \times 10^{-4}}$$

$$\boxed{\text{Change in Crowbar Current} = 39.24\%}$$

49.



Total Energy drawn from V_{DD} during the interval $[t_1, t_2]$ is:

$$E = V_{DD} \times I_{Peak} \times \frac{t_2 - t_1}{2}$$

In a periode the total energy is:

$$E_{tot} = 2 \times V_{DD} \times I_{Peak} \times \frac{t_2 - t_1}{2}$$

$$P_{av} = V_{DD} I_{Peak} (t_2 - t_1) f_1$$

$$\text{Slope of input voltage} = \frac{0.9V_{DD} - 0.1V_{DD}}{t_r} = \frac{0.8V_{DD}}{t_r}$$

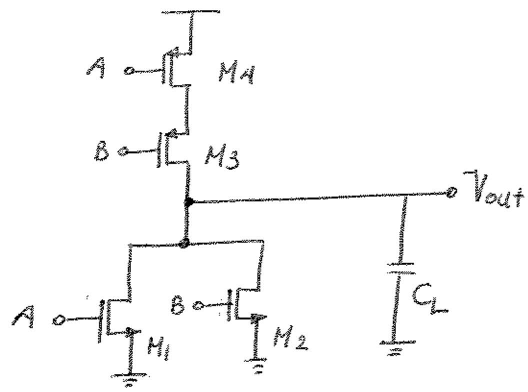
$$(t_2 - t_1) = \frac{(V_{DD} - V_{TH1} - |V_{TH2}|)}{0.8V_{DD}} \times t_r$$

$$P_{av} = V_{DD} \times \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 \left(\frac{V_{DD}}{2} - V_{TH1}\right)^2 \times \frac{(V_{DD} - V_{TH1} - |V_{TH2}|)}{0.8V_{DD}} t_r \times f_1$$

$$P_{av} = \frac{1}{1.6} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 \left(\frac{V_{DD}}{2} - V_{TH1}\right)^2 (V_{DD} - V_{TH1} - |V_{TH2}|) f_1 \cdot t_r$$

$$\boxed{P_{av} = 1.4 \times 10^{-5} \left(\frac{W}{L}\right)_1 \times t_r \times f_1}$$

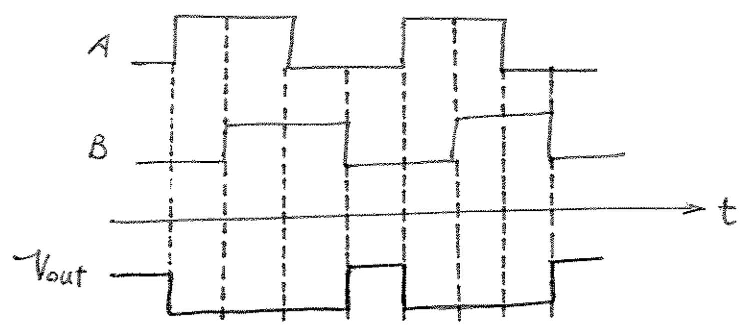
50.



$C_L = 20\,fF$

$f_1 = 500\,MHz$

$P_{av} = ?$

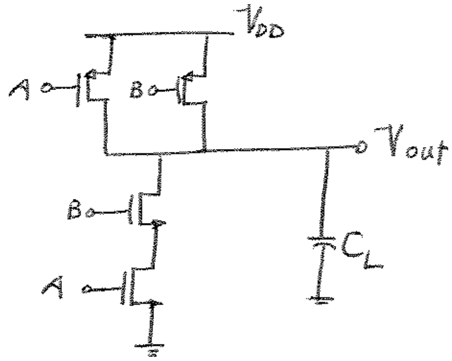


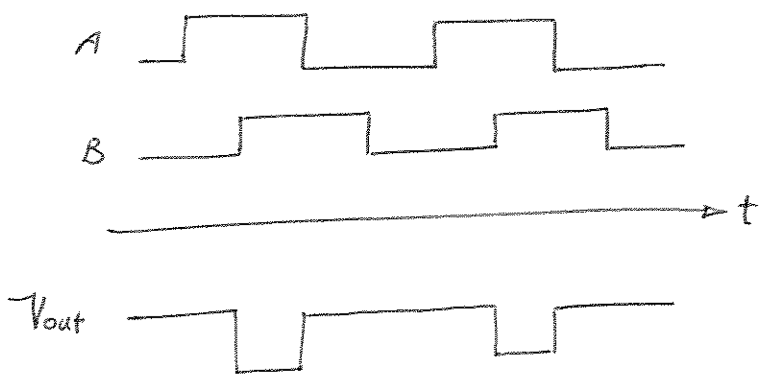
$$P_{av} = f_{in} C_L V_{DD}^2$$

$$= 500\times10^{6} \times 20\times10^{-15} \times (1.8)^2$$

$$\boxed{P_{av} = 3.24\times10^{-5}\,W}$$

51.



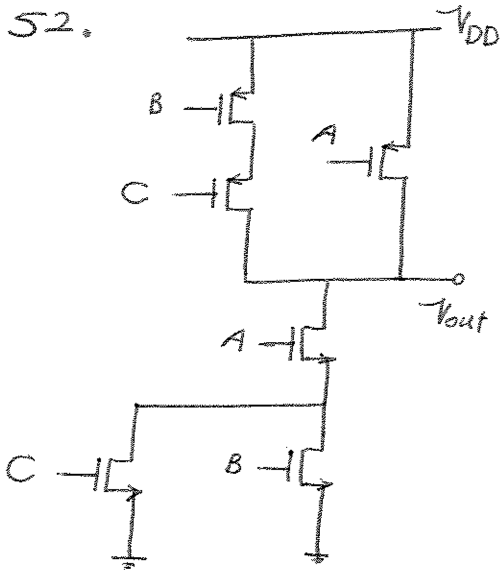


$$P_{av} = f_{in}\, C_L\, V_{DD}^2$$

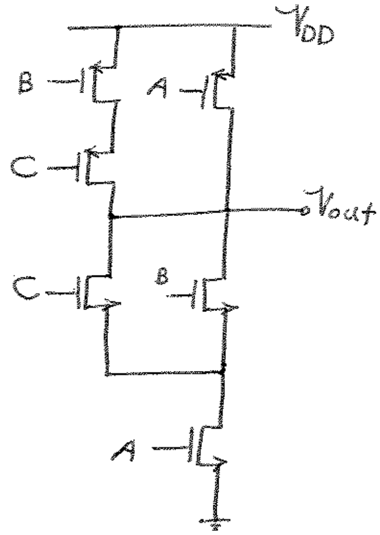
$$= 500\times10^{6} \times 20\times10^{-15} \times (1.8)^2$$

$$\boxed{P_{av} = 3.24\times10^{-5}\,W}$$

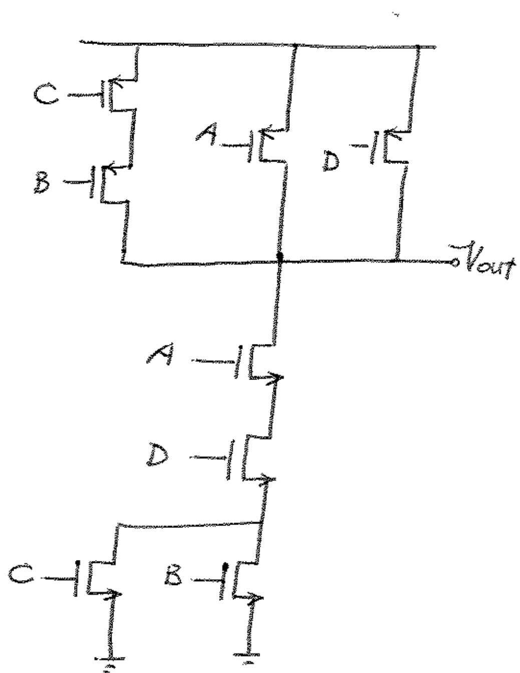
52.



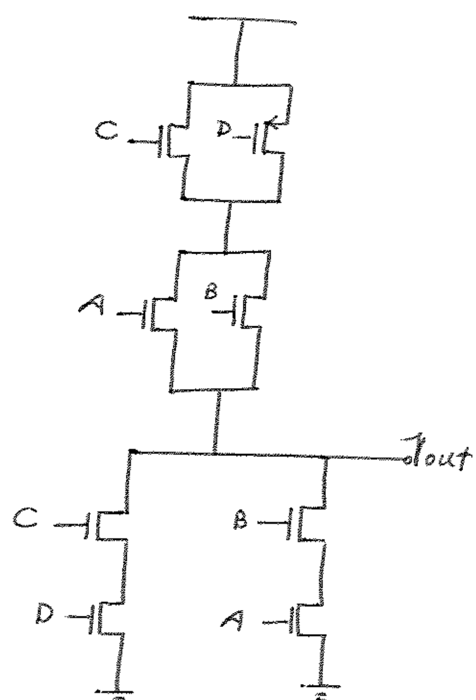
$V_{out} = \overline{(B+C)A}$



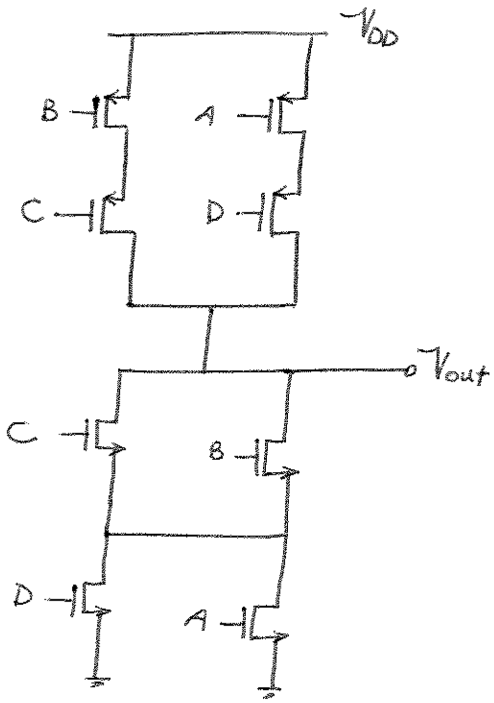
$V_{out} = \overline{(B+C) \cdot A}$



$V_{out} = \overline{(B+C)\, D \cdot A}$

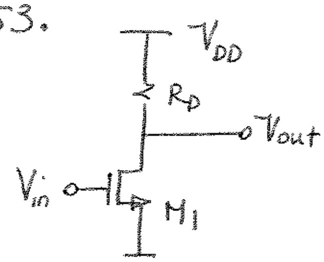


$V_{out} = \overline{A \cdot B + C \cdot D}$



$$V_{out} = \overline{(A+D)\cdot(B+C)}$$

53.



$P_{static} = 0.5\,mW$

$V_{OL} = 100\,mV$

$$\frac{(V_{DD}-V_{OL})^2}{R_D} + V_{OL} \times \frac{V_{DD}-V_{OL}}{R_D} = 0.5 \times 10^{-3}$$

$$\frac{(1.8-0.1)^2}{R_D} + 0.1 \times \frac{1.8-0.1}{R_D} = 0.5 \times 10^{-3}$$

$$\frac{1}{R_D} \times 3.06 = 0.5 \times 10^{-3}$$

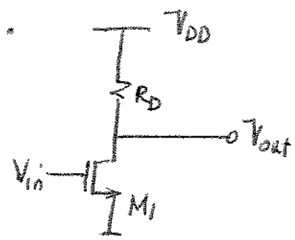
$$\boxed{R_D = 6120\,\Omega}$$

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left[2(V_{DD}-V_{TH1})V_{OL} - V_{OL}^2\right] = \frac{V_{DD}-V_{OL}}{R_D}$$

$$\frac{1}{2} \times 100 \times 10^{-6} \times \left(\frac{W}{L}\right)_1 \times \left[2(1.8-0.4)0.1 - 0.1^2\right] = \frac{1.8-0.1}{6120}$$

$$\boxed{\left(\frac{W}{L}\right)_1 = \frac{3.7}{0.18}}$$

54.



$P_{Static} = 0.25\ mW$

$NM_L = 600\ mV$

Small signal gain $= -g_m R_D$

$$g_m = \mu_n C_{ox} (W/L)(V_{GS} - V_{TH})$$

$$\mu_n C_{ox} \frac{W}{L} (V_{IL} - V_{TH1}) R_D = 1$$

$$V_{IL} = \frac{1}{\mu_n C_{ox} \left(\frac{W}{L}\right) R_D} + V_{TH1}$$

$$NM_L = V_{IL} = \frac{1}{\mu_n C_{ox} \left(\frac{W}{L}\right) R_D} + V_{TH1}$$

$$\frac{1}{\mu_n C_{ox} \left(\frac{W}{L}\right) R_D} = (NM_L - V_{TH1}) \rightarrow \left(\frac{W}{L}\right) R_D = \frac{1}{\mu_n C_{ox} (NM_L - V_{TH1})}$$

$$\frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right) \left[2(V_{DD} - V_{TH1}) V_{OL} - V_{OL}^2\right] = \frac{V_{DD} - V_{OL}}{R_D}$$

$$\frac{1}{2} \cancel{\mu_n C_{ox}} \frac{1}{\cancel{\mu_n C_{ox}} (NM_L - V_{TH1})} \times \left[2(V_{DD} - V_{TH1}) V_{OL} - V_{OL}^2\right] = (V_{DD} - V_{OL})$$

$$2(V_{DD} - V_{TH1}) V_{OL} - V_{OL}^2 = 2(NM_L - V_{TH1})(V_{DD} - V_{OL})$$

$$V_{OL}^2 - 2(V_{DD} - V_{TH1}) V_{OL} - 2(NM_L - V_{TH1}) V_{OL} + 2(NM_L - V_{TH1}) V_{DD} = 0$$

$$V_{OL}^2 - 2(V_{DD} + NM_L - 2V_{TH1}) V_{OL} + 2(NM_L - V_{TH1}) V_{DD} = 0$$

$$V_{OL}^2 - 3.2 V_{OL} + 0.72 = 0$$

$$\boxed{V_{OL} = 0.2435}$$

$$\frac{(V_{DD}-V_{OL})^2}{R_D} + V_{OL} \times \frac{V_{DD}-V_{OL}}{R_D} = 0.25\times10^{-3}$$

$$\frac{(1.8-0.24)^2 + 0.24\times(1.8-0.24)}{R_D} = 0.25\times10^{-3}$$

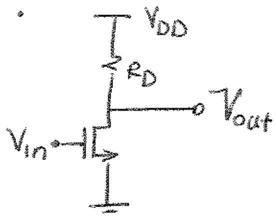
$$\boxed{R_D = 11206.55\ \Omega}$$

$$\left(\frac{W}{L}\right) = \frac{1}{\mu_n C_{ox}(NM_L - V_{TH1})R_D}$$

$$\left(\frac{W}{L}\right) = \frac{1}{100\times10^{-6}(0.6-0.4)\,11206.55}$$

$$\boxed{\left(\frac{W}{L}\right) = \frac{0.8}{0.18}}$$

55.



$V_{OL} = 100mV$ $\qquad$ $P_{av} = 0.25mW$

$$\frac{(V_{DD}-V_{OL})^2}{R_D} + \frac{V_{OL}(V_{DD}-V_{OL})}{R_D} = P_{av}$$

$$\frac{(1.8-0.1)^2 + 0.1 \times (1.8-0.1)}{0.25 \times 10^{-3}} = R_D$$

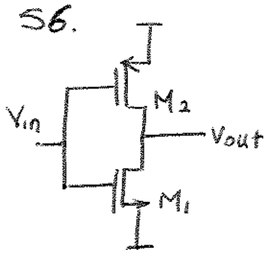
$$\boxed{R_D = 12240}$$

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)\left[2(V_{DD}-V_{TH1})V_{OL} - V_{OL}^2\right] = \frac{V_{DD}-V_{OL}}{R_D}$$

$$\frac{1}{2} \times 100 \times 10^{-6} \times \left(\frac{W}{L}\right) \times \left[2(1.8-0.4) \times 0.1 - 0.1^2\right] = \frac{1.8-0.1}{12240}$$

$$\boxed{\left(\frac{W}{L}\right) = \frac{1.85}{0.18}}$$

56.



$V_{in} = V_{out} = 0.8V$, $I_{D1} = I_{D2} = 0.5\,mA$

$\lambda_n = 0.1\,V^{-1}$

$\lambda_p = 0.2\,V^{-1}$

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{in} - V_{TH1})^2 (1 + \lambda_N V_{out}) = I_{D1}$$

$$\frac{1}{2} \times 100 \times 10^{-6} \times \left(\frac{W}{L}\right)_1 (0.8 - 0.4)^2 (1 + 0.1 \times 0.8) = 0.5 \times 10^{-3}$$

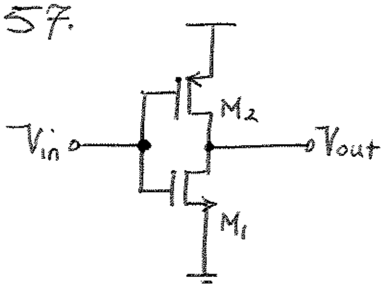
$$\boxed{\left(\frac{W}{L}\right)_1 = \frac{10.4}{0.18}}$$

$$\frac{1}{2}\mu_P C_{ox}\left(\frac{W}{L}\right)_2 (V_{DD} - V_{in} - |V_{TH2}|)^2 \left[1 + \lambda_P (V_{DD} - V_{out})\right] = I_{D2}$$

$$\frac{1}{2} \times 50 \times 10^{-6} \times \left(\frac{W}{L}\right)_2 (1.8 - 0.8 - 0.5)^2 \left[1 + 0.2 \times (1.8 - 0.8)\right] = 0.5 \times 10^{-3}$$

$$\boxed{\left(\frac{W}{L}\right)_2 = \frac{12}{0.18}}$$

57.



$NM_L = NM_H = 0.7\,V$

NM_L : M_1 in Saturation and M_2 in triode

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{in} - V_{TH1})^2 = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 \left[2(V_{DD} - V_{in} - |V_{TH2}|)(V_{DD} - V_{out}) - (V_{DD} - V_{out})^2\right] \quad (1)$$

Differentiating both sides with respect to V_{in}

$$2\mu_n\left(\frac{W}{L}\right)_1 (V_{in} - V_{TH1}) = \mu_p\left(\frac{W}{L}\right)_2 \left[-2(V_{DD} - V_{out}) - 2(V_{DD} - V_{in} - |V_{TH2}|)\frac{\partial V_{out}}{\partial V_{in}} + 2(V_{DD} - V_{out})\frac{\partial V_{out}}{\partial V_{in}}\right]$$

(a) $V_{in} = V_{IL}$, $\dfrac{\partial V_{out}}{\partial V_{in}} = -1$

$$\mu_n\left(\frac{W}{L}\right)_1 (V_{IL} - V_{TH1}) = \mu_p\left(\frac{W}{L}\right)_2 \left(2V_{OH} - V_{IL} - |V_{TH2}| - V_{DD}\right) \quad (2)$$

obtaining V_{OH} from (2), substituting in (1), we arrive at

$$V_{IL} = \frac{2\sqrt{a}\,(V_{DD} - V_{TH1} - |V_{TH2}|)}{(a-1)\sqrt{a+3}} - \frac{V_{DD} - aV_{TH1} - |V_{TH2}|}{a-1}$$

$$a = \frac{\mu_n\left(\frac{W}{L}\right)_1}{\mu_p\left(\frac{W}{L}\right)_2}$$

NM_H, M_1 in triode and M_2 in Saturation

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1\left[2(V_{in}-V_{TH1})V_{out}-V_{out}^2\right]=\frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2\left(V_{DD}-V_{in}-|V_{TH2}|\right)^2$$

Differentiating both sides with respect to V_{in}:

$$\mu_n\left(\frac{W}{L}\right)_1\left[2V_{out}+2(V_{in}-V_{TH1})\frac{\partial V_{out}}{\partial V_{in}}-2V_{out}\frac{\partial V_{out}}{\partial V_{in}}\right]=2\mu_p\left(\frac{W}{L}\right)_2\times$$

$$\left(V_{in}-V_{DD}-|V_{TH2}|\right)$$

Assuming $\frac{\partial V_{out}}{\partial V_{in}}=-1$, $V_{in}=V_{IH}$, and $V_{out}=V_{OL}$ obtaining

$$V_{IH}=\frac{2a\left(V_{DD}-V_{TH1}-|V_{TH2}|\right)}{(a-1)\sqrt{1+3a}}-\frac{V_{DD}-aV_{TH1}-|V_{TH2}|}{a-1}$$

$$V_{IL}=NM_L=0.7$$

$$V_{IH}=V_{DD}-NM_H=1.8-0.7=1.1$$

$$0.7=\frac{2\sqrt{a}\,(1.8-0.4-0.5)}{(a-1)\sqrt{a+3}}-\frac{1.8-0.4a-0.5}{a-1}$$

$$0.7(a-1)=\frac{1.8\sqrt{a}}{\sqrt{a+3}}-\frac{1.3-0.4a}{1}$$

$$0.7a-0.7+1.3-0.4a=\sqrt{\frac{a}{a+3}}\times 1.8$$

$$\frac{0.6+0.3a}{1.8}=\sqrt{\frac{a}{a+3}}\longrightarrow a^3+7a^2-20a+12=0$$

$$a=\begin{cases}-9.3\\ 1.3\\ 1\end{cases}\longrightarrow \boxed{a=1.3}$$

$$1.1 = \frac{2a(1.8-0.4-0.5)}{(a-1)\sqrt{1+3a}} - \frac{1.8-0.4a-0.5}{a-1}$$

$$1.1(a-1) = \frac{1.8a}{\sqrt{1+3a}} - 1.3 + 0.4a$$

$$1.1a - 1.1 + 1.3 - 0.4a = \frac{1.8a}{\sqrt{1+3a}}$$

$$0.2 + 0.7a = \frac{1.8a}{\sqrt{1+3a}}$$

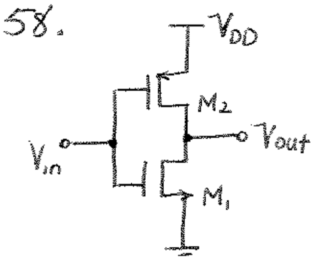
$$147a^3 - 191a^2 + 40a + 4 = 0 \longrightarrow \begin{cases} a_1 = 1 \\ a_2 = 0.37 \\ a_3 = -0.073 \end{cases} \quad a_2 = 0.37 \longrightarrow \boxed{a = 0.37}$$

No it is not possible to design a CMOS inverter with $NM_L = NM_H = 0.7$.

The reason is that each value of $a = \frac{\mu_n C_{ox} (W/L)_1}{\mu_p C_{ox} (W/L)_2}$ specifies a unique set of noise margins (NM_L, NM_H).

Remember, the relative strength of NMOS and PMOS determines the noise margins interdependently.

58.



$T_{PLH} = T_{PHL} = 100\,ps$

$C_L = 50 fF$

T_{PLH}

$$|I_{D2}| = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 (V_{DD} - |V_{TH2}|)^2$$

$$V_{out}(t) = \frac{|I_{D2}|}{C_L} t$$

$$= \frac{1}{2}\mu_p \frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_2 (V_{DD} - |V_{TH2}|)^2 t.$$

$$T_{PLH1} = \frac{2|V_{TH2}|C_L}{\mu_p C_{ox}\left(\frac{W}{L}\right)_2 (V_{DD} - |V_{TH2}|)^2}.$$

$$|I_{D2}| = C_L \frac{dV_{out}}{dt}$$

$$\frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 \left[2(V_{DD} - |V_{TH2}|)(V_{DD} - V_{out}) - (V_{DD} - V_{out})^2\right] = C_L \frac{dV_{out}}{dt}$$

$$\frac{dV_{out}}{2(V_{DD} - |V_{TH2}|)(V_{DD} - V_{out}) - (V_{DD} - V_{out})^2} = \frac{1}{2}\mu_p \frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_2 dt$$

$$\frac{-1}{2(V_{DD} - |V_{TH2}|)} \ln \frac{V_{DD} - V_{out}}{V_{DD} - 2|V_{TH2}| + V_{out}}\Bigg|_{V_{out} = |V_{TH2}|}^{V_{out} = V_{DD}/2} = \frac{1}{2}\mu_p \frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_2 T_{PLH2}$$

$$T_{PLH2} = \frac{C_L}{\mu_p C_{ox}\left(\frac{W}{L}\right)_2 (V_{DD} - |V_{TH2}|)} \ln\left(3 - 4\frac{|V_{TH2}|}{V_{DD}}\right)$$

$$T_{PLH} = T_{PLH1} + T_{PLH2}$$

$$= \frac{C_L}{\mu_p C_{ox} \left(\frac{W}{L}\right)_2 (V_{DD} - |V_{TH2}|)} \left[\frac{2|V_{TH2}|}{V_{DD} - |V_{TH2}|} + Ln\left(3 - 4\frac{|V_{TH2}|}{V_{DD}}\right) \right]$$

$$100 \times 10^{-12} = \frac{50 \times 10^{-15}}{50 \times 10^{-6} \left(\frac{W}{L}\right)_2 (1.8 - 0.5)} \left[\frac{2 \times 0.5}{1.8 - 0.5} + Ln\left(3 - 4\frac{0.5}{1.8}\right) \right]$$

$$\boxed{\left(\frac{W}{L}\right)_2 = \frac{1.9}{0.18}}$$

T_{PHL}

$$T_{PHL1} = \frac{2 V_{TH1} C_L}{\mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{DD} - V_{TH1})^2}$$

$$\frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 \left[2(V_{DD} - V_{TH1}) V_{out} - V_{out}^2 \right] = -C_L \frac{dV_{out}}{dt}$$

$$\frac{-1}{2(V_{DD} - V_{TH1})} Ln \frac{V_{out}}{2(V_{DD} - V_{TH1}) - V_{out}} \Bigg|_{V_{out} = V_{DD} - V_{TH1}}^{V_{out} = V_{DD}/2} = \frac{1}{2} \mu_n C_{ox}/C_L \left(\frac{W}{L}\right)_1 T_{PHL2}.$$

$$T_{PHL2} = \frac{C_L}{\mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{DD} - V_{TH1})} Ln\left(3 - 4\frac{V_{TH1}}{V_{DD}}\right)$$

$$T_{PHL} = T_{PHL1} + T_{PHL2} = \frac{C_L}{\mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{DD} - V_{TH1})} \left[\frac{2V_{TH1}}{V_{DD} - V_{TH1}} + Ln\left(3 - 4\frac{V_{TH1}}{V_{DD}}\right) \right]$$

$$100 \times 10^{-12} = \frac{50 \times 10^{-15}}{100 \times 10^{-6} \times \left(\frac{W}{L}\right)_1 \times (1.8 - 0.4)} \times \left[\frac{2 \times 0.4}{1.8 - 0.4} + Ln\left(3 - 4 \times \frac{0.4}{1.8}\right) \right], \boxed{\left(\frac{W}{L}\right)_1 = \frac{0.85}{0.18}}$$