Chapter 31

1. (a) All the energy in the circuit resides in the capacitor when it has its maximum charge. The current is then zero. If Q is the maximum charge on the capacitor, then the total energy is

$$U = \frac{Q^2}{2C} = \frac{\left(2.90\times10^{-6}\,\mathrm{C}\right)^2}{2\left(3.60\times10^{-6}\,\mathrm{F}\right)} = 1.17\times10^{-6}\,\mathrm{J}.$$

(b) When the capacitor is fully discharged, the current is a maximum and all the energy resides in the inductor. If I is the maximum current, then $U = LI^2/2$ leads to

$$I = \sqrt{\frac{2U}{L}} = \sqrt{\frac{2\left(1.168\times10^{-6}\,\mathrm{J}\right)}{75\times10^{-3}\,\mathrm{H}}} = 5.58\times10^{-3}\,\mathrm{A}.$$

2. (a) We recall the fact that the period is the reciprocal of the frequency. It is helpful to refer also to Fig. 31-1. The values of t when plate A will again have maximum positive charge are multiples of the period:

$$t_A = nT = \frac{n}{f} = \frac{n}{2.00\times10^3\,\mathrm{Hz}} = n\left(5.00\,\mu\mathrm{s}\right),$$

where n = 1, 2, 3, 4, $\ldots$. The earliest time is (n = 1) $t_A = 5.00\,\mu$s.

(b) We note that it takes $t = \frac{1}{2}T$ for the charge on the other plate to reach its maximum positive value for the first time (compare steps a and e in Fig. 31-1). This is when plate A acquires its most negative charge. From that time onward, this situation will repeat once every period. Consequently,

$$t = \frac{1}{2}T + (n-1)T = \frac{1}{2}\left(2n-1\right)T = \frac{\left(2n-1\right)}{2f} = \frac{\left(2n-1\right)}{2\left(2\times10^3\,\mathrm{Hz}\right)} = \left(2n-1\right)\left(2.50\,\mu\mathrm{s}\right),$$

where n = 1, 2, 3, 4, $\ldots$. The earliest time is (n = 1) $t = 2.50\,\mu$s.

(c) At $t = \frac{1}{4}T$, the current and the magnetic field in the inductor reach maximum values for the first time (compare steps a and c in Fig. 31-1). Later this will repeat every half-period (compare steps c and g in Fig. 31-1). Therefore,

$$t_L = \frac{T}{4} + \frac{(n-1)T}{2} = (2n-1)\frac{T}{4} = (2n-1)(1.25\,\mu\text{s}),$$

where n = 1, 2, 3, 4, …. The earliest time is (n = 1) $t = 1.25\,\mu$s.

3. (a) The period is T = 4(1.50 μs) = 6.00 μs.

(b) The frequency is the reciprocal of the period: $f = \frac{1}{T} = \frac{1}{6.00\,\mu\text{s}} = 1.67 \times 10^5\ \text{Hz}.$

(c) The magnetic energy does not depend on the direction of the current (since $U_B \propto i^2$), so this will occur after one-half of a period, or 3.00 μs.

4. We find the capacitance from $U = \frac{1}{2}Q^2/C$:

$$C = \frac{Q^2}{2U} = \frac{(1.60 \times 10^{-6}\ \text{C})^2}{2(140 \times 10^{-6}\ \text{J})} = 9.14 \times 10^{-9}\ \text{F}.$$

5. According to $U = \frac{1}{2}LI^2 = \frac{1}{2}Q^2/C$, the current amplitude is

$$I = \frac{Q}{\sqrt{LC}} = \frac{3.00 \times 10^{-6}\ \text{C}}{\sqrt{(1.10 \times 10^{-3}\ \text{H})(4.00 \times 10^{-6}\ \text{F})}} = 4.52 \times 10^{-2}\ \text{A}.$$

6. (a) The angular frequency is

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{F/x}{m}} = \sqrt{\frac{8.0\ \text{N}}{(2.0 \times 10^{-13}\ \text{m})(0.50\ \text{kg})}} = 89\ \text{rad/s}.$$

(b) The period is 1/f and $f = \omega/2\pi$. Therefore, $T = \frac{2\pi}{\omega} = \frac{2\pi}{89\ \text{rad/s}} = 7.0 \times 10^{-2}\ \text{s}.$

(c) From $\omega = (LC)^{-1/2}$, we obtain

$$C = \frac{1}{\omega^2 L} = \frac{1}{(89\ \text{rad/s})^2(5.0\ \text{H})} = 2.5 \times 10^{-5}\ \text{F}.$$

7. **THINK** This problem explores the analogy between an oscillating LC system and an oscillating mass–spring system.

EXPRESS Table 31-1 provides a comparison of energies in the two systems. From the table, we see the following correspondences:

$$x \leftrightarrow q, \quad k \leftrightarrow \frac{1}{C}, \quad m \leftrightarrow L, \quad v = \frac{dx}{dt} \leftrightarrow \frac{dq}{dt} = i,$$
$$\frac{1}{2}kx^2 \leftrightarrow \frac{q^2}{2C}, \quad \frac{1}{2}mv^2 \leftrightarrow \frac{1}{2}Li^2.$$

ANALYZE (a) The mass m corresponds to the inductance, so m = 1.25 kg.

(b) The spring constant k corresponds to the reciprocal of the capacitance, $1/C$. Since the total energy is given by $U = Q^2/2C$, where Q is the maximum charge on the capacitor and C is the capacitance, we have

$$C = \frac{Q^2}{2U} = \frac{\left(175\times 10^{-6}\text{ C}\right)^2}{2\left(5.70\times 10^{-6}\text{ J}\right)} = 2.69\times 10^{-3}\text{ F}$$

and

$$k = \frac{1}{2.69\times 10^{-3}\text{ m/N}} = 372 \text{ N/m}.$$

(c) The maximum displacement corresponds to the maximum charge, so $x_{\max} = 1.75\times 10^{-4}$ m.

(d) The maximum speed $v_{\max}$ corresponds to the maximum current. The maximum current is

$$I = Q\omega = \frac{Q}{\sqrt{LC}} = \frac{175\times 10^{-6}\text{ C}}{\sqrt{(1.25\text{ H})\left(2.69\times 10^{-3}\text{ F}\right)}} = 3.02\times 10^{-3}\text{ A}.$$

Consequently, $v_{\max} = 3.02 \times 10^{-3}$ m/s.

LEARN The correspondences suggest that an oscillating LC system is mathematically equivalent to an oscillating mass–spring system. The electrical mechanical analogy can also be seen by comparing their angular frequencies of oscillation:

$$\omega = \sqrt{\frac{k}{m}} \text{ (mass-spring system)}, \quad \omega = \frac{1}{\sqrt{LC}} \text{ (}LC\text{ circuit)}$$

8. We apply the loop rule to the entire circuit:

$$\varepsilon_{\text{total}} = \varepsilon_{L_1} + \varepsilon_{C_1} + \varepsilon_{R_1} + \cdots = \sum_j \left(\varepsilon_{L_j} + \varepsilon_{C_j} + \varepsilon_{R_j}\right) = \sum_j \left(L_j \frac{di}{dt} + \frac{q}{C_j} + iR_j\right) = L\frac{di}{dt} + \frac{q}{C} + iR$$

with

$$L = \sum_j L_j, \quad \frac{1}{C} = \sum_j \frac{1}{C_j}, \quad R = \sum_j R_j$$

and we require $\varepsilon_{\text{total}} = 0$. This is equivalent to the simple *LRC* circuit shown in Fig. 31-27(b).

9. The time required is $t = T/4$, where the period is given by $T = 2\pi/\omega = 2\pi\sqrt{LC}.$ Consequently,

$$t = \frac{T}{4} = \frac{2\pi\sqrt{LC}}{4} = \frac{2\pi\sqrt{(0.050\,\text{H})(4.0\times10^{-6}\,\text{F})}}{4} = 7.0\times10^{-4}\,\text{s}.$$

10. We find the inductance from $f = \omega/2\pi = \left(2\pi\sqrt{LC}\right)^{-1}$.

$$L = \frac{1}{4\pi^2 f^2 C} = \frac{1}{4\pi^2\left(10\times10^{3}\,\text{Hz}\right)^2\left(6.7\times10^{-6}\,\text{F}\right)} = 3.8\times10^{-5}\,\text{H}.$$

11. **THINK** The frequency of oscillation f in an *LC* circuit is related to the inductance L and capacitance C by $f = 1/2\pi\sqrt{LC}$.

EXPRESS Since $f \sim 1/\sqrt{C}$, the smaller value of C gives the larger value of f, while the larger value of C gives the smaller value of f. Consequently, $f_{\text{max}} = 1/2\pi\sqrt{LC_{\text{min}}}$, and $f_{\text{min}} = 1/2\pi\sqrt{LC_{\text{max}}}$.

ANALYZE (a) The ratio of the maximum frequency to the minimum frequency is

$$\frac{f_{\text{max}}}{f_{\text{min}}} = \frac{\sqrt{C_{\text{max}}}}{\sqrt{C_{\text{min}}}} = \frac{\sqrt{365\,\text{pF}}}{\sqrt{10\,\text{pF}}} = 6.0.$$

(b) An additional capacitance C is chosen so the desired ratio of the frequencies is

$$r = \frac{1.60\,\text{MHz}}{0.54\,\text{MHz}} = 2.96.$$

Since the additional capacitor is in parallel with the tuning capacitor, its capacitance adds to that of the tuning capacitor. If C is in picofarads (pF), then

$$\frac{\sqrt{C + 365\,\text{pF}}}{\sqrt{C + 10\,\text{pF}}} = 2.96.$$

The solution for C is

$$C=\frac{(365\,\text{pF})-(2.96)^2(10\,\text{pF})}{(2.96)^2-1}=36\,\text{pF}.$$

(c) We solve $f=1/2\pi\sqrt{LC}$ for *L*. For the minimum frequency, *C* = 365 pF + 36 pF = 401 pF and *f* = 0.54 MHz. Thus, the inductance is

$$L=\frac{1}{(2\pi)^2Cf^2}=\frac{1}{(2\pi)^2(401\times10^{-12}\text{ F})(0.54\times10^{6}\text{ Hz})^2}=2.2\times10^{-4}\text{ H}.$$

LEARN One could also use the maximum frequency condition to solve for the inductance of the coil in (d). The capacitance is *C* = 10 pF + 36 pF = 46 pF and *f* = 1.60 MHz, so

$$L=\frac{1}{(2\pi)^2Cf^2}=\frac{1}{(2\pi)^2(46\times10^{-12}\text{ F})(1.60\times10^{6}\text{ Hz})^2}=2.2\times10^{-4}\text{ H}.$$

12. (a) Since the percentage of energy stored in the electric field of the capacitor is $(1-75.0\%)=25.0\%$, then

$$\frac{U_E}{U}=\frac{q^2/2C}{Q^2/2C}=25.0\%$$

which leads to $q/Q=\sqrt{0.250}=0.500.$

(b) From

$$\frac{U_B}{U}=\frac{Li^2/2}{LI^2/2}=75.0\%,$$

we find $i/I=\sqrt{0.750}=0.866.$

13. (a) The charge (as a function of time) is given by $q=Q\sin\omega t$, where *Q* is the maximum charge on the capacitor and ω is the angular frequency of oscillation. A sine function was chosen so that *q* = 0 at time *t* = 0. The current (as a function of time) is

$$i=\frac{dq}{dt}=\omega Q\cos\omega t,$$

and at *t* = 0, it is *I* = ωQ. Since $\omega=1/\sqrt{LC},$

$$Q=I\sqrt{LC}=(2.00\,\text{A})\sqrt{(3.00\times10^{-3}\text{ H})(2.70\times10^{-6}\text{ F})}=1.80\times10^{-4}\text{ C}.$$

(b) The energy stored in the capacitor is given by

$$U_E = \frac{q^2}{2C} = \frac{Q^2 \sin^2 \omega t}{2C}$$

and its rate of change is

$$\frac{dU_E}{dt} = \frac{Q^2 \omega \sin \omega t \cos \omega t}{C}$$

We use the trigonometric identity $\cos \omega t \sin \omega t = \frac{1}{2}\sin(2\omega t)$ to write this as

$$\frac{dU_E}{dt} = \frac{\omega Q^2}{2C}\sin(2\omega t).$$

The greatest rate of change occurs when $\sin(2\omega t) = 1$ or $2\omega t = \pi/2$ rad. This means

$$t = \frac{\pi}{4\omega} = \frac{\pi}{4}\sqrt{LC} = \frac{\pi}{4}\sqrt{(3.00\times10^{-3}\,\text{H})(2.70\times10^{-6}\,\text{F})} = 7.07\times10^{-5}\,\text{s}.$$

(c) Substituting $\omega = 2\pi/T$ and $\sin(2\omega t) = 1$ into $dU_E/dt = (\omega Q^2/2C)\sin(2\omega t)$, we obtain

$$\left(\frac{dU_E}{dt}\right)_{\max} = \frac{2\pi Q^2}{2TC} = \frac{\pi Q^2}{TC}.$$

Now $T = 2\pi\sqrt{LC} = 2\pi\sqrt{(3.00\times10^{-3}\ \text{H})(2.70\times10^{-6}\ \text{F})} = 5.655\times10^{-4}\,\text{s}$, so

$$\left(\frac{dU_E}{dt}\right)_{\max} = \frac{\pi\left(1.80\times10^{-4}\ \text{C}\right)^2}{\left(5.655\times10^{-4}\ \text{s}\right)\left(2.70\times10^{-6}\ \text{F}\right)} = 66.7\,\text{W}.$$

We note that this is a positive result, indicating that the energy in the capacitor is indeed increasing at $t = T/8$.

14. The capacitors C_1 and C_2 can be used in four different ways: (1) C_1 only; (2) C_2 only; (3) C_1 and C_2 in parallel; and (4) C_1 and C_2 in series.

(a) The smallest oscillation frequency is

$$f_3 = \frac{1}{2\pi\sqrt{L(C_1+C_2)}} = \frac{1}{2\pi\sqrt{(1.0\times10^{-2}\ \text{H})(2.0\times10^{-6}\ \text{F}+5.0\times10^{-6}\ \text{F})}}.$$
$$= 6.0\times10^{2}\ \text{Hz}$$

(b) The second smallest oscillation frequency is

$$f_1 = \frac{1}{2\pi\sqrt{LC_1}} = \frac{1}{2\pi\sqrt{(1.0\times10^{-2}\,\text{H})(5.0\times10^{-6}\,\text{F})}} = 7.1\times10^2\,\text{Hz}.$$

(c) The second largest oscillation frequency is

$$f_2 = \frac{1}{2\pi\sqrt{LC_2}} = \frac{1}{2\pi\sqrt{(1.0\times10^{-2}\,\text{H})(2.0\times10^{-6}\,\text{F})}} = 1.1\times10^3\,\text{Hz}.$$

(d) The largest oscillation frequency is

$$f_4 = \frac{1}{2\pi\sqrt{LC_1C_2/(C_1+C_2)}} = \frac{1}{2\pi}\sqrt{\frac{2.0\times10^{-6}\,\text{F}+5.0\times10^{-6}\,\text{F}}{(1.0\times10^{-2}\,\text{H})(2.0\times10^{-6}\,\text{F})(5.0\times10^{-6}\,\text{F})}} = 1.3\times10^3\,\text{Hz}.$$

15. (a) The maximum charge is

$$Q = CV_{\text{max}} = (1.0\times10^{-9}\text{ F})(3.0\text{ V}) = 3.0\times10^{-9}\text{ C}.$$

(b) From $U = \frac{1}{2}LI^2 = \frac{1}{2}Q^2/C$ we get

$$I = \frac{Q}{\sqrt{LC}} = \frac{3.0\times10^{-9}\text{ C}}{\sqrt{(3.0\times10^{-3}\text{ H})(1.0\times10^{-9}\text{ F})}} = 1.7\times10^{-3}\text{ A}.$$

(c) When the current is at a maximum, the magnetic energy is at a maximum also:

$$U_{B,\text{max}} = \frac{1}{2}LI^2 = \frac{1}{2}(3.0\times10^{-3}\text{ H})(1.7\times10^{-3}\text{ A})^2 = 4.5\times10^{-9}\text{ J}.$$

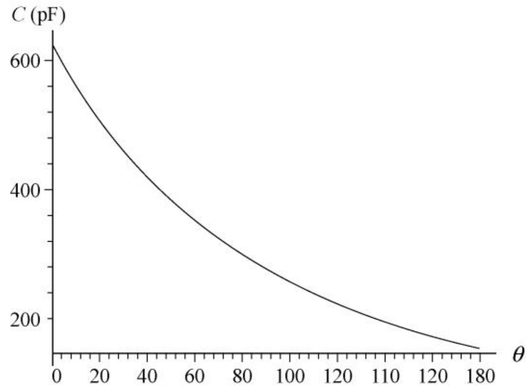
16. The linear relationship between θ (the knob angle in degrees) and frequency f is

$$f = f_0\left(1+\frac{\theta}{180^\circ}\right) \Rightarrow \theta = 180^\circ\left(\frac{f}{f_0}-1\right)$$

where $f_0 = 2\times10^5$ Hz. Since $f = \omega/2\pi = 1/2\pi\sqrt{LC}$, we are able to solve for C in terms of θ:

$$C = \frac{1}{4\pi^2 L f_0^2 (1+\theta/180^\circ)^2} = \frac{81}{400000\pi^2(180^\circ+\theta)^2}$$

with SI units understood. After multiplying by 10^{12} (to convert to picofarads), this is plotted next:



17. (a) After the switch is thrown to position *b* the circuit is an *LC* circuit. The angular frequency of oscillation is $\omega = 1/\sqrt{LC}$. Consequently,

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{\left(54.0\times10^{-3}\text{ H}\right)\left(6.20\times10^{-6}\text{ F}\right)}} = 275\text{ Hz.}$$

(b) When the switch is thrown, the capacitor is charged to V = 34.0 V and the current is zero. Thus, the maximum charge on the capacitor is

$$Q = VC = (34.0\text{ V})(6.20\times10^{-6}\text{ F}) = 2.11\times10^{-4}\text{ C.}$$

The current amplitude is

$$I = \omega Q = 2\pi fQ = 2\pi\left(275\text{ Hz}\right)\left(2.11\times10^{-4}\text{ C}\right) = 0.365\text{ A.}$$

18. (a) From $V = IX_C$ we find $\omega = I/CV$. The period is then $T = 2\pi/\omega = 2\pi CV/I$ = 46.1 μs.

(b) The maximum energy stored in the capacitor is

$$U_E = \frac{1}{2}CV^2 = \frac{1}{2}(2.20\times10^{-7}\text{ F})(0.250\text{ V})^2 = 6.88\times10^{-9}\text{ J}.$$

(c) The maximum energy stored in the inductor is also $U_B = LI^2/2 = 6.88\text{ nJ}$.

(d) We apply Eq. 30-35 as $V = L(di/dt)_{max}$. We can substitute $L = CV^2/I^2$ (combining what we found in part (a) with Eq. 31-4) into Eq. 30-35 (as written above) and solve for $(di/dt)_{max}$. Our result is

$$\left(\frac{di}{dt}\right)_{max} = \frac{V}{L} = \frac{V}{CV^2/I^2} = \frac{I^2}{CV} = \frac{(7.50\times10^{-3}\text{ A})^2}{(2.20\times10^{-7}\text{ F})(0.250\text{ V})} = 1.02\times10^3\text{ A/s}.$$

(e) The derivative of $U_B = \frac{1}{2}Li^2$ leads to

$$\frac{dU_B}{dt} = LI^2\omega \sin\omega t\cos\omega t = \frac{1}{2}LI^2\omega\sin 2\omega t\ .$$

Therefore, $\left(\dfrac{dU_B}{dt}\right)_{\max} = \dfrac{1}{2}LI^2\omega = \dfrac{1}{2}IV = \dfrac{1}{2}(7.50\times10^{-3}\ \text{A})(0.250\ \text{V}) = 0.938\ \text{mW}.$

19. The loop rule, for just two devices in the loop, reduces to the statement that the magnitude of the voltage across one of them must equal the magnitude of the voltage across the other. Consider that the capacitor has charge q and a voltage (which we'll consider positive in this discussion) $V = q/C$. Consider at this moment that the current in the inductor at this moment is directed in such a way that the capacitor charge is increasing (so $i = +dq/dt$). Equation 30-35 then produces a positive result equal to the V across the capacitor: $V = -L(di/dt)$, and we interpret the fact that $-di/dt > 0$ in this discussion to mean that $d(dq/dt)/dt = d^2q/dt^2 < 0$ represents a "deceleration" of the charge-buildup process on the capacitor (since it is approaching its maximum value of charge). In this way we can "check" the signs in Eq. 31-11 (which states $q/C = -L\, d^2q/dt^2$) to make sure we have implemented the loop rule correctly.

20. (a) We use $U = \frac{1}{2}LI^2 = \frac{1}{2}Q^2/C$ to solve for L:

$$L = \frac{1}{C}\left(\frac{Q}{I}\right)^2 = \frac{1}{C}\left(\frac{CV_{\max}}{I}\right)^2 = C\left(\frac{V_{\max}}{I}\right)^2 = \left(4.00\times10^{-6}\,\text{F}\right)\left(\frac{1.50\,\text{V}}{50.0\times10^{-3}\text{A}}\right)^2 = 3.60\times10^{-3}\,\text{H}.$$

(b) Since $f = \omega/2\pi$, the frequency is

$$f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{\left(3.60\times10^{-3}\,\text{H}\right)\left(4.00\times10^{-6}\,\text{F}\right)}} = 1.33\times10^{3}\,\text{Hz}.$$

(c) Referring to Fig. 31-1, we see that the required time is one-fourth of a period (where the period is the reciprocal of the frequency). Consequently,

$$t = \frac{1}{4}T = \frac{1}{4f} = \frac{1}{4\left(1.33\times10^{3}\ \text{Hz}\right)} = 1.88\times10^{-4}\ \text{s}.$$

21. (a) We compare this expression for the current with $i = I\sin(\omega t+\phi_0)$. Setting $(\omega t+\phi) = 2500t + 0.680 = \pi/2$, we obtain $t = 3.56\times10^{-4}$ s.

(b) Since $\omega = 2500$ rad/s $= (LC)^{-1/2}$,

$$L = \frac{1}{\omega^2 C} = \frac{1}{(2500\,\text{rad/s})^2(64.0\times10^{-6}\,\text{F})} = 2.50\times10^{-3}\,\text{H}.$$

(c) The energy is

$$U = \frac{1}{2}LI^2 = \frac{1}{2}(2.50\times10^{-3}\,\text{H})(1.60\,\text{A})^2 = 3.20\times10^{-3}\,\text{J}.$$

22. For the first circuit $\omega = (L_1C_1)^{-1/2}$, and for the second one $\omega = (L_2C_2)^{-1/2}$. When the two circuits are connected in series, the new frequency is

$$\omega' = \frac{1}{\sqrt{L_{\text{eq}}C_{\text{eq}}}} = \frac{1}{\sqrt{(L_1+L_2)C_1C_2/(C_1+C_2)}} = \frac{1}{\sqrt{(L_1C_1C_2+L_2C_2C_1)/(C_1+C_2)}}$$
$$= \frac{1}{\sqrt{L_1C_1}}\frac{1}{\sqrt{(C_1+C_2)/(C_1+C_2)}} = \omega,$$

where we use $\omega^{-1} = \sqrt{L_1C_1} = \sqrt{L_2C_2}$.

23. (a) The total energy U is the sum of the energies in the inductor and capacitor:

$$U = U_E + U_B = \frac{q^2}{2C} + \frac{i^2L}{2} = \frac{(3.80\times10^{-6}\,\text{C})^2}{2(7.80\times10^{-6}\,\text{F})} + \frac{(9.20\times10^{-3}\,\text{A})^2(25.0\times10^{-3}\,\text{H})}{2} = 1.98\times10^{-6}\,\text{J}.$$

(b) We solve $U = Q^2/2C$ for the maximum charge:

$$Q = \sqrt{2CU} = \sqrt{2(7.80\times10^{-6}\,\text{F})(1.98\times10^{-6}\,\text{J})} = 5.56\times10^{-6}\,\text{C}.$$

(c) From $U = I^2L/2$, we find the maximum current:

$$I = \sqrt{\frac{2U}{L}} = \sqrt{\frac{2(1.98\times10^{-6}\,\text{J})}{25.0\times10^{-3}\,\text{H}}} = 1.26\times10^{-2}\,\text{A}.$$

(d) If q_0 is the charge on the capacitor at time $t = 0$, then $q_0 = Q\cos\phi$ and

$$\phi = \cos^{-1}\left(\frac{q}{Q}\right) = \cos^{-1}\left(\frac{3.80\times10^{-6}\,\text{C}}{5.56\times10^{-6}\,\text{C}}\right) = \pm 46.9°.$$

For $\phi = +46.9°$ the charge on the capacitor is decreasing, for $\phi = -46.9°$ it is increasing. To check this, we calculate the derivative of q with respect to time, evaluated for $t = 0$.

We obtain $-\omega Q \sin \phi$, which we wish to be positive. Since sin(+46.9°) is positive and sin(–46.9°) is negative, the correct value for increasing charge is $\phi = -46.9°$.

(e) Now we want the derivative to be negative and $\sin \phi$ to be positive. Thus, we take $\phi = +46.9°$.

24. The charge q after N cycles is obtained by substituting $t = NT = 2\pi N/\omega'$ into Eq. 31-25:

$$\begin{aligned} q &= Qe^{-Rt/2L}\cos(\omega' t + \phi) = Qe^{-RNT/2L}\cos\left[\omega'(2\pi N/\omega') + \phi\right] \\ &= Qe^{-RN\left(2\pi\sqrt{L/C}\right)/2L}\cos(2\pi N + \phi) \\ &= Qe^{-N\pi R\sqrt{C/L}}\cos\phi. \end{aligned}$$

We note that the initial charge (setting $N = 0$ in the above expression) is $q_0 = Q \cos \phi$, where $q_0 = 6.2\ \mu C$ is given (with 3 significant figures understood). Consequently, we write the above result as $q_N = q_0 \exp\left(-N\pi R\sqrt{C/L}\right)$.

(a) For $N = 5$, $q_5 = (6.2\mu\text{C})\exp\left(-5\pi(7.2\Omega)\sqrt{0.0000032\,\text{F}/12\,\text{H}}\right) = 5.85\mu\text{C}.$

(b) For $N = 10$, $q_{10} = (6.2\mu\text{C})\exp\left(-10\pi(7.2\Omega)\sqrt{0.0000032\,\text{F}/12\,\text{H}}\right) = 5.52\mu\text{C}.$

(c) For $N = 100$, $q_{100} = (6.2\mu\text{C})\exp\left(-100\pi(7.2\Omega)\sqrt{0.0000032\,\text{F}/12\,\text{H}}\right) = 1.93\mu\text{C}.$

25. Since $\omega \approx \omega'$, we may write $T = 2\pi/\omega$ as the period and $\omega = 1/\sqrt{LC}$ as the angular frequency. The time required for 50 cycles (with 3 significant figures understood) is

$$\begin{aligned} t &= 50T = 50\left(\frac{2\pi}{\omega}\right) = 50\left(2\pi\sqrt{LC}\right) = 50\left(2\pi\sqrt{(220\times10^{-3}\,\text{H})(12.0\times10^{-6}\,\text{F})}\right) \\ &= 0.5104\,\text{s}. \end{aligned}$$

The maximum charge on the capacitor decays according to $q_{\max} = Qe^{-Rt/2L}$ (this is called the *exponentially decaying amplitude* in Section 31-5), where Q is the charge at time $t = 0$ (if we take $\phi = 0$ in Eq. 31-25). Dividing by Q and taking the natural logarithm of both sides, we obtain

$$\ln\left(\frac{q_{\max}}{Q}\right) = -\frac{Rt}{2L}$$

which leads to

$$R = -\frac{2L}{t}\ln\left(\frac{q_{max}}{Q}\right) = -\frac{2\left(220\times10^{-3}\ \text{H}\right)}{0.5104\,\text{s}}\ln\left(0.99\right) = 8.66\times10^{-3}\ \Omega\,.$$

26. The assumption stated at the end of the problem is equivalent to setting $\phi = 0$ in Eq. 31-25. Since the maximum energy in the capacitor (each cycle) is given by $q_{max}^2/2C$, where q_{max} is the maximum charge (during a given cycle), then we seek the time for which

$$\frac{q_{max}^2}{2C} = \frac{1}{2}\frac{Q^2}{2C} \;\Rightarrow\; q_{max} = \frac{Q}{\sqrt{2}}.$$

Now q_{max} (referred to as the *exponentially decaying amplitude* in Section 31-5) is related to Q (and the other parameters of the circuit) by

$$q_{max} = Qe^{-Rt/2L} \Rightarrow \ln\left(\frac{q_{max}}{Q}\right) = -\frac{Rt}{2L}.$$

Setting $q_{max} = Q/\sqrt{2}$, we solve for t:

$$t = -\frac{2L}{R}\ln\left(\frac{q_{max}}{Q}\right) = -\frac{2L}{R}\ln\left(\frac{1}{\sqrt{2}}\right) = \frac{L}{R}\ln 2\,.$$

The identities $\ln(1/\sqrt{2}) = -\ln\sqrt{2} = -\frac{1}{2}\ln 2$ were used to obtain the final form of the result.

27. **THINK** With the presence of a resistor in the *RLC* circuit, oscillation is damped, and the total electromagnetic energy of the system is no longer conserved, as some energy is transferred to thermal energy in the resistor.

EXPRESS Let t be a time at which the capacitor is fully charged in some cycle and let $q_{max\,1}$ be the charge on the capacitor then. The energy in the capacitor at that time is

$$U(t) = \frac{q_{max1}^2}{2C} = \frac{Q^2}{2C}e^{-Rt/L}$$

where

$$q_{max1} = Qe^{-Rt/2L}$$

(see the discussion of the *exponentially decaying amplitude* in Section 31-5). One period later the charge on the fully charged capacitor is

$$q_{max\,2} = Qe^{-R(t+T)2/L}$$

where $T = \dfrac{2\pi}{\omega'}$, and the energy is

$$U(t+T) = \frac{q_{\max 2}^2}{2C} = \frac{Q^2}{2C} e^{-R(t+T)/L}.$$

ANALYZE The fractional loss in energy is

$$\frac{|\Delta U|}{U} = \frac{U(t) - U(t+T)}{U(t)} = \frac{e^{-Rt/L} - e^{-R(t+T)/L}}{e^{-Rt/L}} = 1 - e^{-RT/L}.$$

Assuming that RT/L is very small compared to 1 (which would be the case if the resistance is small), we expand the exponential (see Appendix E). The first few terms are:

$$e^{-RT/L} \approx 1 - \frac{RT}{L} + \frac{R^2T^2}{2L^2} + \cdots .$$

If we approximate $\omega \approx \omega'$, then we can write T as $2\pi/\omega$. As a result, we obtain

$$\frac{|\Delta U|}{U} \approx 1 - \left(1 - \frac{RT}{L} + \cdots\right) \approx \frac{RT}{L} = \frac{2\pi R}{\omega L}.$$

LEARN The ratio $|\Delta U|/U$ can be rewritten as

$$\frac{|\Delta U|}{U} = \frac{2\pi}{Q}$$

where $Q = \omega L/R$ (not to confuse Q with charge) is called the "quality factor" of the oscillating circuit. A high-Q circuit has low resistance and hence, low fractional energy loss.

28. (a) We use $I = \varepsilon/X_c = \omega_d C\varepsilon$:

$$I = \omega_d C\varepsilon_m = 2\pi f_d C\varepsilon_m = 2\pi(1.00\times10^3\,\text{Hz})(1.50\times10^{-6}\,\text{F})(30.0\text{ V}) = 0.283\text{ A}.$$

(b) $I = 2\pi(8.00 \times 10^3\text{ Hz})(1.50 \times 10^{-6}\text{ F})(30.0\text{ V}) = 2.26\text{ A}$.

29. (a) The current amplitude I is given by $I = V_L/X_L$, where $X_L = \omega_d L = 2\pi f_d L$. Since the circuit contains only the inductor and a sinusoidal generator, $V_L = \varepsilon_m$. Therefore,

$$I = \frac{V_L}{X_L} = \frac{\varepsilon_m}{2\pi f_d L} = \frac{30.0\text{V}}{2\pi(1.00\times10^3\,\text{Hz})(50.0\times10^{-3}\text{H})} = 0.0955\text{A} = 95.5\text{ mA}.$$

(b) The frequency is now eight times larger than in part (a), so the inductive reactance X_L is eight times larger and the current is one-eighth as much. The current is now

$$I = (0.0955 \text{ A})/8 = 0.0119 \text{ A} = 11.9 \text{ mA}.$$

30. (a) The current through the resistor is

$$I = \frac{\varepsilon_m}{R} = \frac{30.0\,\text{V}}{50.0\,\Omega} = 0.600 \text{ A} .$$

(b) Regardless of the frequency of the generator, the current is the same, $I = 0.600$ A .

31. (a) The inductive reactance for angular frequency ω_d is given by $X_L = \omega_d L$, and the capacitive reactance is given by $X_C = 1/\omega_d C$. The two reactances are equal if $\omega_d L = 1/\omega_d C$, or $\omega_d = 1/\sqrt{LC}$. The frequency is

$$f_d = \frac{\omega_d}{2\pi} = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(6.0\times10^{-3}\,\text{H})(10\times10^{-6}\text{F})}} = 6.5\times10^{2} \text{ Hz}.$$

(b) The inductive reactance is

$$X_L = \omega_d L = 2\pi f_d L = 2\pi(650 \text{ Hz})(6.0 \times 10^{-3} \text{ H}) = 24\ \Omega.$$

The capacitive reactance has the same value at this frequency.

(c) The natural frequency for free LC oscillations is $f = \omega/2\pi = 1/2\pi\sqrt{LC}$, the same as we found in part (a).

32. (a) The circuit consists of one generator across one inductor; therefore, $\varepsilon_m = V_L$. The current amplitude is

$$I = \frac{\varepsilon_m}{X_L} = \frac{\varepsilon_m}{\omega_d L} = \frac{25.0 \text{ V}}{(377 \text{ rad/s})(12.7 \text{ H})} = 5.22\times10^{-3} \text{ A} .$$

(b) When the current is at a maximum, its derivative is zero. Thus, Eq. 30-35 gives $\varepsilon_L = 0$ at that instant. Stated another way, since $\varepsilon(t)$ and $i(t)$ have a 90° phase difference, then $\varepsilon(t)$ must be zero when $i(t) = I$. The fact that $\phi = 90° = \pi/2$ rad is used in part (c).

(c) Consider Eq. 31-28 with $\varepsilon = -\varepsilon_m/2$. In order to satisfy this equation, we require $\sin(\omega_d t) = -1/2$. Now we note that the problem states that ε is increasing *in magnitude*, which (since it is already negative) means that it is becoming more negative. Thus, differentiating Eq. 31-28 with respect to time (and demanding the result be negative) we must also require $\cos(\omega_d t) < 0$. These conditions imply that ωt must equal $(2n\pi - 5\pi/6)$ [n = integer]. Consequently, Eq. 31-29 yields (for all values of n)

$$i = I\sin\left(2n\pi - \frac{5\pi}{6} - \frac{\pi}{2}\right) = (5.22\times10^{-3}\,\text{A})\left(\frac{\sqrt{3}}{2}\right) = 4.51\times10^{-3}\ \text{A}\ .$$

33. **THINK** Our circuit consists of an ac generator that produces an alternating current, as well as a load that could be purely resistive, capacitive, or inductive. The nature of the load can be determined by the phase angle between the current and the emf.

EXPRESS The generator emf and the current are given by

$$\varepsilon = \varepsilon_m \sin(\omega_d - \pi/4), \quad i(t) = I\sin(\omega_d - 3\pi/4).$$

The expressions show that the emf is maximum when $\sin(\omega_d t - \pi/4) = 1$ or

$$\omega_d t - \pi/4 = (\pi/2) \pm 2n\pi \quad [n = \text{integer}].$$

Similarly, the current is maximum when $\sin(\omega_d t - 3\pi/4) = 1$, or

$$\omega_d t - 3\pi/4 = (\pi/2) \pm 2n\pi \quad [n = \text{integer}].$$

ANALYZE (a) The first time the emf reaches its maximum after $t = 0$ is when $\omega_d t - \pi/4 = \pi/2$ (that is, $n = 0$). Therefore,

$$t = \frac{3\pi}{4\omega_d} = \frac{3\pi}{4(350\ \text{rad/s})} = 6.73\times10^{-3}\ \text{s}\ .$$

(b) The first time the current reaches its maximum after $t = 0$ is when $\omega_d t - 3\pi/4 = \pi/2$, as in part (a) with $n = 0$. Therefore,

$$t = \frac{5\pi}{4\omega_d} = \frac{5\pi}{4(350\ \text{rad/s})} = 1.12\times10^{-2}\ \text{s}.$$

(c) The current lags the emf by $+\pi/2$ rad, so the circuit element must be an inductor.

(d) The current amplitude I is related to the voltage amplitude V_L by $V_L = IX_L$, where X_L is the inductive reactance, given by $X_L = \omega_d L$. Furthermore, since there is only one element in the circuit, the amplitude of the potential difference across the element must be the same as the amplitude of the generator emf: $V_L = \varepsilon_m$. Thus, $\varepsilon_m = I\omega_d L$ and

$$L = \frac{\varepsilon_m}{I\omega_d} = \frac{30.0\ \text{V}}{(620\times10^{-3}\text{A})(350\ \text{rad/s})} = 0.138\ \text{H}.$$

LEARN The current in the circuit can be rewritten as

$$i(t) = I\sin\left(\omega_d - \frac{3\pi}{4}\right) = I\sin\left(\omega_d - \frac{\pi}{4} - \phi\right)$$

where $\phi = +\pi/2$. In a purely inductive circuit, the current lags the voltage by $90°$.

34. (a) The circuit consists of one generator across one capacitor; therefore, $\varepsilon_m = V_C$. Consequently, the current amplitude is

$$I = \frac{\varepsilon_m}{X_C} = \omega C\varepsilon_m = (377\ \text{rad/s})(4.15\times10^{-6}\,\text{F})(25.0\ \text{V}) = 3.91\times10^{-2}\,\text{A}\ .$$

(b) When the current is at a maximum, the charge on the capacitor is changing at its largest rate. This happens not when it is fully charged ($\pm q_{max}$), but rather as it passes through the (momentary) states of being uncharged ($q = 0$). Since $q = CV$, then the voltage across the capacitor (and at the generator, by the loop rule) is zero when the current is at a maximum. Stated more precisely, the time-dependent emf $\varepsilon(t)$ and current $i(t)$ have a $\phi = -90°$ phase relation, implying $\varepsilon(t) = 0$ when $i(t) = I$. The fact that $\phi = -90° = -\pi/2$ rad is used in part (c).

(c) Consider Eq. 32-28 with $\varepsilon = -\frac{1}{2}\varepsilon_m$. In order to satisfy this equation, we require $\sin(\omega_d t) = -1/2$. Now we note that the problem states that ε is increasing *in magnitude*, which (since it is already negative) means that it is becoming more negative. Thus, differentiating Eq. 32-28 with respect to time (and demanding the result be negative) we must also require $\cos(\omega_d t) < 0$. These conditions imply that ωt must equal $(2n\pi - 5\pi/6)$ [n = integer]. Consequently, Eq. 31-29 yields (for all values of n)

$$i = I\sin\left(2n\pi - \frac{5\pi}{6} + \frac{\pi}{2}\right) = (3.91\times10^{-3}\,\text{A})\left(-\frac{\sqrt{3}}{2}\right) = -3.38\times10^{-2}\,\text{A},$$

or $|i| = 3.38\times10^{-2}$ A.

35. The resistance of the coil is related to the reactances and the phase constant by Eq. 31-65. Thus,

$$\frac{X_L - X_C}{R} = \frac{\omega_d L - 1/\omega_d C}{R} = \tan\phi\ ,$$

which we solve for R:

$$R = \frac{1}{\tan\phi}\left(\omega_d L - \frac{1}{\omega_d C}\right) = \frac{1}{\tan 75°}\left[(2\pi)(930\,\text{Hz}(8.8\times10^{-2}\,\text{H}) - \frac{1}{(2\pi)(930\,\text{Hz})(0.94\times10^{-6}\,\text{F}}\right]$$
$$= 89\,\Omega.$$

36. (a) The circuit has a resistor and a capacitor (but no inductor). Since the capacitive reactance decreases with frequency, then the asymptotic value of Z must be the resistance: $R = 500\ \Omega$.

(b) We describe three methods here (each using information from different points on the graph):

method 1: At $\omega_d = 50$ rad/s, we have $Z \approx 700\ \Omega$, which gives $C = (\omega_d\sqrt{Z^2 - R^2}\,)^{-1} = 41\ \mu$F.

method 2: At $\omega_d = 50$ rad/s, we have $X_C \approx 500\ \Omega$, which gives $C = (\omega_d X_C)^{-1} = 40\ \mu$F.

method 3: At $\omega_d = 250$ rad/s, we have $X_C \approx 100\ \Omega$, which gives $C = (\omega_d X_C)^{-1} = 40\ \mu$F.

37. The rms current in the motor is

$$I_{\text{rms}} = \frac{\varepsilon_{\text{rms}}}{Z} = \frac{\varepsilon_{\text{rms}}}{\sqrt{R^2 + X_L^2}} = \frac{420\,\text{V}}{\sqrt{(45.0\,\Omega)^2 + (32.0\,\Omega)^2}} = 7.61\,\text{A}.$$

38. (a) The graph shows that the resonance angular frequency is 25000 rad/s, which means (using Eq. 31-4)

$$C = (\omega^2 L)^{-1} = [(25000)^2 \times 200 \times 10^{-6}]^{-1} = 8.0\ \mu\text{F}.$$

(b) The graph also shows that the current amplitude at resonance is 4.0 A, but at resonance the impedance Z becomes purely resistive ($Z = R$) so that we can divide the emf amplitude by the current amplitude at resonance to find R: 8.0/4.0 = 2.0 Ω.

39. (a) Now $X_L = 0$, while $R = 200\ \Omega$ and $X_C = 1/2\pi f_d C = 177\ \Omega$. Therefore, the impedance is

$$Z = \sqrt{R^2 + X_C^2} = \sqrt{(200\,\Omega)^2 + (177\,\Omega)^2} = 267\,\Omega.$$

(b) The phase angle is

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{0 - 177\,\Omega}{200\,\Omega}\right) = -41.5°$$

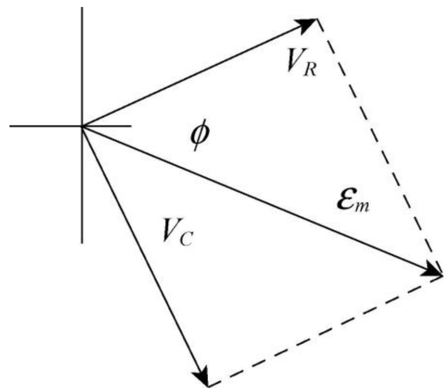
(c) The current amplitude is

$$I = \frac{\varepsilon_m}{Z} = \frac{36.0\ \text{V}}{267\,\Omega} = 0.135\ \text{A}\ .$$

(d) We first find the voltage amplitudes across the circuit elements:

$$V_R = IR = (0.135\,\text{A})(200\Omega) \approx 27.0\,\text{V}$$
$$V_C = IX_C = (0.135\,\text{A})(177\Omega) \approx 23.9\,\text{V}$$

The circuit is capacitive, so I leads ε_m. The phasor diagram is drawn to scale next.



40. A phasor diagram very much like Fig. 31-14(d) leads to the condition:

$$V_L - V_C = (6.00\text{ V})\sin(30^\circ) = 3.00\text{ V}.$$

With the magnitude of the capacitor voltage at 5.00 V, this gives a inductor voltage magnitude equal to 8.00 V. Since the capacitor and inductor voltage phasors are 180° out of phase, the potential difference across the inductor is -8.00 V.

41. **THINK** We have a series *RLC* circuit. Since *R*, *L*, and *C* are in series, the same current is driven in all three of them.

EXPRESS The capacitive and the inductive reactances can be written as

$$X_C = \frac{1}{\omega_d C} = \frac{1}{2\pi f_d C}, \quad X_L = \omega_d L = 2\pi f_d L.$$

The impedance of the circuit is $Z = \sqrt{R^2 + (X_L - X_C)^2}$, and the current amplitude is given by $I = \varepsilon_m / Z$.

ANALYZE (a) Substituting the values given, we find the capacitive reactance to be

$$X_C = \frac{1}{2\pi f_d C} = \frac{1}{2\pi(60.0\,\text{Hz})(70.0\times10^{-6}\,\text{F})} = 37.9\ \Omega.$$

Similarly, the inductive reactance is

$$X_L = 2\pi f_d L = 2\pi(60.0\,\text{Hz})(230\times10^{-3}\,\text{H}) = 86.7\ \Omega.$$

Thus, the impedance is

$$Z=\sqrt{R^2+(X_L-X_C)^2}=\sqrt{(200\ \Omega)^2+(37.9\ \Omega-86.7\ \Omega)^2}=206\ \Omega.$$

(b) The phase angle is

$$\phi=\tan^{-1}\left(\frac{X_L-X_C}{R}\right)=\tan^{-1}\left(\frac{86.7\ \Omega-37.9\ \Omega}{200\ \Omega}\right)=13.7°.$$

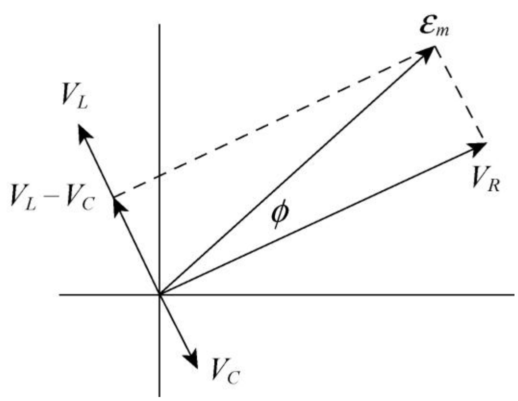
(c) The current amplitude is

$$I=\frac{\varepsilon_m}{Z}=\frac{36.0\ \text{V}}{206\Omega}=0.175\,\text{A}.$$

(d) We first find the voltage amplitudes across the circuit elements:

$$V_R=IR=(0.175\ \text{A})(200\ \ \Omega)=35.0\ \text{V}$$
$$V_L=IX_L=(0.175\ \text{A})(86.7\ \ \Omega)=15.2\ \text{V}$$
$$V_C=IX_C=(0.175\ \text{A})(37.9\ \ \Omega)=6.62\,\text{V}$$

Note that $X_L>X_C$, so that ε_m leads I. The phasor diagram is drawn to scale below.



LEARN The circuit in this problem is more inductive since $X_L>X_C$. The phase angle is positive, so the current lags behind the applied emf.

42. (a) Since $Z=\sqrt{R^2+X_L^2}$ and $X_L=\omega_d L$, then as $\omega_d\to 0$ we find $Z\to R = 40\ \Omega$.

(b) $L = X_L/\omega_d = slope = 60$ mH.

43. (a) Now $X_C = 0$, while $R = 200\ \Omega$ and

$$X_L = \omega L = 2\pi f_d L = 86.7\ \Omega$$

both remain unchanged. Therefore, the impedance is

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{(200\ \Omega)^2 + (86.7\ \Omega)^2} = 218\ \Omega\ .$$

(b) The phase angle is, from Eq. 31-65,

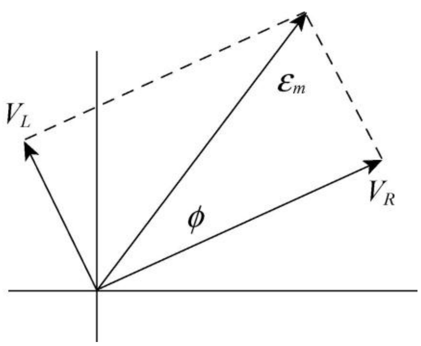
$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{86.7\,\Omega - 0}{200\ \Omega}\right) = 23.4°\ .$$

(c) The current amplitude is now found to be $I = \frac{\varepsilon_m}{Z} = \frac{36.0\ \text{V}}{218\ \Omega} = 0.165\ \text{A}$.

(d) We first find the voltage amplitudes across the circuit elements:

$$V_R = IR = (0.165\ \text{A})(200\Omega) \approx 33\text{V}$$
$$V_L = IX_L = (0.165\text{A})(86.7\Omega) \approx 14.3\text{V}.$$

This is an inductive circuit, so ε_m leads I. The phasor diagram is drawn to scale next.



44. (a) The capacitive reactance is

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(400\ \text{Hz})(24.0\times10^{-6}\text{F})} = 16.6\ \Omega\ .$$

(b) The impedance is

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + (2\pi fL - X_C)^2}$$
$$= \sqrt{(220\,\Omega)^2 + [2\pi(400\ \text{Hz})(150\times10^{-3}\ \text{H}) - 16.6\ \Omega]^2} = 422\ \Omega\ .$$

(c) The current amplitude is

$$I = \frac{\varepsilon_m}{Z} = \frac{220\,\text{V}}{422\,\Omega} = 0.521\ \text{A}\ .$$

(d) Now $X_C \propto C_{\text{eq}}^{-1}$. Thus, X_C increases as C_{eq} decreases.

(e) Now $C_{\text{eq}} = C/2$, and the new impedance is

$$Z = \sqrt{(220\ \Omega)^2 + [2\pi(400\ \text{Hz})(150\times10^{-3}\ \text{H}) - 2(16.6\ \Omega)]^2} = 408\ \Omega < 422\ \Omega\,.$$

Therefore, the impedance decreases.

(f) Since $I \propto Z^{-1}$, it increases.

45. (a) Yes, the voltage amplitude across the inductor can be much larger than the amplitude of the generator emf.

(b) The amplitude of the voltage across the inductor in an *RLC* series circuit is given by $V_L = IX_L = I\omega_d L$. At resonance, the driving angular frequency equals the natural angular frequency: $\omega_d = \omega = 1/\sqrt{LC}$. For the given circuit

$$X_L = \frac{L}{\sqrt{LC}} = \frac{1.0\ \text{H}}{\sqrt{(1.0\ \text{H})(1.0\times10^{-6}\text{F})}} = 1000\ \Omega\,.$$

At resonance the capacitive reactance has this same value, and the impedance reduces simply: $Z = R$. Consequently,

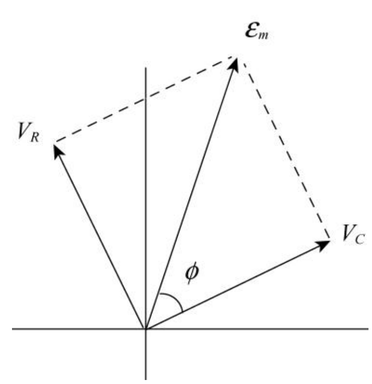
$$I = \left.\frac{\varepsilon_m}{Z}\right|_{\text{resonance}} = \frac{\varepsilon_m}{R} = \frac{10\ \text{V}}{10\ \Omega} = 1.0\ \text{A}\,.$$

The voltage amplitude across the inductor is therefore

$$V_L = IX_L = (1.0\,\text{A})(1000\ \Omega) = 1.0\times10^3\ \text{V}$$

which is much larger than the amplitude of the generator emf.

46. (a) A sketch of the phasor diagram is shown to the right.



(b) We have $I R = I X_C$, or

$$I R = I X_C \;\rightarrow\; R = \frac{1}{\omega_d C}$$

which yields

$$f = \frac{\omega_d}{2\pi} = \frac{1}{2\pi RC} = \frac{1}{2\pi(50.0\,\Omega)(2.00\times10^{-5}\ \text{F})} = 159\ \text{Hz}\,.$$

(c) $\phi = \tan^{-1}(-V_C/V_R) = -45°$.

(d) ω_d = 1/*RC* =1.00 $\times 10^3$ rad/s.

(e) $I = (12\text{ V})/\sqrt{R^2 + X_C^2} = 6/(25\sqrt{2}) \approx 170$ mA.

47. **THINK** In a driven *RLC* circuit, the current amplitude is maximum at resonance, where the driven angular frequency is equal to the natural angular frequency.

EXPRESS For a given amplitude ε_m of the generator emf, the current amplitude is given by

$$I = \frac{\varepsilon_m}{Z} = \frac{\varepsilon_m}{\sqrt{R^2 + (\omega_d L - 1/\omega_d C)^2}}.$$

To explicitly show that *I* is maximum when $\omega_d = \omega = 1/\sqrt{LC}$, we differentiate *I* with respect to ω_d and set the derivative to zero:

$$\frac{dI}{d\omega_d} = -(E)_m[R^2 + (\omega_d L - 1/\omega_d C)^2]^{-3/2}\left(\omega_d L - \frac{1}{\omega_d C}\right)\left(L + \frac{1}{\omega_d^2 C}\right).$$

The only factor that can equal zero is when $\omega_d L - (1/\omega_d C)$, or $\omega_d = 1/\sqrt{LC} = \omega$.

ANALYZE (a) For this circuit, the driving angular frequency is

$$\omega_d = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(1.00\text{ H})(20.0\times 10^{-6}\text{ F})}} = 224\text{ rad/s}.$$

(b) When $\omega_d = \omega$, the impedance is *Z* = *R*, and the current amplitude is

$$I = \frac{\varepsilon_m}{R} = \frac{30.0\text{ V}}{5.00\ \Omega} = 6.00\text{ A}.$$

(c) We want to find the (positive) values of ω_d for which $I = \varepsilon_m / 2R$:

$$\frac{\varepsilon_m}{\sqrt{R^2 + (\omega_d L - 1/\omega_d C)^2}} = \frac{\varepsilon_m}{2R}.$$

This may be rearranged to give

$$\left(\omega_d L - \frac{1}{\omega_d C}\right)^2 = 3R^2.$$

Taking the square root of both sides (acknowledging the two ± roots) and multiplying by $\omega_d C$, we obtain

$$\omega_d^2(LC) \pm \omega_d\left(\sqrt{3}CR\right) - 1 = 0.$$

Using the quadratic formula, we find the smallest positive solution

$$\begin{aligned}\omega_2 &= \frac{-\sqrt{3}CR + \sqrt{3C^2R^2 + 4LC}}{2LC} = \frac{-\sqrt{3}(20.0\times10^{-6}\ \text{F})(5.00\ \Omega)}{2(1.00\ \text{H})(20.0\times10^{-6}\ \text{F})} \\ &\quad + \frac{\sqrt{3(20.0\times10^{-6}\ \text{F})^2(5.00\ \Omega)^2 + 4(1.00\ \text{H})(20.0\times10^{-6}\ \text{F})}}{2(1.00\ \text{H})(20.0\times10^{-6}\ \text{F})} \\ &= 219\ \text{rad/s}.\end{aligned}$$

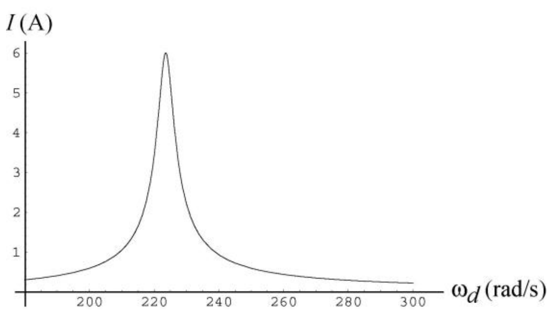
(d) The largest positive solution

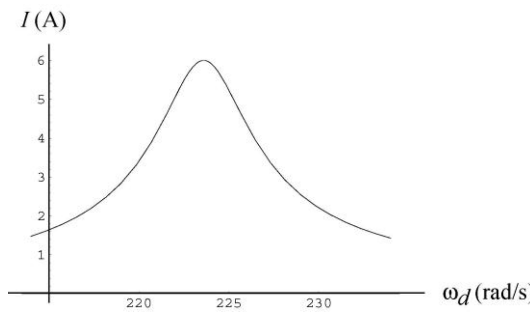
$$\begin{aligned}\omega_1 &= \frac{+\sqrt{3}CR + \sqrt{3C^2R^2 + 4LC}}{2LC} = \frac{+\sqrt{3}(20.0\times10^{-6}\ \text{F})(5.00\,\Omega)}{2(1.00\ \text{H})(20.0\times10^{-6}\ \text{F})} \\ &\quad + \frac{\sqrt{3(20.0\times10^{-6}\ \text{F})^2(5.00\ \Omega)^2 + 4(1.00\ \text{H})(20.0\times10^{-6}\text{F})}}{2(1.00\ \text{H})(20.0\times10^{-6}\ \text{F})} \\ &= 228\ \text{rad/s}.\end{aligned}$$

(e) The fractional width is

$$\frac{\omega_1 - \omega_2}{\omega} = \frac{228\ \text{rad/s} - 219\ \text{rad/s}}{224\ \text{rad/s}} = 0.040.$$

LEARN The current amplitude as a function of ω_d is plotted below.





We see that I is a maximum at $\omega_d = \omega = 224$ rad/s, and is at half maximum (3 A) at 219 rad/s and 228 rad/s.

48. (a) With both switches closed (which effectively removes the resistor from the circuit), the impedance is just equal to the (net) reactance and is equal to

$$X_{\text{net}} = (12\text{ V})/(0.447\text{ A}) = 26.85\ \Omega.$$

With switch 1 closed but switch 2 open, we have the same (net) reactance as just discussed, but now the resistor is part of the circuit; using Eq. 31-65 we find

$$R = \frac{X_{\text{net}}}{\tan\phi} = \frac{26.85\,\Omega}{\tan 15°} = 100\,\Omega\,.$$

(b) For the first situation described in the problem (both switches open) we can reverse our reasoning of part (a) and find

$$X_{\text{net first}} = R\tan\phi' = (100\ \Omega)\tan(-30.9°) = -59.96\ \Omega.$$

We observe that the effect of switch 1 implies

$$X_C = X_{\text{net}} - X_{\text{net first}} = 26.85\ \Omega - (-59.96\ \Omega) = 86.81\ \Omega.$$

Then Eq. 31-39 leads to $C = 1/\omega X_C = 30.6\ \mu\text{F}$.

(c) Since $X_{\text{net}} = X_L - X_C$, then we find $L = X_L/\omega = 301$ mH .

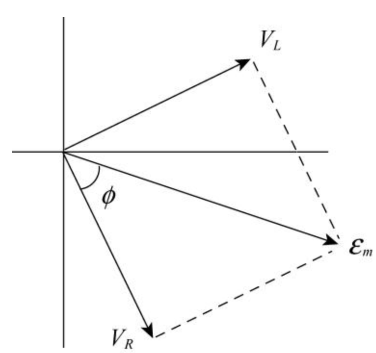
49. (a) Since $L_{\text{eq}} = L_1 + L_2$ and $C_{\text{eq}} = C_1 + C_2 + C_3$ for the circuit, the resonant frequency is

$$\begin{aligned}\omega &= \frac{1}{2\pi\sqrt{L_{\text{eq}}C_{\text{eq}}}} = \frac{1}{2\pi\sqrt{(L_1+L_2)(C_1+C_2+C_3)}} \\ &= \frac{1}{2\pi\sqrt{\left(1.70\times10^{-3}\text{ H}+2.30\times10^{-3}\text{ H}\right)\left(4.00\times10^{-6}\text{ F}+2.50\times10^{-6}\text{ F}+3.50\times10^{-6}\text{ F}\right)}} \\ &= 796\,\text{Hz}.\end{aligned}$$

(b) The resonant frequency does not depend on R so it will not change as R increases.

(c) Since $\omega \propto (L_1 + L_2)^{-1/2}$, it will decrease as L_1 increases.

(d) Since $\omega \propto C_{\text{eq}}^{-1/2}$ and C_{eq} decreases as C_3 is removed, ω will increase.



50. (a) A sketch of the phasor diagram is shown to the right.

(b) We have $V_R = V_L$, which implies

$$I\,R = I\,X_L \ \rightarrow\ R = \omega_d L$$

which yields $f = \omega_d/2\pi = R/2\pi L = 318$ Hz.

(c) $\phi = \tan^{-1}(V_L/V_R) = +45°$.

(d) $\omega_d = R/L = 2.00\times10^3$ rad/s.

(e) $I = (6\text{ V})/\sqrt{R^2 + X_L^2} = 3/(40\sqrt{2}) \approx 53.0$ mA.

51. **THINK** In a driven *RLC* circuit, the current amplitude is maximum at resonance, where the driven angular frequency is equal to the natural angular frequency. It then falls off rapidly away from resonance.

EXPRESS We use the expressions found in Problem 31-47:

$$\omega_1 = \frac{+\sqrt{3}CR + \sqrt{3C^2R^2 + 4LC}}{2LC}, \quad \omega_2 = \frac{-\sqrt{3}CR + \sqrt{3C^2R^2 + 4LC}}{2LC} \ .$$

The resonance angular frequency is $\omega = 1/\sqrt{LC}$.

ANALYZE Thus, the fractional half width is

$$\frac{\Delta\omega_d}{\omega} = \frac{\omega_1 - \omega_2}{\omega} = \frac{2\sqrt{3}CR\sqrt{LC}}{2LC} = R\sqrt{\frac{3C}{L}}.$$

LEARN Note that the value of $\Delta\omega_d/\omega$ increases linearly with *R*; that is, the larger the resistance, the broader the peak. As an example, the data of Problem 31-47 gives

$$\frac{\Delta\omega_d}{\omega} = (5.00\ \Omega)\sqrt{\frac{3(20.0\times10^{-6}\text{ F})}{1.00\,\text{H}}} = 3.87\times10^{-2}.$$

This is in agreement with the result of Problem 31-47. The method used there, however, gives only one significant figure since two numbers close in value are subtracted ($\omega_1 - \omega_2$). Here the subtraction is done algebraically, and three significant figures are obtained.

52. Since the impedance of the voltmeter is large, it will not affect the impedance of the circuit when connected in parallel with the circuit. So the reading will be 100 V in all three cases.

53. **THINK** Energy is supplied by the 120 V rms ac line to keep the air conditioner running.

EXPRESS The impedance of the circuit is $Z = \sqrt{R^2 + (X_L - X_C)^2}$, and the average rate of energy delivery is

$$P_{\text{avg}} = I_{\text{rms}}^2 R = \left(\frac{\varepsilon_{\text{rms}}}{Z}\right)^2 R = \frac{\varepsilon_{\text{rms}}^2 R}{Z^2}.$$

ANALYZE (a) Substituting the values given, the impedance is

$$Z = \sqrt{(12.0\ \Omega)^2 + (1.30\ \Omega - 0)^2} = 12.1\ \Omega.$$

(b) The average rate at which energy has been supplied is

$$P_{\text{avg}} = \frac{\varepsilon_{\text{rms}}^2 R}{Z^2} = \frac{(120\ \text{V})^2 (12.0\ \Omega)}{(12.07\ \Omega)^2} = 1.186\times10^3\ \text{W} \approx 1.19\times10^3\ \text{W}.$$

LEARN In a steady-state operation, the total energy stored in the capacitor and the inductor stays constant. Thus, the net energy transfer is from the generator to the resistor, where electromagnetic energy is dissipated in the form of thermal energy.

54. The amplitude (peak) value is

$$V_{\text{max}} = \sqrt{2}V_{\text{rms}} = \sqrt{2}(100\ \text{V}) = 141\ \text{V}.$$

55. The average power dissipated in resistance R when the current is alternating is given by $P_{\text{avg}} = I_{\text{rms}}^2 R,$ where I_{rms} is the root-mean-square current. Since $I_{\text{rms}} = I/\sqrt{2}$, where I is the current amplitude, this can be written $P_{\text{avg}} = I^2R/2$. The power dissipated in the same resistor when the current i_d is direct is given by $P = i_d^2 R.$ Setting the two powers equal to each other and solving, we obtain

$$i_d = \frac{I}{\sqrt{2}} = \frac{2.60\ \text{A}}{\sqrt{2}} = 1.84\ \text{A}.$$

56. (a) The power consumed by the light bulb is $P = I^2R/2$. So we must let $P_{\text{max}}/P_{\text{min}} = (I/I_{\text{min}})^2 = 5$, or

$$\left(\frac{I}{I_{\text{min}}}\right)^2 = \left(\frac{\varepsilon_m / Z_{\text{min}}}{\varepsilon_m / Z_{\text{max}}}\right)^2 = \left(\frac{Z_{\text{max}}}{Z_{\text{min}}}\right)^2 = \left(\frac{\sqrt{R^2 + (\omega L_{\text{max}})^2}}{R}\right)^2 = 5.$$

We solve for L_{max}:

$$L_{\text{max}} = \frac{2R}{\omega} = \frac{2(120\ \text{V})^2 / 1000\ \text{W}}{2\pi(60.0\ \text{Hz})} = 7.64\times10^{-2}\ \text{H}.$$

(b) Yes, one could use a variable resistor.

(c) Now we must let

$$\left(\frac{R_{\text{max}}+R_{\text{bulb}}}{R_{\text{bulb}}}\right)^2=5,$$

or

$$R_{\text{max}}=\left(\sqrt{5}-1\right)R_{\text{bulb}}=\left(\sqrt{5}-1\right)\frac{(120\,\text{V})^2}{1000\,\text{W}}=17.8\ \Omega.$$

(d) This is not done because the resistors would consume, rather than temporarily store, electromagnetic energy.

57. We shall use

$$P_{\text{avg}}=\frac{\varepsilon_m^2 R}{2Z^2}=\frac{\varepsilon_m^2 R}{2\left[R^2+\left(\omega_d L-1/\omega_d C\right)^2\right]}.$$

where $Z=\sqrt{R^2+\left(\omega_d L-1/\omega_d C\right)^2}$ is the impedance.

(a) Considered as a function of C, P_{avg} has its largest value when the factor $R^2+\left(\omega_d L-1/\omega_d C\right)^2$ has the smallest possible value. This occurs for $\omega_d L=1/\omega_d C$, or

$$C=\frac{1}{\omega_d^2 L}=\frac{1}{(2\pi)^2(60.0\,\text{Hz})^2\left(60.0\times10^{-3}\ \text{H}\right)}=1.17\times10^{-4}\ \text{F}.$$

The circuit is then at resonance.

(b) In this case, we want Z^2 to be as large as possible. The impedance becomes large without bound as C becomes very small. Thus, the smallest average power occurs for C = 0 (which is not very different from a simple open switch).

(c) When $\omega_d L = 1/\omega_d C$, the expression for the average power becomes

$$P_{\text{avg}}=\frac{\varepsilon_m^2}{2R},$$

so the maximum average power is in the resonant case and is equal to

$$P_{\text{avg}}=\frac{(30.0\,\text{V})^2}{2(5.00\,\Omega)}=90.0\,\text{W}.$$

(d) At maximum power, the reactances are equal: $X_L = X_C$. The phase angle ϕ in this case may be found from

$$\tan\phi = \frac{X_L - X_C}{R} = 0,$$

which implies $\phi = 0°$.

(e) At maximum power, the power factor is cos ϕ = cos 0° = 1.

(f) The minimum average power is P_{avg} = 0 (as it would be for an open switch).

(g) On the other hand, at minimum power $X_C \propto 1/C$ is infinite, which leads us to set $\tan\phi = -\infty$. In this case, we conclude that ϕ = –90°.

(h) At minimum power, the power factor is cos ϕ = cos(–90°) = 0.

58. This circuit contains no reactances, so $\varepsilon_{rms} = I_{rms}R_{total}$. Using Eq. 31-71, we find the average dissipated power in resistor R is

$$P_R = I_{rms}^2 R = \left(\frac{\varepsilon_m}{r+R}\right)^2 R.$$

In order to maximize P_R we set the derivative equal to zero:

$$\frac{dP_R}{dR} = \frac{\varepsilon_m^2\left[(r+R)^2 - 2(r+R)R\right]}{(r+R)^4} = \frac{\varepsilon_m^2(r-R)}{(r+R)^3} = 0 \quad\Rightarrow\quad R = r$$

59. (a) The rms current is

$$\begin{aligned} I_{rms} &= \frac{\varepsilon_{rms}}{Z} = \frac{\varepsilon_{rms}}{\sqrt{R^2 + (2\pi fL - 1/2\pi fC)^2}} \\ &= \frac{75.0\text{V}}{\sqrt{(15.0\Omega)^2 + \left\{2\pi(550\text{Hz})(25.0\text{mH}) - 1/\left[2\pi(550\text{Hz})(4.70\mu\text{F})\right]\right\}^2}} \\ &= 2.59\text{A}. \end{aligned}$$

(b) The rms voltage across R is $V_{ab} = I_{rms}R = (2.59\,\text{A})(15.0\,\Omega) = 38.8\,\text{V}$.

(c) The rms voltage across C is

$$V_{bc} = I_{rms}X_C = \frac{I_{rms}}{2\pi fC} = \frac{2.59\text{A}}{2\pi(550\,\text{Hz})(4.70\mu\text{F})} = 159\,\text{V}.$$

(d) The rms voltage across L is

$$V_{cd} = I_{\rm rms} X_L = 2\pi I_{\rm rms} fL = 2\pi(2.59\,{\rm A})(550\ {\rm Hz})(25.0\,{\rm mH}) = 224\ {\rm V}\,.$$

(e) The rms voltage across *C* and *L* together is

$$V_{bd} = |V_{bc} - V_{cd}| = |159.5\,{\rm V} - 223.7\,{\rm V}| = 64.2\ {\rm V}\,.$$

(f) The rms voltage across *R*, *C*, and *L* together is

$$V_{ad} = \sqrt{V_{ab}^2 + V_{bd}^2} = \sqrt{(38.8\,{\rm V})^2 + (64.2\,{\rm V})^2} = 75.0\,{\rm V}\,.$$

(g) For the resistor *R*, the power dissipated is $P_R = \dfrac{V_{ab}^2}{R} = \dfrac{(38.8\,{\rm V})^2}{15.0\,\Omega} = 100\,{\rm W}.$

(h) No energy dissipation in *C*.

(i) No energy dissipation in *L*.

60. The current in the circuit satisfies $i(t) = I \sin(\omega_d t - \phi)$, where

$$\begin{aligned} I &= \frac{\varepsilon_m}{Z} = \frac{\varepsilon_m}{\sqrt{R^2 + (\omega_d L - 1/\omega_d C)^2}} \\ &= \frac{45.0\ {\rm V}}{\sqrt{(16.0\,\Omega)^2 + \left\{(3000\,{\rm rad/s})(9.20\,{\rm mH}) - 1/\left[(3000\,{\rm rad/s})(31.2\,\mu{\rm F})\right]\right\}^2}} \\ &= 1.93\,{\rm A} \end{aligned}$$

and

$$\begin{aligned} \phi &= \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{\omega_d L - 1/\omega_d C}{R}\right) \\ &= \tan^{-1}\left[\frac{(3000\,{\rm rad/s})(9.20\,{\rm mH})}{16.0\,\Omega} - \frac{1}{(3000\,{\rm rad/s})(16.0\,\Omega)(31.2\,\mu{\rm F})}\right] \\ &= 46.5°. \end{aligned}$$

(a) The power supplied by the generator is

$$\begin{aligned} P_g &= i(t)\varepsilon(t) = I\sin(\omega_d t - \phi)\,\varepsilon_m \sin\omega_d t \\ &= (1.93\,{\rm A})(45.0\,{\rm V})\sin\left[(3000\,{\rm rad/s})(0.442\ {\rm ms})\right]\sin\left[(3000\,{\rm rad/s})(0.442\ {\rm ms}) - 46.5°\right] \\ &= 41.4\ {\rm W}. \end{aligned}$$

(b) With

$$v_c(t) = V_c \sin(\omega_d t - \phi - \pi/2) = -V_c \cos(\omega_d t - \phi)$$

where $V_c = I/\omega_d C,$ the rate at which the energy in the capacitor changes is

$$\begin{aligned}
P_c &= \frac{d}{dt}\left(\frac{q^2}{2C}\right) = i\frac{q}{C} = iv_c \\
&= -I\sin(\omega_d t - \phi)\left(\frac{I}{\omega_d C}\right)\cos(\omega_d t - \phi) = -\frac{I^2}{2\omega_d C}\sin\left[2(\omega_d t - \phi)\right] \\
&= -\frac{(1.93\,\text{A})^2}{2(3000\,\text{rad/s})(31.2\times10^{-6}\,\text{F})}\sin\left[2(3000\,\text{rad/s})(0.442\,\text{ms}) - 2(46.5°)\right] \\
&= -17.0\text{ W}.
\end{aligned}$$

(c) The rate at which the energy in the inductor changes is

$$\begin{aligned}
P_L &= \frac{d}{dt}\left(\frac{1}{2}Li^2\right) = Li\frac{di}{dt} = LI\sin(\omega_d t - \phi)\frac{d}{dt}\left[I\sin(\omega_d t - \phi)\right] = \frac{1}{2}\omega_d LI^2\sin\left[2(\omega_d t - \phi)\right] \\
&= \frac{1}{2}(3000\,\text{rad/s})(1.93\,\text{A})^2(9.20\,\text{mH})\sin\left[2(3000\,\text{rad/s})(0.442\,\text{ms}) - 2(46.5°)\right] \\
&= 44.1\text{ W}.
\end{aligned}$$

(d) The rate at which energy is being dissipated by the resistor is

$$\begin{aligned}
P_R &= i^2R = I^2R\sin^2(\omega_d t - \phi) = (1.93\,\text{A})^2(16.0\,\Omega)\sin^2\left[(3000\,\text{rad/s})(0.442\,\text{ms}) - 46.5°\right] \\
&= 14.4\text{ W}.
\end{aligned}$$

(e) Equal. $P_L + P_R + P_c = 44.1\text{W} - 17.0\text{ W} + 14.4\text{ W} = 41.5\text{ W} = P_g.$

61. **THINK** We have an ac generator connected to a "black box," whose load is of the form of an *RLC* circuit. Given the functional forms of the emf and the current in the circuit, we can deduce the nature of the load.

EXPRESS In general, the driving emf and the current can be written as

$$\varepsilon(t) = \varepsilon_m \sin\omega_d t, \quad i(t) = I\sin(\omega_d t - \phi).$$

Thus, we have $\varepsilon_m = 75$ V, I = 1.20 A, and $\phi = -42°$ for this circuit. The power factor of the circuit is simply given by $\cos\phi$.

ANALYZE (a) With $\phi = -42.0°$, we obtain cos ϕ = cos(– 42.0°) = 0.743.

(b) Since the phase constant is negative, $\phi < 0$, $\omega t - \phi > \omega t$. The current leads the emf.

(c) The phase constant is related to the reactance difference by $\tan\phi = (X_L - X_C)/R$. We have

$$\tan\phi = \tan(-42.0°) = -0.900,$$

a negative number. Therefore, $X_L - X_C$ is negative, which implies that $X_C > X_L$. The circuit in the box is predominantly capacitive.

(d) If the circuit were in resonance, X_L would be the same as X_C, then $\tan\phi$ would be zero, and ϕ would be zero as well. Since ϕ is not zero, we conclude the circuit is not in resonance.

(e) Since $\tan\phi$ is negative and finite, neither the capacitive reactance nor the resistance is zero. This means the box must contain a capacitor and a resistor.

(f) The inductive reactance may be zero, so there need not be an inductor.

(g) Yes, there is a resistor.

(h) The average power is

$$P_{\text{avg}} = \frac{1}{2}\varepsilon_m I\cos\phi = \frac{1}{2}(75.0\,\text{V})(1.20\,\text{A})(0.743) = 33.4\,\text{W}.$$

(i) The answers above depend on the frequency only through the phase constant ϕ, which is given. If values were given for R, L, and C, then the value of the frequency would also be needed to compute the power factor.

LEARN The phase constant ϕ allows us to calculate the power factor and deduce the nature of the load in the circuit. In (f) we stated that the inductance may be set to zero. If there is an inductor, then its reactance must be smaller than the capacitive reactance, $X_L < X_C$.

62. We use Eq. 31-79 to find

$$V_s = V_p\left(\frac{N_s}{N_p}\right) = (100\,\text{V})\left(\frac{500}{50}\right) = 1.00\times 10^3\,\text{V}.$$

63. **THINK** The transformer in this problem is a step-down transformer.

EXPRESS If N_p is the number of primary turns, and N_s is the number of secondary turns, then the step-down voltage in the secondary circuit is

$$V_s = V_p\left(\frac{N_s}{N_p}\right).$$

By Ohm's law, the current in the secondary circuit is given by $I_s = V_s / R_s$.

ANALYZE (a) The step-down voltage is

$$V_s = V_p\left(\frac{N_s}{N_p}\right) = (120\,\text{V})\left(\frac{10}{500}\right) = 2.4\,\text{V}.$$

(b) The current in the secondary is $I_s = \dfrac{V_s}{R_s} = \dfrac{2.4\,\text{V}}{15\,\Omega} = 0.16\,\text{A}.$

We find the primary current from Eq. 31-80:

$$I_p = I_s\left(\frac{N_s}{N_p}\right) = (0.16\,\text{A})\left(\frac{10}{500}\right) = 3.2\times10^{-3}\,\text{A}.$$

(c) As shown above, the current in the secondary is $I_s = 0.16\,\text{A}.$

LEARN In a transformer, the voltages and currents in the secondary circuit are related to that in the primary circuit by

$$V_s = V_p\left(\frac{N_s}{N_p}\right), \qquad I_s = I_p\left(\frac{N_p}{N_s}\right).$$

64. For step-up transformer:

(a) The smallest value of the ratio V_s / V_p is achieved by using T_2T_3 as primary and T_1T_3 as secondary coil: V_{13}/V_{23} = (800 + 200)/800 = 1.25.

(b) The second smallest value of the ratio V_s / V_p is achieved by using T_1T_2 as primary and T_2T_3 as secondary coil: V_{23}/V_{13} = 800/200 = 4.00.

(c) The largest value of the ratio V_s / V_p is achieved by using T_1T_2 as primary and T_1T_3 as secondary coil: V_{13}/V_{12} = (800 + 200)/200 = 5.00.

For the step-down transformer, we simply exchange the primary and secondary coils in each of the three cases above.

(d) The smallest value of the ratio V_s / V_p is 1/5.00 = 0.200.

(e) The second smallest value of the ratio V_s / V_p is 1/4.00 = 0.250.

(f) The largest value of the ratio V_s / V_p is 1/1.25 = 0.800.

65. (a) The rms current in the cable is $I_{\text{rms}} = P / V_t = 250 \times 10^3 \text{W} / \left(80 \times 10^3 \text{ V}\right) = 3.125 \text{A}$. Therefore, the rms voltage drop is $\Delta V = I_{\text{rms}} R = (3.125 \text{A})(2)(0.30 \Omega) = 1.9 \text{ V}$.

(b) The rate of energy dissipation is $P_d = I_{\text{rms}}^2 R = (3.125 \text{A})(2)(0.60 \Omega) = 5.9 \text{W}$.

(c) Now $I_{\text{rms}} = 250 \times 10^3 \text{ W} / \left(8.0 \times 10^3 \text{ V}\right) = 31.25 \text{A}$, so $\Delta V = (31.25 \text{A})(0.60 \Omega) = 19 \text{V}$.

(d) $P_d = (3.125 \text{A})^2 (0.60 \Omega) = 5.9 \times 10^2 \text{ W}$.

(e) $I_{\text{rms}} = 250 \times 10^3 \text{ W} / \left(0.80 \times 10^3 \text{ V}\right) = 312.5 \text{ A}$, so $\Delta V = (312.5 \text{A})(0.60 \Omega) = 1.9 \times 10^2 \text{ V}$.

(f) $P_d = (312.5 \text{A})^2 (0.60 \ \Omega) = 5.9 \times 10^4 \text{W}$.

66. (a) The amplifier is connected across the primary windings of a transformer and the resistor R is connected across the secondary windings.

(b) If I_s is the rms current in the secondary coil then the average power delivered to R is $P_{\text{avg}} = I_s^2 R$. Using $I_s = \left(N_p / N_s\right) I_p$, we obtain

$$P_{\text{avg}} = \left(\frac{I_p N_p}{N_s}\right)^2 R.$$

Next, we find the current in the primary circuit. This is effectively a circuit consisting of a generator and two resistors in series. One resistance is that of the amplifier (r), and the other is the equivalent resistance R_{eq} of the secondary circuit. Therefore,

$$I_p = \frac{\varepsilon_{\text{rms}}}{r + R_{\text{eq}}} = \frac{\varepsilon_{\text{rms}}}{r + \left(N_p / N_s\right)^2 R}$$

where Eq. 31-82 is used for R_{eq}. Consequently,

$$P_{\text{avg}} = \frac{\varepsilon^2 (N_p / N_s)^2 R}{\left[r + (N_p / N_s)^2 R\right]^2}.$$

Now, we wish to find the value of N_p/N_s such that P_{avg} is a maximum. For brevity, let $x = (N_p/N_s)^2$. Then

$$P_{\text{avg}} = \frac{\varepsilon^2 Rx}{(r + xR)^2},$$

and the derivative with respect to x is

$$\frac{dP_{\text{avg}}}{dx} = \frac{\varepsilon^2 R(r - xR)}{(r + xR)^3}.$$

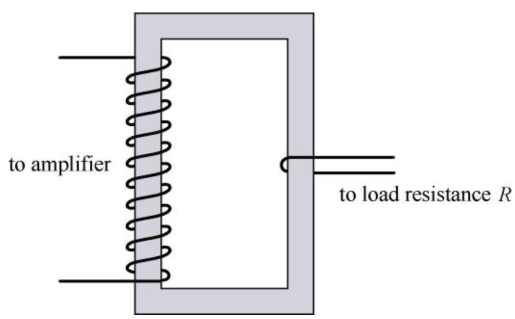
This is zero for

$$x = r/R = (1000\,\Omega)/(10\,\Omega) = 100.$$

We note that for small x, P_{avg} increases linearly with x, and for large x it decreases in proportion to $1/x$. Thus $x = r/R$ is indeed a maximum, not a minimum. Recalling $x = (N_p/N_s)^2$, we conclude that the maximum power is achieved for

$$N_p/N_s = \sqrt{x} = 10.$$

The diagram that follows is a schematic of a transformer with a ten to one turns ratio. An actual transformer would have many more turns in both the primary and secondary coils.



67. (a) Let $\omega t - \pi/4 = \pi/2$ to obtain $t = 3\pi/4\omega = 3\pi/\left[4(350\,\text{rad/s})\right] = 6.73\times10^{-3}$ s.

(b) Let $\omega t + \pi/4 = \pi/2$ to obtain $t = \pi/4\omega = \pi/\left[4(350\,\text{rad/s})\right] = 2.24\times10^{-3}$ s.

(c) Since i leads ε in phase by $\pi/2$, the element must be a capacitor.

(d) We solve C from $X_C = (\omega C)^{-1} = \varepsilon_m/I$:

$$C = \frac{I}{\varepsilon_m \omega} = \frac{6.20\times10^{-3}\text{ A}}{(30.0\text{ V})(350\,\text{rad/s})} = 5.90\times10^{-5}\text{ F}.$$

68. (a) We observe that ω_d = 12566 rad/s. Consequently, X_L = 754 Ω and X_C = 199 Ω. Hence, Eq. 31-65 gives

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = 1.22 \text{ rad} .$$

(b) We find the current amplitude from Eq. 31-60:

$$I = \frac{\varepsilon_m}{\sqrt{R^2 + (X_L - X_C)^2}} = 0.288 \text{ A} .$$

69. (a) Using $\omega = 2\pi f$, $X_L = \omega L$, $X_C = 1/\omega C$ and $\tan(\phi) = (X_L - X_C)/R$, we find

$$\phi = \tan^{-1}[(16.022 - 33.157)/40.0] = -0.40473 \approx -0.405 \text{ rad}.$$

(b) Equation 31-63 gives $I = 120/\sqrt{40^2 + (16\text{-}33)^2} = 2.7576 \approx 2.76$ A.

(c) $X_C > X_L \Rightarrow$ capacitive.

70. (a) We find L from $X_L = \omega L = 2\pi fL$:

$$f = \frac{X_L}{2\pi L} = \frac{1.30\times10^3\,\Omega}{2\pi\left(45.0\times10^{-3}\,\text{H}\right)} = 4.60\times10^3\,\text{Hz}.$$

(b) The capacitance is found from $X_C = (\omega C)^{-1} = (2\pi fC)^{-1}$:

$$C = \frac{1}{2\pi fX_C} = \frac{1}{2\pi\left(4.60\times10^3\,\text{Hz}\right)\left(1.30\times10^3\,\Omega\right)} = 2.66\times10^{-8}\,\text{F}.$$

(c) Noting that $X_L \propto f$ and $X_C \propto f^{-1}$, we conclude that when f is doubled, X_L doubles and X_C reduces by half. Thus,

$$X_L = 2(1.30 \times 10^3\ \Omega) = 2.60 \times 10^3\ \Omega .$$

(d) $X_C = 1.30 \times 10^3\ \Omega/2 = 6.50 \times 10^2\ \Omega$.

71. (a) The impedance is Z = (80.0 V)/(1.25 A) = 64.0 Ω.

(b) We can write $\cos\phi = R/Z$. Therefore,

$$R = (64.0\ \Omega)\cos(0.650 \text{ rad}) = 50.9\ \Omega.$$

(c) Since the current leads the emf, the circuit is capacitive.

72. (a) From Eq. 31-65, we have

$$\phi = \tan^{-1}\left(\frac{V_L - V_C}{V_R}\right) = \tan^{-1}\left(\frac{V_L - (V_L / 1.50)}{(V_L / 2.00)}\right)$$

which becomes $\tan^{-1}(2/3) = 33.7°$ or 0.588 rad.

(b) Since $\phi > 0$, it is inductive ($X_L > X_C$).

(c) We have $V_R = IR = 9.98$ V, so that $V_L = 2.00V_R = 20.0$ V and $V_C = V_L/1.50 = 13.3$ V. Therefore, from Eq. 31-60, we have

$$\varepsilon_m = \sqrt{V_R^2 + (V_L - V_C)^2} = \sqrt{(9.98\text{ V})^2 + (20.0\text{ V} - 13.3\text{ V})^2} = 12.0\text{ V}.$$

73. (a) From Eq. 31-4, we have $L = (\omega^2 C)^{-1} = ((2\pi f)^2 C)^{-1} = 2.41\ \mu$H.

(b) The total energy is the maximum energy on either device (see Fig. 31-4). Thus, we have $U_{\max} = \frac{1}{2}LI^2 = 21.4$ pJ.

(c) Of several methods available to do this part, probably the one most "in the spirit" of this problem (considering the energy that was calculated in part (b)) is to appeal to $U_{\max} = \frac{1}{2}Q^2/C$ (from Chapter 26) to find the maximum charge: $Q = \sqrt{2CU_{\max}} = 82.2$ nC.

74. (a) Equation 31-4 directly gives $1/\sqrt{LC} \approx 5.77\times10^3$ rad/s.

(b) Equation 16-5 then yields $T = 2\pi/\omega = 1.09$ ms.

(c) Although we do not show the graph here, we describe it: it is a cosine curve with amplitude 200 μC and period given in part (b).

75. (a) The impedance is $Z = \dfrac{\varepsilon_m}{I} = \dfrac{125\text{ V}}{3.20\text{ A}} = 39.1\ \Omega.$

(b) From $V_R = IR = \varepsilon_m \cos\phi$, we get

$$R = \frac{\varepsilon_m \cos\phi}{I} = \frac{(125\text{ V})\cos(0.982\text{ rad})}{3.20\text{ A}} = 21.7\ \Omega.$$

(c) Since $X_L - X_C \propto \sin\phi = \sin(-0.982\text{ rad})$, we conclude that $X_L < X_C$. The circuit is predominantly capacitive.

76. (a) Equation 31-39 gives $f = \omega/2\pi = (2\pi C X_C)^{-1} = 8.84$ kHz.

(b) Because of its inverse relationship with frequency, the reactance will go down by a factor of 2 when f increases by a factor of 2. The answer is $X_C = 6.00\ \Omega$.

77. **THINK** The three-phase generator has three ac voltages that are 120° out of phase with each other.

EXPRESS To calculate the potential difference between any two wires, we use the following trigonometric identity:

$$\sin\alpha - \sin\beta = 2\sin\left[(\alpha-\beta)/2\right]\cos\left[(\alpha+\beta)/2\right],$$

where α and β are any two angles.

ANALYZE (a) We consider the following combinations: $\Delta V_{12} = V_1 - V_2$, $\Delta V_{13} = V_1 - V_3$, and $\Delta V_{23} = V_2 - V_3$. For ΔV_{12},

$$\Delta V_{12} = A\sin(\omega_d t) - A\sin(\omega_d t - 120°) = 2A\sin\left(\frac{120°}{2}\right)\cos\left(\frac{2\omega_d t - 120°}{2}\right) = \sqrt{3}A\cos(\omega_d t - 60°)$$

where $\sin 60° = \sqrt{3}/2$. Similarly,

$$\begin{aligned}\Delta V_{13} &= A\sin(\omega_d t) - A\sin(\omega_d t - 240°) = 2A\sin\left(\frac{240°}{2}\right)\cos\left(\frac{2\omega_d t - 240°}{2}\right)\\ &= \sqrt{3}A\cos(\omega_d t - 120°)\end{aligned}$$

and

$$\begin{aligned}\Delta V_{23} &= A\sin(\omega_d t - 120°) - A\sin(\omega_d t - 240°) = 2A\sin\left(\frac{120°}{2}\right)\cos\left(\frac{2\omega_d t - 360°}{2}\right)\\ &= \sqrt{3}A\cos(\omega_d t - 180°).\end{aligned}$$

All three expressions are sinusoidal functions of t with angular frequency ω_d.

(b) We note that each of the above expressions has an amplitude of $\sqrt{3}A$.

LEARN A three-phase generator provides a smoother flow of power than a single-phase generator.

78. (a) The effective resistance R_{eff} satisfies $I_{\text{rms}}^2 R_{\text{eff}} = P_{\text{mechanical}}$, or

$$R_{\text{eff}} = \frac{P_{\text{mechanical}}}{I_{\text{rms}}^2} = \frac{(0.100\ \text{hp})(746\ \text{W/hp})}{(0.650\ \text{A})^2} = 177\ \Omega.$$

(b) This is not the same as the resistance R of its coils, but just the effective resistance for power transfer from electrical to mechanical form. In fact $I^2_{\text{rms}}R$ would not give $P_{\text{mechanical}}$ but rather the rate of energy loss due to thermal dissipation.

79. **THINK** The total energy in the LC circuit is the sum of electrical energy stored in the capacitor, and the magnetic energy stored in the inductor. Energy is conserved.

EXPRESS Let U_E be the electrical energy in the capacitor and U_B be the magnetic energy in the inductor. The total energy is $U = U_E + U_B$. When $U_E = 0.500U_B$ (at time t), then $U_B = 2.00U_E$ and $U = U_E + U_B = 3.00U_E$. Now, U_E is given by $q^2/2C$, where q is the charge on the capacitor at time t. The total energy U is given by $Q^2/2C$, where Q is the maximum charge on the capacitor.

ANALYZE (a) Thus,

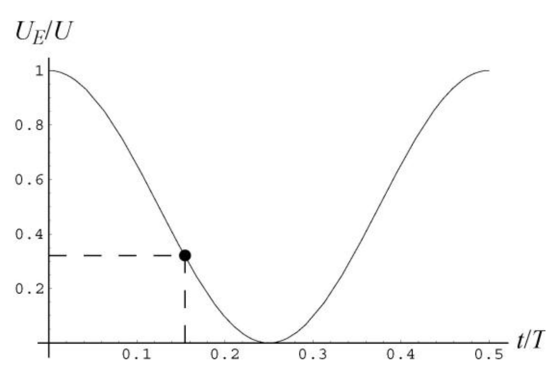
$$\frac{Q^2}{2C} = \frac{3.00q^2}{2C} \quad \Rightarrow \quad q = \frac{Q}{\sqrt{3.00}} = 0.577Q\,.$$

(b) If the capacitor is fully charged at time $t = 0$, then the time-dependent charge on the capacitor is given by $q = Q\cos\omega t$. This implies that the condition $q = 0.577Q$ is satisfied when $\cos\omega t = 0.557$, or $\omega t = 0.955$ rad. Since $\omega = 2\pi/T$ (where T is the period of oscillation), $t = 0.955T/2\pi = 0.152T$, or $t/T = 0.152$.

LEARN The fraction of total energy that is of electrical nature at a given time t is given by

$$\frac{U_E}{U} = \frac{(Q^2/2C)\cos^2\omega t}{Q^2/2C} = \cos^2\omega t = \cos^2\left(\frac{2\pi t}{T}\right).$$

A plot of U_E/U as a function of t/T is given below.



From the plot, we see that $U_E/U = 1/3$ at $t/T = 0.152$.

80. (a) The reactances are as follows:

$$X_L = 2\pi f_d L = 2\pi(400 \text{ Hz})(0.0242 \text{ H}) = 60.82\ \Omega$$
$$X_C = (2\pi f_d C)^{-1} = [2\pi(400 \text{ Hz})(1.21\times10^{-5} \text{ F})]^{-1} = 32.88\ \Omega$$
$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(20.0\ \Omega)^2 + (60.82\ \Omega - 32.88\ \Omega)^2} = 34.36\ \Omega\,.$$

With $\varepsilon = 90.0$ V, we have

$$I = \frac{\varepsilon}{Z} = \frac{90.0 \text{ V}}{34.36\ \Omega} = 2.62 \text{ A} \ \Rightarrow\ I_{\text{rms}} = \frac{I}{\sqrt{2}} = \frac{2.62 \text{ A}}{\sqrt{2}} = 1.85 \text{ A}\,.$$

Therefore, the rms potential difference across the resistor is $V_{R\,\text{rms}} = I_{\text{rms}} R$ = 37.0 V.

(b) Across the capacitor, the rms potential difference is $V_{C\,\text{rms}} = I_{\text{rms}} X_C$ = 60.9 V.

(c) Similarly, across the inductor, the rms potential difference is $V_{L\,\text{rms}} = I_{\text{rms}} X_L$ = 113 V.

(d) The average rate of energy dissipation is $P_{\text{avg}} = (I_{\text{rms}})^2 R$ = 68.6 W.

81. **THINK** Since the current lags the generator emf, the phase angle is positive and the circuit is more inductive than capacitive.

EXPRESS Let V_L be the maximum potential difference across the inductor, V_C be the maximum potential difference across the capacitor, and V_R be the maximum potential difference across the resistor. The phase constant is given by

$$\phi = \tan^{-1}\left(\frac{V_L - V_C}{V_R}\right).$$

The maximum emf is related to the current amplitude by $\varepsilon_m = IZ$, where Z is the impedance.

ANALYZE (a) With $V_C = V_L / 2.00$ and $V_R = V_L / 2.00$, we find the phase constant to be

$$\phi = \tan^{-1}\left(\frac{V_L - V_L / 2.00}{V_L / 2.00}\right) = \tan^{-1}(1.00) = 45.0°.$$

(b) The resistance is related to the impedance by $R = Z\cos\phi$. Thus,

$$R = \frac{\varepsilon_m \cos\phi}{I} = \frac{(30.0\text{ V})(\cos 45°)}{300\times10^{-3}\text{ A}} = 70.7\ \Omega.$$

LEARN With R and I known, the inductive and capacitive reactances are, respectively, $X_L = 2.00R = 141\ \Omega$, and $X_C = R = 70.7\ \Omega$. Similarly, the impedance of the circuit is

$$Z = \frac{\mathcal{E}_m}{I} = (30.0\text{ V})/(300\times 10^{-3}\text{ A}) = 100\,\Omega\,.$$

82. From $U_{\max} = \frac{1}{2}LI^2$ we get I = 0.115 A.

83. From Eq. 31-4 we get $f = 1/2\pi\sqrt{LC}$ = 1.84 kHz.

84. (a) With a phase constant of 45º the (net) reactance must equal the resistance in the circuit, which means the circuit impedance becomes

$$Z = R\sqrt{2} \;\Rightarrow\; R = Z/\sqrt{2} = 707\ \Omega.$$

(b) Since f = 8000 Hz, then $\omega_d = 2\pi(8000)$ rad/s. The net reactance (which, as observed, must equal the resistance) is therefore

$$X_L - X_C = \omega_d L - (\omega_d C)^{-1} = 707\ \Omega.$$

We are also told that the resonance frequency is 6000 Hz, which (by Eq. 31-4) means

$$C = \frac{1}{\omega^2 L} = \frac{1}{(2\pi f)^2 L} = \frac{1}{4\pi^2 f^2 L} = \frac{1}{4\pi^2(6000\text{ Hz})^2 L}\,.$$

Substituting this for C in our previous expression (for the net reactance) we obtain an equation that can be solved for the self-inductance. Our result is L = 32.2 mH.

(c) $C = ((2\pi(6000))^2 L)^{-1}$ = 21.9 nF.

85. **THINK** The current and the charge undergo sinusoidal oscillations in the LC circuit. Energy is conserved.

EXPRESS The angular frequency oscillation is related to the capacitance C and inductance L by $\omega = 1/\sqrt{LC}$. The electrical energy and magnetic energy in the circuit as a function of time are given by

$$U_E = \frac{q^2}{2C} = \frac{Q^2}{2C}\cos^2(\omega t + \phi)$$

$$U_B = \frac{1}{2}Li^2 = \frac{1}{2}L\omega^2 Q^2 \sin^2(\omega t + \phi) = \frac{Q^2}{2C}\sin^2(\omega t + \phi).$$

The maximum value of U_E is $Q^2/2C$, which is the total energy in the circuit, U. Similarly, the maximum value of U_B is also $Q^2/2C$, which can also be written as $LI^2/2$ using $I = \omega Q$.

ANALYZE (a) Using the fact that $\omega = 2\pi f$, the inductance is

$$L = \frac{1}{\omega^2 C} = \frac{1}{4\pi^2 f^2 C} = \frac{1}{4\pi^2 \left(10.4\times10^3\ \text{Hz}\right)^2 \left(340\times10^{-6}\ \text{F}\right)} = 6.89\times10^{-7}\ \text{H}.$$

(b) The total energy may be calculated from the inductor (when the current is at maximum):

$$U = \frac{1}{2}LI^2 = \frac{1}{2}\left(6.89\times10^{-7}\ \text{H}\right)\left(7.20\times10^{-3}\ \text{A}\right)^2 = 1.79\times10^{-11}\ \text{J}.$$

(c) We solve for Q from $U = \frac{1}{2}Q^2/C$:

$$Q = \sqrt{2CU} = \sqrt{2\left(340\times10^{-6}\ \text{F}\right)\left(1.79\times10^{-11}\ \text{J}\right)} = 1.10\times10^{-7}\ \text{C}.$$

LEARN Figure 31-4 of the textbook illustrates the oscillations of electrical and magnetic energies. The total energy $U = U_E + U_B = Q^2/2C$ remains constant. When U_E is maximum, U_B is zero, and vice versa.

86. From Eq. 31-60, we have $(220\ \text{V}/3.00\ \text{A})^2 = R^2 + X_L^2 \ \Rightarrow\ X_L = 69.3\,\Omega$.

87. When the switch is open, we have a series LRC circuit involving just the one capacitor near the upper right corner. Equation 31-65 leads to

$$\frac{\omega_d L - \dfrac{1}{\omega_d C}}{R} = \tan\phi_o = \tan(-20°) = -\tan 20°.$$

Now, when the switch is in position 1, the equivalent capacitance in the circuit is $2C$. In this case, we have

$$\frac{\omega_d L - \dfrac{1}{2\omega_d C}}{R} = \tan\phi_1 = \tan 10.0°.$$

Finally, with the switch in position 2, the circuit is simply an LC circuit with current amplitude

$$I_2 = \frac{\varepsilon_m}{Z_{LC}} = \frac{\varepsilon_m}{\sqrt{\left(\omega_d L - \dfrac{1}{\omega_d C}\right)^2}} = \frac{\varepsilon_m}{\dfrac{1}{\omega_d C} - \omega_d L}$$

where we use the fact that $(\omega_d C)^{-1} > \omega_d L$ in simplifying the square root (this fact is evident from the description of the first situation, when the switch was open). We solve for *L*, *R* and *C* from the three equations above, and the results are as follows:

(a) $R = \dfrac{-\varepsilon_m}{I_2 \tan\phi_o} = \dfrac{-120\text{V}}{(2.00\,\text{A})\tan(-20.0°)} = 165\,\Omega$,

(b) $L = \dfrac{\varepsilon_m}{\omega_d I_2}\left(1 - 2\dfrac{\tan\phi_1}{\tan\phi_o}\right) = \dfrac{120\,\text{V}}{2\pi(60.0\text{ Hz})(2.00\text{ A})}\left(1 - 2\dfrac{\tan 10.0°}{\tan(-20.0°)}\right) = 0.313\text{ H}$,

(c) and

$$C = \frac{I_2}{2\omega_d \varepsilon_m\left(1 - \tan\phi_1/\tan\phi_0\right)} = \frac{2.00\text{ A}}{2(2\pi)(60.0\text{ Hz})(120\text{ V})\left(1 - \tan 10.0°/\tan(-20.0°)\right)}$$
$$= 1.49\times10^{-5}\text{ F.}$$

88. (a) Eqs. 31-4 and 31-14 lead to

$$Q = \frac{1}{\omega} = I\sqrt{LC} = 1.27\times10^{-6}\text{C}\ .$$

(b) We choose the phase constant in Eq. 31-12 to be $\phi = -\pi/2$, so that i_0 = I in Eq. 31-15). Thus, the energy in the capacitor is

$$U_E = \frac{q^2}{2C} = \frac{Q^2}{2C}(\sin\omega t)^2\ .$$

Differentiating and using the fact that 2 sin θ cos θ = sin 2θ, we obtain

$$\frac{dU_E}{dt} = \frac{Q^2}{2C}\omega\sin 2\omega t\ .$$

We find the maximum value occurs whenever sin $2\omega t = 1$, which leads (with *n* = *odd* integer) to

$$t = \frac{1}{2\omega}\frac{n\pi}{2} = \frac{n\pi}{4\omega} = \frac{n\pi}{4}\sqrt{LC} = 8.31\times10^{-5}\text{s},\ 2.49\times10^{-4}\text{s},\ \ldots\ .$$

The earliest time is $t = 8.31\times10^{-5}$s.

(c) Returning to the above expression for dU_E/dt with the requirement that $\sin 2\omega t = 1$, we obtain

$$\left(\frac{dU_E}{dt}\right)_{\max} = \frac{Q^2}{2C}\omega = \frac{\left(I\sqrt{LC}\right)^2}{2C}\frac{I}{\sqrt{LC}} = \frac{I^2}{2}\sqrt{\frac{L}{C}} = 5.44\times10^{-3}\,\text{J/s}\,.$$

89. **THINK** In this problem, we demonstrate that in a driven *RLC* circuit, the energies stored in the capacitor and the inductor stay constant; however, energy is transferred from the driving emf device to the resistor.

EXPRESS The energy stored in the capacitor is given by $U_E = q^2/2C$. Similarly, the energy stored in the inductor is $U_B = \frac{1}{2}Li^2$. The rate of energy supply by the driving emf device is $P_\varepsilon = i\varepsilon$, where $i = I\sin(\omega_d - \phi)$ and $\varepsilon = \varepsilon_m \sin\omega_d t$. The rate with which energy dissipates in the resistor is $P_R = i^2R$.

ANALYZE (a) Since the charge q is a periodic function of t with period T, so must be U_E. Consequently, U_E will not be changed over one complete cycle. Actually, U_E has period $T/2$, which does not alter our conclusion.

(b) Since the current i is a periodic function of t with period T, so must be U_B.

(c) The energy supplied by the emf device over one cycle is

$$\begin{aligned} U_\varepsilon &= \int_0^T P_\varepsilon dt = I\varepsilon_m\int_0^T \sin(\omega_d t-\phi)\sin(\omega_d t)dt = I\varepsilon_m\int_0^T [\sin\omega_d t\cos\phi - \cos\omega_d t\sin\phi]\sin(\omega_d t)dt \\ &= \frac{T}{2}I\varepsilon_m\cos\phi, \end{aligned}$$

where we have used

$$\int_0^T \sin^2(\omega_d t)dt = \frac{T}{2}, \qquad \int_0^T \sin(\omega_d t)\cos(\omega_d t)dt = 0.$$

(d) Over one cycle, the energy dissipated in the resistor is

$$U_R = \int_0^T P_R dt = I^2R\int_0^T \sin^2(\omega_d t - \phi)dt = \frac{T}{2}I^2R.$$

(e) Since $\varepsilon_m I\cos\phi = \varepsilon_m I\left(V_R/\varepsilon_m\right) = \varepsilon_m I\left(IR/\varepsilon_m\right) = I^2R$, the two quantities are indeed the same.

LEARN In solving for (c) and (d), we could have used Eqs. 31-74 and 31-71: By doing so, we find the energy supplied by the generator to be

$$P_{\text{avg}}T = \left(I_{\text{rms}}\varepsilon_{\text{rms}}\cos\phi\right)T = \left(\frac{1}{2}T\right)\varepsilon_m I\cos\phi$$

where we substitute $I_{\text{rms}} = I/\sqrt{2}$ and $\varepsilon_{\text{rms}} = \varepsilon_m/\sqrt{2}$. Similarly, the energy dissipated by the resistor is

$$P_{\text{avg,resistor}}\, T = \left(I_{\text{rms}} V_R\right) T = I_{\text{rms}}\left(I_{\text{rms}} R\right) T = \left(\frac{1}{2} T\right) I^2 R.$$

The same results are obtained without any integration.

90. From Eq. 31-4, we have $C = (\omega^2 L)^{-1} = ((2\pi f)^2 L)^{-1} = 1.59\ \mu\text{F}$.

91. Resonance occurs when the inductive reactance equals the capacitive reactance. Reactances of a certain type add (in series) just like resistances. Thus, since the resonance ω values are the same for both circuits, we have for each circuit:

$$\omega L_1 = \frac{1}{\omega C_1}, \qquad \omega L_2 = \frac{1}{\omega C_2}$$

and adding these equations we find

$$\omega\left(L_1 + L_2\right) = \frac{1}{\omega}\left(\frac{1}{C_1} + \frac{1}{C_2}\right).$$

Since $L_{\text{eq}} = L_1 + L_2$ and $C_{\text{eq}}^{-1} = (C_1^{-1} + C_2^{-1})$,

$$\omega L_{\text{eq}} = \frac{1}{\omega C_{\text{eq}}} \quad \Rightarrow \quad \text{resonance in the combined circuit.}$$

92. When switch S_1 is closed and the others are open, the inductor is essentially out of the circuit and what remains is an RC circuit. The time constant is $\tau_C = RC$. When switch S_2 is closed and the others are open, the capacitor is essentially out of the circuit. In this case, what we have is an LR circuit with time constant $\tau_L = L/R$. Finally, when switch S_3 is closed and the others are open, the resistor is essentially out of the circuit and what remains is an LC circuit that oscillates with period $T = 2\pi\sqrt{LC}$. Substituting $L = R\tau_L$ and $C = \tau_C/R$, we obtain $T = 2\pi\sqrt{\tau_C \tau_L}$.

93. (a) We note that we obtain the maximum value in Eq. 31-28 when we set

$$t = \frac{\pi}{2\omega_d} = \frac{1}{4f} = \frac{1}{4(60)} = 0.00417\ \text{s}$$

or 4.17 ms. The result is $\varepsilon_m \sin(\pi/2) = \varepsilon_m \sin(90°) = 36.0\ \text{V}$.

(b) At $t = 4.17$ ms, the current is

$$i = I\sin(\omega_d t - \phi) = I\sin(90° - (-24.3°)) = (0.164\,\text{A})\cos(24.3°)$$
$$= 0.1495\,\text{A} \approx 0.150\text{ A}.$$

Ohm's law directly gives

$$v_R = iR = (0.1495\,\text{A})(200\,\Omega) = 29.9\,\text{V}.$$

(c) The capacitor voltage phasor is 90° less than that of the current. Thus, at t = 4.17 ms, we obtain

$$v_C = I\sin(90° - (-24.3°) - 90°)X_C = IX_C\sin(24.3°) = (0.164\,\text{A})(177\,\Omega)\sin(24.3°)$$
$$= 11.9\,\text{V}.$$

(d) The inductor voltage phasor is 90° more than that of the current. Therefore, at t = 4.17 ms, we find

$$v_L = I\sin(90° - (-24.3°) + 90°)X_L = -IX_L\sin(24.3°) = -(0.164\,\text{A})(86.7\,\Omega)\sin(24.3°)$$
$$= -5.85\,\text{V}.$$

(e) Our results for parts (b), (c) and (d) add to give 36.0 V, the same as the answer for part (a).

Chapter 32

1. We use $\sum_{n=1}^{6} \Phi_{Bn} = 0$ to obtain

$$\Phi_{B6} = -\sum_{n=1}^{5} \Phi_{Bn} = -(-1\,\text{Wb} + 2\,\text{Wb} - 3\,\text{Wb} + 4\,\text{Wb} - 5\,\text{Wb}) = +3\,\text{Wb}\ .$$

2. (a) The flux through the top is +(0.30 T)πr^2 where r = 0.020 m. The flux through the bottom is +0.70 mWb as given in the problem statement. Since the *net* flux must be zero then the flux through the sides must be negative and exactly cancel the total of the previously mentioned fluxes. Thus (in magnitude) the flux though the sides is 1.1 mWb.

(b) The fact that it is negative means it is inward.

3. **THINK** Gauss' law for magnetism states that the net magnetic flux through any closed surface is zero.

EXPRESS Mathematically, Gauss' law for magnetism is expressed as $\oint \vec{B}\cdot d\vec{A} = 0$. Now, our Gaussian surface has the shape of a right circular cylinder with two end caps and a curved surface. Thus,

$$\oint \vec{B}\cdot d\vec{A} = \Phi_1 + \Phi_2 + \Phi_C,$$

where Φ_1 is the magnetic flux through the first end cap, Φ_2 is the magnetic flux through the second end cap, and Φ_C is the magnetic flux through the curved surface. Over the first end the magnetic field is inward, so the flux is Φ_1 = –25.0 μWb. Over the second end the magnetic field is uniform, normal to the surface, and outward, so the flux is $\Phi_2 = AB = \pi r^2 B$, where A is the area of the end and r is the radius of the cylinder.

ANALYZE (a) Substituting the values given, the flux through the second end is

$$\Phi_2 = \pi(0.120\,\text{m})^2(1.60\times10^{-3}\,\text{T}) = +7.24\times10^{-5}\,\text{Wb} = +72.4\,\mu\text{Wb}.$$

Since the three fluxes must sum to zero,

$$\Phi_C = -\Phi_1 - \Phi_2 = 25.0\,\mu\text{Wb} - 72.4\,\mu\text{Wb} = -47.4\,\mu\text{Wb}.$$

Thus, the magnitude is $|\Phi_C| = 47.4\,\mu\text{Wb}$.

(b) The minus sign in Φ_C indicates that the flux is inward through the curved surface.

LEARN Gauss' law for magnetism implies that magnetic monopoles do not exist; the simplest magnetic structure is a magnetic dipole (having a north pole and a south pole).

4. From Gauss' law for magnetism, the flux through S_1 is equal to that through S_2, the portion of the *xz* plane that lies within the cylinder. Here the normal direction of S_2 is $+y$. Therefore,

$$\Phi_B(S_1) = \Phi_B(S_2) = \int_{-r}^{r} B(x)L\,dx = 2\int_{-r}^{r} B_{\text{left}}(x)L\,dx = 2\int_{-r}^{r} \frac{\mu_0 i}{2\pi}\frac{1}{2r-x}L\,dx = \frac{\mu_0 iL}{\pi}\ln 3\,.$$

5. **THINK** Changing electric flux induces a magnetic field.

EXPRESS Consider a circle of radius *r* between the plates, with its center on the axis of the capacitor. Since there is no current between the capacitor plates, the Ampere-Maxwell's law reduces to

$$\oint \vec{B}\cdot d\vec{A} = \mu_0\varepsilon_0 \frac{d\Phi_E}{dt}\,,$$

where $\vec{B}$ is the magnetic field at points on the circle, and Φ_E is the electric flux through the circle. Since the $\vec{B}$ field on the circle is in the tangential direction, and $\Phi_E = AE = \pi R^2 E,$ where *R* is the radius of the capacitor, we have

$$2\pi rB = \mu_0\varepsilon_0\pi R^2\frac{dE}{dt}$$

or

$$B = \frac{\mu_0\varepsilon_0 R^2}{2r}\frac{dE}{dt} \qquad (r \ge R).$$

ANALYZE Solving for *dE*/*dt*, we obtain

$$\frac{dE}{dt} = \frac{2Br}{\mu_0\varepsilon_0 R^2} = \frac{2\left(2.0\times10^{-7}\ \text{T}\right)\left(6.0\times10^{-3}\ \text{m}\right)}{\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(3.0\times10^{-3}\text{m}\right)^2} = 2.4\times10^{13}\,\frac{\text{V}}{\text{m}\cdot\text{s}}.$$

LEARN Outside the capacitor, the induced magnetic field decreases with increased radial distance *r*, from a maximum value at the plate edge $r = R$.

6. The integral of the field along the indicated path is, by Eq. 32-18 and Eq. 32-19, equal to

$$\mu_0 i_d\left(\frac{\text{enclosed area}}{\text{total area}}\right) = \mu_0(0.75\ \text{A})\frac{(4.0\ \text{cm})(2.0\ \text{cm})}{12\ \text{cm}^2} = 52\ \text{nT}\cdot\text{m}\,.$$

7. (a) Inside we have (by Eq. 32-16) $B = \mu_0 i_d r_1 / 2\pi R^2$, where $r_1 = 0.0200$ m, $R = 0.0300$ m, and the displacement current is given by Eq. 32-38 (in SI units):

$$i_d = \varepsilon_0 \frac{d\Phi_E}{dt} = (8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(3.00\times10^{-3}\ \text{V/m}\cdot\text{s}) = 2.66\times10^{-14}\ \text{A}\,.$$

Thus, we find

$$B = \frac{\mu_0 i_d r_1}{2\pi R^2} = \frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(2.66\times10^{-14}\ \text{A})(0.0200\ \text{m})}{2\pi(0.0300\ \text{m})^2} = 1.18\times10^{-19}\ \text{T}\,.$$

(b) Outside we have (by Eq. 32-17) $B = \mu_0 i_d / 2\pi r_2$ where r_2 = 0.0500 cm. Here we obtain

$$B = \frac{\mu_0 i_d}{2\pi r_2} = \frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(2.66\times10^{-14}\ \text{A})}{2\pi(0.0500\ \text{m})} = 1.06\times10^{-19}\ \text{T}$$

8. (a) Application of Eq. 32-3 along the circle referred to in the second sentence of the problem statement (and taking the derivative of the flux expression given in that sentence) leads to

$$B(2\pi r) = \varepsilon_0\mu_0\left(0.60\ \text{V}\cdot\text{m/s}\right)\frac{r}{R}\,.$$

Using r = 0.0200 m (which, in any case, cancels out) and R = 0.0300 m, we obtain

$$B = \frac{\varepsilon_0\mu_0(0.60\ \text{V}\cdot\text{m/s})}{2\pi R} = \frac{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(0.60\ \text{V}\cdot\text{m/s})}{2\pi(0.0300\ \text{m})}$$
$$= 3.54\times10^{-17}\ \text{T}\,.$$

(b) For a value of r larger than R, we must note that the flux enclosed has already reached its full amount (when r = R in the given flux expression). Referring to the equation we wrote in our solution of part (a), this means that the final fraction (r/R) should be replaced with unity. On the left hand side of that equation, we set r = 0.0500 m and solve. We now find

$$B = \frac{\varepsilon_0\mu_0(0.60\ \text{V}\cdot\text{m/s})}{2\pi r} = \frac{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(0.60\ \text{V}\cdot\text{m/s})}{2\pi(0.0500\ \text{m})}$$
$$= 2.13\times10^{-17}\ \text{T}\,.$$

9. (a) Application of Eq. 32-7 with A = πr^2 (and taking the derivative of the field expression given in the problem) leads to

$$B(2\pi r) = \varepsilon_0\mu_0\pi r^2\left(0.00450\ \text{V/m}\cdot\text{s}\right).$$

For r = 0.0200 m, this gives

$$\begin{aligned} B &= \frac{1}{2}\varepsilon_0\mu_0 r(0.00450 \text{ V/m}\cdot\text{s}) \\ &= \frac{1}{2}(8.85\times10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2)(4\pi\times10^{-7} \text{ T}\cdot\text{m/A})(0.0200 \text{ m})(0.00450 \text{ V/m}\cdot\text{s}) \\ &= 5.01\times10^{-22} \text{ T}. \end{aligned}$$

(b) With $r > R$, the expression above must replaced by

$$B(2\pi r) = \varepsilon_0\mu_0\pi R^2\left(0.00450 \text{ V/m}\cdot\text{s}\right).$$

Substituting r = 0.050 m and R = 0.030 m, we obtain $B = 4.51 \times 10^{-22}$ T.

10. (a) Here, the enclosed electric flux is found by integrating

$$\Phi_E = \int_0^r E\, 2\pi r dr = t(0.500 \text{ V/m}\cdot\text{s})(2\pi)\int_0^r \left(1-\frac{r}{R}\right) r dr = t\pi\left(\frac{1}{2}r^2 - \frac{r^3}{3R}\right)$$

with SI units understood. Then (after taking the derivative with respect to time) Eq. 32-3 leads to

$$B(2\pi r) = \varepsilon_0\mu_0\pi\left(\frac{1}{2}r^2 - \frac{r^3}{3R}\right).$$

For r = 0.0200 m and R = 0.0300 m, this gives $B = 3.09 \times 10^{-20}$ T.

(b) The integral shown above will no longer (since now $r > R$) have r as the upper limit; the upper limit is now R. Thus,

$$\Phi_E = t\pi\left(\frac{1}{2}R^2 - \frac{R^3}{3R}\right) = \frac{1}{6}t\pi R^2.$$

Consequently, Eq. 32-3 becomes

$$B(2\pi r) = \frac{1}{6}\varepsilon_0\mu_0\pi R^2$$

which for r = 0.0500 m, yields

$$B = \frac{\varepsilon_0\mu_0 R^2}{12r} = \frac{(8.85\times10^{-12}\ )(4\pi\times10^{-7})(0.030)^2}{12(0.0500)} = 1.67\times10^{-20} \text{ T}.$$

11. (a) Noting that the magnitude of the electric field (assumed uniform) is given by $E = V/d$ (where d = 5.0 mm), we use the result of part (a) in Sample Problem 32.01 – "Magnetic field induced by changing electric field:"

$$B = \frac{\mu_0 \varepsilon_0 r}{2}\frac{dE}{dt} = \frac{\mu_0 \varepsilon_0 r}{2d}\frac{dV}{dt} \qquad (r \le R).$$

We also use the fact that the time derivative of sin (ωt) (where $\omega = 2\pi f = 2\pi(60) \approx 377$/s in this problem) is $\omega\cos(\omega t)$. Thus, we find the magnetic field as a function of r (for $r \le R$; note that this neglects "fringing" and related effects at the edges):

$$B = \frac{\mu_0 \varepsilon_0 r}{2d} V_{\max}\omega\cos(\omega t) \quad \Rightarrow \quad B_{\max} = \frac{\mu_0 \varepsilon_0 r V_{\max}\omega}{2d}$$

where $V_{\max}$ = 150 V. This grows with r until reaching its highest value at $r = R$ = 30 mm:

$$B_{\max}\Big|_{r=R} = \frac{\mu_0 \varepsilon_0 R V_{\max}\omega}{2d} = \frac{(4\pi\times10^{-7}\ \mathrm{H/m})(8.85\times10^{-12}\ \mathrm{F/m})(30\times10^{-3}\mathrm{m})(150\,\mathrm{V})(377/\mathrm{s})}{2(5.0\times10^{-3}\mathrm{m})}$$
$$= 1.9\times10^{-12}\,\mathrm{T}.$$

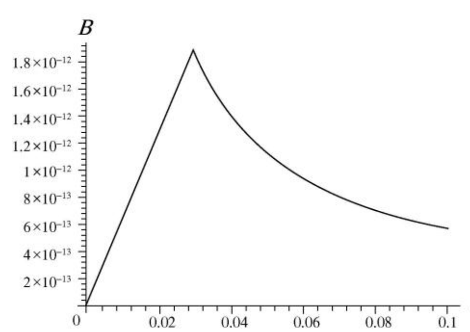
(b) For $r \le 0.03$ m, we use the expression

$$B_{\max} = \mu_0\varepsilon_0 r V_{\max}\omega/2d$$

found in part (a) (note the $B \propto r$ dependence), and for $r \ge 0.03$ m we perform a similar calculation starting with the result of part (b) in Sample Problem 32.01 — "Magnetic field induced by changing electric field:"

$$B_{\max} = \left(\frac{\mu_0\varepsilon_0 R^2}{2r}\frac{dE}{dt}\right)_{\max} = \left(\frac{\mu_0\varepsilon_0 R^2}{2rd}\frac{dV}{dt}\right)_{\max} = \left(\frac{\mu_0\varepsilon_0 R^2}{2rd}V_{\max}\omega\cos(\omega t)\right)_{\max}$$
$$= \frac{\mu_0\varepsilon_0 R^2 V_{\max}\omega}{2rd} \qquad (\text{for } r \ge R)$$

(note the $B \propto r^{-1}$ dependence — see also Eqs. 32-16 and 32-17). The plot, with SI units understood, is shown to the right.



12. From Sample Problem 32.01 — "Magnetic field induced by changing electric field," we know that $B \propto r$ for $r \le R$ and $B \propto r^{-1}$ for $r \ge R$. So the maximum value of B occurs at $r = R$, and there are two possible values of r at which the magnetic field is 75% of $B_{\max}$. We denote these two values as r_1 and r_2, where $r_1 < R$ and $r_2 > R$.

(a) Inside the capacitor, $0.75\ B_{\max}/B_{\max} = r_1/R$, or $r_1 = 0.75\ R = 0.75$ (40 mm) =30 mm.

(b) Outside the capacitor, $0.75\ B_{\max}/B_{\max} = (r_2/R)^{-1}$, or

$$r_2 = R/0.75 = 4R/3 = (4/3)(40\text{ mm}) = 53\text{ mm}.$$

(c) From Eqs. 32-15 and 32-17,

$$B_{\max} = \frac{\mu_0 i_d}{2\pi R} = \frac{\mu_0 i}{2\pi R} = \frac{\left(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A}\right)\left(6.0\,\text{A}\right)}{2\pi\left(0.040\,\text{m}\right)} = 3.0\times10^{-5}\,\text{T}.$$

13. Let the area plate be *A* and the plate separation be *d*. We use Eq. 32-10:

$$i_d = \varepsilon_0 \frac{d\Phi_E}{dt} = \varepsilon_0 \frac{d}{dt}\left(AE\right) = \varepsilon_0 A \frac{d}{dt}\left(\frac{V}{d}\right) = \frac{\varepsilon_0 A}{d}\left(\frac{dV}{dt}\right),$$

or

$$\frac{dV}{dt} = \frac{i_d d}{\varepsilon_0 A} = \frac{i_d}{C} = \frac{1.5\,\text{A}}{2.0\times10^{-6}\,\text{F}} = 7.5\times10^{5}\,\text{V/s}.$$

Therefore, we need to change the voltage difference across the capacitor at the rate of 7.5×10^5 V/s.

14. Consider an area *A*, normal to a uniform electric field $\vec{E}$. The displacement current density is uniform and normal to the area. Its magnitude is given by $J_d = i_d/A$. For this situation, $i_d = \varepsilon_0 A(dE/dt)$, so

$$J_d = \frac{1}{A}\varepsilon_0 A \frac{dE}{dt} = \varepsilon_0 \frac{dE}{dt}.$$

15. **THINK** The displacement current is related to the changing electric flux by $i_d = \varepsilon_0 (d\Phi_E/dt)$.

EXPRESS Let *A* be the area of a plate and *E* be the magnitude of the electric field between the plates. The field between the plates is uniform, so $E = V/d$, where *V* is the potential difference across the plates and *d* is the plate separation.

ANALYZE Thus, the displacement current is

$$i_d = \varepsilon_0 \frac{d\Phi_E}{dt} = \varepsilon_0 \frac{d(EA)}{dt} = \varepsilon_0 A \frac{dE}{dt} = \frac{\varepsilon_0 A}{d}\frac{dV}{dt}.$$

Now, $\varepsilon_0 A/d$ is the capacitance *C* of a parallel-plate capacitor (not filled with a dielectric), so

$$i_d = C\frac{dV}{dt}.$$

LEARN The real current charging the capacitor is

$$i = \frac{dq}{dt} = \frac{d}{dt}(CV) = C\frac{dV}{dt}.$$

Thus, we see that $i = i_d$.

16. We use Eq. 32-14: $i_d = \varepsilon_0 A(dE/dt)$. Note that, in this situation, A is the area over which a changing electric field is present. In this case $r > R$, so $A = \pi R^2$. Thus,

$$\frac{dE}{dt} = \frac{i_d}{\varepsilon_0 A} = \frac{i_d}{\varepsilon_0 \pi R^2} = \frac{2.0\,\text{A}}{\pi\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(0.10\,\text{m}\right)^2} = 7.2\times10^{12}\,\frac{\text{V}}{\text{m}\cdot\text{s}}.$$

17. (a) Using Eq. 27-10, we find $E = \rho J = \dfrac{\rho i}{A} = \dfrac{\left(1.62\times10^{-8}\,\Omega\cdot\text{m}\right)\left(100\,\text{A}\right)}{5.00\times10^{-6}\,\text{m}^2} = 0.324\,\text{V/m}.$

(b) The displacement current is

$$i_d = \varepsilon_0\frac{d\Phi_E}{dt} = \varepsilon_0 A\frac{dE}{dt} = \varepsilon_0 A\frac{d}{dt}\left(\frac{\rho i}{A}\right) = \varepsilon_0\rho\frac{di}{dt} = \left(8.85\times10^{-12}\,\text{F/m}\right)\left(1.62\times10^{-8}\,\Omega\right)\left(2000\,\text{A/s}\right)$$
$$= 2.87\times10^{-16}\,\text{A}.$$

(c) The ratio of fields is $\dfrac{B\left(\text{due to } i_d\right)}{B\left(\text{due to } i\right)} = \dfrac{\mu_0 i_d/2\pi r}{\mu_0 i/2\pi r} = \dfrac{i_d}{i} = \dfrac{2.87\times10^{-16}\,\text{A}}{100\,\text{A}} = 2.87\times10^{-18}.$

18. From Eq. 28-11, we have $i = (\varepsilon / R)\, e^{-t/\tau}$ since we are ignoring the self-inductance of the capacitor. Equation 32-16 gives

$$B = \frac{\mu_0 i_d r}{2\pi R^2}.$$

Furthermore, Eq. 25-9 yields the capacitance

$$C = \frac{\varepsilon_0 \pi (0.05\text{ m})^2}{0.003\text{ m}} = 2.318\times10^{-11}\,\text{F},$$

so that the capacitive time constant is

$$\tau = (20.0\times10^{6}\,\Omega)(2.318\times10^{-11}\,\text{F}) = 4.636\times10^{-4}\,\text{s}.$$

At $t = 250\times10^{-6}$ s, the current is

$$i = \frac{12.0\text{ V}}{20.0\text{ x }10^6\,\Omega}\ e^{-t/\tau} = 3.50\times10^{-7}\,\text{A}.$$

Since $i = i_d$ (see Eq. 32-15) and r = 0.0300 m, then (with plate radius R = 0.0500 m) we find

$$B = \frac{\mu_0 i_d r}{2\pi R^2} = \frac{(4\pi\times 10^{-7}\ \mathrm{T\cdot m/A})(3.50\times 10^{-7}\ \mathrm{A})(0.030\ \mathrm{m})}{2\pi(0.050\ \mathrm{m})^2} = 8.40\times 10^{-13}\ \mathrm{T}.$$

19. (a) Equation 32-16 (with Eq. 26-5) gives, with $A = \pi R^2$,

$$B = \frac{\mu_0 i_d r}{2\pi R^2} = \frac{\mu_0 J_d A r}{2\pi R^2} = \frac{\mu_0 J_d (\pi R^2) r}{2\pi R^2} = \frac{1}{2}\mu_0 J_d r$$
$$= \frac{1}{2}(4\pi\times 10^{-7}\ \mathrm{T\cdot m/A})(6.00\ \mathrm{A/m^2})(0.0200\ \mathrm{m}) = 75.4\ \mathrm{nT}.$$

(b) Similarly, Eq. 32-17 gives $B = \dfrac{\mu_0 i_d}{2\pi r} = \dfrac{\mu_0 J_d \pi R^2}{2\pi r} = 67.9\ \mathrm{nT}$.

20. (a) Equation 32-16 gives $B = \dfrac{\mu_0 i_d r}{2\pi R^2} = 2.22\ \mu\mathrm{T}$.

(b) Equation 32-17 gives $B = \dfrac{\mu_0 i_d}{2\pi r} = 2.00\ \mu\mathrm{T}$.

21. (a) Equation 32-11 applies (though the last term is zero) but we must be careful with $i_{d,\mathrm{enc}}$. It is the enclosed portion of the displacement current, and if we related this to the displacement current density J_d, then

$$i_{d\ \mathrm{enc}} = \int_0^r J_d\, 2\pi r\, dr = (4.00\ \mathrm{A/m^2})(2\pi)\int_0^r (1 - r/R)\, r\, dr = 8\pi\left(\frac{1}{2}r^2 - \frac{r^3}{3R}\right)$$

with SI units understood. Now, we apply Eq. 32-17 (with i_d replaced with $i_{d,\mathrm{enc}}$) or start from scratch with Eq. 32-11, to get $B = \dfrac{\mu_0 i_{d\ \mathrm{enc}}}{2\pi r} = 27.9\ \mathrm{nT}$.

(b) The integral shown above will no longer (since now $r > R$) have r as the upper limit; the upper limit is now R. Thus,

$$i_{d\ \mathrm{enc}} = i_d = 8\pi\left(\frac{1}{2}R^2 - \frac{R^3}{3R}\right) = \frac{4}{3}\pi R^2.$$

Now Eq. 32-17 gives $B = \dfrac{\mu_0 i_d}{2\pi r} = 15.1\ \mathrm{nT}$.

22. (a) Eq. 32-11 applies (though the last term is zero) but we must be careful with $i_{d,\mathrm{enc}}$. It is the enclosed portion of the displacement current. Thus Eq. 32-17 (which derives from Eq. 32-11) becomes, with i_d replaced with $i_{d,\mathrm{enc}}$,

$$B = \frac{\mu_0 i_{d\,\text{enc}}}{2\pi r} = \frac{\mu_0 (3.00\text{ A})(r/R)}{2\pi r}$$

which yields (after canceling *r*, and setting R = 0.0300 m) B = 20.0 μT.

(b) Here i_d = 3.00 A, and we get $B = \dfrac{\mu_0 i_d}{2\pi r} = 12.0\ \mu\text{T}$.

23. **THINK** The electric field between the plates in a parallel-plate capacitor is changing, so there is a nonzero displacement current $i_d = \varepsilon_0 (d\Phi_E/dt)$ between the plates.

EXPRESS Let A be the area of a plate and E be the magnitude of the electric field between the plates. The field between the plates is uniform, so $E = V/d$, where V is the potential difference across the plates and d is the plate separation. The current into the positive plate of the capacitor is

$$i = \frac{dq}{dt} = \frac{d}{dt}(CV) = C\frac{dV}{dt} = \frac{\varepsilon_0 A}{d}\frac{d(Ed)}{dt} = \varepsilon_0 A \frac{dE}{dt} = \varepsilon_0 \frac{d\Phi_E}{dt},$$

which is the same as the displacement current.

ANALYZE (a) Thus, at any instant the displacement current i_d in the gap between the plates equals the conduction current i in the wires: $i_d = i$ = 2.0 A.

(b) The rate of change of the electric field is

$$\frac{dE}{dt} = \frac{1}{\varepsilon_0 A}\left(\varepsilon_0 \frac{d\Phi_E}{dt}\right) = \frac{i_d}{\varepsilon_0 A} = \frac{2.0\,\text{A}}{\left(8.85\times10^{-12}\ \text{F/m}\right)\left(1.0\,\text{m}\right)^2} = 2.3\times10^{11}\ \frac{\text{V}}{\text{m}\cdot\text{s}}.$$

(c) The displacement current through the indicated path is

$$i_d' = i_d\left(\frac{d^2}{L^2}\right) = (2.0\ \text{A})\left(\frac{0.50\text{m}}{1.0\text{m}}\right)^2 = 0.50\ \text{A}.$$

(d) The integral of the field around the indicated path is

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 i_d' = \left(1.26\times10^{-16}\ \text{H/m}\right)\left(0.50\,\text{A}\right) = 6.3\times10^{-7}\ \text{T}\cdot\text{m}.$$

LEARN the displacement through the dashed path is proportional to the area encircled by the path since the displacement current is uniformly distributed over the full plate area.

24. (a) From Eq. 32-10,

$$i_d = \varepsilon_0 \frac{d\Phi_E}{dt} = \varepsilon_0 A \frac{dE}{dt} \varepsilon_0 A \frac{d}{dt}\Big[\left(4.0\times10^5\right)-\left(6.0\times10^4 t\right)\Big] = -\varepsilon_0 A\left(6.0\times10^4\ \text{V/m}\cdot\text{s}\right)$$
$$= -\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(4.0\times10^{-2}\text{m}^2\right)\left(6.0\times10^4\ \text{V/m}\cdot\text{s}\right)$$
$$= -2.1\times10^{-8}\,\text{A}.$$

Thus, the magnitude of the displacement current is $|i_d| = 2.1\times10^{-8}\,\text{A}$.

(b) The negative sign in i_d implies that the direction is downward.

(c) If one draws a counterclockwise circular loop s around the plates, then according to Eq. 32-18,

$$\oint_s \vec{B}\cdot d\vec{s} = \mu_0 i_d < 0,$$

which means that $\vec{B}\cdot d\vec{s} < 0$. Thus $\vec{B}$ must be clockwise.

25. (a) We use $\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{\text{enclosed}}$ to find

$$B = \frac{\mu_0 I_{\text{enclosed}}}{2\pi r} = \frac{\mu_0\left(J_d \pi r^2\right)}{2\pi r} = \frac{1}{2}\mu_0 J_d r = \frac{1}{2}\left(1.26\times10^{-6}\ \text{H/m}\right)\left(20\,\text{A/m}^2\right)\left(50\times10^{-3}\text{m}\right)$$
$$= 6.3\times10^{-7}\,\text{T}.$$

(b) From $i_d = J_d \pi r^2 = \varepsilon_0 \dfrac{d\Phi_E}{dt} = \varepsilon_0 \pi r^2 \dfrac{dE}{dt}$, we get

$$\frac{dE}{dt} = \frac{J_d}{\varepsilon_0} = \frac{20\,\text{A/m}^2}{8.85\times10^{-12}\ \text{F/m}} = 2.3\times10^{12}\ \frac{\text{V}}{\text{m}\cdot\text{s}}.$$

26. (a) Since $i = i_d$ (Eq. 32-15) then the portion of displacement current enclosed is

$$i_{d,\text{enc}} = i\frac{\pi\left(R/3\right)^2}{\pi R^2} = \frac{i}{9} = 1.33\,\text{A}.$$

(b) We see from Sample Problem 32.01 — "Magnetic field induced by changing electric field" that the maximum field is at $r = R$ and that (in the interior) the field is simply proportional to r. Therefore,

$$\frac{B}{B_{\max}} = \frac{3.00\,\text{mT}}{12.0\,\text{mT}} = \frac{r}{R}$$

which yields $r = R/4 = (1.20\ \text{cm})/4 = 0.300\ \text{cm}$.

(c) We now look for a solution in the exterior region, where the field is inversely proportional to r (by Eq. 32-17):

$$\frac{B}{B_{\max}} = \frac{3.00\,\text{mT}}{12.0\,\text{mT}} = \frac{R}{r}$$

which yields $r = 4R = 4(1.20\text{ cm}) = 4.80$ cm.

27. (a) In region a of the graph,

$$|i_d| = \varepsilon_0 \left|\frac{d\Phi_E}{dt}\right| = \varepsilon_0 A\left|\frac{dE}{dt}\right| = \left(8.85\times10^{-12}\text{ F/m}\right)\left(1.6\,\text{m}^2\right)\left|\frac{4.5\times10^5\text{ N/C}-6.0\times10^5\text{ N/C}}{4.0\times10^{-6}\text{s}}\right| = 0.71\text{A}.$$

(b) $i_d \propto dE/dt = 0$.

(c) In region c of the graph,

$$|i_d| = \varepsilon_0 A\left|\frac{dE}{dt}\right| = \left(8.85\times10^{-12}\text{ F/m}\right)\left(1.6\text{m}^2\right)\left|\frac{-4.0\times10^5\text{ N/C}}{2.0\times10^{-6}\text{s}}\right| = 2.8\text{A}.$$

28. (a) Figure 32-35 indicates that $i = 4.0$ A when $t = 20$ ms. Thus,

$$B_i = \mu_0 i/2\pi r = 0.089\text{ mT}.$$

(b) Figure 32-35 indicates that $i = 8.0$ A when $t = 40$ ms. Thus, $B_i \approx 0.18$ mT.

(c) Figure 32-35 indicates that $i = 10$ A when $t > 50$ ms. Thus, $B_i \approx 0.220$ mT.

(d) Equation 32-4 gives the displacement current in terms of the time-derivative of the electric field: $i_d = \varepsilon_0 A(dE/dt)$, but using Eq. 26-5 and Eq. 26-10 we have $E = \rho i/A$ (in terms of the real current); therefore, $i_d = \varepsilon_0\rho(di/dt)$. For $0 < t < 50$ ms, Fig. 32-35 indicates that $di/dt = 200$ A/s. Thus,

$$B_{id} = \mu_0 i_d/2\pi r = 6.4\times10^{-22}\text{ T}.$$

(e) As in (d), $B_{id} = \mu_0 i_d/2\pi r = 6.4\times10^{-22}$ T.

(f) Here $di/dt = 0$, so (by the reasoning in the previous step) $B = 0$.

(g) By the right-hand rule, the direction of $\vec{B}_i$ at $t = 20$ s is out of the page.

(h) By the right-hand rule, the direction of $\vec{B}_{id}$ at $t = 20$ s is out of the page.

29. (a) At any instant the displacement current i_d in the gap between the plates equals the conduction current i in the wires. Thus $i_{\max} = i_{d\,\max} = 7.60\ \mu$A.

(b) Since $i_d = \varepsilon_0 (d\Phi_E/dt)$, we have

$$\left(\frac{d\Phi_E}{dt}\right)_{\max} = \frac{i_{d\,\max}}{\varepsilon_0} = \frac{7.60\times10^{-6}\,\text{A}}{8.85\times10^{-12}\,\text{F/m}} = 8.59\times10^{5}\,\text{V}\cdot\text{m/s}.$$

(c) Let the area plate be A and the plate separation be d. The displacement current is

$$i_d = \varepsilon_0\frac{d\Phi_E}{dt} = \varepsilon_0\frac{d}{dt}(AE) = \varepsilon_0 A\frac{d}{dt}\left(\frac{V}{d}\right) = \frac{\varepsilon_0 A}{d}\left(\frac{dV}{dt}\right).$$

Now the potential difference across the capacitor is the same in magnitude as the emf of the generator, so $V = \varepsilon_m \sin \omega t$ and $dV/dt = \omega\varepsilon_m \cos \omega t$. Thus, $i_d = (\varepsilon_0 A\omega\varepsilon_m / d)\cos\omega t$ and $i_{d\,\max} = \varepsilon_0 A\omega\varepsilon_m / d$. This means

$$d = \frac{\varepsilon_0 A\omega\varepsilon_m}{i_{d\,\max}} = \frac{(8.85\times10^{-12}\,\text{F/m})\pi(0.180\,\text{m})^2(130\,\text{rad/s})(220\,\text{V})}{7.60\times10^{-6}\,\text{A}} = 3.39\times10^{-3}\,\text{m},$$

where $A = \pi R^2$ was used.

(d) We use the Ampere-Maxwell law in the form $\oint \vec{B}\cdot d\vec{s} = \mu_0 I_d$, where the path of integration is a circle of radius r between the plates and parallel to them. I_d is the displacement current through the area bounded by the path of integration. Since the displacement current density is uniform between the plates, $I_d = (r^2/R^2)i_d$, where i_d is the total displacement current between the plates and R is the plate radius. The field lines are circles centered on the axis of the plates, so $\vec{B}$ is parallel to $d\vec{s}$. The field has constant magnitude around the circular path, so $\oint \vec{B}\cdot d\vec{s} = 2\pi rB$. Thus,

$$2\pi rB = \mu_0\left(\frac{r^2}{R^2}\right)i_d \quad\Rightarrow\quad B = \frac{\mu_0 i_d r}{2\pi R^2}.$$

The maximum magnetic field is given by

$$B_{\max} = \frac{\mu_0 i_{d\,\max} r}{2\pi R^2} = \frac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(7.6\times10^{-6}\,\text{A})(0.110\text{m})}{2\pi(0.180\,\text{m})^2} = 5.16\times10^{-12}\,\text{T}.$$

30. (a) The flux through Arizona is

$$\Phi = -B_r A = -(43\times10^{-6}\,\text{T})(295{,}000\,\text{km}^2)(10^3\,\text{m/km})^2 = -1.3\times10^{7}\,\text{Wb},$$

inward. By Gauss' law this is equal to the negative value of the flux Φ' through the rest of the surface of the Earth. So $\Phi' = 1.3 \times 10^7$ Wb.

(b) The direction is outward.

31. The horizontal component of the Earth's magnetic field is given by $B_h = B\cos\phi_i$, where B is the magnitude of the field and ϕ_i is the inclination angle. Thus

$$B = \frac{B_h}{\cos\phi_i} = \frac{16\,\mu\text{T}}{\cos 73°} = 55\,\mu\text{T}\ .$$

32. (a) The potential energy of the atom in association with the presence of an external magnetic field $\vec{B}_{\text{ext}}$ is given by Eqs. 32-31 and 32-32:

$$U = -\mu_{\text{orb}} \cdot \vec{B}_{\text{ext}} = -\mu_{\text{orb},z} B_{\text{ext}} = -m_\ell \mu_B B_{\text{ext}}\,.$$

For level E_1 there is no change in energy as a result of the introduction of $\vec{B}_{\text{ext}}$, so $U \propto m_\ell$ = 0, meaning that $m_\ell = 0$ for this level.

(b) For level E_2 the single level splits into a triplet (i.e., three separate ones) in the presence of $\vec{B}_{\text{ext}}$, meaning that there are three different values of m_ℓ. The middle one in the triplet is unshifted from the original value of E_2 so its m_ℓ must be equal to 0. The other two in the triplet then correspond to $m_\ell = -1$ and $m_\ell = +1$, respectively.

(c) For any pair of adjacent levels in the triplet, $|\Delta m_\ell| = 1$. Thus, the spacing is given by

$$\Delta U = |\Delta(-m_\ell \mu_B B)| = |\Delta m_\ell|\,\mu_B B = \mu_B B = (9.27\times10^{-24}\ \text{J/T})(0.50\text{T}) = 4.64\times10^{-24}\ \text{J.}$$

33. **THINK** An electron in an atom has both orbital angular momentum and spin angular momentum; the *z* components of the angular momenta are quantized.

EXPRESS The *z* component of the orbital angular momentum is give by

$$L_{\text{orb},z} = \frac{m_\ell h}{2\pi}$$

where h is the Planck constant and m_ℓ is the orbital magnetic quantum number. The corresponding *z* component of the orbital magnetic dipole moment is

$$\mu_{\text{orb},z} = -m_\ell \mu_{\text{B}}$$

where $\mu_B = eh/4\pi m$ is the Bohr magneton. When placed in an external field $\vec{B}_{\text{ext}}$, the energy associated with the orientation of $\vec{\mu}_{\text{orb}}$ is given by

$$U = -\vec{\mu}_{\text{orb}} \cdot \vec{B}_{\text{ext}}.$$

ANALYZE (a) Since $m_\ell = 0$, $L_{\text{orb},z} = m_\ell h/2\pi = 0$.

(b) Since $m_\ell = 0$, $\mu_{\text{orb},z} = -m_\ell \mu_B = 0$.

(c) Since $m_\ell = 0$, then from Eq. 32-32, $U = -\mu_{\text{orb},z}B_{\text{ext}} = -m_\ell \mu_B B_{\text{ext}} = 0$.

(d) Regardless of the value of m_ℓ, we find for the spin part

$$U = -\mu_{s,z}B = \pm\mu_B B = \pm\left(9.27\times10^{-24}\text{ J/T}\right)\left(35\,\text{mT}\right) = \pm3.2\times10^{-25}\text{ J} .$$

(e) Now $m_\ell = -3$, so

$$L_{\text{orb},z} = \frac{m_\ell h}{2\pi} = \frac{(-3)\left(6.63\times10^{-27}\text{ J}\cdot\text{s}\right)}{2\pi} = -3.16\times10^{-34}\text{ J}\cdot\text{s} \approx -3.2\times10^{-34}\text{ J}\cdot\text{s}$$

(f) and $\mu_{\text{orb},z} = -m_\ell\mu_B = -(-3)\left(9.27\times10^{-24}\text{ J/T}\right) = 2.78\times10^{-23}\text{ J/T} \approx 2.8\times10^{-23}\text{ J/T}$.

(g) The potential energy associated with the electron's orbital magnetic moment is now

$$U = -\mu_{\text{orb},z}B_{\text{ext}} = -\left(2.78\times10^{-23}\text{ J/T}\right)\left(35\times10^{-3}\text{ T}\right) = -9.7\times10^{-25}\text{ J}.$$

(h) On the other hand, the potential energy associated with the electron spin, being independent of m_ℓ, remains the same: $\pm3.2 \times 10^{-25}$ J.

LEARN Spin is an intrinsic angular momentum that is not associated with the motion of the electron. Its z component is quantized, and can be written as

$$S_z = \frac{m_s h}{2\pi}$$

where $m_s = \pm 1/2$ is the spin magnetic quantum number.

34. We use Eq. 32-27 to obtain

$$\Delta U = -\Delta(\mu_{s,z}B) = -B\Delta\mu_{s,z},$$

where $\mu_{s,z} = \pm eh/4\pi m_e = \pm\mu_B$ (see Eqs. 32-24 and 32-25). Thus,

$$\Delta U = -B\ \mu_B - (-\mu_B)\ = 2\mu_B B = 2(9.27\times10^{-24}\ \text{J/T})(0.25\text{T}) = 4.6\times10^{-24}\ \text{J}\ .$$

35. We use Eq. 32-31: $\mu_{\text{orb},\,z} = -m_\ell\,\mu_B$.

(a) For m_ℓ = 1, $\mu_{\text{orb},z}$ = –(1) (9.3 × 10^{-24} J/T) = –9.3 × 10^{-24} J/T.

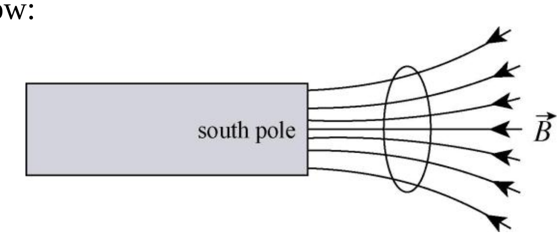
(b) For m_ℓ = –2, $\mu_{\text{orb},z}$ = –(–2) (9.3 × 10^{-24} J/T) = 1.9 × 10^{-23} J/T.

36. Combining Eq. 32-27 with Eqs. 32-22 and 32-23, we see that the energy difference is

$$\Delta U = 2\mu_B B$$

where μ_B is the Bohr magneton (given in Eq. 32-25). With ΔU = 6.00 × 10^{-25} J, we obtain B = 32.3 mT.

37. (a) A sketch of the field lines (due to the presence of the bar magnet) in the vicinity of the loop is shown below:



(b) The primary conclusion of Section 32-9 is two-fold: $\vec{u}$ is opposite to $\vec{B}$, and the effect of $\vec{F}$ is to move the material toward regions of smaller $|\vec{B}|$ values. The direction of the magnetic moment vector (of our loop) is toward the right in our sketch, or in the +*x* direction.

(c) The direction of the current is clockwise (from the perspective of the bar magnet).

(d) Since the size of $|\vec{B}|$ relates to the "crowdedness" of the field lines, we see that $\vec{F}$ is toward the right in our sketch, or in the +*x* direction.

38. An electric field with circular field lines is induced as the magnetic field is turned on. Suppose the magnetic field increases linearly from zero to B in time t. According to Eq. 31-27, the magnitude of the electric field at the orbit is given by

$$E = \left(\frac{r}{2}\right)\frac{dB}{dt} = \left(\frac{r}{2}\right)\frac{B}{t}\ ,$$

where r is the radius of the orbit. The induced electric field is tangent to the orbit and changes the speed of the electron, the change in speed being given by

$$\Delta v = at = \frac{eE}{m_e}t = \left(\frac{e}{m_e}\right)\left(\frac{r}{2}\right)\left(\frac{B}{t}\right)t = \frac{erB}{2m_e} .$$

The average current associated with the circulating electron is $i = ev/2\pi r$ and the dipole moment is

$$\mu = Ai = \left(\pi r^2\right)\left(\frac{ev}{2\pi r}\right) = \frac{1}{2}evr .$$

The change in the dipole moment is

$$\Delta\mu = \frac{1}{2}er\Delta v = \frac{1}{2}er\left(\frac{erB}{2m_e}\right) = \frac{e^2r^2B}{4m_e} .$$

39. For the measurements carried out, the largest ratio of the magnetic field to the temperature is (0.50 T)/(10 K) = 0.050 T/K. Look at Fig. 32-14 to see if this is in the region where the magnetization is a linear function of the ratio. It is quite close to the origin, so we conclude that the magnetization obeys Curie's law.

40. (a) From Fig. 32-14 we estimate a slope of B/T = 0.50 T/K when $M/M_{\max}$ = 50%. So

$$B = 0.50\text{ T} = (0.50\text{ T/K})(300\text{ K}) = 1.5\times10^2\text{ T}.$$

(b) Similarly, now $B/T \approx 2$ so $B = (2)(300) = 6.0\times10^2$ T.

(c) Except for very short times and in very small volumes, these values are not attainable in the lab.

41. **THINK** As defined in Eq. 32-38, magnetization is the dipole moment per unit volume.

EXPRESS Let M be the magnetization and $\mathcal{V}$ be the volume of the cylinder ($\mathcal{V} = \pi r^2 L$, where r is the radius of the cylinder and L is its length). The dipole moment is given by $\mu = M\mathcal{V}$.

ANALYZE Substituting the values given, we obtain

$$\mu = M\pi r^2 L = \left(5.30\times10^3\text{ A/m}\right)\pi\left(0.500\times10^{-2}\text{ m}\right)^2\left(5.00\times10^{-2}\text{ m}\right) = 2.08\times10^{-2}\text{ J/T} .$$

LEARN In a sample with N atoms, the magnetization reaches maximum, or saturation, when all the dipoles are completely aligned, leading to $M_{\max} = N\mu/\mathcal{V}$.

42. Let

$$K = \frac{3}{2}kT = \left|\vec{\mu}\cdot\vec{B} - \left(-\vec{\mu}\cdot\vec{B}\right)\right| = 2\mu B$$

which leads to

$$T = \frac{4\mu B}{3k} = \frac{4\left(1.0\times10^{-23}\ \text{J/T}\right)\left(0.50\ \text{T}\right)}{3\left(1.38\times10^{-23}\ \text{J/K}\right)} = 0.48\ \text{K}\ .$$

43. (a) A charge e traveling with uniform speed v around a circular path of radius r takes time $T = 2\pi r/v$ to complete one orbit, so the average current is

$$i = \frac{e}{T} = \frac{ev}{2\pi r}.$$

The magnitude of the dipole moment is this multiplied by the area of the orbit:

$$\mu = \frac{ev}{2\pi r}\pi r^2 = \frac{evr}{2}.$$

Since the magnetic force with magnitude evB is centripetal, Newton's law yields $evB = m_e v^2/r$, so $r = m_e v / eB$. Thus,

$$\mu = \frac{1}{2}(ev)\left(\frac{m_e v}{eB}\right) = \left(\frac{1}{B}\right)\left(\frac{1}{2}m_e v^2\right) = \frac{K_e}{B}.$$

The magnetic force $-e\vec{v}\times\vec{B}$ must point toward the center of the circular path. If the magnetic field is directed out of the page (defined to be $+z$ direction), the electron will travel counterclockwise around the circle. Since the electron is negative, the current is in the opposite direction, clockwise and, by the right-hand rule for dipole moments, the dipole moment is into the page, or in the $-z$ direction. That is, the dipole moment is directed opposite to the magnetic field vector.

(b) We note that the charge canceled in the derivation of $\mu = K_e/B$. Thus, the relation $\mu = K_i/B$ holds for a positive ion.

(c) The direction of the dipole moment is $-z$, opposite to the magnetic field.

(d) The magnetization is given by $M = \mu_e n_e + \mu_i n_i$, where μ_e is the dipole moment of an electron, n_e is the electron concentration, μ_i is the dipole moment of an ion, and n_i is the ion concentration. Since $n_e = n_i$, we may write n for both concentrations. We substitute $\mu_e = K_e/B$ and $\mu_i = K_i/B$ to obtain

$$M = \frac{n}{B}\left(K_e + K_i\right) = \frac{5.3\times10^{21}\text{m}^{-3}}{1.2\text{T}}\left(6.2\times10^{-20}\text{J}+7.6\times10^{-21}\text{J}\right) = 3.1\times10^{2}\ \text{A/m}.$$

44. Section 32-10 explains the terms used in this problem and the connection between M and μ. The graph in Fig. 32-39 gives a slope of

$$\frac{M/M_{\text{max}}}{B_{\text{ext}}/T}=\frac{0.15}{0.20\ \text{T/K}}=0.75\ \text{K/T}\ .$$

Thus we can write

$$\frac{\mu}{\mu_{\text{max}}}=(0.75\ \text{K/T})\frac{0.800\ \text{T}}{2.00\ \text{K}}=0.30\,.$$

45. **THINK** According to statistical mechanics, the probability of a magnetic dipole moment placed in an external magnetic field having energy U is $P=e^{-U/kT}$, where k is the Boltzmann's constant.

EXPRESS The orientation energy of a dipole in a magnetic field is given by $U=-\vec{\mu}\cdot\vec{B}$. So if a dipole is parallel with $\vec{B}$, then $U=-\mu B$; however, $U=+\mu B$ if the alignment is anti-parallel. We use the notation $P(\mu)=e^{\mu B/kT}$ for the probability of a dipole that is parallel to $\vec{B}$, and $P(-\mu)=e^{-\mu B/kT}$ for the probability of a dipole that is anti-parallel to the field. The magnetization may be thought of as a "weighted average" in terms of these probabilities.

ANALYZE (a) With N atoms per unit volume, we find the magnetization to be

$$M=\frac{N\mu P(\mu)-N\mu P(-\mu)}{P(\mu)+P(-\mu)}=\frac{N\mu\left(e^{\mu B/kT}-e^{-\mu B/kT}\right)}{e^{\mu B/kT}+e^{-\mu B/kT}}=N\mu\tanh\left(\frac{\mu B}{kT}\right).$$

(b) For $\mu B \ll kT$ (that is, $\mu B/kT \ll 1$) we have $e^{\pm\mu B/kT}\approx 1\pm\mu B/kT$, so

$$M=N\mu\tanh\left(\frac{\mu B}{kT}\right)\approx\frac{N\mu\,(1+\mu B/kT)-(1-\mu B/kT)}{(1+\mu B/kT)+(1-\mu B/kT)}=\frac{N\mu^2 B}{kT}\ .$$

(c) For $\mu B \gg kT$ we have $\tanh(\mu B/kT)\approx 1$, so $M=N\mu\tanh\left(\dfrac{\mu B}{kT}\right)\approx N\mu$.

(d) One can easily plot the tanh function using, for instance, a graphical calculator. One can then note the resemblance between such a plot and Fig. 32-14. By adjusting the parameters used in one's plot, the curve in Fig. 32-14 can reliably be fit with a tanh function.

LEARN As can be seen from Fig. 32-14, the magnetization M is linear in B/kT in the regime $B/T \ll 1$. On the other hand, when $B \gg T$, M approaches a constant.

46. From Eq. 29-37 (see also Eq. 29-36) we write the torque as $\tau = -\mu B_h \sin\theta$ where the minus indicates that the torque opposes the angular displacement θ (which we will assume is small and in radians). The small angle approximation leads to $\tau \approx -\mu B_h \theta$, which is an indicator for simple harmonic motion (see section 16-5, especially Eq. 16-22). Comparing with Eq. 16-23, we then find the period of oscillation is

$$T = 2\pi\sqrt{\frac{I}{\mu B_h}}$$

where I is the rotational inertial that we asked to solve for. Since the frequency is given as 0.312 Hz, then the period is $T = 1/f = 1/(0.312\text{ Hz}) = 3.21$ s. Similarly, $B_h = 18.0 \times 10^{-6}$ T and $\mu = 6.80 \times 10^{-4}$ J/T. The above relation then yields $I = 3.19 \times 10^{-9}\ \text{kg}\cdot\text{m}^2$.

47. **THINK** In this problem, we model the Earth's magnetic dipole moment with a magnetized iron sphere.

EXPRESS If the magnetization of the sphere is saturated, the total dipole moment is $\mu_{\text{total}} = N\mu$, where N is the number of iron atoms in the sphere and μ is the dipole moment of an iron atom. We wish to find the radius of an iron sphere with N iron atoms. The mass of such a sphere is Nm, where m is the mass of an iron atom. It is also given by $4\pi\rho R^3/3$, where ρ is the density of iron and R is the radius of the sphere. Thus $Nm = 4\pi\rho R^3/3$ and

$$N = \frac{4\pi\rho R^3}{3m}.$$

We substitute this into $\mu_{\text{total}} = N\mu$ to obtain

$$\mu_{\text{total}} = \frac{4\pi\rho R^3\mu}{3m} \quad\Rightarrow\quad R = \left(\frac{3m\mu_{\text{total}}}{4\pi\rho\mu}\right)^{1/3}.$$

ANALYZE (a) The mass of an iron atom is

$$m = 56\,\text{u} = (56\,\text{u})(1.66\times10^{-27}\ \text{kg/u}) = 9.30\times10^{-26}\ \text{kg}.$$

Therefore, the radius of the iron sphere is

$$R = \left[\frac{3(9.30\times10^{-26}\,\text{kg})(8.0\times10^{22}\ \text{J/T})}{4\pi(14\times10^{3}\ \text{kg/m}^3)(2.1\times10^{-23}\ \text{J/T})}\right]^{1/3} = 1.8\times10^{5}\,\text{m}.$$

(b) The volume of the sphere is $V_s = \frac{4\pi}{3}R^3 = \frac{4\pi}{3}\left(1.82\times10^5\,\text{m}\right)^3 = 2.53\times10^{16}\,\text{m}^3$ and the volume of the Earth is

$$V_E = \frac{4\pi}{3}R_E^3 = \frac{4\pi}{3}\left(6.37\times10^6\,\text{m}\right)^3 = 1.08\times10^{21}\,\text{m}^3,$$

so the fraction of the Earth's volume that is occupied by the sphere is

$$\frac{V_s}{V_E} = \frac{2.53\times10^{16}\,\text{m}^3}{1.08\times10^{21}\,\text{m}^3} = 2.3\times10^{-5}.$$

LEARN The finding that $V_s \ll V_E$ makes it unlikely that our simple model of a magnetized iron sphere could explain the origin of Earth's magnetization.

48. (a) The number of iron atoms in the iron bar is

$$N = \frac{\left(7.9\,\text{g/cm}^3\right)\left(5.0\,\text{cm}\right)\left(1.0\,\text{cm}^2\right)}{\left(55.847\,\text{g/mol}\right)/\left(6.022\times10^{23}/\text{mol}\right)} = 4.3\times10^{23}.$$

Thus the dipole moment of the iron bar is

$$\mu = \left(2.1\times10^{-23}\,\text{J/T}\right)\left(4.3\times10^{23}\right) = 8.9\,\text{A}\cdot\text{m}^2.$$

(b) $\tau = \mu B \sin 90° = (8.9\ \text{A}\cdot\text{m}^2)(1.57\ \text{T}) = 13\ \text{N}\cdot\text{m}$.

49. **THINK** Exchange coupling is a quantum phenomenon in which electron spins of one atom interact with those of neighboring atoms.

EXPRESS The field of a dipole along its axis is given by Eq. 30-29:

$$B = \frac{\mu_0}{2\pi}\frac{\mu}{z^3},$$

where μ is the dipole moment and z is the distance from the dipole. The energy of a magnetic dipole $\vec{\mu}$ in a magnetic field $\vec{B}$ is given by

$$U = -\vec{\mu}\cdot\vec{B} = -\mu B\cos\phi,$$

where ϕ is the angle between the dipole moment and the field.

ANALYZE (a) Thus, the magnitude of the magnitude field at a distance 10 nm away from the atom is

$$B=\frac{\left(4\pi\times10^{-7}\ \mathrm{T\cdot m/A}\right)\left(1.5\times10^{-23}\ \mathrm{J/T}\right)}{2\pi\left(10\times10^{-9}\ \mathrm{m}\right)}=3.0\times10^{-6}\ \mathrm{T}.$$

(b) The energy required to turn it end-for-end (from $\phi = 0°$ to $\phi = 180°$) is

$$\Delta U = 2\mu B = 2\left(1.5\times10^{-23}\ \mathrm{J/T}\right)\left(3.0\times10^{-6}\mathrm{T}\right) = 9.0\times10^{-29}\,\mathrm{J} = 5.6\times10^{-10}\ \mathrm{eV}.$$

(c) The mean kinetic energy of translation at room temperature is about 0.04 eV. Thus, if dipole-dipole interactions were responsible for aligning dipoles, collisions would easily randomize the directions of the moments and they would not remain aligned.

LEARN The persistent alignment of magnetic dipole moments despite the randomizing tendency due to thermal agitation is what gives the ferromagnetic materials their permanent magnetism.

50. (a) Equation 29-36 gives

$$\tau = \mu_{\text{rod}} B \sin\theta = (2700\ \mathrm{A/m})(0.06\ \mathrm{m})\pi(0.003\ \mathrm{m})^2(0.035\ \mathrm{T})\sin(68°) = 1.49\times10^{-4}\ \mathrm{N\cdot m}.$$

We have used the fact that the volume of a cylinder is its length times its (circular) cross sectional area.

(b) Using Eq. 29-38, we have

$$\begin{aligned}\Delta U &= -\mu_{\text{rod}} B(\cos\theta_f - \cos\theta_i)\\ &= -(2700\ \mathrm{A/m})(0.06\ \mathrm{m})\pi(0.003\mathrm{m})^2(0.035\mathrm{T})[\cos(34°) - \cos(68°)]\\ &= -72.9\ \mu\mathrm{J}.\end{aligned}$$

51. The saturation magnetization corresponds to complete alignment of all atomic dipoles and is given by $M_{\text{sat}} = \mu n$, where n is the number of atoms per unit volume and μ is the magnetic dipole moment of an atom. The number of nickel atoms per unit volume is $n = \rho/m$, where ρ is the density of nickel. The mass of a single nickel atom is calculated using $m = M/N_A$, where M is the atomic mass of nickel and N_A is Avogadro's constant. Thus,

$$\begin{aligned}n&=\frac{\rho N_A}{M}=\frac{\left(8.90\,\mathrm{g/cm^3}\right)\left(6.02\times10^{23}\ \mathrm{atoms/mol}\right)}{58.71\mathrm{g/mol}}=9.126\times10^{22}\ \mathrm{atoms/cm^3}\\ &=9.126\times10^{28}\ \mathrm{atoms/m^3}.\end{aligned}$$

The dipole moment of a single atom of nickel is

$$\mu=\frac{M_{\text{sat}}}{n}=\frac{4.70\times10^{5}\ \mathrm{A/m}}{9.126\times10^{28}\mathrm{m^3}}=5.15\times10^{-24}\,\mathrm{A\cdot m^2}.$$

52. The Curie temperature for iron is 770°C. If x is the depth at which the temperature has this value, then 10°C + (30°C/km)x = 770°C. Therefore,

$$x = \frac{770°\,\mathrm{C} - 10°\,\mathrm{C}}{30°\,\mathrm{C/km}} = 25\,\mathrm{km}.$$

53. (a) The magnitude of the toroidal field is given by $B_0 = \mu_0 n i_p$, where n is the number of turns per unit length of toroid and i_p is the current required to produce the field (in the absence of the ferromagnetic material). We use the average radius (r_{avg} = 5.5 cm) to calculate n:

$$n = \frac{N}{2\pi r_{\text{avg}}} = \frac{400\ \text{turns}}{2\pi(5.5\times10^{-2}\,\mathrm{m})} = 1.16\times10^3\ \text{turns/m}\ .$$

Thus,

$$i_p = \frac{B_0}{\mu_0 n} = \frac{0.20\times10^{-3}\ \mathrm{T}}{(4\pi\times10^{-7}\,\mathrm{T\cdot m/A})(1.16\times10^3\,/\,\mathrm{m})} = 0.14\ \mathrm{A}\ .$$

(b) If Φ is the magnetic flux through the secondary coil, then the magnitude of the emf induced in that coil is $\varepsilon = N(d\Phi/dt)$ and the current in the secondary is $i_s = \varepsilon/R$, where R is the resistance of the coil. Thus,

$$i_s = \left(\frac{N}{R}\right)\frac{d\Phi}{dt}.$$

The charge that passes through the secondary when the primary current is turned on is

$$q = \int i_s\ dt = \frac{N}{R}\int\frac{d\Phi}{dt}\,dt = \frac{N}{R}\int_0^{\Phi}\ d\Phi = \frac{N\Phi}{R}\ .$$

The magnetic field through the secondary coil has magnitude $B = B_0 + B_M = 801B_0$, where B_M is the field of the magnetic dipoles in the magnetic material. The total field is perpendicular to the plane of the secondary coil, so the magnetic flux is $\Phi = AB$, where A is the area of the Rowland ring (the field is inside the ring, not in the region between the ring and coil). If r is the radius of the ring's cross section, then $A = \pi r^2$. Thus,

$$\Phi = 801\pi r^2 B_0\ .$$

The radius r is (6.0 cm – 5.0 cm)/2 = 0.50 cm and

$$\Phi = 801\pi(0.50\times10^{-2}\ \mathrm{m})^2(0.20\times10^{-3}\mathrm{T}) = 1.26\times10^{-5}\ \mathrm{Wb}\ .$$

Consequently, $q = \dfrac{50(1.26\times10^{-5}\ \mathrm{Wb})}{8.0\ \Omega} = 7.9\times10^{-5}\ \mathrm{C}$.

54. (a) At a distance r from the center of the Earth, the magnitude of the magnetic field is given by

$$B = \frac{\mu_0 \mu}{4\pi r^3}\sqrt{1+3\sin^2\lambda_m}\ ,$$

where μ is the Earth's dipole moment and λ_m is the magnetic latitude. The ratio of the field magnitudes for two different distances at the same latitude is

$$\frac{B_2}{B_1} = \frac{r_1^3}{r_2^3}\ .$$

With B_1 being the value at the surface and B_2 being half of B_1, we set r_1 equal to the radius R_e of the Earth and r_2 equal to $R_e + h$, where h is altitude at which B is half its value at the surface. Thus,

$$\frac{1}{2} = \frac{R_e^3}{\left(R_e + h\right)^3}\ .$$

Taking the cube root of both sides and solving for h, we get

$$h = \left(2^{1/3} - 1\right)R_e = \left(2^{1/3} - 1\right)\left(6370\,\text{km}\right) = 1.66\times10^3\,\text{km}.$$

(b) For maximum B, we set $\sin\lambda_m = 1.00$. Also, $r = 6370$ km – 2900 km = 3470 km. Thus,

$$B_{\max} = \frac{\mu_0\mu}{4\pi r^3}\sqrt{1+3\sin^2\lambda_m} = \frac{\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(8.00\times10^{22}\,\text{A}\cdot\text{m}^2\right)}{4\pi\left(3.47\times10^6\,\text{m}\right)^3}\sqrt{1+3\left(1.00\right)^2}$$
$$= 3.83\times10^{-4}\,\text{T}.$$

(c) The angle between the magnetic axis and the rotational axis of the Earth is 11.5°, so $\lambda_m = 90.0° - 11.5° = 78.5°$ at Earth's geographic north pole. Also $r = R_e = 6370$ km. Thus,

$$B = \frac{\mu_0\mu}{4\pi R_E^3}\sqrt{1+3\sin^2\lambda_m} = \frac{\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(8.0\times10^{22}\ \text{J/T}\right)\sqrt{1+3\sin^2 78.5°}}{4\pi\left(6.37\times10^6\,\text{m}\right)^3}$$
$$= 6.11\times10^{-5}\,\text{T}.$$

(d) $\phi_i = \tan^{-1}\left(2\tan 78.5°\right) = 84.2°$.

(e) A plausible explanation to the discrepancy between the calculated and measured values of the Earth's magnetic field is that the formulas we used are based on dipole approximation, which does not accurately represent the Earth's actual magnetic field distribution on or near its surface. (Incidentally, the dipole approximation becomes more reliable when we calculate the Earth's magnetic field far from its center.)

55. (a) From $\mu = iA = i\pi R_e^2$ we get

$$i = \frac{\mu}{\pi R_e^2} = \frac{8.0\times10^{22}\,\mathrm{J/T}}{\pi(6.37\times10^6\ \mathrm{m})^2} = 6.3\times10^8\,\mathrm{A}\ .$$

(b) Yes, because far away from the Earth the fields of both the Earth itself and the current loop are dipole fields. If these two dipoles cancel each other out, then the net field will be zero.

(c) No, because the field of the current loop is not that of a magnetic dipole in the region close to the loop.

56. (a) The period of rotation is $T = 2\pi/\omega$, and in this time all the charge passes any fixed point near the ring. The average current is $i = q/T = q\omega/2\pi$ and the magnitude of the magnetic dipole moment is

$$\mu = iA = \frac{q\omega}{2\pi}\pi r^2 = \frac{1}{2}q\omega r^2\ .$$

(b) We curl the fingers of our right hand in the direction of rotation. Since the charge is positive, the thumb points in the direction of the dipole moment. It is the same as the direction of the angular momentum vector of the ring.

57. The interacting potential energy between the magnetic dipole of the compass and the Earth's magnetic field is

$$U = -\vec{\mu}\cdot\vec{B}_e = -\mu B_e\cos\theta,$$

where θ is the angle between $\vec{\mu}$ and $\vec{B}_e$. For small angle θ,

$$U(\theta) = -\mu B_e\cos\theta \approx -\mu B_e\left(1-\frac{\theta^2}{2}\right) = \frac{1}{2}\kappa\theta^2 - \mu B_e$$

where $\kappa = \mu B_e$. Conservation of energy for the compass then gives

$$\frac{1}{2}I\left(\frac{d\theta}{dt}\right)^2 + \frac{1}{2}\kappa\theta^2 = \text{const.}$$

This is to be compared with the following expression for the mechanical energy of a spring-mass system:

$$\frac{1}{2}m\left(\frac{dx}{dt}\right)^2 + \frac{1}{2}kx^2 = \text{const.}\,,$$

which yields $\omega = \sqrt{k/m}$. So by analogy, in our case

$$\omega = \sqrt{\frac{\kappa}{I}} = \sqrt{\frac{\mu B_e}{I}} = \sqrt{\frac{\mu B_e}{ml^2/12}} ,$$

which leads to

$$\mu = \frac{ml^2\omega^2}{12B_e} = \frac{(0.050\,\text{kg})(4.0\times10^{-2}\ \text{m})^2(45\,\text{rad/s})^2}{12(16\times10^{-6}\ \text{T})} = 8.4\times10^2\ \text{J/T} .$$

58. (a) Equation 30-22 gives $B = \dfrac{\mu_0 i r}{2\pi R^2} = 222\ \mu\text{T}$.

(b) Equation 30-19 (or Eq. 30-6) gives $B = \dfrac{\mu_0 i}{2\pi r} = 167\ \mu\text{T}$.

(c) As in part (b), we obtain a field of $B = \dfrac{\mu_0 i}{2\pi r} = 22.7\ \mu\text{T}$.

(d) Equation 32-16 (with Eq. 32-15) gives $B = \dfrac{\mu_0 i_d r}{2\pi R^2} = 1.25\ \mu\text{T}$.

(e) As in part (d), we get $B = \dfrac{\mu_0 i_d r}{2\pi R^2} = 3.75\ \mu\text{T}$.

(f) Equation 32-17 yields B = 22.7 μT.

(g) Because the displacement current in the gap is spread over a larger cross-sectional area, values of B within that area are relatively small. Outside that cross-sectional area, the two values of B are identical.

59. (a) We use the result of part (a) in Sample Problem 32.01 — "Magnetic field induced by changing electric field:"

$$B = \frac{\mu_0\varepsilon_0 r}{2}\frac{dE}{dt} \qquad (\text{for } r \le R) ,$$

where r = 0.80R , and

$$\frac{dE}{dt} = \frac{d}{dt}\left(\frac{V}{d}\right) = \frac{1}{d}\frac{d}{dt}\left(V_0 e^{-t/\tau}\right) = -\frac{V_0}{\tau d}e^{-t/\tau} .$$

Here V_0 = 100 V. Thus,

$$B(t)=\left(\frac{\mu_0\varepsilon_0 r}{2}\right)\left(-\frac{V_0}{\tau d}e^{-t/\tau}\right)=-\frac{\mu_0\varepsilon_0 V_0 r}{2\tau d}e^{-t/\tau}$$

$$=-\frac{(4\pi\times10^{-7}\,\mathrm{T\cdot m/A})(8.85\times10^{-12}\,\frac{\mathrm{C}^2}{\mathrm{N\cdot m}^2})(100\,\mathrm{V})(0.80)(16\,\mathrm{mm})}{2(12\times10^{-3}\,\mathrm{s})(5.0\,\mathrm{mm})}e^{-t/12\,\mathrm{ms}}$$

$$=-(1.2\times10^{-13}\,\mathrm{T})e^{-t/12\,\mathrm{ms}}\ .$$

The magnitude is $|B(t)|=(1.2\times10^{-13}\,\mathrm{T})e^{-t/12\,\mathrm{ms}}$.

(b) At time $t = 3\tau$, $B(t) = -(1.2 \times 10^{-13}\ \mathrm{T})e^{-3\tau/\tau} = -5.9 \times 10^{-15}$ T, with a magnitude $|B(t)|=$ 5.9×10^{-15} T.

60. (a) From Eq. 32-1, we have

$$(\Phi_B)_{\mathrm{in}}=(\Phi_B)_{\mathrm{out}}=0.0070\,\mathrm{Wb}+(0.40\,\mathrm{T})(\pi r^2)=9.2\times10^{-3}\,\mathrm{Wb}.$$

Thus, the magnetic of the magnetic flux is 9.2 mWb.

(b) The flux is inward.

61. **THINK** The Earth's magnetic field at a given latitude has both horizontal and vertical components.

EXPRESS Let B_h and B_v be the horizontal and vertical components of the Earth's magnetic field, respectively. Since B_h and B_v are perpendicular to each other, the Pythagorean theorem leads to $B=\sqrt{B_h^2+B_v^2}$. The tangent of the inclination angle is given by $\tan\phi_i = B_v / B_h$.

ANALYZE (a) Substituting the expression given in the problem statement, we have

$$B=\sqrt{B_h^2+B_v^2}=\sqrt{\left(\frac{\mu_0\mu}{4\pi r^3}\cos\lambda_m\right)^2+\left(\frac{\mu_0\mu}{2\pi r^3}\sin\lambda_m\right)^2}=\frac{\mu_0\mu}{4\pi r^3}\sqrt{\cos^2\lambda_m+4\sin^2\lambda_m}$$

$$=\frac{\mu_0\mu}{4\pi r^3}\sqrt{1+3\sin^2\lambda_m},$$

where $\cos^2\lambda_m + \sin^2\lambda_m = 1$ was used.

(b) The inclination ϕ_i is related to λ_m by $\tan\phi_i=\dfrac{B_v}{B_h}=\dfrac{(\mu_0\mu/2\pi r^3)\sin\lambda_m}{(\mu_0\mu/4\pi r^3)\cos\lambda_m}=2\tan\lambda_m.$

LEARN At the magnetic equator ($\lambda_m = 0$), $\phi_i = 0°$, and the field is

$$B=\frac{\mu_0\mu}{4\pi r^3}=\frac{\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(8.00\times10^{22}\,\text{A}\cdot\text{m}^2\right)}{4\pi\left(6.37\times10^6\,\text{m}\right)^3}=3.10\times10^{-5}\,\text{T}.$$

62. (a) At the magnetic equator ($\lambda_m = 0$), the field is

$$B=\frac{\mu_0\mu}{4\pi r^3}=\frac{\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(8.00\times10^{22}\,\text{A}\cdot\text{m}^2\right)}{4\pi\left(6.37\times10^6\,\text{m}\right)^3}=3.10\times10^{-5}\,\text{T}.$$

(b) $\phi_i = \tan^{-1}(2\tan\lambda_m) = \tan^{-1}(0) = 0°$.

(c) At $\lambda_m = 60.0°$, we find

$$B=\frac{\mu_0\mu}{4\pi r^3}\sqrt{1+3\sin^2\lambda_m}=\left(3.10\times10^{-5}\right)\sqrt{1+3\sin^2 60.0°}=5.59\times10^{-5}\,\text{T}.$$

(d) $\phi_i = \tan^{-1}(2\tan 60.0°) = 73.9°$.

(e) At the north magnetic pole ($\lambda_m = 90.0°$), we obtain

$$B=\frac{\mu_0\mu}{4\pi r^3}\sqrt{1+3\sin^2\lambda_m}=\left(3.10\times10^{-5}\right)\sqrt{1+3(1.00)^2}=6.20\times10^{-5}\,\text{T}.$$

(f) $\phi_i = \tan^{-1}(2\tan 90.0°) = 90.0°$.

63. Let R be the radius of a capacitor plate and r be the distance from axis of the capacitor. For points with $r \le R$, the magnitude of the magnetic field is given by

$$B=\frac{\mu_0\varepsilon_0 r}{2}\frac{dE}{dt},$$

and for $r \ge R$, it is

$$B=\frac{\mu_0\varepsilon_0 R^2}{2r}\frac{dE}{dt}.$$

The maximum magnetic field occurs at points for which $r = R$, and its value is given by either of the formulas above:

$$B_{\max}=\frac{\mu_0\varepsilon_0 R}{2}\frac{dE}{dt}.$$

There are two values of r for which $B = B_{\max}/2$: one less than R and one greater.

(a) To find the one that is less than *R*, we solve

$$\frac{\mu_0\varepsilon_0 r}{2}\frac{dE}{dt}=\frac{\mu_0\varepsilon_0 R}{4}\frac{dE}{dt}$$

for *r*. The result is $r = R/2 = (55.0\text{ mm})/2 = 27.5\text{ mm}$.

(b) To find the one that is greater than *R*, we solve

$$\frac{\mu_0\varepsilon_0 R^2}{2r}\frac{dE}{dt}=\frac{\mu_0\varepsilon_0 R}{4}\frac{dE}{dt}$$

for *r*. The result is $r = 2R = 2(55.0\text{ mm}) = 110\text{ mm}$.

64. (a) Again from Fig. 32-14, for $M/M_{\text{max}} = 50\%$ we have $B/T = 0.50$. So $T = B/0.50 = 2/0.50 = 4$ K.

(b) Now $B/T = 2.0$, so $T = 2/2.0 = 1$ K.

65. Let the area of each circular plate be *A* and that of the central circular section be *a*. Then

$$\frac{A}{a}=\frac{\pi R^2}{\pi\left(R/2\right)^2}=4\ .$$

Thus, from Eqs. 32-14 and 32-15 the total discharge current is given by $i = i_d = 4(2.0\text{ A}) = 8.0\text{ A}$.

66. Ignoring points where the determination of the slope is problematic, we find the interval of largest $|\Delta\vec{E}|/\Delta t$ is 6 μs < *t* < 7 μs. During that time, we have, from Eq. 32-14,

$$i_d=\varepsilon_0 A\frac{|\Delta\vec{E}|}{\Delta t}=(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(2.0\,\text{m}^2)(2.0\times10^6\ \text{V/m})=3.5\times10^{-5}\,\text{A}.$$

67. (a) Using Eq. 32-13 but noting that the capacitor is being *discharged*, we have

$$\frac{d\,|\vec{E}\,|}{dt}=-\frac{i}{\varepsilon_0 A}=-\frac{5.0\text{ A}}{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(0.0080\text{ m})^2}=-8.8\times10^{15}\ \text{V/m}\cdot\text{s}\ .$$

(b) Assuming a perfectly uniform field, even so near to an edge (which is consistent with the fact that fringing is neglected in Section 32-4), we follow part (a) of Sample Problem 32.02 — "Treating a changing electric field as a displacement current" and relate the (absolute value of the) line integral to the portion of displacement current enclosed:

$$\left|\oint \vec{B}\cdot d\vec{s}\right| = \mu_0 i_{d,\text{enc}} = \mu_0\left(\frac{WH}{L^2}i\right) = 5.9\times10^{-7}\,\text{Wb/m}.$$

68. (a) Using Eq. 32-31, we find

$$\mu_{\text{orb},z} = -3\mu_B = -2.78 \times 10^{-23}\ \text{J/T}.$$

That these are acceptable units for magnetic moment is seen from Eq. 32-32 or Eq. 32-27; they are equivalent to A·m^2.

(b) Similarly, for $m_\ell = -4$ we obtain $\mu_{\text{orb},z}$ = 3.71 × 10^{-23} J/T.

69. (a) Since the field lines of a bar magnet point toward its South pole, then the $\vec{B}$ arrows in one's sketch should point generally toward the left and also towards the central axis.

(b) The sign of $\vec{B}\cdot d\vec{A}$ for every $d\vec{A}$ on the side of the paper cylinder is negative.

(c) No, because Gauss' law for magnetism applies to an *enclosed* surface only. In fact, if we include the top and bottom of the cylinder to form an enclosed surface *S* then $\oint_s \vec{B}\cdot d\vec{A} = 0$ will be valid, as the flux through the open end of the cylinder near the magnet is positive.

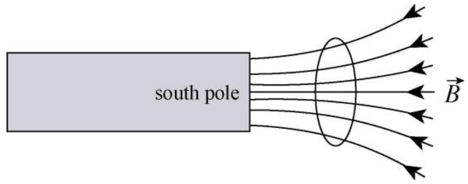
70. (a) From Eq. 21-3,

$$E = \frac{e}{4\pi\varepsilon_0 r^2} = \frac{\left(1.60\times10^{-19}\ \text{C}\right)\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)}{\left(5.2\times10^{-11}\ \text{m}\right)^2} = 5.3\times10^{11}\ \text{N/C}\ .$$

(b) We use Eq. 29-28: $B = \dfrac{\mu_0}{2\pi}\dfrac{\mu_p}{r^3} = \dfrac{\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(1.4\times10^{-26}\ \text{J/T}\right)}{2\pi\left(5.2\times10^{-11}\ \text{m}\right)^3} = 2.0\times10^{-2}\ \text{T}$.

(c) From Eq. 32-30, $\dfrac{\mu_{\text{orb}}}{\mu_p} = \dfrac{eh/4\pi m_e}{\mu_p} = \dfrac{\mu_B}{\mu_p} = \dfrac{9.27\times10^{-24}\ \text{J/T}}{1.4\times10^{-26}\ \text{J/T}} = 6.6\times10^{2}$.

71. (a) A sketch of the field lines (due to the presence of the bar magnet) in the vicinity of the loop is shown below:



(b) For paramagnetic materials, the dipole moment $\vec{\mu}$ is in the same direction as $\vec{B}$. From the above figure, $\vec{\mu}$ points in the $-x$ direction.

(c) Form the right-hand rule, since $\vec{\mu}$ points in the $-x$ direction, the current flows counterclockwise, from the perspective of the bar magnet.

(d) The effect of $\vec{F}$ is to move the material toward regions of larger $|\vec{B}|$ values. Since the size of $|\vec{B}|$ relates to the "crowdedness" of the field lines, we see that $\vec{F}$ is toward the left, or $-x$.

72. (a) Inside the gap of the capacitor, $B_1 = \mu_0 i_d\, r_1/2\pi R^2$ (Eq. 32-16); outside the gap the magnetic field is $B_2 = \mu_0 i_d/2\pi r_2$ (Eq. 32-17). Consequently, $B_2 = B_1 R^2/r_1 r_2 = 16.7$ nT.

(b) The displacement current is $i_d = 2\pi B_1 R^2/\mu_0 r_1 = 5.00$ mA.

73. **THINK** The z component of the orbital angular momentum is give by $L_{\text{orb},z} = m_\ell h/2\pi$, where h is the Planck constant and m_ℓ is the orbital magnetic quantum number.

EXPRESS The "limit" for m_ℓ is 3. This means that the allowed values of m_ℓ are: $0, \pm 1, \pm 2,$ and $\pm 3.$

ANALYZE (a) The number of different m_ℓ's is 2(3) + 1 = 7. Since $L_{\text{orb},z} \propto m_\ell$, there are a total of seven different values of $L_{\text{orb},z}$.

(b) Similarly, since $\mu_{\text{orb},z} \propto m_\ell$, there are also a total of seven different values of $\mu_{\text{orb},z}$.

(c) The greatest allowed value of $L_{\text{orb},z}$ is given by $|m_\ell|_{\max} h/2\pi = 3h/2\pi$.

(d) Similar to part (c), since $\mu_{\text{orb},z} = -m_\ell \mu_B$, the greatest allowed value of $\mu_{\text{orb},z}$ is given by $|m_\ell|_{\max}\mu_B = 3eh/4\pi m_e$.

(e) From Eqs. 32-23 and 32-29 the z component of the net angular momentum of the electron is given by

$$L_{\text{net},z} = L_{\text{orb},z} + L_{s,z} = \frac{m_\ell h}{2\pi} + \frac{m_s h}{2\pi}.$$

For the maximum value of $L_{\text{net},z}$ let $m_\ell = [m_\ell]_{\max} = 3$ and $m_s = \frac{1}{2}$. Thus

$$L_{\text{net},z}\big|_{\max} = \left(3 + \frac{1}{2}\right)\frac{h}{2\pi} = \frac{3.5h}{2\pi}.$$

(f) Since the maximum value of $L_{\text{net},z}$ is given by $[m_J]_{\max}h/2\pi$ with $[m_J]_{\max} = 3.5$ (see the last part above), the number of allowed values for the z component of $L_{\text{net},z}$ is given by $2[m_J]_{\max} + 1 = 2(3.5) + 1 = 8$.

LEARN As we shall see in Chapter 40, the allowed values of m_ℓ range from $-\ell$ to $+\ell$, where ℓ is called the orbital quantum number.

74. The definition of displacement current is Eq. 32-10, and the formula of greatest convenience here is Eq. 32-17:

$$i_d = \frac{2\pi r\ B}{\mu_0} = \frac{2\pi\left(0.0300\,\text{m}\right)\left(2.00\times10^{-6}\,\text{T}\right)}{4\pi\times10^{-7}\,\text{T}\cdot\text{m/A}} = 0.300\ \text{A}\ .$$

75. (a) The complete set of values are

$$\{-4, -3, -2, -1, 0, +1, +2, +3, +4\} \quad\Rightarrow\quad \text{nine values in all.}$$

(b) The maximum value is $4\mu_B = 3.71 \times 10^{-23}$ J/T.

(c) Multiplying our result for part (b) by 0.250 T gives $U = +9.27 \times 10^{-24}$ J.

(d) Similarly, for the lower limit, $U = -9.27 \times 10^{-24}$ J.

76. (a) The z component of the orbital magnetic dipole moment is

$$\mu_{\text{orb},z} = -m_\ell \mu_B$$

where $\mu_B = eh/4\pi m = 9.27\times10^{-24}\,\text{J/T}$ is the Bohr magneton. For $m_l = 3$, we have

$$\mu_{\text{orb},z} = -m_\ell\mu_B - (3)(9.27\times10^{-24}\ \text{J/T}) = -2.78\times10^{-23}\ \text{J/T}.$$

(b) Similarly, for $m_l = -4$, the result is

$$\mu_{\text{orb},z} = -m_\ell\mu_B - (-4)(9.27\times10^{-24}\ \text{J/T}) = 3.71\times10^{-23}\ \text{J/T}.$$

Chapter 33

1. Since $\Delta\lambda \ll \lambda$, we find Δf is equal to

$$\left|\Delta\left(\frac{c}{\lambda}\right)\right| \approx \frac{c\Delta\lambda}{\lambda^2} = \frac{(3.0\times10^8\text{ m/s})(0.0100\times10^{-9}\text{ m})}{(632.8\times10^{-9}\text{ m})^2} = 7.49\times10^9\text{ Hz.}$$

2. (a) The frequency of the radiation is

$$f = \frac{c}{\lambda} = \frac{3.0\times10^8\text{ m/s}}{(1.0\times10^5)(6.4\times10^6\text{ m})} = 4.7\times10^{-3}\text{ Hz.}$$

(b) The period of the radiation is

$$T = \frac{1}{f} = \frac{1}{4.7\times10^{-3}\text{ Hz}} = 212\text{ s} = 3\min 32\text{ s.}$$

3. (a) From Fig. 33-2 we find the smaller wavelength in question to be about 515 nm.

(b) Similarly, the larger wavelength is approximately 610 nm.

(c) From Fig. 33-2 the wavelength at which the eye is most sensitive is about 555 nm.

(d) Using the result in (c), we have

$$f = \frac{c}{\lambda} = \frac{3.00\times10^8\text{ m/s}}{555\text{ nm}} = 5.41\times10^{14}\text{ Hz}.$$

(e) The period is $T = 1/f = (5.41 \times 10^{14}\text{ Hz})^{-1} = 1.85 \times 10^{-15}$ s.

4. In air, light travels at roughly $c = 3.0 \times 10^8$ m/s. Therefore, for $t = 1.0$ ns, we have a distance of

$$d = ct = (3.0\times10^8\text{ m/s})(1.0\times10^{-9}\text{ s}) = 0.30\text{ m.}$$

5. **THINK** The frequency of oscillation of the current in the LC circuit of the generator is $f = 1/2\pi\sqrt{LC}$, where C is the capacitance and L is the inductance. This frequency is the same as the frequency of an electromagnetic wave.

EXPRESS If f is the frequency and λ is the wavelength of an electromagnetic wave, then $f\lambda = c$. Thus,

$$\frac{\lambda}{2\pi\sqrt{LC}} = c.$$

ANALYZE The solution for L is

$$L = \frac{\lambda^2}{4\pi^2 Cc^2} = \frac{\left(550\times10^{-9}\text{ m}\right)^2}{4\pi^2\left(17\times10^{-12}\text{ F}\right)\left(2.998\times10^{8}\text{ m/s}\right)^2} = 5.00\times10^{-21}\text{ H}.$$

This is exceedingly small.

LEARN The frequency is

$$f = \frac{c}{\lambda} = \frac{3.0\times10^{8}\text{ m/s}}{550\times10^{-9}\text{ m}} = 5.45\times10^{14}\text{ Hz}.$$

The EM wave is in the visible spectrum.

6. The emitted wavelength is

$$\lambda = \frac{c}{f} = 2\pi c\sqrt{LC} = 2\pi\left(2.998\times10^{8}\text{ m/s}\right)\sqrt{\left(0.253\times10^{-6}\text{ H}\right)\left(25.0\times10^{-12}\text{ F}\right)} = 4.74\text{ m}.$$

7. The intensity is the average of the Poynting vector:

$$I = S_{\text{avg}} = \frac{cB_m^2}{2\mu_0} = \frac{\left(3.0\times10^{8}\text{ m/s}\right)\left(1.0\times10^{-4}\text{ T}\right)^2}{2\left(1.26\times10^{-6}\text{ H/m}\right)^2} = 1.2\times10^{6}\text{ W/m}^2.$$

8. The intensity of the signal at Proxima Centauri is

$$I = \frac{P}{4\pi r^2} = \frac{1.0\times10^{6}\text{ W}}{4\pi\left(4.3\text{ly}\right)\left(9.46\times10^{15}\text{ m/ly}\right)^2} = 4.8\times10^{-29}\text{ W/m}^2.$$

9. If P is the power and Δt is the time interval of one pulse, then the energy in a pulse is

$$E = P\Delta t = \left(100\times10^{12}\text{ W}\right)\left(1.0\times10^{-9}\text{ s}\right) = 1.0\times10^{5}\text{ J}.$$

10. (a) Setting $v = c$ in the wave relation $kv = \omega = 2\pi f$, we find $f = 1.91\times10^{8}$ Hz.

(b) $E_{\text{rms}} = E_m/\sqrt{2} = B_m/c\sqrt{2} = 18.2$ V/m.

(c) $I = (E_{\text{rms}})^2/c\mu_o = 0.878$ W/m^2.

11. (a) The amplitude of the magnetic field is

$$B_m = \frac{E_m}{c} = \frac{2.0\,\text{V/m}}{2.998\times10^{8}\,\text{m/s}} = 6.67\times10^{-9}\,\text{T} \approx 6.7\times10^{-9}\,\text{T}.$$

(b) Since the $\vec{E}$-wave oscillates in the z direction and travels in the x direction, we have $B_x = B_z = 0$. So, the oscillation of the magnetic field is parallel to the y axis.

(c) The direction ($+x$) of the electromagnetic wave propagation is determined by $\vec{E}\times\vec{B}$. If the electric field points in $+z$, then the magnetic field must point in the $-y$ direction.

With SI units understood, we may write

$$B_y = B_m \cos\left[\pi\times10^{15}\left(t-\frac{x}{c}\right)\right] = \frac{2.0\cos\left[10^{15}\pi\left(t-x/c\right)\right]}{3.0\times10^{8}}$$
$$= \left(6.7\times10^{-9}\right)\cos\left[10^{15}\pi\left(t-\frac{x}{c}\right)\right]$$

12. (a) The amplitude of the magnetic field in the wave is

$$B_m = \frac{E_m}{c} = \frac{5.00\,\text{V/m}}{2.998\times10^{8}\,\text{m/s}} = 1.67\times10^{-8}\,\text{T}.$$

(b) The intensity is the average of the Poynting vector:

$$I = S_{\text{avg}} = \frac{E_m^2}{2\mu_0 c} = \frac{\left(5.00\,\text{V/m}\right)^2}{2\left(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A}\right)\left(2.998\times10^{8}\,\text{m/s}\right)} = 3.31\times10^{-2}\,\text{W/m}^2.$$

13. (a) We use $I = E_m^2/2\mu_0 c$ to calculate E_m:

$$E_m = \sqrt{2\mu_0 I_c} = \sqrt{2\left(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A}\right)\left(1.40\times10^{3}\,\text{W/m}^2\right)\left(2.998\times10^{8}\,\text{m/s}\right)}$$
$$= 1.03\times10^{3}\,\text{V/m}.$$

(b) The magnetic field amplitude is therefore

$$B_m = \frac{E_m}{c} = \frac{1.03\times10^{4}\,\text{V/m}}{2.998\times10^{8}\,\text{m/s}} = 3.43\times10^{-6}\,\text{T}.$$

14. From the equation immediately preceding Eq. 33-12, we see that the maximum value of $\partial B/\partial t$ is ωB_m. We can relate B_m to the intensity:

$$B_m = \frac{E_m}{c} = \frac{\sqrt{2c\mu_0 I}}{c},$$

and relate the intensity to the power P (and distance r) using Eq. 33-27. Finally, we relate ω to wavelength λ using $\omega = kc = 2\pi c/\lambda$. Putting all this together, we obtain

$$\left(\frac{\partial B}{\partial t}\right)_{\max} = \sqrt{\frac{2\mu_0 P}{4\pi c}}\,\frac{2\pi c}{\lambda r} = 3.44\times 10^6\ \text{T/s}.$$

15. (a) The average rate of energy flow per unit area, or intensity, is related to the electric field amplitude E_m by $I = E_m^2/2\mu_0 c$, so

$$\begin{aligned} E_m &= \sqrt{2\mu_0 cI} = \sqrt{2\left(4\pi\times 10^{-7}\ \text{H/m}\right)\left(2.998\times 10^8\ \text{m/s}\right)\left(10\times 10^{-6}\ \text{W/m}^2\right)} \\ &= 8.7\times 10^{-2}\ \text{V/m}. \end{aligned}$$

(b) The amplitude of the magnetic field is given by

$$B_m = \frac{E_m}{c} = \frac{8.7\times 10^{-2}\ \text{V/m}}{2.998\times 10^8\ \text{m/s}} = 2.9\times 10^{-10}\ \text{T}.$$

(c) At a distance r from the transmitter, the intensity is $I = P/2\pi r^2$, where P is the power of the transmitter over the hemisphere having a surface area $2\pi r^2$. Thus

$$P = 2\pi r^2 I = 2\pi\left(10\times 10^3\ \text{m}\right)^2\left(10\times 10^{-6}\ \text{W/m}^2\right) = 6.3\times 10^3\ \text{W}.$$

16. (a) The power received is

$$P_r = \left(1.0\times 10^{-12}\ \text{W}\right)\frac{\pi(300\ \text{m})^2/4}{4\pi\left(6.37\times 10^6\ \text{m}\right)^2} = 1.4\times 10^{-22}\ \text{W}.$$

(b) The power of the source would be

$$P = 4\pi r^2 I = 4\pi\left[\left(2.2\times 10^4\ \text{ly}\right)\left(9.46\times 10^{15}\ \text{m/ly}\right)\right]^2\left[\frac{1.0\times 10^{-12}\ \text{W}}{4\pi\left(6.37\times 10^6\ \text{m}\right)^2}\right] = 1.1\times 10^{15}\ \text{W}.$$

17. (a) The magnetic field amplitude of the wave is

$$B_m = \frac{E_m}{c} = \frac{2.0\,\text{V/m}}{2.998\times10^8\,\text{m/s}} = 6.7\times10^{-9}\,\text{T}.$$

(b) The intensity is

$$I = \frac{E_m^2}{2\mu_0 c} = \frac{(2.0\,\text{V/m})^2}{2(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(2.998\times10^8\,\text{m/s})} = 5.3\times10^{-3}\,\text{W/m}^2.$$

(c) The power of the source is

$$P = 4\pi r^2 I_{\text{avg}} = 4\pi(10\,\text{m})^2\left(5.3\times10^{-3}\,\text{W/m}^2\right) = 6.7\,\text{W}.$$

18. Equation 33-27 suggests that the slope in an intensity versus inverse-square-distance graph (I plotted versus r^{-2}) is $P/4\pi$. We estimate the slope to be about 20 (in SI units), which means the power is $P = 4\pi(30) \approx 2.5\times10^2$ W.

19. **THINK** The plasma completely reflects all the energy incident on it, so the radiation pressure is given by $p_r = 2I/c$, where I is the intensity.

EXPRESS The intensity is $I = P/A$, where P is the power and A is the area intercepted by the radiation.

ANALYZE Thus, the radiation pressure is

$$p_r = \frac{2I}{c} = \frac{2P}{Ac} = \frac{2\left(1.5\times10^9\,\text{W}\right)}{\left(1.00\times10^{-6}\,\text{m}^2\right)\left(2.998\times10^8\,\text{m/s}\right)} = 1.0\times10^7\,\text{Pa}.$$

LEARN In the case of total absorption, the radiation pressure would be $p_r = I/c$, a factor of 2 smaller than the case of total reflection.

20. (a) The radiation pressure produces a force equal to

$$F_r = p_r\left(\pi R_e^2\right) = \left(\frac{I}{c}\right)\left(\pi R_e^2\right) = \frac{\pi\left(1.4\times10^3\,\text{W/m}^2\right)\left(6.37\times10^6\,\text{m}\right)^2}{2.998\times10^8\,\text{m/s}} = 6.0\times10^8\,\text{N}.$$

(b) The gravitational pull of the Sun on the Earth is

$$\begin{aligned} F_{\text{grav}} &= \frac{GM_sM_e}{d_{es}^2} = \frac{\left(6.67\times10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(2.0\times10^{30}\,\text{kg}\right)\left(5.98\times10^{24}\,\text{kg}\right)}{\left(1.5\times10^{11}\,\text{m}\right)^2} \\ &= 3.6\times10^{22}\,\text{N}, \end{aligned}$$

which is much greater than F_r.

21. Since the surface is perfectly absorbing, the radiation pressure is given by $p_r = I/c$, where I is the intensity. Since the bulb radiates uniformly in all directions, the intensity a distance r from it is given by $I = P/4\pi r^2$, where P is the power of the bulb. Thus

$$p_r = \frac{P}{4\pi r^2 c} = \frac{500\,\text{W}}{4\pi(1.5\ \text{m})^2\,(2.998\times10^8\ \text{m/s})} = 5.9\times10^{-8}\ \text{Pa}.$$

22. The radiation pressure is

$$p_r = \frac{I}{c} = \frac{10\,\text{W/m}^2}{2.998\times10^8\ \text{m/s}} = 3.3\times10^{-8}\ \text{Pa}.$$

23. (a) The upward force supplied by radiation pressure in this case (Eq. 33-32) must be equal to the magnitude of the pull of gravity (mg). For a sphere, the "projected" area (which is a factor in Eq. 33-32) is that of a circle $A = \pi r^2$ (not the entire surface area of the sphere) and the volume (needed because the mass is given by the density multiplied by the volume: $m = \rho V$) is $V = 4\pi r^3/3$. Finally, the intensity is related to the power P of the light source and another area factor $4\pi R^2$, given by Eq. 33-27. In this way, with $\rho = 1.9\times10^4$ kg/m^3, equating the forces leads to

$$P = 4\pi R^2 c\left(\rho\frac{4\pi r^3 g}{3}\right)\frac{1}{\pi r^2} = 4.68\times10^{11}\ \text{W}.$$

(b) Any chance disturbance could move the sphere from being directly above the source, and then the two force vectors would no longer be along the same axis.

24. We require $F_{\text{grav}} = F_r$ or

$$G\frac{mM_s}{d_{es}^2} = \frac{2IA}{c},$$

and solve for the area A:

$$A = \frac{cGmM_s}{2Id_{es}^2} = \frac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(1500\ \text{kg})(1.99\times10^{30}\ \text{kg})(2.998\times10^8\ \text{m/s})}{2(1.40\times10^3\ \text{W/m}^2)(1.50\times10^{11}\ \text{m})^2}$$
$$= 9.5\times10^5\ \text{m}^2 = 0.95\ \text{km}^2.$$

25. **THINK** In this problem we relate radiation pressure to energy density in the incident beam.

EXPRESS Let f be the fraction of the incident beam intensity that is reflected. The fraction absorbed is $1 - f$. The reflected portion exerts a radiation pressure of

$$p_r = \frac{2fI_0}{c}$$

and the absorbed portion exerts a radiation pressure of

$$p_a = \frac{(1-f)I_0}{c},$$

where I_0 is the incident intensity. The factor 2 enters the first expression because the momentum of the reflected portion is reversed. The total radiation pressure is the sum of the two contributions:

$$p_{\text{total}} = p_r + p_a = \frac{2fI_0 + (1-f)I_0}{c} = \frac{(1+f)I_0}{c}.$$

ANALYZE To relate the intensity and energy density, we consider a tube with length ℓ and cross-sectional area A, lying with its axis along the propagation direction of an electromagnetic wave. The electromagnetic energy inside is $U = uA\ell$, where u is the energy density. All this energy passes through the end in time $t = \ell / c$, so the intensity is

$$I = \frac{U}{At} = \frac{uA\ell c}{A\ell} = uc.$$

Thus $u = I/c$. The intensity and energy density are positive, regardless of the propagation direction. For the partially reflected and partially absorbed wave, the intensity just outside the surface is

$$I = I_0 + fI_0 = (1+f)I_0,$$

where the first term is associated with the incident beam and the second is associated with the reflected beam. Consequently, the energy density is

$$u = \frac{I}{c} = \frac{(1+f)I_0}{c},$$

the same as radiation pressure.

LEARN In the case of total reflection, $f = 1$, and $p_{\text{total}} = p_r = 2I_0 / c$. On the other hand, the energy density is $u = I / c = 2I_0 / c$, which is the same as p_{total}. Similarly, for total absorption, $f = 0$, $p_{\text{total}} = p_a = I_0 / c$, and since $I = I_0$, we have $u = I / c = I_0 / c$, which again is the same as p_{total}.

26. The mass of the cylinder is $m = \rho(\pi D^2 / 4)H$, where D is the diameter of the cylinder. Since it is in equilibrium

$$F_{\text{net}} = mg - F_r = \frac{\pi H D^2 g \rho}{4} - \left(\frac{\pi D^2}{4}\right)\left(\frac{2I}{c}\right) = 0.$$

We solve for H:

$$\begin{aligned} H &= \frac{2I}{gc\rho} = \left(\frac{2P}{\pi D^2/4}\right)\frac{1}{gc\rho} \\ &= \frac{2(4.60\,\text{W})}{[\pi(2.60\times10^{-3}\,\text{m})^2/4](9.8\,\text{m/s}^2)(3.0\times10^8\,\text{m/s})(1.20\times10^3\,\text{kg/m}^3)} \\ &= 4.91\times10^{-7}\,\text{m}. \end{aligned}$$

27. **THINK** Electromagnetic waves travel at speed of light, and carry both linear momentum and energy.

EXPRESS The speed of the electromagnetic wave is $c = \lambda f$, where λ is the wavelength and f is the frequency of the wave. The angular frequency is $\omega = 2\pi f$, and the angular wave number is $k = 2\pi/\lambda$. The magnetic field amplitude is related to the electric field amplitude by $B_m = E_m/c$. The intensity of the wave is given by Eq. 33-26:

$$I = \frac{1}{c\mu_0}E_{\text{rms}}^2 = \frac{1}{2c\mu_0}E_m^2.$$

ANALYZE (a) With λ = 3.0 m, the frequency of the wave is

$$f = \frac{c}{\lambda} = \frac{2.998\times10^8\ \text{m/s}}{3.0\ \text{m}} = 1.0\times10^8\ \text{Hz}.$$

(b) From the value of f obtained in (a), we find the angular frequency to be

$$\omega = 2\pi f = 2\pi(1.0\times10^8\ \text{Hz}) = 6.3\times10^8\ \text{rad/s}.$$

(c) The corresponding angular wave number is

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{3.0\ \text{m}} = 2.1\ \text{rad/m}.$$

(d) With E_m = 300 V/m, the magnetic field amplitude is

$$B_m = \frac{E_m}{c} = \frac{300\,\text{V/m}}{2.998\times10^8\,\text{m/s}} = 1.0\times10^{-6}\,\text{T}.$$

(e) Since $\vec{E}$ is in the positive y direction, $\vec{B}$ must be in the positive z direction so that their cross product $\vec{E}\times\vec{B}$ points in the positive x direction (the direction of propagation).

(f) The intensity of the wave is

$$I=\frac{E_m^2}{2\mu_0 c}=\frac{(300\,\text{V/m})^2}{2(4\pi\times10^{-7}\,\text{H/m})(2.998\times10^8\,\text{m/s})}=119\,\text{W/m}^2\approx1.2\times10^2\,\text{W/m}^2.$$

(g) Since the sheet is perfectly absorbing, the rate per unit area with which momentum is delivered to it is I/c, so

$$\frac{dp}{dt}=\frac{IA}{c}=\frac{(119\ \text{W/m}^2)(2.0\ \text{m}^2)}{2.998\times10^8\ \text{m/s}}=8.0\times10^{-7}\ \text{N}.$$

(h) The radiation pressure is

$$p_r=\frac{dp/dt}{A}=\frac{8.0\times10^{-7}\ \text{N}}{2.0\ \text{m}^2}=4.0\times10^{-7}\ \text{Pa}.$$

LEARN The energy density is given by

$$u=\frac{I}{c}=\frac{119\ \text{W/m}^2}{2.998\times10^8\ \text{m/s}}=4.0\times10^{-7}\ \text{J/m}^3$$

which is the same as the radiation pressure p_r.

28. (a) Assuming complete absorption, the radiation pressure is

$$p_r=\frac{I}{c}=\frac{1.4\times10^3\ \text{W/m}^2}{3.0\times10^8\ \text{m/s}}=4.7\times10^{-6}\ \text{N/m}^2\,.$$

(b) We compare values by setting up a ratio:

$$\frac{p_r}{p_0}=\frac{4.7\times10^{-6}\ \text{N/m}^2}{1.0\times10^5\ \text{N/m}^2}=4.7\times10^{-11}.$$

29. **THINK** The laser beam carries both energy and momentum. The total momentum of the spaceship and light is conserved.

EXPRESS If the beam carries energy U away from the spaceship, then it also carries momentum $p = U/c$ away. By momentum conservation, this is the magnitude of the momentum acquired by the spaceship. If P is the power of the laser, then the energy carried away in time t is $U = Pt$.

ANALYZE We note that there are 86400 seconds in a day. Thus, $p = Pt/c$ and, if m is mass of the spaceship, its speed is

$$v = \frac{p}{m} = \frac{Pt}{mc} = \frac{(10\times10^3\ \text{W})(86400\ \text{s})}{(1.5\times10^3\ \text{kg})(2.998\times10^8\ \text{m/s})} = 1.9\times10^{-3}\ \text{m/s}.$$

LEARN As expected, the speed of the spaceship is proportional to the power of the laser beam.

30. (a) We note that the cross-section area of the beam is $\pi d^2/4$, where d is the diameter of the spot ($d = 2.00\lambda$). The beam intensity is

$$I = \frac{P}{\pi d^2/4} = \frac{5.00\times10^{-3}\ \text{W}}{\pi\left[(2.00)(633\times10^{-9}\ \text{m})\right]^2/4} = 3.97\times10^9\ \text{W/m}^2.$$

(b) The radiation pressure is

$$p_r = \frac{I}{c} = \frac{3.97\times10^9\ \text{W/m}^2}{2.998\times10^8\ \text{m/s}} = 13.2\ \text{Pa}.$$

(c) In computing the corresponding force, we can use the power and intensity to eliminate the area (mentioned in part (a)). We obtain

$$F_r = \left(\frac{\pi d^2}{4}\right)p_r = \left(\frac{P}{I}\right)p_r = \frac{(5.00\times10^{-3}\ \text{W})(13.2\ \text{Pa})}{3.97\times10^9\ \text{W/m}^2} = 1.67\times10^{-11}\ \text{N}.$$

(d) The acceleration of the sphere is

$$a = \frac{F_r}{m} = \frac{F_r}{\rho(\pi d^3/6)} = \frac{6(1.67\times10^{-11}\ \text{N})}{\pi(5.00\times10^3\ \text{kg/m}^3)[(2.00)(633\times10^{-9}\ \text{m})]^3}$$
$$= 3.14\times10^3\ \text{m/s}^2.$$

31. We shall assume that the Sun is far enough from the particle to act as an isotropic point source of light.

(a) The forces that act on the dust particle are the radially outward radiation force $\vec{F}_r$ and the radially inward (toward the Sun) gravitational force $\vec{F}_g$. Using Eqs. 33-32 and 33-27, the radiation force can be written as

$$F_r = \frac{IA}{c} = \frac{P_S}{4\pi r^2}\frac{\pi R^2}{c} = \frac{P_S R^2}{4r^2 c},$$

where R is the radius of the particle, and $A = \pi R^2$ is the cross-sectional area. On the other hand, the gravitational force on the particle is given by Newton's law of gravitation (Eq. 13-1):

$$F_g = \frac{GM_S m}{r^2} = \frac{GM_S \rho(4\pi R^3/3)}{r^2} = \frac{4\pi GM_S \rho R^3}{3r^2},$$

where $m = \rho(4\pi R^3/3)$ is the mass of the particle. When the two forces balance, the particle travels in a straight path. The condition that $F_r = F_g$ implies

$$\frac{P_S R^2}{4r^2 c} = \frac{4\pi GM_S \rho R^3}{3r^2},$$

which can be solved to give

$$R = \frac{3P_S}{16\pi c\rho GM_S} = \frac{3(3.9\times10^{26}\ \text{W})}{16\pi(3\times10^8\ \text{m/s})(3.5\times10^3\ \text{kg/m}^3)(6.67\times10^{-11}\ \text{m}^3/\text{kg}\cdot\text{s}^2)(1.99\times10^{30}\ \text{kg})}$$
$$= 1.7\times10^{-7}\ \text{m}.$$

(b) Since F_g varies with R^3 and F_r varies with R^2, if the radius R is larger, then $F_g > F_r$, and the path will be curved toward the Sun (like path 3).

32. After passing through the first polarizer the initial intensity I_0 reduces by a factor of 1/2. After passing through the second one it is further reduced by a factor of $\cos^2(\pi - \theta_1 - \theta_2) = \cos^2(\theta_1 + \theta_2)$. Finally, after passing through the third one it is again reduced by a factor of $\cos^2(\pi - \theta_2 - \theta_3) = \cos^2(\theta_2 + \theta_3)$. Therefore,

$$\frac{I_f}{I_0} = \frac{1}{2}\cos^2(\theta_1+\theta_2)\cos^2(\theta_2+\theta_3) = \frac{1}{2}\cos^2(50°+50°)\cos^2(50°+50°)$$
$$= 4.5\times10^{-4}.$$

Thus, 0.045% of the light's initial intensity is transmitted.

33. **THINK** Unpolarized light becomes polarized when it is sent through a polarizing sheet. In this problem, three polarizing sheets are involved, we work through the system sheet by sheet, applying either the one-half rule or the cosine-squared rule.

EXPRESS Let I_0 be the intensity of the unpolarized light that is incident on the first polarizing sheet. The transmitted intensity is, by one-half rule, $I_1 = \frac{1}{2}I_0$, and the direction of polarization of the transmitted light is $\theta_1 = 40°$ *counterclockwise* from the y axis in the diagram. For the second sheet (and the third one as well), we apply the cosine-squared rule:

$$I_2 = I_1 \cos^2 \theta_2'$$

where θ_2' is the angle between the direction of polarization that is incident on that sheet and the polarizing direction of the sheet.

ANALYZE The polarizing direction of the second sheet is θ_2 = 20° *clockwise* from the *y* axis, so $\theta_2' = 40° + 20° = 60°$. The transmitted intensity is

$$I_2 = I_1 \cos^2 60° = \frac{1}{2} I_0 \cos^2 60°,$$

and the direction of polarization of the transmitted light is 20° clockwise from the *y* axis. The polarizing direction of the third sheet is θ_3 = 40° *counterclockwise* from the *y* axis. Consequently, the angle between the direction of polarization of the light incident on that sheet and the polarizing direction of the sheet is 20° + 40° = 60°. The transmitted intensity is

$$I_3 = I_2 \cos^2 60° = \frac{1}{2} I_0 \cos^4 60° = 3.1 \times 10^{-2} I_0.$$

Thus, 3.1% of the light's initial intensity is transmitted.

LEARN When two polarizing sheets are crossed ($\theta = 90°$), no light passes through and the transmitted intensity is zero.

34. In this case, we replace $I_0 \cos^2 70°$ by $\frac{1}{2} I_0$ as the intensity of the light after passing through the first polarizer. Therefore,

$$I_f = \frac{1}{2} I_0 \cos^2 (90° - 70°) = \frac{1}{2}(43\ \mathrm{W/m^2})(\cos^2 20°) = 19\ \mathrm{W/m^2}.$$

35. The angle between the direction of polarization of the light incident on the first polarizing sheet and the polarizing direction of that sheet is θ_1 = 70°. If I_0 is the intensity of the incident light, then the intensity of the light transmitted through the first sheet is

$$I_1 = I_0 \cos^2 \theta_1 = (43\ \mathrm{W/m^2}) \cos^2 70° = 5.03\ \mathrm{W/m^2}.$$

The direction of polarization of the transmitted light makes an angle of 70° with the vertical and an angle of θ_2 = 20° with the horizontal. θ_2 is the angle it makes with the polarizing direction of the second polarizing sheet. Consequently, the transmitted intensity is

$$I_2 = I_1 \cos^2 \theta_2 = (5.03\ \mathrm{W/m^2}) \cos^2 20° = 4.4\ \mathrm{W/m^2}.$$

36. (a) The fraction of light that is transmitted by the glasses is

$$\frac{I_f}{I_0}=\frac{E_f^2}{E_0^2}=\frac{E_v^2}{E_v^2+E_h^2}=\frac{E_v^2}{E_v^2+(2.3E_v)^2}=0.16.$$

(b) Since now the horizontal component of $\vec{E}$ will pass through the glasses,

$$\frac{I_f}{I_0}=\frac{E_h^2}{E_v^2+E_h^2}=\frac{(2.3E_v)^2}{E_v^2+(2.3E_v)^2}=0.84.$$

37. **THINK** A polarizing sheet can change the direction of polarization of the incident beam since it allows only the component that is parallel to its polarization direction to pass.

EXPRESS The 90° rotation of the polarization direction cannot be done with a single sheet. If a sheet is placed with its polarizing direction at an angle of 90° to the direction of polarization of the incident radiation, no radiation is transmitted.

ANALYZE (a) The 90° rotation of the polarization direction can be done with two sheets. We place the first sheet with its polarizing direction at some angle θ, between 0 and 90°, to the direction of polarization of the incident radiation. Place the second sheet with its polarizing direction at 90° to the polarization direction of the incident radiation. The transmitted radiation is then polarized at 90° to the incident polarization direction. The intensity is

$$I=I_0\cos^2\theta\cos^2(90°-\theta)=I_0\cos^2\theta\sin^2\theta,$$

where I_0 is the incident radiation. If θ is not 0 or 90°, the transmitted intensity is not zero.

(b) Consider n sheets, with the polarizing direction of the first sheet making an angle of $\theta = 90°/n$ relative to the direction of polarization of the incident radiation. The polarizing direction of each successive sheet is rotated 90°/n in the same sense from the polarizing direction of the previous sheet. The transmitted radiation is polarized, with its direction of polarization making an angle of 90° with the direction of polarization of the incident radiation. The intensity is

$$I=I_0\cos^{2n}(90°/n).$$

We want the smallest integer value of n for which this is greater than $0.60I_0$. We start with n = 2 and calculate $\cos^{2n}(90°/n)$. If the result is greater than 0.60, we have obtained the solution. If it is less, increase n by 1 and try again. We repeat this process, increasing n by 1 each time, until we have a value for which $\cos^{2n}(90°/n)$ is greater than 0.60. The first one will be n = 5.

LEARN The intensities associated with n = 1 to 5 are:

$$\begin{aligned}
I_{n=1} &= I_0 \cos^2(90°) = 0 \\
I_{n=2} &= I_0 \cos^4(45°) = I_0/4 = 0.25 I_0 \\
I_{n=3} &= I_0 \cos^6(30°) = 0.422 I_0 \\
I_{n=4} &= I_0 \cos^8(22.5°) = 0.531 I_0 \\
I_{n=5} &= I_0 \cos^{10}(18°) = 0.605 I_0
\end{aligned}$$

38. We note the points at which the curve is zero (θ_2 = 0° and 90°) in Fig. 33-43. We infer that sheet 2 is perpendicular to one of the other sheets at θ_2 = 0°, and that it is perpendicular to the *other* of the other sheets when θ_2 = 90°. Without loss of generality, we choose θ_1 = 0°, θ_3 = 90°. Now, when θ_2 = 30°, it will be $\Delta\theta$ = 30° relative to sheet 1 and $\Delta\theta'$ = 60° relative to sheet 3. Therefore,

$$\frac{I_f}{I_i} = \frac{1}{2}\cos^2(\Delta\theta)\cos^2(\Delta\theta') = 9.4\% .$$

39. (a) Since the incident light is unpolarized, half the intensity is transmitted and half is absorbed. Thus the transmitted intensity is I = 5.0 mW/m^2. The intensity and the electric field amplitude are related by $I = E_m^2/2\mu_0 c$, so

$$\begin{aligned}
E_m &= \sqrt{2\mu_0 c I} = \sqrt{2(4\pi\times10^{-7}\ \text{H/m})(3.00\times10^{8}\ \text{m/s})(5.0\times10^{-3}\ \text{W/m}^2)} \\
&= 1.9\ \text{V/m}.
\end{aligned}$$

(b) The radiation pressure is $p_r = I_a/c$, where I_a is the absorbed intensity. Thus

$$p_r = \frac{5.0\times10^{-3}\ \text{W/m}^2}{3.00\times10^{8}\ \text{m/s}} = 1.7\times10^{-11}\ \text{Pa}.$$

40. We note the points at which the curve is zero (θ_2 = 60° and 140°) in Fig. 33-44. We infer that sheet 2 is perpendicular to one of the other sheets at θ_2 = 60°, and that it is perpendicular to the *other* of the other sheets when θ_2 = 140°. Without loss of generality, we choose θ_1 = 150°, θ_3 = 50°. Now, when θ_2 = 90°, it will be $|\Delta\theta|$ = 60° relative to sheet 1 and $|\Delta\theta'|$ = 40° relative to sheet 3. Therefore,

$$\frac{I_f}{I_i} = \frac{1}{2}\cos^2(\Delta\theta)\cos^2(\Delta\theta') = 7.3\% .$$

41. As the polarized beam of intensity I_0 passes the first polarizer, its intensity is reduced to $I_0\cos^2\theta$. After passing through the second polarizer, which makes a 90° angle with the first filter, the intensity is

$$I = (I_0\cos^2\theta)\sin^2\theta = I_0/10$$

which implies $\sin^2\theta\cos^2\theta = 1/10$, or $\sin\theta\cos\theta = \sin 2\theta/2 = 1/\sqrt{10}$. This leads to $\theta = 70°$ or 20°.

42. We examine the point where the graph reaches zero: $\theta_2 = 160°$. Since the polarizers must be "crossed" for the intensity to vanish, then $\theta_1 = 160° - 90° = 70°$. Now we consider the case $\theta_2 = 90°$ (which is hard to judge from the graph). Since θ_1 is still equal to 70°, then the angle between the polarizers is now $\Delta\theta = 20°$. Accounting for the "automatic" reduction (by a factor of one-half) whenever unpolarized light passes through any polarizing sheet, then our result is

$$\tfrac{1}{2}\cos^2(\Delta\theta) = 0.442 \approx 44\%.$$

43. Let I_0 be the intensity of the incident beam and f be the fraction that is polarized. Thus, the intensity of the polarized portion is fI_0. After transmission, this portion contributes $fI_0\cos^2\theta$ to the intensity of the transmitted beam. Here θ is the angle between the direction of polarization of the radiation and the polarizing direction of the filter. The intensity of the unpolarized portion of the incident beam is $(1-f)I_0$ and after transmission, this portion contributes $(1-f)I_0/2$ to the transmitted intensity. Consequently, the transmitted intensity is

$$I = fI_0\cos^2\theta + \frac{1}{2}(1-f)I_0.$$

As the filter is rotated, $\cos^2\theta$ varies from a minimum of 0 to a maximum of 1, so the transmitted intensity varies from a minimum of

$$I_{\min} = \frac{1}{2}(1-f)I_0$$

to a maximum of

$$I_{\max} = fI_0 + \frac{1}{2}(1-f)I_0 = \frac{1}{2}(1+f)I_0.$$

The ratio of $I_{\max}$ to $I_{\min}$ is

$$\frac{I_{\max}}{I_{\min}} = \frac{1+f}{1-f}.$$

Setting the ratio equal to 5.0 and solving for f, we get $f = 0.67$.

44. We apply Eq. 33-40 (once) and Eq. 33-42 (twice) to obtain

$$I = \frac{1}{2}I_0\cos^2\theta_2\cos^2(90° - \theta_2).$$

Using trig identities, we rewrite this as $\frac{I}{I_0} = \frac{1}{8}\sin^2(2\theta_2)$.

(a) Therefore we find $\theta_2 = \frac{1}{2}\sin^{-1}\sqrt{0.40} = 19.6°$.

(b) Since the first expression we wrote is symmetric under the exchange $\theta_2 \leftrightarrow 90° - \theta_2$, we see that the angle's complement, 70.4°, is also a solution.

45. Note that the normal to the refracting surface is vertical in the diagram. The angle of refraction is $\theta_2 = 90°$ and the angle of incidence is given by $\tan\theta_1 = L/D$, where D is the height of the tank and L is its width. Thus

$$\theta_1 = \tan^{-1}\left(\frac{L}{D}\right) = \tan^{-1}\left(\frac{1.10\text{ m}}{0.850\text{ m}}\right) = 52.31°.$$

The law of refraction yields

$$n_1 = n_2\frac{\sin\theta_2}{\sin\theta_1} = (1.00)\left(\frac{\sin 90°}{\sin 52.31°}\right) = 1.26,$$

where the index of refraction of air was taken to be unity.

46. (a) For the angles of incidence and refraction to be equal, the graph in Fig. 33-47(b) would consist of a "$y = x$" line at 45º in the plot. Instead, the curve for material 1 falls under such a "$y = x$" line, which tells us that all refraction angles are less than incident ones. With $\theta_2 < \theta_1$ Snell's law implies $n_2 > n_1$.

(b) Using the same argument as in (a), the value of n_2 for material 2 is also greater than that of water (n_1).

(c) It's easiest to examine the topmost point of each curve. With θ_2 = 90º and θ_1 = ½(90º), and with n_2 = 1.33 (Table 33-1), we find n_1 = 1.9 from Snell's law.

(d) Similarly, with θ_2 = 90º and θ_1 = ¾(90º), we obtain n_1 = 1.4.

47. The law of refraction states

$$n_1\sin\theta_1 = n_2\sin\theta_2.$$

We take medium 1 to be the vacuum, with $n_1 = 1$ and $\theta_1 = 32.0°$. Medium 2 is the glass, with $\theta_2 = 21.0°$. We solve for n_2:

$$n_2 = n_1 \frac{\sin\theta_1}{\sin\theta_2} = (1.00)\left(\frac{\sin 32.0°}{\sin 21.0°}\right) = 1.48.$$

48. (a) For the angles of incidence and refraction to be equal, the graph in Fig. 33-48(b) would consist of a "$y = x$" line at 45º in the plot. Instead, the curve for material 1 falls under such a "$y = x$" line, which tells us that all refraction angles are less than incident ones. With $\theta_2 < \theta_1$ Snell's law implies $n_2 > n_1$.

(b) Using the same argument as in (a), the value of n_2 for material 2 is also greater than that of water (n_1).

(c) It's easiest to examine the right end-point of each curve. With θ_1 = 90º and θ_2 = ¾(90º), and with n_1 = 1.33 (Table 33-1) we find, from Snell's law, n_2 = 1.4 for material 1.

(d) Similarly, with θ_1 = 90º and θ_2 = ½(90º), we obtain n_2 = 1.9.

49. The angle of incidence for the light ray on mirror B is 90° – θ. So the outgoing ray r' makes an angle 90° – (90° – θ) = θ with the vertical direction, and is antiparallel to the incoming one. The angle between i and r' is therefore 180°.

50. (a) From $n_1\sin\theta_1 = n_2\sin\theta_2$ and $n_2\sin\theta_2 = n_3\sin\theta_3$, we find $n_1\sin\theta_1 = n_3\sin\theta_3$. This has a simple implication: that $\theta_1 = \theta_3$ when $n_1 = n_3$. Since we are given θ_1 = 40º in Fig. 33-50(a), then we look for a point in Fig. 33-50(b) where θ_3 = 40º. This seems to occur at n_3 = 1.6, so we infer that n_1 = 1.6.

(b) Our first step in our solution to part (a) shows that information concerning n_2 disappears (cancels) in the manipulation. Thus, we cannot tell; we need more information.

(c) From 1.6sin70° = 2.4sinθ_3 we obtain θ_3 = 39°.

51. (a) Approximating n = 1 for air, we have

$$n_1 \sin\theta_1 = (1)\sin\theta_5 \quad \Rightarrow \quad 56.9° = \theta_5$$

and with the more accurate value for n_{air} in Table 33-1, we obtain 56.8°.

(b) Equation 33-44 leads to

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 = n_3 \sin\theta_3 = n_4 \sin\theta_4$$

so that

$$\theta_4 = \sin^{-1}\left(\frac{n_1}{n_4}\sin\theta_1\right) = 35.3°.$$

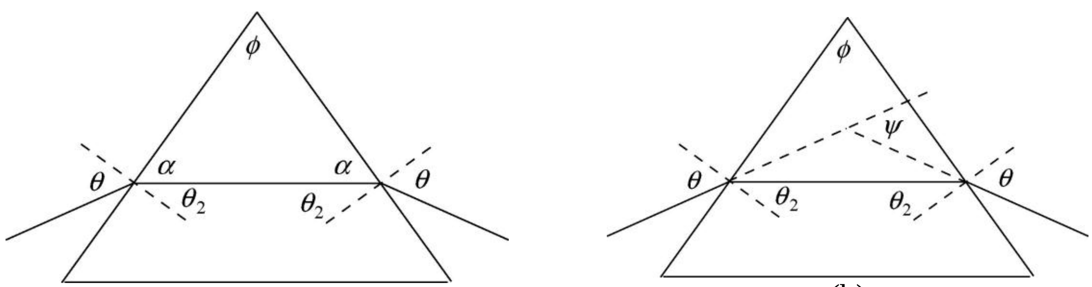
52. (a) A simple implication of Snell's law is that $\theta_2 = \theta_1$ when $n_1 = n_2$. Since the angle of incidence is shown in Fig. 33-52(a) to be 30º, we look for a point in Fig. 33-52(b) where θ_2 = 30º. This seems to occur when n_2 = 1.7. By inference, then, n_1 = 1.7.

(b) From 1.7sin(60º) = 2.4sin(θ_2) we get θ_2 = 38°.

53. **THINK** The angle with which the light beam emerges from the triangular prism depends on the index of refraction of the prism.

EXPRESS Consider diagram (a) shown next. The incident angle is θ and the angle of refraction is θ_2. Since $\theta_2 + \alpha = 90°$ and $\phi + 2\alpha = 180°$, we have

$$\theta_2 = 90° - \alpha = 90° - \frac{1}{2}\left(180° - \phi\right) = \frac{\phi}{2}.$$



(a) (b)

ANALYZE Next, examine diagram (b) and consider the triangle formed by the two normals and the ray in the interior. One can show that ψ is given by

$$\psi = 2(\theta - \theta_2) .$$

Upon substituting $\phi/2$ for θ_2, we obtain $\psi = 2(\theta - \phi/2)$ which yields $\theta = (\phi + \psi)/2$. Thus, using the law of refraction, we find the index of refraction of the prism to be

$$n = \frac{\sin\theta}{\sin\theta_2} = \frac{\sin\frac{1}{2}(\phi+\psi)}{\sin\frac{1}{2}\phi}.$$

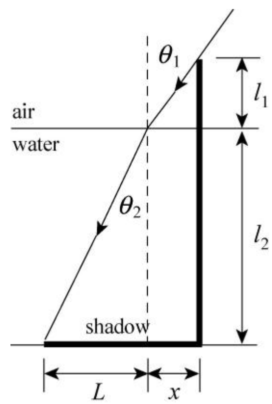
LEARN The angle ψ is called the deviation angle. Physically, it represents the total angle through which the beam has turned while passing through the prism. This angle is minimum when the beam passes through the prism "symmetrically," as it does in this case. Knowing the value of ϕ and ψ allows us to determine the value of n for the prism material.

54. (a) Snell's law gives n_{air} sin(50º) = n_{2b} sin θ_{2b} and n_{air} sin(50º) = n_{2r} sin θ_{2r} where we use subscripts b and r for the blue and red light rays. Using the common approximation for air's index (n_{air} = 1.0) we find the two angles of refraction to be 30.176° and 30.507°. Therefore, $\Delta\theta$ = 0.33°.

(b) Both of the refracted rays emerge from the other side with the same angle (50°) with which they were incident on the first side (generally speaking, light comes into a block at the same angle that it emerges with from the opposite parallel side). There is thus no difference (the difference is 0°) and thus there is no dispersion in this case.

55. **THINK** Light is refracted at the air–water interface. To calculate the length of the shadow of the pole, we first calculate the angle of refraction using the Snell's law.

EXPRESS Consider a ray that grazes the top of the pole, as shown in the diagram below.



Here θ_1 = 90° – θ = 90° –55° = 35°, $\ell_1 = 0.50$ m, and $\ell_2 = 1.50$ m. The length of the shadow is $d = x + L$.

ANALYZE The distance x is given by

$$x = \ell_1 \tan\theta_1 = (0.50\ \text{m})\tan 35° = 0.35\ \text{m}.$$

According to the law of refraction, $n_2 \sin\theta_2 = n_1 \sin\theta_1$. We take $n_1 = 1$ and $n_2 = 1.33$ (from Table 33-1). Then,

$$\theta_2 = \sin^{-1}\left(\frac{\sin\theta_1}{n_2}\right) = \sin^{-1}\left(\frac{\sin 35.0°}{1.33}\right) = 25.55°.$$

L is given by

$$L = \ell_2 \tan\theta_2 = (1.50\ \text{m})\tan 25.55° = 0.72\ \text{m}.$$

Thus, the length of the shadow is d = 0.35 m + 0.72 m = 1.07 m.

LEARN If the pole were empty with no water, then $\theta_1 = \theta_2$ and the length of the shadow would be

$$d' = \ell_1 \tan\theta_1 + \ell_2 \tan\theta_1 = (\ell_1 + \ell_2)\tan\theta_1$$

by simple geometric consideration.

56. (a) We use subscripts *b* and *r* for the blue and red light rays. Snell's law gives

$$\theta_{2b} = \sin^{-1}\left(\frac{1}{1.343}\sin(70°)\right) = 44.403°$$
$$\theta_{2r} = \sin^{-1}\left(\frac{1}{1.331}\sin(70°)\right) = 44.911°$$

for the refraction angles at the first surface (where the normal axis is vertical). These rays strike the second surface (where *A* is) at complementary angles to those just calculated (since the normal axis is horizontal for the second surface). Taking this into consideration, we again use Snell's law to calculate the second refractions (with which the light re-enters the air):

$$\theta_{3b} = \sin^{-1}[1.343\sin(90° - \theta_{2b})] = 73.636°$$
$$\theta_{3r} = \sin^{-1}[1.331\sin(90° - \theta_{2r})] = 70.497°$$

which differ by 3.1° (thus giving a rainbow of angular width 3.1°).

(b) Both of the refracted rays emerge from the bottom side with the same angle (70°) with which they were incident on the topside (the occurrence of an intermediate reflection [from side 2] does not alter this overall fact: light comes into the block at the same angle that it emerges with from the opposite parallel side). There is thus no difference (the difference is 0°) and thus there is no rainbow in this case.

57. Reference to Fig. 33-24 may help in the visualization of why there appears to be a "circle of light" (consider revolving that picture about a vertical axis). The depth and the radius of that circle (which is from point *a* to point *f* in that figure) is related to the tangent of the angle of incidence. Thus, the diameter *D* of the circle in question is

$$D = 2h\tan\theta_c = 2h\tan\left[\sin^{-1}\left(\frac{1}{n_w}\right)\right] = 2(80.0\,\text{cm})\tan\left[\sin^{-1}\left(\frac{1}{1.33}\right)\right] = 182\text{ cm}.$$

58. The critical angle is $\theta_c = \sin^{-1}\left(\frac{1}{n}\right) = \sin^{-1}\left(\frac{1}{1.8}\right) = 34°.$

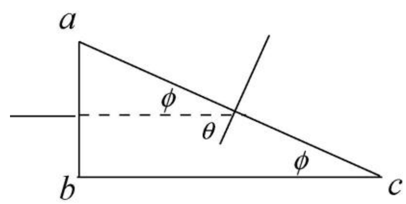
59. **THINK** Total internal reflection happens when the angle of incidence exceeds a critical angle such that Snell's law gives $\sin\theta_2 > 1$.

EXPRESS When light reaches the interfaces between two materials with indices of refraction n_1 and n_2, if $n_1 > n_2$, and the incident angle exceeds a critical value given by

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right),$$

then total internal reflection will occur.

In our case, the incident light ray is perpendicular to the face *ab*. Thus, no refraction occurs at the surface *ab*, so the angle of incidence at surface *ac* is $\theta = 90° - \phi$, as shown in the figure below.



ANALYZE (a) For total internal reflection at the second surface, $n_g \sin(90° - \phi)$ must be greater than n_a. Here n_g is the index of refraction for the glass and n_a is the index of refraction for air. Since $\sin(90° - \phi) = \cos\phi$, we want the largest value of ϕ for which $n_g \cos\phi \geq n_a$. Recall that $\cos\phi$ decreases as ϕ increases from zero. When ϕ has the largest value for which total internal reflection occurs, then $n_g \cos\phi = n_a$, or

$$\phi = \cos^{-1}\left(\frac{n_a}{n_g}\right) = \cos^{-1}\left(\frac{1}{1.52}\right) = 48.9°.$$

The index of refraction for air is taken to be unity.

(b) We now replace the air with water. If $n_w = 1.33$ is the index of refraction for water, then the largest value of ϕ for which total internal reflection occurs is

$$\phi = \cos^{-1}\left(\frac{n_w}{n_g}\right) = \cos^{-1}\left(\frac{1.33}{1.52}\right) = 29.0°.$$

LEARN Total internal reflection cannot occur if the incident light is in the medium with lower index of refraction. With $\theta_c = \sin^{-1}(n_2/n_1)$, we see that the larger the ratio n_2/n_1, the larger the value of θ_c.

60. (a) The condition (in Eq. 33-44) required in the critical angle calculation is $\theta_3 = 90°$. Thus (with $\theta_2 = \theta_c$, which we don't compute here),

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 = n_3 \sin\theta_3$$

leads to $\theta_1 = \theta = \sin^{-1} n_3/n_1 = 54.3°$.

(b) Yes. Reducing θ leads to a reduction of θ_2 so that it becomes less than the critical angle; therefore, there will be some transmission of light into material 3.

(c) We note that the complement of the angle of refraction (in material 2) is the critical angle. Thus,

$$n_1 \sin\theta = n_2 \cos\theta_c = n_2\sqrt{1-\left(\frac{n_3}{n_2}\right)^2} = \sqrt{n_2^2 - n_3^2}$$

leading to $\theta = 51.1°$.

(d) No. Reducing θ leads to an increase of the angle with which the light strikes the interface between materials 2 and 3, so it becomes greater than the critical angle. Therefore, there will be no transmission of light into material 3.

61. (a) We note that the complement of the angle of refraction (in material 2) is the critical angle. Thus,

$$n_1 \sin\theta = n_2 \cos\theta_c = n_2\sqrt{1-\left(\frac{n_3}{n_2}\right)^2} = \sqrt{n_2^2 - n_3^2}$$

leading to $\theta = 26.8°$.

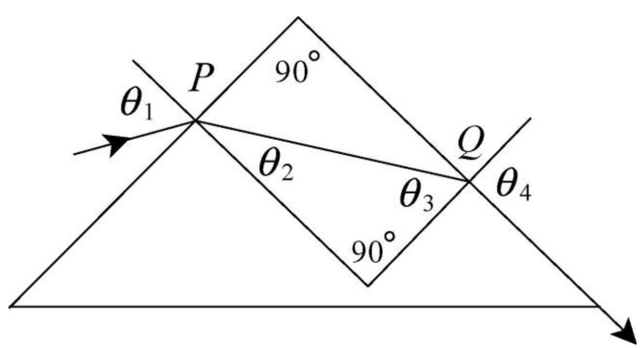
(b) Increasing θ leads to a decrease of the angle with which the light strikes the interface between materials 2 and 3, so it becomes greater than the critical angle; therefore, there will be some transmission of light into material 3.

62. (a) Reference to Fig. 33-24 may help in the visualization of why there appears to be a "circle of light" (consider revolving that picture about a vertical axis). The depth and the radius of that circle (which is from point *a* to point *f* in that figure) is related to the tangent of the angle of incidence. The diameter of the circle in question is given by $d = 2h \tan \theta_c$. For water $n = 1.33$, so Eq. 33-47 gives $\sin \theta_c = 1/1.33$, or $\theta_c = 48.75°$. Thus,

$$d = 2h\tan\theta_c = 2(2.00 \text{ m})(\tan 48.75°) = 4.56 \text{ m}.$$

(b) The diameter *d* of the circle will increase if the fish descends (increasing *h*).

63. (a) A ray diagram is shown below.



Let θ_1 be the angle of incidence and θ_2 be the angle of refraction at the first surface. Let θ_3 be the angle of incidence at the second surface. The angle of refraction there is θ_4 = 90°. The law of refraction, applied to the second surface, yields $n \sin \theta_3 = \sin \theta_4 = 1$. As shown in the diagram, the normals to the surfaces at *P* and *Q* are perpendicular to each other. The interior angles of the triangle formed by the ray and the two normals must sum to 180°, so $\theta_3 = 90° - \theta_2$ and

$$\sin\theta_3 = \sin(90° - \theta_2) = \cos\theta_2 = \sqrt{1 - \sin^2\theta_2}.$$

According to the law of refraction, applied at *Q*, $n\sqrt{1-\sin^2\theta_2} = 1$. The law of refraction, applied to point *P*, yields $\sin \theta_1 = n \sin \theta_2$, so $\sin \theta_2 = (\sin \theta_1)/n$ and

$$n\sqrt{1 - \frac{\sin^2\theta_1}{n^2}} = 1.$$

Squaring both sides and solving for *n*, we get

$$n = \sqrt{1 + \sin^2\theta_1}.$$

(b) The greatest possible value of $\sin^2 \theta_1$ is 1, so the greatest possible value of *n* is $n_{\max} = \sqrt{2} = 1.41$.

(c) For a given value of *n*, if the angle of incidence at the first surface is greater than θ_1, the angle of refraction there is greater than θ_2 and the angle of incidence at the second face is less than θ_3 (= 90° – θ_2). That is, it is less than the critical angle for total internal reflection, so light leaves the second surface and emerges into the air.

(d) If the angle of incidence at the first surface is less than θ_1, the angle of refraction there is less than θ_2 and the angle of incidence at the second surface is greater than θ_3. This is greater than the critical angle for total internal reflection, so all the light is reflected at *Q*.

64. (a) We refer to the entry point for the original incident ray as point *A* (which we take to be on the left side of the prism, as in Fig. 33-53), the prism vertex as point *B*, and the point where the interior ray strikes the right surface of the prism as point *C*. The angle between line *AB* and the interior ray is β (the complement of the angle of refraction at the first surface), and the angle between the line *BC* and the interior ray is α (the complement of its angle of incidence when it strikes the second surface). When the incident ray is at the minimum angle for which light is able to exit the prism, the light exits along the second face. That is, the angle of refraction at the second face is 90°, and the angle of incidence there for the interior ray is the critical angle for total internal reflection. Let θ_1 be the angle of incidence for the original incident ray and θ_2 be the angle of refraction at the first face, and let θ_3 be the angle of incidence at the second face. The law of refraction, applied to point *C*, yields $n \sin \theta_3 = 1$, so

$$\sin\theta_3 = 1/n = 1/1.60 = 0.625 \Rightarrow \theta_3 = 38.68°.$$

The interior angles of the triangle *ABC* must sum to 180°, so $\alpha + \beta = 120°$. Now, $\alpha = 90° - \theta_3 = 51.32°$, so $\beta = 120° - 51.32° = 69.68°$. Thus, $\theta_2 = 90° - \beta = 21.32°$. The law of refraction, applied to point *A*, yields

$$\sin\theta_1 = n\sin\theta_2 = 1.60\sin 21.32° = 0.5817.$$

Thus $\theta_1 = 35.6°$.

(b) We apply the law of refraction to point *C*. Since the angle of refraction there is the same as the angle of incidence at *A*, $n\sin\theta_3 = \sin\theta_1$. Now, $\alpha + \beta = 120°$, $\alpha = 90° - \theta_3$, and $\beta = 90° - \theta_2$, as before. This means $\theta_2 + \theta_3 = 60°$. Thus, the law of refraction leads to

$$\sin\theta_1 = n\sin\left(60° - \theta_2\right) \Rightarrow \sin\theta_1 = n\sin 60°\cos\theta_2 - n\cos 60°\sin\theta_2$$

where the trigonometric identity

$$\sin(A - B) = \sin A\cos B - \cos A\sin B$$

is used. Next, we apply the law of refraction to point *A*:

$$\sin\theta_1 = n\sin\theta_2 \Rightarrow \sin\theta_2 = \left(1/n\right)\sin\theta_1$$

which yields $\cos\theta_2 = \sqrt{1-\sin^2\theta_2} = \sqrt{1-\left(1/n^2\right)\sin^2\theta_1}$. Thus,

$$\sin\theta_1 = n\sin 60°\sqrt{1-\left(1/n\right)^2\sin^2\theta_1} - \cos 60°\sin\theta_1$$

or

$$\left(1+\cos 60°\right)\sin\theta_1 = \sin 60°\sqrt{n^2 - \sin^2\theta_1}.$$

Squaring both sides and solving for $\sin\theta_1$, we obtain

$$\sin\theta_1 = \frac{n\sin 60°}{\sqrt{\left(1+\cos 60°\right)^2 + \sin^2 60°}} = \frac{1.60\sin 60°}{\sqrt{\left(1+\cos 60°\right)^2 + \sin^2 60°}} = 0.80$$

and $\theta_1 = 53.1°$.

65. When examining Fig. 33-61, it is important to note that the angle (measured from the central axis) for the light ray in air, θ, is not the angle for the ray in the glass core, which we denote θ'. The law of refraction leads to

$$\sin\theta' = \frac{1}{n_1}\sin\theta$$

assuming $n_{\text{air}} = 1$. The angle of incidence for the light ray striking the coating is the complement of θ', which we denote as θ'_{comp}, and recall that

$$\sin\theta'_{\text{comp}} = \cos\theta' = \sqrt{1-\sin^2\theta'}.$$

In the critical case, θ'_{comp} must equal θ_c specified by Eq. 33-47. Therefore,

$$\frac{n_2}{n_1} = \sin\theta'_{\text{comp}} = \sqrt{1-\sin^2\theta'} = \sqrt{1-\left(\frac{1}{n_1}\sin\theta\right)^2}$$

which leads to the result: $\sin\theta = \sqrt{n_1^2 - n_2^2}$. With n_1 = 1.58 and n_2 = 1.53, we obtain

$$\theta = \sin^{-1}\left(1.58^2 - 1.53^2\right) = 23.2°.$$

66. (a) We note that the upper-right corner is at an angle (measured from the point where the light enters, and measured relative to a normal axis established at that point the normal at that point would be horizontal in Fig. 33-62) is at $\tan^{-1}(2/3)$ = 33.7º. The angle of refraction is given by

$$n_{\text{air}} \sin 40° = 1.56 \sin\theta_2$$

which yields θ_2 = 24.33º if we use the common approximation n_{air} = 1.0, and yields θ_2 = 24.34º if we use the more accurate value for n_{air} found in Table 33-1. The value is less than 33.7º, which means that the light goes to side 3.

(b) The ray strikes a point on side 3, which is 0.643 cm below that upper-right corner, and then (using the fact that the angle is symmetrical upon reflection) strikes the top surface (side 2) at a point 1.42 cm to the left of that corner. Since 1.42 cm is certainly less than 3 cm we have a self-consistency check to the effect that the ray does indeed strike side 2 as its second reflection (if we had gotten 3.42 cm instead of 1.42 cm, then the situation would be quite different).

(c) The normal axes for sides 1 and 3 are both horizontal, so the angle of incidence (in the plastic) at side 3 is the same as the angle of refraction was at side 1. Thus,

$$1.56 \sin 24.3° = n_{\text{air}} \sin\theta_{\text{air}} \quad\Rightarrow\quad \theta_{\text{air}} = 40°.$$

(d) It strikes the top surface (side 2) at an angle (measured from the normal axis there, which in this case would be a vertical axis) of 90º $-\ \theta_2$ = 66º, which is much greater than the critical angle for total internal reflection ($\sin^{-1}(n_{air}/1.56) = 39.9^o$). Therefore, no refraction occurs when the light strikes side 2.

(e) In this case, we have

$$n_{air} \sin 70^o = 1.56 \sin \theta_2$$

which yields $\theta_2 = 37.04^o$ if we use the common approximation $n_{air} = 1.0$, and yields $\theta_2 = 37.05^o$ if we use the more accurate value for n_{air} found in Table 33-1. This is greater than the 33.7º mentioned above (regarding the upper-right corner), so the ray strikes side 2 instead of side 3.

(f) After bouncing from side 2 (at a point fairly close to that corner) it goes to side 3.

(g) When it bounced from side 2, its angle of incidence (because the normal axis for side 2 is orthogonal to that for side 1) is $90^o - \theta_2 = 53^o$, which is much greater than the critical angle for total internal reflection (which, again, is $\sin^{-1}(n_{air}/1.56) = 39.9^o$). Therefore, no refraction occurs when the light strikes side 2.

(h) For the same reasons implicit in the calculation of part (c), the refracted ray emerges from side 3 with the same angle (70°) that it entered side 1. We see that the occurrence of an intermediate reflection (from side 2) does not alter this overall fact: light comes into the block at the same angle that it emerges with from the opposite parallel side.

67. (a) In the notation of this problem, Eq. 33-47 becomes

$$\theta_c = \sin^{-1}\frac{n_3}{n_2}$$

which yields $n_3 = 1.39$ for $\theta_c = \phi = 60°$.

(b) Applying Eq. 33-44 to the interface between material 1 and material 2, we have

$$n_2 \sin 30° = n_1 \sin \theta$$

which yields $\theta = 28.1°$.

(c) Decreasing θ will increase ϕ and thus cause the ray to strike the interface (between materials 2 and 3) at an angle larger than θ_c. Therefore, no transmission of light into material 3 can occur.

68. (a) We use Eq. 33-49: $\theta_B = \tan^{-1} n_w = \tan^{-1}(1.33) = 53.1°$.

(b) Yes, since n_w depends on the wavelength of the light.

69. **THINK** A reflected wave will be fully polarized if it strikes the boundary at the Brewster angle.

EXPRESS The angle of incidence for which reflected light is fully polarized is given by Eq. 33-48:

$$\theta_B = \tan^{-1}\left(\frac{n_2}{n_1}\right)$$

where n_1 is the index of refraction for the medium of incidence and n_2 is the index of refraction for the second medium. The angle θ_B is called the Brewster angle.

ANALYZE With $n_1 = 1.33$ and $n_2 = 1.53$, we obtain

$$\theta_B = \tan^{-1}(n_2/n_1) = \tan^{-1}(1.53/1.33) = 49.0°.$$

LEARN In general, reflected light is partially polarized, having components both parallel and perpendicular to the plane of incidence. However, it can be completely polarized when incident at the Brewster angle.

70. Since the layers are parallel, the angle of refraction regarding the first surface is the same as the angle of incidence regarding the second surface (as is suggested by the notation in Fig. 33-64). We recall that as part of the derivation of Eq. 33-49 (Brewster's angle), the refracted angle is the complement of the incident angle:

$$\theta_2 = (\theta_1)_c = 90° - \theta_1.$$

We apply Eq. 33-49 to both refractions, setting up a product:

$$\left(\frac{n_2}{n_1}\right)\left(\frac{n_3}{n_2}\right) = (\tan\theta_{B1\to 2})(\tan\theta_{B2\to 3}) \quad \Rightarrow \quad \frac{n_3}{n_1} = (\tan\theta_1)(\tan\theta_2).$$

Now, since θ_2 is the complement of θ_1 we have

$$\tan\theta_2 = \tan(\theta_1)_c = \frac{1}{\tan\theta_1}.$$

Therefore, the product of tangents cancel and we obtain $n_3/n_1 = 1$. Consequently, the third medium is air: $n_3 = 1.0$.

71. **THINK** All electromagnetic waves, including visible light, travel at the same speed c in vacuum.

EXPRESS The time for light to travel a distance d in free space is $t = d/c$, where c is the speed of light (3.00×10^8 m/s).

ANALYZE (a) We take d to be 150 km = 150 × 10^3 m. Then,

$$t = \frac{d}{c} = \frac{150\times10^3\text{ m}}{3.00\times10^8\text{ m/s}} = 5.00\times10^{-4}\text{ s.}$$

(b) At full moon, the Moon and Sun are on opposite sides of Earth, so the distance traveled by the light is

$$d = (1.5\times10^8\text{ km}) + 2\,(3.8\times10^5\text{ km}) = 1.51\times10^8\text{ km} = 1.51\times10^{11}\text{ m.}$$

The time taken by light to travel this distance is

$$t = \frac{d}{c} = \frac{1.51\times10^{11}\text{ m}}{3.00\times10^8\text{ m/s}} = 500\text{ s} = 8.4\text{ min.}$$

(c) We take d to be 2(1.3 × 10^9 km) = 2.6 × 10^{12} m. Then,

$$t = \frac{d}{c} = \frac{2.6\times10^{12}\text{ m}}{3.00\times10^8\text{ m/s}} = 8.7\times10^3\text{ s} = 2.4\text{ h.}$$

(d) We take d to be 6500 ly and the speed of light to be 1.00 ly/y. Then,

$$t = \frac{d}{c} = \frac{6500\text{ ly}}{1.00\text{ ly/y}} = 6500\text{ y.}$$

The explosion took place in the year 1054 – 6500 = –5446 or 5446 B.C.

LEARN Since the speed c is constant, the travel time is proportional to the distance. The radio signals at 150 km away reach you almost instantly.

72. (a) The expression $E_y = E_m \sin(kx - \omega t)$ fits the requirement "at point P … [it] is decreasing with time" if we imagine P is just to the right ($x > 0$) of the coordinate origin (but at a value of x less than $\pi/2k = \lambda/4$ which is where there would be a maximum, at t = 0). It is important to bear in mind, in this description, that the wave is moving to the right. Specifically, $x_P = (1/k)\sin^{-1}(1/4)$ so that E_y = (1/4) E_m at t = 0, there. Also, E_y = 0 with our choice of expression for E_y. Therefore, part (a) is answered simply by solving for x_P. Since $k = 2\pi f/c$ we find

$$x_P = \frac{c}{2\pi f}\sin^{-1}\left(\frac{1}{4}\right) = 30.1\text{ nm}.$$

(b) If we proceed to the right on the x axis (still studying this "snapshot" of the wave at t = 0) we find another point where E_y = 0 at a distance of one-half wavelength from the previous point where $E_y = 0$. Thus (since $\lambda = c/f$) the next point is at $x = \frac{1}{2}\lambda = \frac{1}{2}c/f$ and is consequently a distance $c/2f - x_P = 345$ nm to the right of P.

73. **THINK** The electric and magnetic components of the electromagnetic waves are always in phase, perpendicular to each other, and perpendicular to the direction of propagation of the wave.

EXPRESS The electric and magnetic fields can be written as sinusoidal functions of position and time as:

$$E = E_m \sin(kx + \omega t), \quad B = B_m \sin(kx + \omega t)$$

where E_m and B_m are the amplitudes of the fields, and ω and k, are the angular frequency and angular wave number of the wave, respectively. The two amplitudes are related by Eq. 33-4: $E_m / B_m = c$, where c is the speed of the wave.

ANALYZE (a) From $kc = \omega$ where $k = 1.00 \times 10^6$ m^{-1}, we obtain $\omega = 3.00 \times 10^{14}$ rad/s. The magnetic field amplitude is, from Eq. 33-5,

$$B_m = E_m/c = (5.00 \text{ V/m})/c = 1.67 \times 10^{-8} \text{ T}.$$

From the argument of the sinusoidal fucntion for E, we see that the direction of propagation is in the –z direction. Since $\vec{E} = E_y\hat{\text{j}}$, and that $\vec{B}$ is perpendicular to $\vec{E}$ and $\vec{E}\times\vec{B}$,, we conclude that the only non-zero component of $\vec{B}$ is B_x, so that we have

$$B_x = (1.67\times10^{-8} \text{ T})\sin[(1.00\times10^6 / \text{m})z + (3.00\times10^{14} / \text{s})t].$$

(b) The wavelength is $\lambda = 2\pi/k = 6.28 \times 10^{-6}$ m.

(c) The period is $T = 2\pi/\omega = 2.09 \times 10^{-14}$ s.

(d) The intensity is

$$I = \frac{1}{c\mu_0}\left(\frac{5.00\,\text{V/m}}{\sqrt{2}}\right)^2 = 0.0332\,\text{W/m}^2 .$$

(e) As noted in part (a), the only nonzero component of $\vec{B}$ is B_x. The magnetic field oscillates along the x axis.

(f) The wavelength found in part (b) places this in the infrared portion of the spectrum.

LEARN Electromagnetic wave is a transverse wave. Knowing the functional form of the electric field allows us to determine the corresponding magnetic field, and vice versa.

74. (a) Let r be the radius and ρ be the density of the particle. Since its volume is $(4\pi/3)r^3$, its mass is $m = (4\pi/3)\rho r^3$. Let R be the distance from the Sun to the particle and let M be the mass of the Sun. Then, the gravitational force of attraction of the Sun on the particle has magnitude

$$F_g = \frac{GMm}{R^2} = \frac{4\pi GM\rho r^3}{3R^2}.$$

If P is the power output of the Sun, then at the position of the particle, the radiation intensity is $I = P/4\pi R^2$, and since the particle is perfectly absorbing, the radiation pressure on it is

$$p_r = \frac{I}{c} = \frac{P}{4\pi R^2 c}.$$

All of the radiation that passes through a circle of radius r and area $A = \pi r^2$, perpendicular to the direction of propagation, is absorbed by the particle, so the force of the radiation on the particle has magnitude

$$F_r = p_r A = \frac{\pi P r^2}{4\pi R^2 c} = \frac{Pr^2}{4R^2 c}.$$

The force is radially outward from the Sun. Notice that both the force of gravity and the force of the radiation are inversely proportional to R^2. If one of these forces is larger than the other at some distance from the Sun, then that force is larger at all distances. The two forces depend on the particle radius r differently: F_g is proportional to r^3 and F_r is proportional to r^2. We expect a small radius particle to be blown away by the radiation pressure and a large radius particle with the same density to be pulled inward toward the Sun. The critical value for the radius is the value for which the two forces are equal. Equating the expressions for F_g and F_r, we solve for r:

$$r = \frac{3P}{16\pi GM\rho c}.$$

(b) According to Appendix C, $M = 1.99 \times 10^{30}$ kg and $P = 3.90 \times 10^{26}$ W. Thus,

$$r = \frac{3(3.90\times10^{26}\ \text{W})}{16\pi(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(1.99\times10^{30}\ \text{kg})(1.0\times10^{3}\ \text{kg}/\text{m}^3)(3.00\times10^{8}\ \text{m}/\text{s})}$$
$$= 5.8\times10^{-7}\ \text{m}.$$

75. **THINK** Total internal reflection happens when the angle of incidence exceeds a critical angle such that Snell's law gives $\sin\theta_2 > 1$.

EXPRESS When light reaches the interfaces between two materials with indices of refraction n_1 and n_2, if $n_1 > n_2$, and the incident angle exceeds a critical value given by

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right),$$

then total internal reflection will occur.

Referring to Fig. 33-65, let $\theta_1 = 45°$ be the angle of incidence at the first surface and θ_2 be the angle of refraction there. Let θ_3 be the angle of incidence at the second surface. The condition for total internal reflection at the second surface is

$$n \sin \theta_3 \geq 1.$$

We want to find the smallest value of the index of refraction n for which this inequality holds. The law of refraction, applied to the first surface, yields

$$n \sin \theta_2 = \sin \theta_1.$$

Consideration of the triangle formed by the surface of the slab and the ray in the slab tells us that $\theta_3 = 90° - \theta_2$. Thus, the condition for total internal reflection becomes

$$1 \leq n \sin(90° - \theta_2) = n \cos \theta_2.$$

Squaring this equation and using $\sin^2 \theta_2 + \cos^2 \theta_2 = 1$, we obtain $1 \leq n^2 (1 - \sin^2 \theta_2)$. Substituting $\sin \theta_2 = (1/n) \sin \theta_1$ now leads to

$$1 \leq n^2\left(1 - \frac{\sin^2\theta_1}{n^2}\right) = n^2 - \sin^2\theta_1.$$

The smallest value of n for which this equation is true is given by $1 = n^2 - \sin^2 \theta_1$. We solve for n:

$$n = \sqrt{1+\sin^2\theta_1} = \sqrt{1+\sin^2 45°} = 1.22.$$

LEARN With $n = 1.22$, we have $\theta_2 = \sin^{-1}[(1/1.22)\sin 45°] = 35°$, which gives $\theta_3 = 90° - 35° = 55°$ as the angle of incidence at the second surface. We can readily verify that $n \sin \theta_3 = (1.22) \sin 55° = 1$, meeting the threshold condition for total internal reflection.

76. Since some of the angles in Fig. 33-66 are measured from vertical axes and some are measured from horizontal axes, we must be very careful in taking differences. For instance, the angle difference between the first polarizer struck by the light and the second is 110º (or 70º depending on how we measure it; it does not matter in the final result whether we put $\Delta\theta_1 = 70º$ or put $\Delta\theta_1 = 110º$). Similarly, the angle difference between the second and the third is $\Delta\theta_2 = 40º$, and between the third and the fourth is $\Delta\theta_3$

= 40º, also. Accounting for the "automatic" reduction (by a factor of one-half) whenever unpolarized light passes through any polarizing sheet, then our result is the incident intensity multiplied by

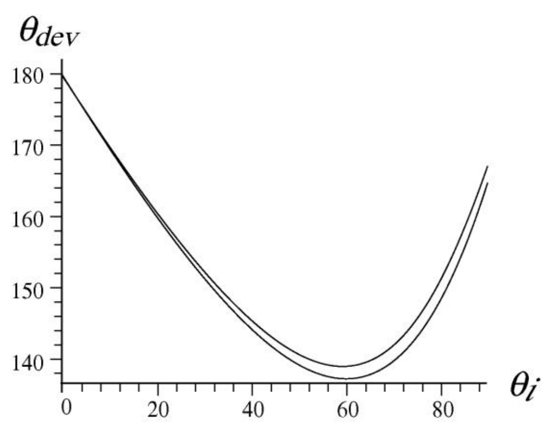
$$\frac{1}{2}\cos^2(\Delta\theta_1)\cos^2(\Delta\theta_2)\cos^2(\Delta\theta_3)\,.$$

Thus, the light that emerges from the system has intensity equal to 0.50 W/m^2.

77. (a) The first contribution to the overall deviation is at the first refraction: $\delta\theta_1 = \theta_i - \theta_r$. The next contribution to the overall deviation is the reflection. Noting that the angle between the ray right before reflection and the axis normal to the back surface of the sphere is equal to θ_r, and recalling the law of reflection, we conclude that the angle by which the ray turns (comparing the direction of propagation before and after the reflection) is $\delta\theta_2 = 180°-2\theta_r$. The final contribution is the refraction suffered by the ray upon leaving the sphere: $\delta\theta_3 = \theta_i - \theta_r$ again. Therefore,

$$\theta_{\text{dev}} = \delta\theta_1 + \delta\theta_2 + \delta\theta_3 = 180° + 2\theta_i - 4\theta_r.$$

(b) We substitute $\theta_r = \sin^{-1}(\frac{1}{n}\sin\theta_i)$ into the expression derived in part (a), using the two given values for n. The higher curve is for the blue light.



(c) We can expand the graph and try to estimate the minimum, or search for it with a more sophisticated numerical procedure. We find that the θ_{dev} minimum for red light is $137.63° \approx 137.6°$, and this occurs at θ_i = 59.52°.

(d) For blue light, we find that the θ_{dev} minimum is $139.35° \approx 139.4°$, and this occurs at θ_i = 59.52°.

(e) The difference in θ_{dev} in the previous two parts is 1.72°.

78. (a) The first contribution to the overall deviation is at the first refraction: $\delta\theta_1 = \theta_i - \theta_r$. The next contribution(s) to the overall deviation is (are) the reflection(s).

Noting that the angle between the ray right before reflection and the axis normal to the back surface of the sphere is equal to θ_r, and recalling the law of reflection, we conclude that the angle by which the ray turns (comparing the direction of propagation before and after [each] reflection) is $\delta\theta_r = 180° - 2\theta_r$. Thus, for k reflections, we have $\delta\theta_2 = k\theta_r$ to account for these contributions. The final contribution is the refraction suffered by the ray upon leaving the sphere: $\delta\theta_3 = \theta_i - \theta_r$ again. Therefore,

$$\theta_{\text{dev}} = \delta\theta_1 + \delta\theta_2 + \delta\theta_3 = 2(\theta_i - \theta_r) + k(180° - 2\theta_r) = k(180°) + 2\theta_i - 2(k+1)\theta_r.$$

(b) For k = 2 and n = 1.331 (given in Problem 33-77), we search for the second-order rainbow angle numerically. We find that the θ_{dev} minimum for red light is $230.37° \approx 230.4°$, and this occurs at θ_i = 71.90°.

(c) Similarly, we find that the second-order θ_{dev} minimum for blue light (for which n = 1.343) is $233.48° \approx 233.5°$, and this occurs at θ_i = 71.52°.

(d) The difference in θ_{dev} in the previous two parts is approximately 3.1°.

(e) Setting k = 3, we search for the third-order rainbow angle numerically. We find that the θ_{dev} minimum for red light is 317.5°, and this occurs at θ_i = 76.88°.

(f) Similarly, we find that the third-order θ_{dev} minimum for blue light is 321.9°, and this occurs at θ_i = 76.62°.

(g) The difference in θ_{dev} in the previous two parts is 4.4°.

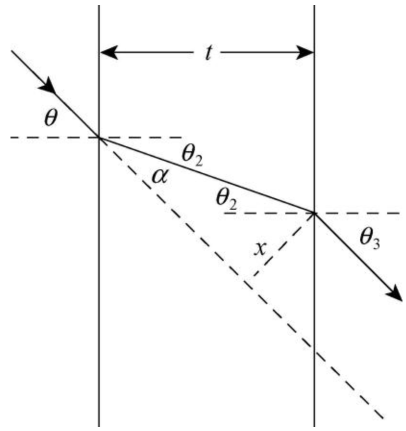
79. **THINK** We apply law of refraction to both interfaces to calculate the sideway displacement.

EXPRESS Let θ be the angle of incidence and θ_2 be the angle of refraction at the left face of the plate. Let n be the index of refraction of the glass. Then, the law of refraction yields

$$\sin\theta = n\sin\theta_2.$$

The angle of incidence at the right face is also θ_2. If θ_3 is the angle of emergence there, then

$$n\sin\theta_2 = \sin\theta_3.$$



ANALYZE (a) Combining the two expressions gives sin θ_3 = sin θ, which implies that $\theta_3 = \theta$. Thus, the emerging ray is parallel to the incident ray.

(b) We wish to derive an expression for x in terms of θ. If D is the length of the ray in the glass, then $D \cos \theta_2 = t$ and $D = t/\cos \theta_2$. The angle α in the diagram equals $\theta - \theta_2$ and

$$x = D \sin \alpha = D \sin (\theta - \theta_2).$$

Thus,

$$x = \frac{t \sin(\theta - \theta_2)}{\cos\theta_2}.$$

If all the angles θ, θ_2, θ_3, and $\theta - \theta_2$ are small and measured in radians, then sin $\theta \approx \theta$, sin $\theta_2 \approx \theta_2$, $\sin(\theta - \theta_2) \approx \theta - \theta_2$, and $\cos \theta_2 \approx 1$. Thus $x \approx t(\theta - \theta_2)$. The law of refraction applied to the point of incidence at the left face of the plate is now $\theta \approx n\theta_2$, so $\theta_2 \approx \theta/n$ and

$$x \approx t\left(\theta - \frac{\theta}{n}\right) = \frac{(n-1)t\theta}{n}.$$

LEARN The thicker the glass, the greater the displacement x. Note in the limit $n = 1$ (no glass), $x = 0$, as expected.

80. (a) The magnitude of the magnetic field is

$$B = \frac{E}{c} = \frac{100\,\text{V/m}}{3.0\times10^8\,\text{m/s}} = 3.3\times10^{-7}\,\text{T}.$$

(b) With $\vec{E}\times\vec{B} = \mu_0\vec{S}$, where $\vec{E} = E\hat{\text{k}}$ and $\vec{S} = S(-\hat{\text{j}})$, one can verify easily that since $\hat{\text{k}}\times(-\hat{\text{i}}) = -\hat{\text{j}}$, $\vec{B}$ has to be in the $-x$ direction.

81. (a) The polarization direction is defined by the electric field (which is perpendicular to the magnetic field in the wave, and also perpendicular to the direction of wave travel). The given function indicates the magnetic field is along the x axis (by the subscript on B)

and the wave motion is along $-y$ axis (see the argument of the sine function). Thus, the electric field direction must be parallel to the z axis.

(b) Since k is given as 1.57×10^7/m, then $\lambda = 2\pi/k = 4.0 \times 10^{-7}$ m, which means $f = c/\lambda = 7.5 \times 10^{14}$ Hz.

(c) The magnetic field amplitude is given as $B_m = 4.0 \times 10^{-6}$ T. The electric field amplitude E_m is equal to B_m divided by the speed of light c. The rms value of the electric field is then E_m divided by $\sqrt{2}$. Equation 33-26 then gives $I = 1.9$ kW/m^2.

82. We apply Eq. 33-40 (once) and Eq. 33-42 (twice) to obtain

$$I = \frac{1}{2} I_0 \cos^2\theta_1' \cos^2\theta_2'$$

where $\theta_1' = 90° - \theta_1 = 60°$ and $\theta_2' = 90° - \theta_2 = 60°$. This yields $I/I_0 = 0.031$.

83. **THINK** The index of refraction encountered by light generally depends on the wavelength of the light.

EXPRESS The critical angle for total internal reflection is given by $\sin\theta_c = 1/n$. With an index of refraction $n = 1.456$ at the red end, the critical angle is $\theta_c = 43.38°$ for red. Similarly, with $n = 1.470$ at the blue end, the critical angle is $\theta_c = 42.86°$ for blue.

ANALYZE (a) An angle of incidence of $\theta_1 = 42.00°$ is less than the critical angles for both red and blue light, so the refracted light is white.

(b) An angle of incidence of $\theta_1 = 43.10°$ is slightly less than the critical angle for red light but greater than the critical angle for blue light, so the refracted light is dominated by red end.

(c) An angle of incidence of $\theta_1 = 44.00°$ is greater than the critical angles for both red and blue light, so there is no refracted light.

LEARN The dependence of the index of refraction of fused quartz on wavelength is shown in Fig. 33-18. From the figure, we see that the index of refraction is greater for a shorter wavelength. Such dependence results in the spreading of light as it enters or leaves quartz, a phenomenon called "chromatic dispersion."

84. Using Eqs. 33-40 and 33-42, we obtain

$$\frac{I_{\text{final}}}{I_0} = \frac{(I_0/2)(\cos^2 45°)(\cos^2 45°)}{I_0} = \frac{1}{8} = 0.125.$$

85. We write $m = \rho\mathcal{V}$ where $\mathcal{V} = 4\pi R^3/3$ is the volume. Plugging this into $F = ma$ and then into Eq. 33-32 (with $A = \pi R^2$, assuming the light is in the form of plane waves), we find

$$\rho\frac{4\pi R^3}{3}a = \frac{I\pi R^2}{c}.$$

This simplifies to

$$a = \frac{3I}{4\rho cR}$$

which yields $a = 1.5 \times 10^{-9}$ m/s^2.

86. Accounting for the "automatic" reduction (by a factor of one-half) whenever unpolarized light passes through any polarizing sheet, then our result is

$$\tfrac{1}{2}(\cos^2(30°))^3 = 0.21.$$

87. **THINK** Since the radar beam is emitted uniformly over a hemisphere, the source power is also the same everywhere within the hemisphere.

EXPRESS The intensity of the beam is given by

$$I = \frac{P}{A} = \frac{P}{2\pi r^2}$$

where $A = 2\pi r^2$ is the area of a hemisphere. The power of the aircraft's reflection is equal to the product of the intensity at the aircraft's location and its cross-sectional area: $P_r = IA_r$. The intensity is related to the amplitude of the electric field by Eq. 33-26: $I = E_{\text{rms}}^2 / c\mu_0 = E_m^2 / 2c\mu_0$.

ANALYZE (a) Substituting the values given we get

$$I = \frac{P}{2\pi r^2} = \frac{180\times10^3\ \text{W}}{2\pi(90\times10^3\ \text{m})^2} = 3.5\times10^{-6}\ \text{W/m}^2.$$

(b) The power of the aircraft's reflection is

$$P_r = IA_r = (3.5\times10^{-6}\ \text{W/m}^2)(0.22\ \text{m}^2) = 7.8\times10^{-7}\ \text{W}.$$

(c) Back at the radar site, the intensity is

$$I_r = \frac{P_r}{2\pi r^2} = \frac{7.8\times10^{-7}\ \text{W}}{2\pi(90\times10^3\ \text{m})^2} = 1.5\times10^{-17}\ \text{W/m}^2.$$

(d) From $I_r = E_m^2 / 2c\mu_0$, we find the amplitude of the electric field to be

$$\begin{aligned} E_m &= \sqrt{2c\mu_0 I_r} = \sqrt{2(3.0\times10^8 \text{ m/s})(4\pi\times10^{-7} \text{ T}\cdot\text{m/A})(1.5\times10^{-17} \text{ W/m}^2)} \\ &= 1.1\times10^{-7} \text{ V/m}. \end{aligned}$$

(e) The rms value of the magnetic field is

$$B_{\text{rms}} = \frac{E_{\text{rms}}}{c} = \frac{E_m}{\sqrt{2}c} = \frac{1.1\times10^{-7} \text{ V/m}}{\sqrt{2}(3.0\times10^8 \text{ m/s})} = 2.5\times10^{-16}\text{T}.$$

LEARN The intensity due to a power source decreases with the square of the distance. Also, as emphasized in Sample Problem — "Light wave: rms values of the electric and magnetic fields," one cannot compare the values of the two fields because they are measured in different units. Both components are on the same basis from the perspective of wave propagation, and they have the same average energy.

88. The amplitude of the magnetic field in the wave is

$$B_m = \frac{E_m}{c} = \frac{3.20\times10^{-4} \text{V/m}}{2.998\times10^8 \text{m/s}} = 1.07\times10^{-12} \text{ T}.$$

89. From Fig. 33-19 we find n_{max} = 1.470 for λ = 400 nm and n_{min} = 1.456 for λ = 700 nm.
(a) The corresponding Brewster's angles are

$$\theta_{\text{B,max}} = \tan^{-1} n_{\text{max}} = \tan^{-1} (1.470) = 55.8°,$$

(b) and $\theta_{\text{B,min}} = \tan^{-1} (1.456) = 55.5°$.

90. (a) Suppose there are a total of N transparent layers (N = 5 in our case). We label these layers from left to right with indices 1, 2, …, N. Let the index of refraction of the air be n_0. We denote the initial angle of incidence of the light ray upon the air-layer boundary as θ_i and the angle of the emerging light ray as θ_f. We note that, since all the boundaries are parallel to each other, the angle of incidence θ_j at the boundary between the j-th and the (j + 1)-th layers is the same as the angle between the transmitted light ray and the normal in the j-th layer. Thus, for the first boundary (the one between the air and the first layer)

$$\frac{n_1}{n_0} = \frac{\sin\theta_i}{\sin\theta_1},$$

for the second boundary

$$\frac{n_2}{n_1} = \frac{\sin\theta_1}{\sin\theta_2},$$

and so on. Finally, for the last boundary

$$\frac{n_0}{n_N} = \frac{\sin\theta_N}{\sin\theta_f},$$

Multiplying these equations, we obtain

$$\left(\frac{n_1}{n_0}\right)\left(\frac{n_2}{n_1}\right)\left(\frac{n_3}{n_2}\right)\cdots\left(\frac{n_0}{n_N}\right) = \left(\frac{\sin\theta_i}{\sin\theta_1}\right)\left(\frac{\sin\theta_1}{\sin\theta_2}\right)\left(\frac{\sin\theta_2}{\sin\theta_3}\right)\cdots\left(\frac{\sin\theta_N}{\sin\theta_f}\right).$$

We see that the L.H.S. of the equation above can be reduced to n_0/n_0 while the R.H.S. is equal to $\sin\theta_i/\sin\theta_f$. Equating these two expressions, we find

$$\sin\theta_f = \left(\frac{n_0}{n_0}\right)\sin\theta_i = \sin\theta_i,$$

which gives $\theta_i = \theta_f$. So for the two light rays in the problem statement, the angle of the emerging light rays are both the same as their respective incident angles. Thus, $\theta_f = 0$ for ray *a*,

(b) and $\theta_f = 20°$ for ray *b*.

(c) In this case, all we need to do is to change the value of n_0 from 1.0 (for air) to 1.5 (for glass). This does not change the result above. That is, we still have $\theta_f = 0$ for ray *a*,

(d) and $\theta_f = 20°$ for ray *b*.

Note that the result of this problem is fairly general. It is independent of the number of layers and the thickness and index of refraction of each layer.

91. (a) At r = 40 m, the intensity is

$$I = \frac{P}{\pi d^2/4} = \frac{P}{\pi(\theta r)^2/4} = \frac{4(3.0\times10^{-3}\,\text{W})}{\pi\left[\left(0.17\times10^{-3}\,\text{rad}\right)\left(40\,\text{m}\right)\right]^2} = 83\,\text{W/m}^2.$$

(b) $P' = 4\pi r^2 I = 4\pi(40\text{m})^2(83\,\text{W/m}^2) = 1.7\times10^6\,\text{W}.$

92. The law of refraction requires that

$$\sin\theta_1/\sin\theta_2 = n_{\text{water}} = \text{const.}$$

We can check that this is indeed valid for any given pair of θ_1 and θ_2. For example, sin 10° / sin 8° = 1.3, and sin 20° / sin 15°30′ = 1.3, etc. Therefore, the index of refraction of water is $n_{\text{water}} = 1.3$.

93. We remind ourselves that when the unpolarized light passes through the first sheet, its intensity is reduced by a factor of 2. Thus, to end up with an overall reduction of one-third, the second sheet must cause a further decrease by a factor of two-thirds (since (1/2)(2/3) = 1/3). Thus, $\cos^2\theta = 2/3 \;\Rightarrow\; \theta = 35°$.

94. (a) The magnitude of the electric field at point *P* is

$$E = \frac{V}{l} = \frac{iR}{l} = (25.0\text{ A})\left(\frac{1.00\,\Omega}{300\text{ m}}\right) = 0.0833\text{ V/m}.$$

The direction of $\vec{E}$ at point *P* is in the +*x* direction, same as the current.

(b) We use Ampere's law: $\oint \vec{B}\cdot d\vec{s} = \mu_0 i$, where the integral is around a closed loop and *i* is the net current through the loop. The magnitude of the magnetic field is

$$B = \frac{\mu_0 i}{2\pi r} = \frac{\left(4\pi\times10^{-7}\text{ T}\cdot\text{m/A}\right)\left(25.0\text{ A}\right)}{2\pi\left(1.25\times10^{-3}\text{m}\right)} = 4.00\times10^{-3}\text{ T}.$$

The direction of $\vec{B}$ at point *P* is in the +*z* direction (out of the page).

(c) From $\vec{S} = \vec{E}\times\vec{B}/\mu_0$, we find the magnitude of the Poynting vector to be

$$S = \frac{EB}{\mu_0} = \frac{(0.0833\text{ V/m})(4.0\times10^{-3}\text{ T})}{2\left(4\pi\times10^{-7}\text{ T}\cdot\text{m/A}\right)} = 265\text{ W/m}^2.$$

(d) Since $\vec{S}$ points in the direction of $\vec{E}\times\vec{B}$, using the right-hand-rule, the direction of $\vec{S}$ at point *P* is in the –*y* direction.

95. (a) For the cylindrical resistor shown in Figure 33-74, the magnetic field is in the $-\hat{\theta}$, or clockwise direction. On the other hand, the electric field is in the same direction as the current, $-\hat{z}$. Since $\vec{S} = \vec{E}\times\vec{B}/\mu_0$, $\vec{S}$ is in the direction of $(-\hat{z})\times(-\hat{\theta}) = -\hat{r}$, or radially inward.

(b) The magnitudes of the electric and magnetic fields are $E = V/l = iR/l$ and $B = \mu_0 i/2\pi a$, respectively. Thus,

$$S=\frac{EB}{\mu_0}=\frac{1}{\mu_0}\left(\frac{iR}{l}\right)\left(\frac{\mu_0 i}{2\pi a}\right)=\frac{i^2R}{2\pi al}.$$

Noting that the magnitude of the Poynting vector S is constant, we have

$$\int\vec{S}\cdot d\vec{A}=SA=\left(\frac{i^2R}{2\pi al}\right)(2\pi al)=i^2R.$$

96. The average rate of energy flow per unit area, or intensity, is related to the electric field amplitude E_m by $I=E_m^2/2\mu_0c$, implying that the rate of energy absorbed is $P_{\text{abs}}=IA=E_m^2A/2\mu_0c$. If all the energy is used to heat up the sheet (converting to its internal energy), then

$$P_{\text{abs}}=\frac{dE_{\text{int}}}{dt}=mc_s\frac{dT}{dt},$$

where c_s is the specific heat of the material. Solving for dT/dt, we find

$$mc_s\frac{dT}{dt}=\frac{E_m^2A}{2\mu_0c}\quad\Rightarrow\quad\frac{dT}{dt}=\frac{E_m^2A}{2mc_s\mu_0c}.$$

97. Let I_0 be the intensity of the unpolarized light that is incident on the first polarizing sheet. The transmitted intensity is, by one-half rule, $I_1=\frac{1}{2}I_0$. For the second sheet, we apply the cosine-squared rule:

$$I_2=I_1\cos^2\theta=\frac{1}{2}I_0\cos^2\theta$$

where θ is the angle between the direction of polarization of the two sheets. With $I_2/I_0=p/100$, we solve for θ and obtain

$$\frac{I_2}{I_0}=\frac{p}{100}=\frac{1}{2}\cos^2\theta\quad\Rightarrow\quad\theta=\cos^{-1}\left(\sqrt{\frac{p}{50}}\right).$$

98. The cross-sectional area of the beam on the surface is $A\cos\theta$. In a time interval Δt, the volume of the beam that's been reflected is $\Delta V=(A\cos\theta)c\Delta t$, and the momentum carried by this volume is $p=(I/c^2)(A\cos\theta)c\Delta t$. Upon being reflected, the change in momentum is

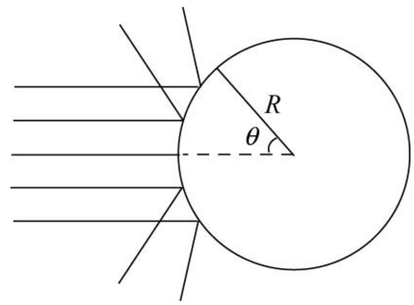
$$\Delta p=2p\cos\theta=2IA\cos^2\theta\Delta t/c$$

Thus, the radiation pressure is

$$p_r=\frac{F_r}{A}=\frac{\Delta p}{A\Delta t}=\frac{2I}{c}\cos^2\theta=p_{r\perp}\cos^2\theta$$

where $p_{r\perp} = 2I/c$ is the radiation pressure when $\theta = 0$.

99. Consider the figure shown to the right. The *y*-component of the force cancels out, and we're left with the *x*-component:



$$dF_x = 2dF\cos\theta = 2(p_r dA)\cos\theta .$$

Using the result from Problem 98: $p_r = (2I/c)\cos^2\theta$, and $dA = RLd\theta$, where L is the length of the cylinder, we obtain

$$\frac{F_x}{L} = \int 2(2I\cos\theta/c)\cos\theta\, Rd\theta = \frac{4IR}{c}\int_0^{\pi/2}\cos^3\theta d\theta = \frac{8IR}{3c}.$$

100. We apply Eq. 33-40 (once) and Eq. 33-42 (twice) to obtain

$$I = \frac{1}{2}I_0\cos^2\theta_1'\cos^2\theta_2'$$

where $\theta_1' = (90° - \theta_1) + \theta_2 = 110°$ is the relative angle between the first and the second polarizing sheets, and $\theta_2' = 90° - \theta_2 = 50°$ is the relative angle between the second and the third polarizing sheets. Thus, we have I/I_0 = 0.024.

101. We apply Eq. 33-40 (once) and Eq. 33-42 (twice) to obtain

$$I = \frac{1}{2}I_0\cos^2\theta'\cos^2\theta''.$$

With $\theta' = \theta_2 - \theta_1$ = 60° – 20° = 40° and $\theta'' = \theta_3 + (\pi/2 - \theta_2)$ = 40° + 30° = 70°, we get I/I_0 = 0.034.

102. We use Eq. 33-33 for the force, where *A* is the area of the reflecting surface (4.0 m^2). The intensity is gotten from Eq. 33-27 where $P = P_S$ is in Appendix C (see also Sample Problem 33-2) and $r = 3.0 \times 10^{11}$ m (given in the problem statement). Our result for the force is 9.2 μN.

103. Eq. 33-5 gives $B = E/c$, which relates the field values at any instant — and so relates rms values to rms values, and amplitude values to amplitude values, as the case may be. Thus, the rms value of the magnetic field is

$$B_{\text{rms}} = (0.200\ \text{V/m})/(3 \times 10^8\ \text{m/s}) = 6.67 \times 10^{-10}\ \text{T},$$

which (upon multiplication by $\sqrt{2}$) yields an amplitude value of magnetic field equal to 9.43×10^{-10} T.

104. (a) The Sun is far enough away that we approximate its rays as "parallel" in this Figure. That is, if the sunray makes angle θ from horizontal when the bird is in one position, then it makes the same angle θ when the bird is any other position. Therefore, its shadow on the ground moves as the bird moves: at 15 m/s.

(b) If the bird is in a position, a distance $x > 0$ from the wall, such that its shadow is on the wall at a distance $0 \geq y \geq h$ from the top of the wall, then it is clear from the Figure that $\tan\theta = y/x$. Thus,

$$\frac{dy}{dt} = \frac{dx}{dt}\tan\theta = (-15 \text{ m/s})\tan 30° = -8.7 \text{ m/s},$$

which means that the distance y (which was measured as a positive number downward from the top of the wall) is shrinking at the rate of 8.7 m/s.

(c) Since $\tan\theta$ grows as $0 \leq \theta < 90°$ increases, then a larger value of $|dy/dt|$ implies a larger value of θ. The Sun is higher in the sky when the hawk glides by.

(d) With $|dy/dt| = 45$ m/s, we find

$$v_{\text{hawk}} = \left|\frac{dx}{dt}\right| = \frac{|dy/dt|}{\tan\theta}$$

so that we obtain $\theta = 72°$ if we assume $v_{\text{hawk}} = 15$ m/s.

105. (a) The wave is traveling in the $-y$ direction (see §16-5 for the significance of the relative sign between the spatial and temporal arguments of the wave function).

(b) Figure 33-5 may help in visualizing this. The direction of propagation (along the y axis) is perpendicular to $\vec{B}$ (presumably along the x axis, since the problem gives B_x and no other component) and both are perpendicular to $\vec{E}$ (which determines the axis of polarization). Thus, the wave is z-polarized.

(c) Since the magnetic field amplitude is $B_m = 4.00\ \mu$T, then (by Eq. 33-5) $E_m = 1199$ V/m $\approx 1.20\times10^3$ V/m. Dividing by $\sqrt{2}$ yields $E_{\text{rms}} = 848$ V/m. Then, Eq. 33-26 gives

$$I = \frac{I}{c\mu_0}E_{\text{rms}}^2 = 1.91\times10^3 \text{ W/m}^2.$$

(d) Since $kc = \omega$ (equivalent to $c = f\lambda$), we have

$$k=\frac{2.00\times 10^{15}}{c}=6.67\times 10^{6}\ \text{m}^{-1}.$$

Summarizing the information gathered so far, we have (with SI units understood)

$$E_z=(1.2\times 10^{3}\ \text{V/m})\sin[(6.67\times 10^{6}/\text{m})y+(2.00\times 10^{15}/\text{s})t].$$

(e) $\lambda = 2\pi/k = 942$ nm.

(f) This is an infrared light.

106. (a) The angle of incidence $\theta_{B,1}$ at B is the complement of the critical angle at A; its sine is

$$\sin\theta_{B,1}=\cos\theta_c=\sqrt{1-\left(\frac{n_3}{n_2}\right)^2}$$

so that the angle of refraction $\theta_{B,2}$ at B becomes

$$\theta_{B,2}=\sin^{-1}\left(\frac{n_2}{n_3}\sqrt{1-\left(\frac{n_3}{n_2}\right)^2}\right)=\sin^{-1}\sqrt{\left(\frac{n_2}{n_3}\right)^2-1}=35.1^\circ .$$

(b) From $n_1 \sin\theta = n_2 \sin\theta_c = n_2(n_3/n_2)$, we find

$$\theta=\sin^{-1}\left(\frac{n_3}{n_1}\right)=49.9^\circ .$$

(c) The angle of incidence $\theta_{A,1}$ at A is the complement of the critical angle at B; its sine is

$$\sin\theta_{A,1}=\cos\theta_c=\sqrt{1-\left(\frac{n_3}{n_2}\right)^2}\ .$$

so that the angle of refraction $\theta_{A,2}$ at A becomes

$$\theta_{A,2}=\sin^{-1}\left(\frac{n_2}{n_3}\sqrt{1-\left(\frac{n_3}{n_2}\right)^2}\right)=\sin^{-1}\sqrt{\left(\frac{n_2}{n_3}\right)^2-1}=35.1^\circ .$$

(d) From

$$n_1\sin\theta=n_2\sin\theta_{A,1}=n_2\sqrt{1-\left(\frac{n_3}{n_2}\right)^2}=\sqrt{n_2^2-n_3^2}\ ,$$

we find

$$\theta = \sin^{-1}\left(\frac{\sqrt{n_2^2 - n_3^2}}{n_1}\right) = 26.1°$$

(e) The angle of incidence $\theta_{B,1}$ at B is the complement of the Brewster angle at A; its sine is

$$\sin\theta_{B,1} = \frac{n_2}{\sqrt{n_2^2 + n_3^2}}$$

so that the angle of refraction $\theta_{B,2}$ at B becomes

$$\theta_{B,2} = \sin^{-1}\left(\frac{n_2^2}{n_3\sqrt{n_2^2 + n_3^2}}\right) = 60.7° .$$

(f) From

$$n_1 \sin\theta = n_2 \sin\theta_{\text{Brewster}} = n_2 \frac{n_3}{\sqrt{n_2^2 + n_3^2}} ,$$

we find

$$\theta = \sin^{-1}\left(\frac{n_2 n_3}{n_1\sqrt{n_2^2 + n_3^2}}\right) = 35.3° .$$

107. (a) and (b) At the Brewster angle, $\theta_{\text{incident}} + \theta_{\text{refracted}} = \theta_B + 32.0° = 90.0°$, so $\theta_B = 58.0°$ and

$$n_{\text{glass}} = \tan\theta_B = \tan 58.0° = 1.60.$$

108. We take the derivative with respect to x of both sides of Eq. 33-11:

$$\frac{\partial}{\partial x}\left(\frac{\partial E}{\partial x}\right) = \frac{\partial^2 E}{\partial x^2} = \frac{\partial}{\partial x}\left(-\frac{\partial B}{\partial t}\right) = -\frac{\partial^2 B}{\partial x \partial t}.$$

Now we differentiate both sides of Eq. 33-18 with respect to t:

$$\frac{\partial}{\partial t}\left(-\frac{\partial B}{\partial x}\right) = -\frac{\partial^2 B}{\partial x \partial t} = \frac{\partial}{\partial t}\left(\varepsilon_0\mu_0 \frac{\partial E}{\partial t}\right) = \varepsilon_0\mu_0 \frac{\partial^2 E}{\partial t^2}.$$

Substituting $\partial^2 E/\partial x^2 = -\partial^2 B/\partial x \partial t$ from the first equation above into the second one, we get

$$\varepsilon_0\mu_0 \frac{\partial^2 E}{\partial t^2} = \frac{\partial^2 E}{\partial x^2} \quad \Rightarrow \quad \frac{\partial^2 E}{\partial t^2} = \frac{1}{\varepsilon_0\mu_0}\frac{\partial^2 E}{\partial x^2} = c^2 \frac{\partial^2 E}{\partial x^2}.$$

Similarly, we differentiate both sides of Eq. 33-11 with respect to *t*

$$\frac{\partial^2 E}{\partial x \partial t} = -\frac{\partial^2 B}{\partial t^2},$$

and differentiate both sides of Eq. 33-18 with respect to *x*

$$-\frac{\partial^2 B}{\partial x^2} = \varepsilon_0 \mu_0 - \frac{\partial^2 E}{\partial x \partial t}.$$

Combining these two equations, we get

$$\frac{\partial^2 B}{\partial t^2} = \frac{1}{\varepsilon_0 \mu_0} \frac{\partial^2 B}{\partial x^2} = c^2 \frac{\partial^2 B}{\partial x^2}.$$

109. (a) From Eq. 33-1,

$$\frac{\partial^2 E}{\partial t^2} = \frac{\partial^2}{\partial t^2} \; E_m \sin(kx - \omega t) \; = -\omega^2 E_m \sin(kx - \omega t),$$

and

$$c^2 \frac{\partial^2 E}{\partial x^2} = c^2 \frac{\partial^2}{\partial x^2} \; E_m \sin(kx - \omega t) \; = -k^2 c^2 \sin(kx - \omega t) = -\omega^2 E_m \sin(kx - \omega t).$$

Consequently,

$$\frac{\partial^2 E}{\partial t^2} = c^2 \frac{\partial^2 E}{\partial x^2}$$

is satisfied. Analogously, one can show that Eq. 33-2 satisfies

$$\frac{\partial^2 B}{\partial t^2} = c^2 \frac{\partial^2 B}{\partial x^2}.$$

(b) From $E = E_m f(kx \pm \omega t)$,

$$\frac{\partial^2 E}{\partial t^2} = E_m \frac{\partial^2 f(kx \pm \omega t)}{\partial t^2} = \omega^2 E_m \left.\frac{d^2 f}{du^2}\right|_{u = kx \pm \omega t}$$

and

$$c^2 \frac{\partial^2 E}{\partial x^2} = c^2 E_m \frac{\partial^2 f(kx \pm \omega t)}{\partial t^2} = c^2 E_m k^2 \left.\frac{d^2 f}{du^2}\right|_{u = kx \pm \omega t}$$

Since $\omega = ck$ the right-hand sides of these two equations are equal. Therefore,

$$\frac{\partial^2 E}{\partial t^2} = c^2 \frac{\partial^2 E}{\partial x^2}.$$

Changing E to B and repeating the derivation above shows that $B = B_m f(kx \pm \omega t)$ satisfies

$$\frac{\partial^2 B}{\partial t^2} = c^2 \frac{\partial^2 B}{\partial x^2}.$$

110. Since intensity is power divided by area (and the area is spherical in the isotropic case), then the intensity at a distance of r = 20 m from the source is

$$I = \frac{P}{4\pi r^2} = 0.040\,\mathrm{W/m^2}.$$

as illustrated in Sample Problem 33-2. Now, in Eq. 33-32 for a totally absorbing area A, we note that the exposed area of the small sphere is that on a flat circle $A = \pi(0.020\text{ m})^2 = 0.0013\text{ m}^2$. Therefore,

$$F = \frac{IA}{c} = \frac{(0.040)(0.0013)}{3\times 10^8} = 1.7\times 10^{-13}\,\mathrm{N}.$$

Chapter 34

1. The bird is a distance d_2 in front of the mirror; the plane of its image is that same distance d_2 behind the mirror. The lateral distance between you and the bird is d_3 = 5.00 m. We denote the distance from the camera to the mirror as d_1, and we construct a right triangle out of d_3 and the distance between the camera and the image plane (d_1 + d_2). Thus, the focus distance is

$$d = \sqrt{(d_1 + d_2)^2 + d_3^2} = \sqrt{(4.30 \text{ m} + 3.30 \text{ m})^2 + (5.00 \text{ m})^2} = 9.10 \text{ m}.$$

2. The image is 10 cm behind the mirror and you are 30 cm in front of the mirror. You must focus your eyes for a distance of 10 cm + 30 cm = 40 cm.

3. The intensity of light from a point source varies as the inverse of the square of the distance from the source. Before the mirror is in place, the intensity at the center of the screen is given by $I_P = A/d^2$, where A is a constant of proportionality. After the mirror is in place, the light that goes directly to the screen contributes intensity I_P, as before. Reflected light also reaches the screen. This light appears to come from the image of the source, a distance d behind the mirror and a distance 3d from the screen. Its contribution to the intensity at the center of the screen is

$$I_r = \frac{A}{(3d)^2} = \frac{A}{9d^2} = \frac{I_P}{9}.$$

The total intensity at the center of the screen is

$$I = I_P + I_r = I_P + \frac{I_P}{9} = \frac{10}{9} I_P.$$

The ratio of the new intensity to the original intensity is I/I_P = 10/9 = 1.11.

4. When S is barely able to see B, the light rays from B must reflect to S off the edge of the mirror. The angle of reflection in this case is 45°, since a line drawn from S to the mirror's edge makes a 45° angle relative to the wall. By the law of reflection, we find

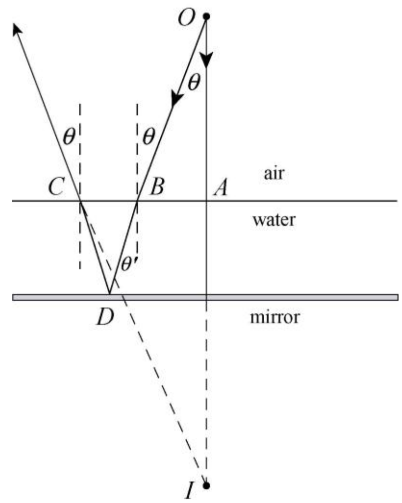
$$\frac{x}{d/2} = \tan 45° = 1 \quad \Rightarrow \quad x = \frac{d}{2} = \frac{3.0\,\text{m}}{2} = 1.5\,\text{m}.$$

5. **THINK** This problem involves refraction at air–water interface and reflection from a plane mirror at the bottom of the pool.

EXPRESS We apply the law of refraction, assuming all angles are in radians:

$$\frac{\sin\theta}{\sin\theta'} = \frac{n_w}{n_{\text{air}}},$$

which in our case reduces to $\theta' \approx \theta/n_w$ (since both θ and θ' are small, and $n_{\text{air}} \approx 1$). We refer to our figure on the right.



The object O is a vertical distance d_1 above the water, and the water surface is a vertical distance d_2 above the mirror. We are looking for a distance d (treated as a positive number) below the mirror where the image I of the object is formed. In the triangle OAB

$$|AB| = d_1 \tan\theta \approx d_1\theta,$$

and in the triangle CBD

$$|BC| = 2d_2 \tan\theta' \approx 2d_2\theta' \approx \frac{2d_2\theta}{n_w}.$$

Finally, in the triangle ACI, we have $|AI| = d + d_2$.

ANALYZE Therefore,

$$d = |AI| - d_2 = \frac{|AC|}{\tan\theta} - d_2 \approx \frac{|AB| + |BC|}{\theta} - d_2 = \left(d_1\theta + \frac{2d_2\theta}{n_w}\right)\frac{1}{\theta} - d_2 = d_1 + \frac{2d_2}{n_w} - d_2$$
$$= 250\,\text{cm} + \frac{2(200\,\text{cm})}{1.33} - 200\,\text{cm} = 351\,\text{cm}.$$

LEARN If the pool were empty without water, then $\theta = \theta'$, and the distance would be $d = d_1 + 2d_2 - d_2 = d_1 + d_2$. This is precisely what we expect from a plane mirror.

6. We note from Fig. 34-34 that $m = \frac{1}{2}$ when p = 5 cm. Thus Eq. 34-7 (the magnification equation) gives us i = –10 cm in that case. Then, by Eq. 34-9 (which applies to mirrors and thin lenses) we find the focal length of the mirror is f = 10 cm. Next, the problem asks us to consider p = 14 cm. With the focal length value already determined, then Eq. 34-9 yields i = 35 cm for this new value of object distance. Then, using Eq. 34-7 again, we find $m = i/p = -2.5$.

7. We use Eqs. 34-3 and 34-4, and note that $m = -i/p$. Thus,

$$\frac{1}{p}-\frac{1}{pm}=\frac{1}{f}=\frac{2}{r}.$$

We solve for *p*: $p=\frac{r}{2}\left(1-\frac{1}{m}\right)=\frac{35.0\,\text{cm}}{2}\left(1-\frac{1}{2.50}\right)=10.5\text{ cm.}$

8. The graph in Fig. 34-35 implies that f = 20 cm, which we can plug into Eq. 34-9 (with p = 70 cm) to obtain i = +28 cm.

9. **THINK** A concave mirror has a positive value of focal length.

EXPRESS For spherical mirrors, the focal length *f* is related to the radius of curvature *r* by $f=r/2$. The object distance *p*, the image distance *i*, and the focal length *f* are related by Eq. 34-4:

$$\frac{1}{p}+\frac{1}{i}=\frac{1}{f}.$$

The value of *i* is positive for a real images, and negative for virtual images.

The corresponding lateral magnification is $m=-i/p$. The value of *m* is positive for upright (not inverted) images, and negative for inverted images. Real images are formed on the same side as the object, while virtual images are formed on the opposite side of the mirror.

ANALYZE (a) With f = +12 cm and p = +18 cm, the radius of curvature is $r = 2f$ = 2(12 cm) = + 24 cm.

(b) The image distance is $i=\frac{pf}{p-f}=\frac{(18\text{ cm})(12\text{ cm})}{18\text{ cm}-12\text{ cm}}=36\text{ cm.}$

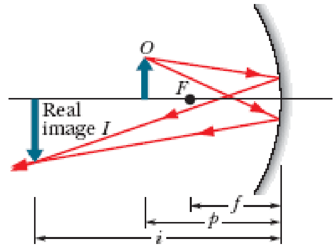
(c) The lateral magnification is $m = -i/p$ = – (36 cm)/(18 cm) = –2.0.

(d) Since the image distance *i* is positive, the image is real (R).

(e) Since the magnification *m* is negative, the image is inverted (I).

(f) A real image is formed on the <u>same</u> side as the object.

LEARN The situation in this problem is similar to that illustrated in Fig. 34-10(c). The object is outside the focal point, and its image is real and inverted.



10. A concave mirror has a positive value of focal length.

(a) Then (with f = +10 cm and p = +15 cm), the radius of curvature is $r = 2f = +20$ cm.

(b) Equation 34-9 yields $i = pf/(p - f)$ = +30 cm.

(c)Then, by Eq. 34-7, $m = -i/p = -2.0$.

(d) Since the image distance computation produced a positive value, the image is real (R).

(e) The magnification computation produced a negative value, so it is inverted (I).

(f) A real image is formed on the <u>same</u> side as the object.

11. **THINK** A convex mirror has a negative value of focal length.

EXPRESS For spherical mirrors, the focal length f is related to the radius of curvature r by $f = r/2$. The object distance p, the image distance i, and the focal length f are related by Eq. 34-4:

$$\frac{1}{p} + \frac{1}{i} = \frac{1}{f}.$$

The value of i is positive for a real images, and negative for virtual images.

The corresponding lateral magnification is

$$m = -\frac{i}{p}.$$

The value of m is positive for upright (not inverted) images, and negative for inverted images. Real images are formed on the same side as the object, while virtual images are formed on the opposite side of the mirror.

ANALYZE (a) With f = –10 cm and p = +8 cm, the radius of curvature is $r = 2f$ = –20 cm.

(b) The image distance is $i = \dfrac{pf}{p - f} = \dfrac{(8\text{ cm})(-10\text{ cm})}{8\text{ cm} - (-10)\text{ cm}} = -4.44$ cm.

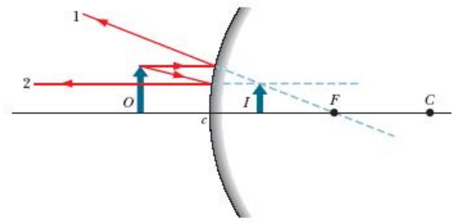
(c) The lateral magnification is $m = -i/p = -(-4.44\text{ cm})/(8.0\text{ cm}) = +0.56$.

(d) Since the image distance is negative, the image is virtual (V).

(e) The magnification m is positive, so the image is upright [not inverted] (NI).

(f) A virtual image is formed on the <u>opposite</u> side of the mirror from the object.

LEARN The situation in this problem is similar to that illustrated in Fig. 34-11(c). The mirror is convex, and its image is virtual and upright.



12. A concave mirror has a positive value of focal length.

(a) Then (with f = +36 cm and p = +24 cm), the radius of curvature is $r = 2f$ = + 72 cm.

(b) Equation 34-9 yields $i = pf/(p-f) = -72$ cm.

(c) Then, by Eq. 34-7, $m = -i/p = +3.0$.

(d) Since the image distance is negative, the image is virtual (V).

(e) The magnification computation produced a positive value, so it is upright [not inverted] (NI).

(f) A virtual image is formed on the <u>opposite</u> side of the mirror from the object.

13. **THINK** A concave mirror has a positive value of focal length.

EXPRESS For spherical mirrors, the focal length f is related to the radius of curvature r by $f = r/2$.

The object distance p, the image distance i, and the focal length f are related by Eq. 34-4:

$$\frac{1}{p}+\frac{1}{i}=\frac{1}{f}.$$

The value of i is positive for real images and negative for virtual images.

The corresponding lateral magnification is $m = -i/p$. The value of m is positive for upright (not inverted) images, and is negative for inverted images. Real images are formed on the same side as the object, while virtual images are formed on the opposite side of the mirror.

ANALYZE With f = +18 cm and p = +12 cm, the radius of curvature is $r = 2f$ = + 36 cm.

(b) Equation 34-9 yields $i = pf/(p-f) = -36$ cm.

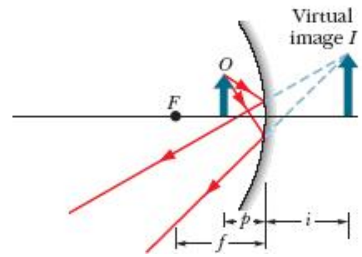
(c) Then, by Eq. 34-7, $m = -i/p = +3.0$.

(d) Since the image distance is negative, the image is virtual (V).

(e) The magnification computation produced a positive value, so it is upright [not inverted] (NI).

(f) A virtual image is formed on the opposite side of the mirror from the object.

LEARN The situation in this problem is similar to that illustrated in Fig. 34-11(a). The mirror is concave, and its image is virtual, enlarged, and upright.



14. A convex mirror has a negative value of focal length.

(a) Then (with $f = -35$ cm and $p = +22$ cm), the radius of curvature is $r = 2f = -70$ cm.

(b) Equation 34-9 yields $i = pf/(p - f) = -14$ cm.

(c) Then, by Eq. 34-7, $m = -i/p = +0.61$.

(d) Since the image distance is negative, the image is virtual (V).

(e) The magnification computation produced a positive value, so it is upright [not inverted] (NI).

(f) The side where a virtual image forms is opposite from the side where the object is.

15. **THINK** A convex mirror has a negative value of focal length.

EXPRESS For spherical mirrors, the focal length f is related to the radius of curvature r by $f = r/2$.

The object distance p, the image distance i, and the focal length f are related by Eq. 34-4:

$$\frac{1}{p} + \frac{1}{i} = \frac{1}{f}.$$

The value of i is positive for a real images, and negative for virtual images.

The corresponding lateral magnification is $m = -i/p$. The value of m is positive for upright (not inverted) images, and is negative for inverted images. Real images are formed on the same side as the object, while virtual images are formed on the opposite side of the mirror.

ANALYZE (a) With f = –8 cm and p = +10 cm, the radius of curvature is $r = 2f = 2(-8$ cm) = –16 cm.

(b) The image distance is $i = \dfrac{pf}{p-f} = \dfrac{(10\text{ cm})(-8\text{ cm})}{10\text{ cm} - (-8)\text{ cm}} = -4.44\text{ cm}.$

(c) The lateral magnification is $m = -i/p = -(-4.44\text{ cm})/(10\text{ cm}) = +0.44$.

(d) Since the image distance is negative, the image is virtual (V).

(e) The magnification m is positive, so the image is upright [not inverted] (NI).

(f) A virtual image is formed on the opposite side of the mirror from the object.

LEARN The situation in this problem is similar to that illustrated in Fig. 34-11(c). The mirror is convex, and its image is virtual and upright.

16. A convex mirror has a negative value of focal length.

(a) Then (with f = –14 cm and p = +17 cm), the radius of curvature is $r = 2f$ = –28 cm.

(b) Equation 34-9 yields $i = pf/(p-f)$ = –7.7 cm.

(c) Then, by Eq. 34-7, $m = -i/p$ = +0.45.

(d) Since the image distance is negative, the image is virtual (V).

(e) The magnification computation produced a positive value, so it is upright [not inverted] (NI).

(f) A virtual image is formed on the opposite side of the mirror from the object.

17. (a) The mirror is concave.

(b) f = +20 cm (positive, because the mirror is concave).

(c) $r = 2f$ = 2(+20 cm) = +40 cm.

(d) The object distance p = +10 cm, as given in the table.

(e) The image distance is $i = (1/f - 1/p)^{-1} = (1/20\text{ cm} - 1/10\text{ cm})^{-1} = -20$ cm.

(f) $m = -i/p = -(-20\text{ cm}/10\text{ cm}) = +2.0$.

(g) The image is virtual (V).

(h) The image is upright or not inverted (NI).

(i) A virtual image is formed on the <u>opposite</u> side of the mirror from the object.

18. (a) Since the image is inverted, we can scan Figs. 34-8, 34-10, and 34-11 in the textbook and find that the mirror must be concave.

(b) This also implies that we must put a minus sign in front of the "0.50" value given for m. To solve for f, we first find $i = -pm = +12$ cm from Eq. 34-6 and plug into Eq. 34-4; the result is $f = +8$ cm.

(c) Thus, $r = 2f = +16$ cm.

(d) $p = +24$ cm, as given in the table.

(e) As shown above, $i = -pm = +12$ cm.

(f) $m = -0.50$, with a minus sign.

(g) The image is real (R), since $i > 0$.

(h) The image is inverted (I), as noted above.

(i) A real image is formed on the <u>same</u> side as the object.

19. (a) Since $r < 0$ then (by Eq. 34-3) $f < 0$, which means the mirror is convex.

(b) The focal length is $f = r/2 = -20$ cm.

(c) $r = -40$ cm, as given in the table.

(d) Equation 34-4 leads to $p = +20$ cm.

(e) $i = -10$ cm, as given in the table.

(f) Equation 34-6 gives $m = +0.50$.

(g) The image is virtual (V).

(h) The image is upright, or not inverted (NI).

(i) A virtual image is formed on the <u>opposite</u> side of the mirror from the object.

20. (a) From Eq. 34-7, we get $i = -mp = +28$ cm, which implies the image is real (R) and on the same side as the object. Since $m < 0$, we know it was inverted (I). From Eq. 34-9,

we obtain $f = ip/(i + p) = +16$ cm, which tells us (among other things) that the mirror is concave.

(b) $f = ip/(i + p) = +16$ cm.

(c) $r = 2f = +32$ cm.

(d) $p = +40$ cm, as given in the table.

(e) $i = -mp = +28$ cm.

(f) $m = -0.70$, as given in the table.

(g) The image is real (R).

(h) The image is inverted (I).

(i) A real image is formed on the <u>same</u> side as the object.

21. (a) Since $f > 0$, the mirror is concave.

(b) $f = +\ 20$ cm, as given in the table.

(c) Using Eq. 34-3, we obtain $r = 2f = +40$ cm.

(d) $p = +\ 10$ cm, as given in the table.

(e) Equation 34-4 readily yields $i = pf/(p - f) = +60$ cm.

(f) Equation 34-6 gives $m = -i/p = -2.0$.

(g) Since $i > 0$, the image is real (R).

(h) Since $m < 0$, the image is inverted (I).

(i) A real image is formed on the <u>same</u> side as the object.

22. (a) Since $0 < m < 1$, the image is upright but smaller than the object. With that in mind, we examine the various possibilities in Figs. 34-8, 34-10, and 34-11, and note that such an image (for reflections from a single mirror) can only occur if the mirror is convex.

(b) Thus, we must put a minus sign in front of the "20" value given for f, that is, $f = -\ 20$ cm.

(c) Equation 34-3 then gives $r = 2f = -40$ cm.

(d) To solve for i and p we must set up Eq. 34-4 and Eq. 34-6 as a simultaneous set and solve for the two unknowns. The results are p = +180 cm = +1.8 m, and

(e) i = –18 cm.

(f) m = 0.10, as given in the table.

(g) The image is virtual (V) since i < 0.

(h) The image is upright, or not inverted (NI), as already noted.

(i) A virtual image is formed on the opposite side of the mirror from the object.

23. **THINK** A positive value for the magnification means that the image is upright (not inverted).

EXPRESS For spherical mirrors, the focal length f is related to the radius of curvature r by $f = r/2$. The object distance p, the image distance i, and the focal length f are related by Eq. 34-4:

$$\frac{1}{p}+\frac{1}{i}=\frac{1}{f}.$$

The value of i is positive for a real images, and negative for virtual images. The corresponding lateral magnification is $m=-i/p.$ The value of m is positive for upright (not inverted) images, and is negative for inverted images. Real images are formed on the same side as the object, while virtual images are formed on the opposite side of the mirror.

ANALYZE (a) The magnification is given by $m=-i/p.$ Since p > 0, a positive value for m means that the image distance (i) is negative, implying a virtual image. A positive magnification of magnitude less than unity is only possible for convex mirrors.

(b) With $i=-mp,$ we may write $p=f(1-1/m)$. For 0 < m < 1, a positive value for p can be obtained only if $f < 0$. Thus, with a minus sign, we have f = –30 cm.

(c) The radius of curvature is $r = 2f$ = –60 cm.

(d) The object distance is $p = f(1 - 1/m)$ = (–30 cm)(1 –1/0.20) = + 120 cm = 1.2 m.

(e) The image distance is $i = -mp$ = –(0.20)(120 cm) = –24 cm.

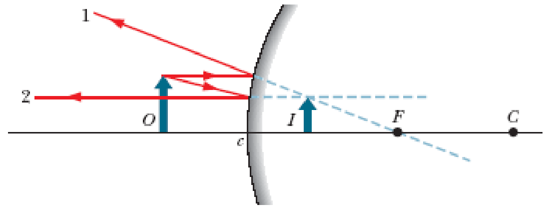
(f) The magnification is m = +0.20, as given in the Table.

(g) As discussed in (a), the image is virtual (V).

(h) As discussed in (a), the image is upright, or not inverted (NI).

(i) A virtual image is formed on the opposite side of the mirror from the object.

LEARN The situation in this problem is similar to that illustrated in Fig. 34-11(c). The mirror is convex, and its image is virtual and upright.



24. (a) Since $m = -1/2 < 0$, the image is inverted. With that in mind, we examine the various possibilities in Figs. 34-8, 34-10, and 34-11, and note that an inverted image (for reflections from a single mirror) can only occur if the mirror is concave (and if $p > f$).

(b) Next, we find i from Eq. 34-6 (which yields $i = mp = 30$ cm) and then use this value (and Eq. 34-4) to compute the focal length; we obtain $f = +20$ cm.

(c) Then, Eq. 34-3 gives $r = 2f = +40$ cm.

(d) $p = 60$ cm, as given in the table.

(e) As already noted, $i = +30$ cm.

(f) $m = -1/2$, as given.

(g) Since $i > 0$, the image is real (R).

(h) As already noted, the image is inverted (I).

(i) A real image is formed on the same side as the object.

25. (a) As stated in the problem, the image is inverted (I), which implies that it is real (R). It also (more directly) tells us that the magnification is equal to a negative value: $m = -0.40$. By Eq. 34-7, the image distance is consequently found to be $i = +12$ cm. Real images don't arise (under normal circumstances) from convex mirrors, so we conclude that this mirror is concave.

(b) The focal length is $f = +8.6$ cm, using Eq. 34-9, $f = +8.6$ cm.

(c) The radius of curvature is $r = 2f = +17.2 \text{ cm} \approx 17$ cm.

(d) $p = +30$ cm, as given in the table.

(e) As noted above, $i = +12$ cm.

(f) Similarly, $m = -0.40$, with a minus sign.

(g) The image is real (R).

(h) The image is inverted (I).

(i) A real image is formed on the <u>same</u> side as the object.

26. (a) We are told that the image is on the same side as the object; this means the image is real (R) and further implies that the mirror is concave.

(b) The focal distance is $f = +20$ cm.

(c) The radius of curvature is $r = 2f = +40$ cm.

(d) $p = +60$ cm, as given in the table.

(e) Equation 34-9 gives $i = pf/(p - f) = +30$ cm.

(f) Equation 34-7 gives $m = -i/p = -0.50$.

(g) As noted above, the image is real (R).

(h) The image is inverted (I) since $m < 0$.

(i) A real image is formed on the <u>same</u> side as the object.

27. (a) The fact that the focal length is given as a negative value means the mirror is convex.

(b) $f = -30$ cm, as given in the Table.

(c) The radius of curvature is $r = 2f = -60$ cm.

(d) Equation 34-9 gives $p = if/(i - f) = +30$ cm.

(e) $i = -15$, as given in the table.

(f) From Eq. 34-7, we get $m = +1/2 = 0.50$.

(g) The image distance is given as a negative value (as it would have to be, since the mirror is convex), which means the image is virtual (V).

(h) Since $m > 0$, the image is upright (not inverted: NI).

(i) The image is on the <u>opposite</u> side of the mirror as the object.

28. (a) The fact that the magnification is 1 means that the mirror is flat (plane).

(b) Flat mirrors (and flat “lenses” such as a window pane) have $f = \infty$ (or $f = -\infty$ since the sign does not matter in this extreme case).

(c) The radius of curvature is $r = 2f = \infty$ (or $r = -\infty$) by Eq. 34-3.

(d) $p = +\,10$ cm, as given in the table.

(e) Equation 34-4 readily yields $i = pf/(p - f) = -10$ cm.

(f) The magnification is $m = -i/p = +1.0$.

(g) The image is virtual (V) since $i < 0$.

(h) The image is upright, or not inverted (NI).

(i) A virtual image is formed on the <u>opposite</u> side of the mirror from the object.

29. **THINK** A convex mirror has a negative value of focal length.

EXPRESS For spherical mirrors, the focal length f is related to the radius of curvature r by $f = r/2$. The object distance p, the image distance i, and the focal length f are related by Eq. 34-4:

$$\frac{1}{p}+\frac{1}{i}=\frac{1}{f}.$$

The value of i is positive for a real images, and negative for virtual images. The corresponding lateral magnification is $m = -i/p$. The value of m is positive for upright (not inverted) images, and is negative for inverted images. Real images are formed on the same side as the object, while virtual images are formed on the opposite side of the mirror.

ANALYZE (a) The mirror is convex, as given.

(b) Since the mirror is convex, the radius of curvature is negative, so $r = -\,40$ cm. Then, the focal length is $f = r/2 = (-40\text{ cm})/2 = -20$ cm.

(c) The radius of curvature is $r = -\,40$ cm.

(d) The fact that the mirror is convex also means that we need to insert a minus sign in front of the “4.0” value given for i, since the image in this case must be virtual. Eq. 34-4 leads to

$$p=\frac{if}{i-f}=\frac{(-4.0\text{ cm})(-20\text{ cm})}{-4.0\text{ cm}-(-20\text{ cm})}=5.0\text{ cm}$$

(e) As noted above, $i = -\,4.0$ cm.

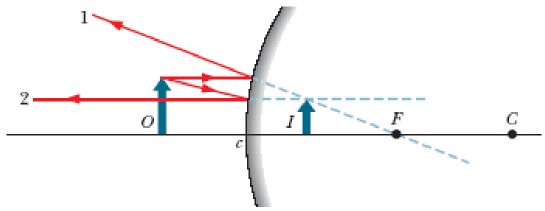
(f) The magnification is $m = -i/p = -(-4.0\text{ cm})/(5.0\text{ cm}) = +0.80$.

(g) The image is virtual (V) since $i < 0$.

(h) The image is upright, or not inverted (NI).

(i) A virtual image is formed on the opposite side of the mirror from the object.

LEARN The situation in this problem is similar to that illustrated in Fig. 34-11(c). The mirror is convex, and its image is virtual and upright.



30. We note that there is "singularity" in this graph (Fig. 34-36) like there was in Fig. 34-35), which tells us that there is no point where $p = f$ (which causes Eq. 34-9 to "blow up"). Since $p > 0$, as usual, then this means that the focal length is not positive. We know it is not a flat mirror since the curve shown does decrease with p, so we conclude it is a convex mirror. We examine the point where $m = 0.50$ and $p = 10$ cm. Combining Eq. 34-7 and Eq. 34-9 we obtain

$$m = -\frac{i}{p} = -\frac{f}{p-f}.$$

This yields $f = -10$ cm (verifying our expectation that the mirror is convex). Now, for $p = 21\text{ cm}$, we find $m = -f/(p-f) = +0.32$.

31. (a) From Eqs. 34-3 and 34-4, we obtain

$$i = \frac{pf}{p-f} = \frac{pr}{2p-r}.$$

Differentiating both sides with respect to time and using $v_O = -dp/dt$, we find

$$v_I = \frac{di}{dt} = \frac{d}{dt}\left(\frac{pr}{2p-r}\right) = \frac{-rv_O(2p-r)+2v_O pr}{(2p-r)^2} = \left(\frac{r}{2p-r}\right)^2 v_O.$$

(b) If $p = 30$ cm, we obtain $v_I = \left[\frac{15\text{ cm}}{2(30\text{ cm})-15\text{ cm}}\right]^2 (5.0\text{ cm/s}) = 0.56\text{ cm/s}.$

(c) If $p = 8.0$ cm, we obtain $v_I = \left[\frac{15\text{ cm}}{2(8.0\text{ cm})-15\text{ cm}}\right]^2 (5.0\text{ cm/s}) = 1.1\times10^3\text{ cm/s}.$

(d) If p = 1.0 cm, we obtain $v_I = \left[\frac{15\,\text{cm}}{2(1.0\,\text{cm})-15\,\text{cm}}\right]^2 (5.0\,\text{cm/s}) = 6.7\,\text{cm/s}.$

32. In addition to n_1 =1.0, we are given (a) n_2 = 1.5, (b) p = +10 cm, and (c) r = +30 cm.

(d) Equation 34-8 yields $i = n_2\left(\frac{n_2-n_1}{r}-\frac{n_1}{p}\right)^{-1} = 1.5\left(\frac{1.5-1.0}{30\text{ cm}}-\frac{1.0}{10\text{ cm}}\right) = -18\text{ cm}.$

(e) The image is virtual (V) and upright since $i<0$.

(f) The object and its image are on the same side. The ray diagram would be similar to Fig. 34-12(c) in the textbook.

33. **THINK** An image is formed by refraction through a spherical surface. A negative value for the image distance implies that the image is virtual.

EXPRESS Let n_1 be the index of refraction of the material where the object is located, n_2 be the index of refraction of the material on the other side of the refracting surface, and r be the radius of curvature of the surface. The image distance i is related to the object distance p by Eq. 34-8:

$$\frac{n_1}{p}+\frac{n_2}{i}=\frac{n_2-n_1}{r}.$$

The value of i is positive for a real images, and negative for virtual images.

ANALYZE In addition to n_1 =1.0, we are given (a) n_2 = 1.5, (b) p = +10 cm, and (d) $i = -13\text{ cm}$.

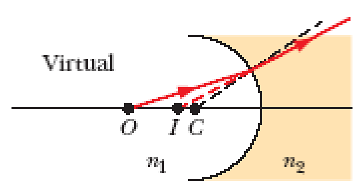
(c) Eq. 34-8 yields

$$r = (n_2-n_1)\left(\frac{n_1}{p}+\frac{n_2}{i}\right)^{-1} = (1.5-1.0)\left(\frac{1.0}{10\text{ cm}}+\frac{1.5}{-13\text{ cm}}\right)^{-1} = -32.5\,\text{cm} \approx -33\text{ cm}.$$

(e) The image is virtual (V) and upright.

(f) The object and its image are on the same side.

LEARN The ray diagram for this problem is similar to the one shown in Fig. 34-12(e). Here refraction always directs the ray away from the central axis; the images are always virtual, regardless of the object distance.



34. In addition to n_1 =1.5, we are given (b) p = +100, (c) r = –30 cm, and (d) $i = +600\,\text{cm}$.

(a) We manipulate Eq. 34-8 to separate the indices:

$$n_2\left(\frac{1}{r}-\frac{1}{i}\right)=\left(\frac{n_1}{p}+\frac{n_1}{r}\right) \;\Rightarrow\; n_2\left(\frac{1}{-30}-\frac{1}{600}\right)=\left(\frac{1.5}{100}+\frac{1.5}{-30}\right) \;\Rightarrow\; n_2(-0.035)=-0.035$$

which implies n_2 = 1.0.

(e) The image is real (R) and inverted.

(f) The object and its image are on the opposite side. The ray diagram would be similar to Fig. 34-12(b) in the textbook.

35. **THINK** An image is formed by refraction through a spherical surface. Whether the image is real or virtual depends on the relative values of n_1 and n_2, and on the geometry.

EXPRESS Let n_1 be the index of refraction of the material where the object is located, n_2 be the index of refraction of the material on the other side of the refracting surface, and r be the radius of curvature of the surface. The image distance i is related to the object distance p by Eq. 34-8:

$$\frac{n_1}{p}+\frac{n_2}{i}=\frac{n_2-n_1}{r}.$$

The value of i is positive for a real images, and negative for virtual images.

ANALYZE In addition to n_1 =1.5, we are also given (a) n_2 = 1.0, (b) p = +70 cm, and (c) r = +30 cm. Notice that $n_2 < n_1$.

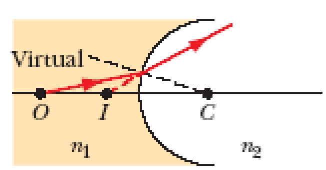
(d) We manipulate Eq. 34-8 to find the image distance:

$$i = n_2\left(\frac{n_2-n_1}{r}-\frac{n_1}{p}\right)^{-1} = 1.0\left(\frac{1.0-1.5}{30\text{ cm}}-\frac{1.5}{70\text{ cm}}\right)^{-1} = -26\text{ cm}.$$

(e) The image is virtual (V) and upright.

(f) The object and its image are on the same side.

LEARN The ray diagram for this problem is similar to the one shown in Fig. 34-12(f). Here refraction always directs the ray away from the central axis; the images are always virtual, regardless of the object distance.



36. In addition to n_1 =1.5, we are given (a) n_2 =1.0, (c) r = –30 cm and (d) $i = -7.5\,\text{cm}$.

(b)We manipulate Eq. 34-8 to find p:

$$p = \frac{n_1}{\frac{n_2 - n_1}{r} - \frac{n_2}{i}} = \frac{1.5}{\frac{1.0-1.5}{-30\text{ cm}} - \frac{1.0}{-7.5\text{ cm}}} = 10\text{ cm}.$$

(e) The image is virtual (V) and upright.

(f) The object and its image are on the same side. The ray diagram would be similar to Fig. 34-12(d) in the textbook.

37. In addition to n_1 =1.5, we are given (a) n_2 =1.0, (b) p = +10 cm, and (d) $i = -6.0\,\text{cm}$.

(c) We manipulate Eq. 34-8 to find r:

$$r = (n_2 - n_1)\left(\frac{n_1}{p} + \frac{n_2}{i}\right)^{-1} = (1.0 - 1.5)\left(\frac{1.5}{10\text{ cm}} + \frac{1.0}{-6.0\text{ cm}}\right)^{-1} = 30\text{ cm}.$$

(e) The image is virtual (V) and upright.

(f) The object and its image are on the same side. The ray diagram would be similar to Fig. 34-12(f) in the textbook, but with the object and the image located closer to the surface.

38. In addition to n_1 =1.0, we are given (a) n_2=1.5, (c) r = +30 cm, and (d) $i = +600$.

(b) Equation 34-8 gives $p = \frac{n_1}{\frac{n_2 - n_1}{r} - \frac{n_2}{i}} = \frac{1.0}{\frac{1.5-1.0}{30\text{ cm}} - \frac{1.5}{600\text{ cm}}} = 71\text{cm}.$

(e) With $i > 0$, the image is real (R) and inverted.

(f) The object and its image are on the opposite side. The ray diagram would be similar to Fig. 34-12(a) in the textbook.

39. (a) We use Eq. 34-8 and note that $n_1 = n_{\text{air}} = 1.00$, $n_2 = n$, $p = \infty$, and $i = 2r$:

$$\frac{1.00}{\infty} + \frac{n}{2r} = \frac{n-1}{r}.$$

We solve for the unknown index: $n = 2.00$.

(b) Now $i = r$ so Eq. 34-8 becomes

$$\frac{n}{r} = \frac{n-1}{r},$$

which is not valid unless $n \to \infty$ or $r \to \infty$. It is impossible to focus at the center of the sphere.

40. We use Eq. 34-8 (and Fig. 34-11(d) is useful), with n_1 = 1.6 and n_2 = 1 (using the rounded-off value for air):

$$\frac{1.6}{p} + \frac{1}{i} = \frac{1-1.6}{r}.$$

Using the sign convention for r stated in the paragraph following Eq. 34-8 (so that $r = -5.0\text{ cm}$), we obtain i = –2.4 cm for objects at p = 3.0 cm. Returning to Fig. 34-38 (and noting the location of the observer), we conclude that the tabletop seems 7.4 cm away.

41. (a) We use Eq. 34-10:

$$f = \left[(n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)\right]^{-1} = \left[(1.5-1)\left(\frac{1}{\infty} - \frac{1}{-20\text{ cm}}\right)\right]^{-1} = +40\text{ cm}.$$

(b) From Eq. 34-9,

$$i = \left(\frac{1}{f} - \frac{1}{p}\right)^{-1} = \left(\frac{1}{40\text{ cm}} - \frac{1}{40\text{ cm}}\right)^{-1} = \infty.$$

42. Combining Eq. 34-7 and Eq. 34-9, we have $m(p - f) = -f$. The graph in Fig. 34-39 indicates that m = 0.5 where p = 15 cm, so our expression yields f = –15 cm. Plugging this back into our expression and evaluating at p = 35 cm yields m = +0.30.

43. We solve Eq. 34-9 for the image distance:

$$i = \left(\frac{1}{f} - \frac{1}{p}\right)^{-1} = \frac{fp}{p-f}.$$

The height of the image is

$$h_i = mh_p = \left(\frac{i}{p}\right)h_p = \frac{fh_p}{p-f} = \frac{(75\text{ mm})(1.80\text{ m})}{27\text{ m} - 0.075\text{ m}} = 5.0\text{ mm}.$$

44. The singularity the graph (where the curve goes to $\pm\infty$) is at p = 30 cm, which implies (by Eq. 34-9) that f = 30 cm > 0 (converging type lens). For p = 100 cm, Eq. 34-9 leads to i = +43 cm.

45. Let the diameter of the Sun be d_s and that of the image be d_i. Then, Eq. 34-5 leads to

$$d_i = |m| d_s = \left(\frac{i}{p}\right) d_s \approx \left(\frac{f}{p}\right) d_s = \frac{\left(20.0\times10^{-2}\,\text{m}\right)(2)\left(6.96\times10^{8}\,\text{m}\right)}{1.50\times10^{11}\,\text{m}} = 1.86\times10^{-3}\,\text{m} = 1.86\ \text{mm}.$$

46. Since the focal length is a constant for the whole graph, then $1/p + 1/i$ = constant. Consider the value of the graph at p = 20 cm; we estimate its value there to be –10 cm. Therefore, 1/20 + 1/(–10) = 1/70 + $1/i_{\text{new}}$. Thus, i_{new} = –16 cm.

47. **THINK** Our lens is of double-convex type. We apply lens maker's equation to analyze the problem.

EXPRESS The lens maker's equation is given by Eq. 34-10:

$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

where f is the focal length, n is the index of refraction, r_1 is the radius of curvature of the first surface encountered by the light and r_2 is the radius of curvature of the second surface. Since one surface has twice the radius of the other and since one surface is convex to the incoming light while the other is concave, set $r_2 = -2r_1$ to obtain

$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} + \frac{1}{2r_1}\right) = \frac{3(n-1)}{2r_1}.$$

ANALYZE (a) We solve for the smaller radius r_1:

$$r_1 = \frac{3(n-1)f}{2} = \frac{3(1.5-1)(60\ \text{mm})}{2} = 45\ \text{mm}.$$

(b) The magnitude of the larger radius is $|r_2| = 2r_1 = 90$ mm.

LEARN An image of an object can be formed with a lens because it can bend the light rays, but the bending is possible only if the index of refraction of the lens is different from that of its surrounding medium.

48. Combining Eq. 34-7 and Eq. 34-9, we have $m(p - f) = -f$. The graph in Fig. 34-42 indicates that m = 2 where p = 5 cm, so our expression yields f = 10 cm. Plugging this back into our expression and evaluating at p = 14 cm yields m = –2.5.

49. **THINK** The image is formed on the screen, so the sum of the object distance and the image distance is equal to the distance between the slide and the screen.

EXPRESS Using Eq. 34-9:

$$\frac{1}{f}=\frac{1}{p}+\frac{1}{i}$$

and noting that $p + i = d = 44$ cm, we obtain $p^2 - dp + df = 0$.

ANALYZE The focal length is $f = 11$ cm. Solving the quadratic equation, we find the solution to p to be

$$p=\frac{1}{2}(d\pm\sqrt{d^2-4df})=22\text{ cm}\pm\frac{1}{2}\sqrt{(44\text{ cm})^2-4(44\text{ cm})(11\text{ cm})}=22\text{ cm}.$$

LEARN Since $p > f$, the object is outside the focal length. The image distance is $i = d - p = 44 - 22 = 22$ cm.

50. We recall that for a converging (C) lens, the focal length value should be positive ($f = +4$ cm).

(a) Equation 34-9 gives $i = pf/(p - f) = +5.3$ cm.

(b) Equation 34-7 gives $m = -i/p = -0.33$.

(c) The fact that the image distance i is a positive value means the image is real (R).

(d) The fact that the magnification is a negative value means the image is inverted (I).

(e) The image is on the opposite side of the object (see Fig. 34-16(a)).

51. We recall that for a converging (C) lens, the focal length value should be positive ($f = +16$ cm).

(a) Equation 34-9 gives $i = pf/(p - f) = -48$ cm.

(b) Equation 34-7 gives $m = -i/p = +4.0$.

(c) The fact that the image distance is a negative value means the image is virtual (V).

(d) A positive value of magnification means the image is not inverted (NI).

(e) The image is on the same side as the object (see Fig. 34-16(b)).

52. We recall that for a converging (C) lens, the focal length value should be positive ($f = +35$ cm).

(a) Equation 34-9 gives $i = pf/(p - f) = -88$ cm.

(b) Equation 34-7 give $m = -i/p = +3.5$.

(c) The fact that the image distance is a negative value means the image is virtual (V).

(d) A positive value of magnification means the image is not inverted (NI).

(e) The image is on the same side as the object (see Fig. 34-16(b)).

53. **THINK** For a diverging (D) lens, the focal length value is negative.

EXPRESS The object distance p, the image distance i, and the focal length f are related by Eq. 34-9:

$$\frac{1}{f}=\frac{1}{p}+\frac{1}{i}.$$

The value of i is positive for a real images, and negative for virtual images. The corresponding lateral magnification is $m=-i/p$. The value of m is positive for upright (not inverted) images, and is negative for inverted images.

ANALYZE For this lens, we have $f = -12$ cm and $p = +8.0$ cm.

(a) The image distance is $i=\dfrac{pf}{p-f}=\dfrac{(8.0\text{ cm})(-12\text{ cm})}{8.0\text{ cm}-(-12)\text{ cm}}=-4.8\text{ cm}.$

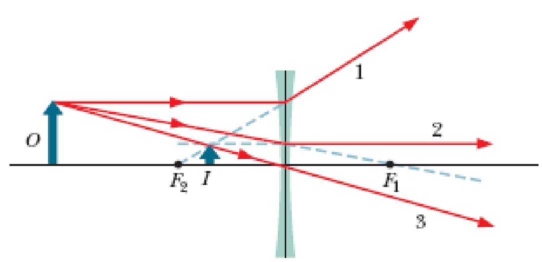
(b) The magnification is $m = -i/p = -(-4.8\text{ cm})/(8.0\text{ cm}) = +0.60$.

(c) The fact that the image distance is a negative value means the image is virtual (V).

(d) A positive value of magnification means the image is not inverted (NI).

(e) The image is on the same side as the object.

LEARN The ray diagram for this problem is similar to the one shown in Fig. 34-16(c). The lens is diverging, forming a virtual image with the same orientation as the object, and on the same side as the object.



54. We recall that for a diverging (D) lens, the focal length value should be negative ($f = -6$ cm).

(a) Equation 34-9 gives $i = pf/(p - f) = -3.8$ cm.

(b) Equation 34-7 gives $m = -i/p = +0.38$.

(c) The fact that the image distance is a negative value means the image is virtual (V).

(d) A positive value of magnification means the image is not inverted (NI).

(e) The image is on the same side as the object (see Fig. 34-16(c)).

55. **THINK** For a diverging (D) lens, the value of the focal length is negative.

EXPRESS The object distance p, the image distance i, and the focal length f are related by Eq. 34-9:

$$\frac{1}{f}=\frac{1}{p}+\frac{1}{i}.$$

The value of i is positive for a real images, and negative for virtual images. The corresponding lateral magnification is $m=-i/p$. The value of m is positive for upright (not inverted) images, and is negative for inverted images.

ANALYZE For this lens, we have f = –14 cm and p = +22.0 cm.

(a) The image distance is $i=\dfrac{pf}{p-f}=\dfrac{(22\text{ cm})(-14\text{ cm})}{22\text{ cm}-(-14)\text{ cm}}=-8.6\text{ cm}.$

(b) The magnification is $m = -i/p = -(-8.6\text{ cm})/(22\text{ cm}) = +0.39$.

(c) The fact that the image distance is a negative value means the image is virtual (V).

(d) A positive value of magnification means the image is not inverted (NI).

(e) The image is on the same side as the object.

LEARN The ray diagram for this problem is similar to the one shown in Fig. 34-16(c). The lens is diverging, forming a virtual image with the same orientation as the object, and on the same side as the object.

56. We recall that for a diverging (D) lens, the focal length value should be negative ($f = -31$ cm).

(a) Equation 34-9 gives $i = pf/(p-f) = -8.7$ cm.

(b) Equation 34-7 gives $m = -i/p = +0.72$.

(c) The fact that the image distance is a negative value means the image is virtual (V).

(d) A positive value of magnification means the image is not inverted (NI).

(e) The image is on the same side as the object (see Fig. 34-16(c)).

57. **THINK** For a converging (C) lens, the focal length value is positive.

EXPRESS The object distance p, the image distance i, and the focal length f are related by Eq. 34-9:

$$\frac{1}{f}=\frac{1}{p}+\frac{1}{i}.$$

The value of i is positive for a real images, and negative for virtual images. The corresponding lateral magnification is $m=-i/p$. The value of m is positive for upright (not inverted) images, and is negative for inverted images.

ANALYZE For this lens, we have f = +20 cm and p = +45.0 cm.

(a) The image distance is $i=\dfrac{pf}{p-f}=\dfrac{(45\text{ cm})(20\text{ cm})}{45\text{ cm}-20\text{ cm}}=+36\text{ cm}.$

(b) The magnification is $m = -i/p=-(+36\text{ cm})/(45\text{ cm})=-0.80.$

(c) The fact that the image distance is a positive value means the image is real (R).

(d) A negative value of magnification means the image is inverted (I).

(e) The image is on the opposite side of the object.

LEARN The ray diagram for this problem is similar to the one shown in Fig. 34-16(a). The lens is converging, forming a real, inverted image on the opposite side of the object.

58. (a) Combining Eq. 34-9 and Eq. 34-10 gives i = –63 cm.

(b) Equation 34-7 gives $m = -i/p$ = +2.2.

(c) The fact that the image distance is a negative value means the image is virtual (V).

(d) A positive value of magnification means the image is not inverted (NI).

(e) The image is on the same side as the object.

59. **THINK** Since r_1 is positive and r_2 is negative, our lens is of double-convex type. We apply lens maker's equation to analyze the problem.

EXPRESS The lens maker's equation is given by Eq. 34-10:

$$\frac{1}{f}=(n-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$$

where f is the focal length, n is the index of refraction, r_1 is the radius of curvature of the first surface encountered by the light and r_2 is the radius of curvature of the second surface. The object distance p, the image distance i, and the focal length f are related by Eq. 34-9:

$$\frac{1}{f}=\frac{1}{p}+\frac{1}{i}.$$

ANALYZE For this lens, we have $r_1 = +30$ cm, $r_2 = -42$ cm, n = 1.55 and $p = +75$ cm.

(a) The focal length is

$$f=\frac{r_1 r_2}{(n-1)(r_2-r_1)}=\frac{(+30\text{ cm})(-42\text{ cm})}{(1.55-1)(-42\text{ cm}-30\text{ cm})}=+31.8\text{ cm}.$$

Thus, the image distance is $i=\dfrac{pf}{p-f}=\dfrac{(75\text{ cm})(31.8\text{ cm})}{75\text{ cm}-31.8\text{ cm}}=+55$ cm.

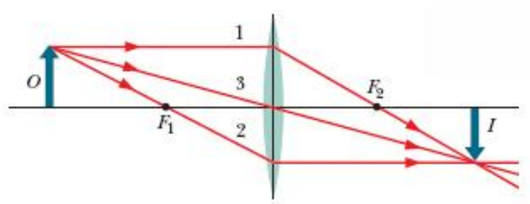
(b) Eq. 34-7 give $m = -i/p = -(55\text{ cm})/(75\text{ cm}) = -0.74$.

(c) The fact that the image distance is a positive value means the image is real (R).

(d) The fact that the magnification is a negative value means the image is inverted (I).

(e) The image is on the side opposite from the object.

LEARN The ray diagram for this problem is similar to the one shown in Fig. 34-16(a). The lens is converging, forming a real, inverted image on the opposite side of the object.



60. (a) Combining Eq. 34-9 and Eq. 34-10 gives $i = -26$ cm.

(b) Equation 34-7 gives $m = -i/p = +4.3$.

(c) The fact that the image distance is a negative value means the image is virtual (V).

(d) A positive value of magnification means the image is not inverted (NI).

(e) The image is on the same side as the object.

61. (a) Combining Eq. 34-9 and Eq. 34-10 gives i = –18 cm.

(b) Equation 34-7 gives $m = -i/p$ = +0.76.

(c) The fact that the image distance is a negative value means the image is virtual (V).

(d) A positive value of magnification means the image is not inverted (NI).

(e) The image is on the same side as the object.

62. (a) Equation 34-10 yields

$$f = \frac{r_1 r_2}{(n-1)(r_2 - r_1)} = +30 \text{ cm}$$

Since $f > 0$, this must be a converging (“C”) lens. From Eq. 34-9, we obtain

$$i = \frac{1}{\frac{1}{f} - \frac{1}{p}} = \frac{1}{\frac{1}{30 \text{ cm}} - \frac{1}{10 \text{ cm}}} = -15 \text{cm}.$$

(b) Equation 34-6 yields $m = -i/p$ = –(–15 cm)/(10 cm) = +1.5.

(c) Since $i < 0$, the image is virtual (V).

(d) Since $m > 0$, the image is upright, or not inverted (NI).

(e) The image is on the same side as the object. The ray diagram is similar to Fig. 34-16(b) of the textbook.

63. (a) Combining Eq. 34-9 and Eq. 34-10 gives i = –30 cm.

(b) Equation 34-7 gives $m = -i/p$ = +0.86.

(c) The fact that the image distance is a negative value means the image is virtual (V).

(d) A positive value of magnification means the image is not inverted (NI).

(e) The image is on the same side as the object.

64. (a) Equation 34-10 yields

$$f = \frac{1}{n-1}\left(1/r_1 - 1/r_2\right)^{-1} = -120 \text{ cm}.$$

Since $f < 0$, this must be a diverging (“D”) lens. From Eq. 34-9, we obtain

$$i=\frac{1}{\frac{1}{f}-\frac{1}{p}}=\frac{1}{\frac{1}{-120\text{ cm}}-\frac{1}{10\text{ cm}}}=-9.2\,\text{cm}.$$

(b) Equation 34-6 yields $m = -i/p = -(-9.2\text{ cm})/(10\text{ cm}) = +0.92$.

(c) Since $i < 0$, the image is virtual (V).

(d) Since $m > 0$, the image is upright, or not inverted (NI).

(e) The image is on the same side as the object. The ray diagram is similar to Fig. 34-16(c) of the textbook.

65. (a) Equation 34-10 yields $f = \frac{1}{n-1}(1/r_1 - 1/r_2)^{-1} = -30$ cm. Since $f < 0$, this must be a diverging ("D") lens. From Eq. 34-9, we obtain

$$i=\frac{1}{\frac{1}{f}-\frac{1}{p}}=\frac{1}{\frac{1}{-30\text{ cm}}-\frac{1}{10\text{ cm}}}=-7.5\,\text{cm}.$$

(b) Equation 34-6 yields $m = -i/p = -(-7.5\text{ cm})/(10\text{ cm}) = +0.75$.

(c) Since $i < 0$, the image is virtual (V).

(d) Since $m > 0$, the image is upright, or not inverted (NI).

(e) The image is on the same side as the object. The ray diagram is similar to Fig. 34-16(c) of the textbook.

66. (a) Combining Eq. 34-9 and Eq. 34-10 gives $i = -9.7$ cm.

(b) Equation 34-7 gives $m = -i/p = +0.54$.

(c) The fact that the image distance is a negative value means the image is virtual (V).

(d) A positive value of magnification means the image is not inverted (NI).

(e) The image is on the same side as the object.

67. (a) Combining Eq. 34-9 and Eq. 34-10 gives $i = +84$ cm.

(b) Equation 34-7 gives $m = -i/p = -1.4$.

(c) The fact that the image distance is a positive value means the image is real (R).

(d) The fact that the magnification is a negative value means the image is inverted (I).

(e) The image is on the side opposite from the object.

68. (a) A convex (converging) lens, since a real image is formed.

(b) Since $i = d - p$ and $i/p = 1/2$,

$$p = \frac{2d}{3} = \frac{2(40.0\text{ cm})}{3} = 26.7\text{ cm}.$$

(c) The focal length is

$$f = \left(\frac{1}{i} + \frac{1}{p}\right)^{-1} = \left(\frac{1}{d/3} + \frac{1}{2d/3}\right)^{-1} = \frac{2d}{9} = \frac{2(40.0\text{ cm})}{9} = 8.89\text{ cm}.$$

69. (a) Since $f > 0$, this is a converging lens (“C”).

(d) Equation 34-9 gives

$$i = \frac{1}{\frac{1}{f} - \frac{1}{p}} = \frac{1}{\frac{1}{10\text{ cm}} - \frac{1}{5.0\text{ cm}}} = -10\text{ cm}.$$

(e) From Eq. 34-6, $m = -(-10\text{ cm})/(5.0\text{ cm}) = +2.0$.

(f) The fact that the image distance i is a negative value means the image is virtual (V).

(g) A positive value of magnification means the image is not inverted (NI).

(h) The image is on the same side as the object.

70. (a) The fact that $m < 1$ and that the image is upright (not inverted: NI) means the lens is of the diverging type (D) (it may help to look at Fig. 34-16 to illustrate this).

(b) A diverging lens implies that $f = -20$ cm, with a minus sign.

(d) Equation 34-9 gives $i = -5.7$ cm.

(e) Equation 34-7 gives $m = -i/p = +0.71$.

(f) The fact that the image distance i is a negative value means the image is virtual (V).

(h) The image is on the same side as the object.

71. (a) Eq. 34-7 yields $i = -mp = -(0.25)(16 \text{ cm}) = -4.0$ cm. Equation 34-9 gives $f = -5.3$ cm, which implies the lens is of the diverging type (D).

(b) From (a), we have $f = -5.3$ cm.

(d) Similarly, $i = -4.0$ cm.

(f) The fact that the image distance i is a negative value means the image is virtual (V).

(g) A positive value of magnification means the image is not inverted (NI).

(h) The image is on the same side as the object.

72. (a) Equation 34-7 readily yields $i = +4.0$ cm. Then Eq. 34-9 gives $f = +3.2$ cm, which implies the lens is of the converging type (C).

(b) From (a), we have $f = +3.2$ cm.

(d) Similarly, $i = +4.0$ cm.

(f) The fact that the image distance is a positive value means the image is real (R).

(g) The fact that the magnification is a negative value means the image is inverted (I).

(h) The image is on the opposite side of the object.

73. (a) Using Eq. 34-6 (which implies the image is inverted) and the given value of p, we find $i = -mp = +5.0$ cm; it is a real image. Equation 34-9 then yields the focal length: $f = +3.3$ cm. Therefore, the lens is of the converging ("C") type.

(b) From (a), we have $f = +3.3$ cm.

(d) Similarly, $i = -mp = +5.0$ cm.

(f) The fact that the image distance is a positive value means the image is real (R).

(g) The fact that the magnification is a negative value means the image is inverted (I).

(h) The image is on the side opposite from the object. The ray diagram is similar to Fig. 34-16(a) of the textbook.

74. (b) Since this is a converging lens ("C") then $f > 0$, so we should put a plus sign in front of the "10" value given for the focal length.

(d) Equation 34-9 gives

$$i=\frac{1}{\frac{1}{f}-\frac{1}{p}}=\frac{1}{\frac{1}{10\text{ cm}}-\frac{1}{20\text{ cm}}}=+20\,\text{cm}.$$

(e) From Eq. 34-6, m = –20/20 = –1.0.

(f) The fact that the image distance is a positive value means the image is real (R).

(g) The fact that the magnification is a negative value means the image is inverted (I).

(h) The image is on the side opposite from the object.

75. **THINK** Since the image is on the same side as the object, it must be a virtual image.

EXPRESS The object distance p, the image distance i, and the focal length f are related by Eq. 34-9:

$$\frac{1}{f}=\frac{1}{p}+\frac{1}{i}.$$

The value of i is positive for a real images, and negative for virtual images. The corresponding lateral magnification is $m=-i/p$. The value of m is positive for upright (not inverted) images, and is negative for inverted images.

ANALYZE (a) Since the image is virtual (on the same side as the object), the image distance i is negative. By substituting $i=fp/(p-f)$ into $m=-i/p$, we obtain

$$m=-\frac{i}{p}=-\frac{f}{p-f}.$$

The fact that the magnification is less than 1.0 implies that f must be negative. This means that the lens is of the diverging ("D") type.

(b) Thus, the focal length is $f=-10$ cm.

(d) The image distance is $i=\dfrac{pf}{p-f}=\dfrac{(5.0\text{ cm})(-10\text{ cm})}{5.0\text{ cm}-(-10\text{ cm})}=-3.3$ cm.

(e) The magnification is $m=-i/p=-(-3.3\text{ cm})/(5.0\text{ cm})=+0.67$.

(f) The fact that the image distance i is a negative value means the image is virtual (V).

(g) A positive value of magnification means the image is not inverted (NI).

LEARN The ray diagram for this problem is similar to the one shown in Fig. 34-16(c). The lens is diverging, forming a virtual image with the same orientation as the object, and on the same side as the object.

76. (a) We are told the magnification is positive and greater than 1. Scanning the single-lens-image figures in the textbook (Figs. 34-16, 34-17, and 34-19), we see that such a magnification (which implies an upright image larger than the object) is only possible if the lens is of the converging ("C") type (and if $p < f$).

(b) We should put a plus sign in front of the "10" value given for the focal length.

(d) Equation 34-9 gives $i = \dfrac{1}{\frac{1}{f} - \frac{1}{p}} = \dfrac{1}{\frac{1}{10\text{ cm}} - \frac{1}{5.0\text{ cm}}} = -10\,\text{cm}.$

(e) $m = -i/p = +2.0$.

(f) The fact that the image distance i is a negative value means the image is virtual (V).

(g) A positive value of magnification means the image is not inverted (NI).

(h) The image is on the same side as the object.

77. **THINK** A positive value for the magnification m means that the image is upright (not inverted). In addition, $m > 1$ indicates that the image is enlarged.

EXPRESS The object distance p, the image distance i, and the focal length f are related by Eq. 34-9:

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{i}.$$

The value of i is positive for a real images, and negative for virtual images. The corresponding lateral magnification is $m = -i/p$. The value of m is positive for upright (not inverted) images, and is negative for inverted images.

ANALYZE (a) Combining Eqs. 34-7 and 34-9, we find the focal length to be

$$f = \frac{p}{1 - 1/m} = \frac{16\text{ cm}}{1 - 1/1.25} = 80\text{ cm}.$$

Since the value of f is positive, the lens is of the converging type (C).

(b) From (a), we have $f = +80$ cm.

(d) The image distance is $i = -mp = -(1.25)(16\text{ cm}) = -20$ cm.

(e) The magnification is $m = +1.25$, as given.

(f) The fact that the image distance i is a negative value means the image is virtual (V).

(g) A positive value of magnification means the image is not inverted (NI).

(h) The image it is on the same side as the object.

LEARN The ray diagram for this problem is similar to the one shown in Fig. 34-16(b). The lens is converging. With the object placed inside the focal point ($p < f$), we have a virtual image with the same orientation as the object, and on the same side as the object.

78. (a) We are told the absolute value of the magnification is 0.5 and that the image was upright (NI). Thus, $m = +0.5$. Using Eq. 34-6 and the given value of p, we find $i = -5.0$ cm; it is a virtual image. Equation 34-9 then yields the focal length: $f = -10$ cm. Therefore, the lens is of the diverging ("D") type.

(b) From (a), we have $f = -10$ cm.

(d) Similarly, $i = -5.0$ cm.

(e) $m = +0.5$, with a plus sign.

(f) The fact that the image distance i is a negative value means the image is virtual (V).

(h) The image is on the same side as the object.

79. (a) The fact that $m > 1$ means the lens is of the converging type (C) (it may help to look at Fig. 34-16 to illustrate this).

(b) A converging lens implies $f = +20$ cm, with a plus sign.

(d) Equation 34-9 then gives $i = -13$ cm.

(e) Equation 34-7 gives $m = -i / p = +1.7$.

(f) The fact that the image distance i is a negative value means the image is virtual (V).

(g) A positive value of magnification means the image is not inverted (NI).

(h) The image is on the same side as the object.

80. (a) The image from lens 1 (which has $f_1 = +15$ cm) is at $i_1 = -30$ cm (by Eq. 34-9). This serves as an "object" for lens 2 (which has $f_2 = +8$ cm) with $p_2 = d - i_1 = 40$ cm. Then Eq. 34-9 (applied to lens 2) yields $i_2 = +10$ cm.

(b) Equation 34-11 yields $M = m_1m_2 = (-i_1/p_1)(-i_2/p_2) = i_1i_2/p_1p_2 = -0.75$.

(c) The fact that the (final) image distance is a positive value means the image is real (R).

(d) The fact that the magnification is a negative value means the image is inverted (I).

(e) The image is on the side opposite from the object (relative to lens 2).

81. (a) The image from lens 1 (which has $f_1 = +8$ cm) is at $i_1 = 24$ cm (by Eq. 34-9). This serves as an "object" for lens 2 (which has $f_2 = +6$ cm) with $p_2 = d - i_1 = 8$ cm. Then Eq. 34-9 (applied to lens 2) yields $i_2 = +24$ cm.

(b) Equation 34-11 yields $M = m_1m_2 = (-i_1/p_1)(-i_2/p_2) = i_1i_2/p_1p_2 = +6.0$.

(c)The fact that the (final) image distance is a positive value means the image is real (R).

(d) The fact that the magnification is positive means the image is not inverted (NI).

(e) The image is on the side opposite from the object (relative to lens 2).

82. (a) The image from lens 1 (which has $f_1 = -6$ cm) is at $i_1 = -3.4$ cm (by Eq. 34-9). This serves as an "object" for lens 2 (which has $f_2 = +6$ cm) with $p_2 = d - i_1 = 15.4$ cm. Then Eq. 34-9 (applied to lens 2) yields $i_2 = +9.8$ cm.

(b) Equation 34-11 yields $M = -0.27$.

(c) The fact that the (final) image distance is a positive value means the image is real (R).

(d) The fact that the magnification is a negative value means the image is inverted (I).

(e) The image is on the side opposite from the object (relative to lens 2).

83. **THINK** In a system with two lenses, the image formed by lens 1 serves the "object" for lens 2.

EXPRESS To analyze two-lens systems, we first ignore lens 2, and apply the standard procedure used for a single-lens system. The object distance p_1, the image distance i_1, and the focal length f_1 are related by:

$$\frac{1}{f_1} = \frac{1}{p_1} + \frac{1}{i_1}.$$

Next, we ignore the lens 1 but treat the image formed by lens 1 as the object for lens 2. The object distance p_2 is the distance between lens 2 and the location of the first image. The location of the final image, i_2, is obtained by solving

$$\frac{1}{f_2} = \frac{1}{p_2} + \frac{1}{i_2}$$

where f_2 is the focal length of lens 2.

ANALYZE (a) Since lens 1 is converging, f_1 = +9 cm, and we find the image distance to be

$$i_1 = \frac{p_1 f_1}{p_1 - f_1} = \frac{(20 \text{ cm})(9 \text{ cm})}{20 \text{ cm} - 9 \text{ cm}} = 16.4 \text{ cm}.$$

This serves as an “object” for lens 2 (which has f_2 = +5 cm) with an object distance given by $p_2 = d - i_1 = -8.4$ cm. The negative sign means that the “object” is behind lens 2. Solving the lens equation, we obtain

$$i_2 = \frac{p_2 f_2}{p_2 - f_2} = \frac{(-8.4 \text{ cm})(5.0 \text{ cm})}{-8.4 \text{ cm} - 5.0 \text{ cm}} = 3.13 \text{ cm}.$$

(b) Te overall magnification is $M = m_1 m_2 = (-i_1 / p_1)(-i_2 / p_2) = i_1 i_2 / p_1 p_2 = -0.31$.

(c) The fact that the (final) image distance is a positive value means the image is real (R).

(d) The fact that the magnification is a negative value means the image is inverted (I).

(e) The image it is on the side opposite from the object (relative to lens 2).

LEARN Since this result involves a negative value for p_2 (and perhaps other “non-intuitive” features), we offer a few words of explanation: lens 1 is converging the rays towards an image (that never gets a chance to form due to the intervening presence of lens 2) that would be real and inverted (and 8.4 cm beyond lens 2’s location). Lens 2, in a sense, just causes these rays to converge a little more rapidly, and causes the image to form a little closer (to the lens system) than if lens 2 were not present.

84. (a) The image from lens 1 (which has f_1 = +12 cm) is at i_1 = +60 cm (by Eq. 34-9). This serves as an “object” for lens 2 (which has f_2 = +10 cm) with $p_2 = d - i_1 = 7$ cm. Then Eq. 34-9 (applied to lens 2) yields $i_2 = -23$ cm.

(b) Equation 34-11 yields $M = m_1 m_2 = (-i_1 / p_1)(-i_2 / p_2) = i_1 i_2 / p_1 p_2 = -13$.

(c) The fact that the (final) image distance is negative means the image is virtual (V).

(d) The fact that the magnification is a negative value means the image is inverted (I).

(e) The image is on the same side as the object (relative to lens 2).

85. (a) The image from lens 1 (which has f_1 = +6 cm) is at i_1 = –12 cm (by Eq. 34-9). This serves as an “object” for lens 2 (which has f_2 = –6 cm) with $p_2 = d - i_1$ = 20 cm. Then Eq. 34-9 (applied to lens 2) yields i_2 = –4.6 cm.

(b) Equation 34-11 yields M = +0.69.

(c) The fact that the (final) image distance is negative means the image is virtual (V).

(d) The fact that the magnification is positive means the image is not inverted (NI).

(e) The image is on the same side as the object (relative to lens 2).

86. (a) The image from lens 1 (which has f_1 = +8 cm) is at i_1 = +24 cm (by Eq. 34-9). This serves as an “object” for lens 2 (which has f_2 = –8 cm) with $p_2 = d - i_1$ = 6 cm. Then Eq. 34-9 (applied to lens 2) yields i_2 = –3.4 cm.

(b) Equation 34-11 yields M = –1.1.

(c) The fact that the (final) image distance is negative means the image is virtual (V).

(d) The fact that the magnification is a negative value means the image is inverted (I).

(e) The image is on the same side as the object (relative to lens 2).

87. (a) The image from lens 1 (which has f_1 = –12 cm) is at i_1 = –7.5 cm (by Eq. 34-9). This serves as an “object” for lens 2 (which has f_2 = –8 cm) with

$$p_2 = d - i_1 = 17.5 \text{ cm}.$$

Then Eq. 34-9 (applied to lens 2) yields i_2 = –5.5 cm.

(b) Equation 34-11 yields M = +0.12.

(c) The fact that the (final) image distance is negative means the image is virtual (V).

(d) The fact that the magnification is positive means the image is not inverted (NI).

(e) The image is on the same side as the object (relative to lens 2).

88. The minimum diameter of the eyepiece is given by

$$d_{\text{ey}} = \frac{d_{\text{ob}}}{m_\theta} = \frac{75 \text{ mm}}{36} = 2.1 \text{ mm}.$$

89. **THINK** The compound microscope shown in Fig. 34-20 consists of an objective and an eyepiece. It’s used for viewing small objects that are very close to the objective.

EXPRESS Let f_{ob} be the focal length of the objective, and f_{ey} be the focal length of the eyepiece. The distance between the two lenses is

$$L = s + f_{ob} + f_{ey},$$

where s is the tube length. The magnification of the objective is

$$m = -\frac{i}{p} = -\frac{s}{f_{ob}}$$

and the angular magnification produced by the eyepiece is $m_\theta = (25\text{ cm})/f_{ey}$.

ANALYZE (a) The tube length is

$$s = L - f_{ob} - f_{ey} = 25.0\text{ cm} - 4.00\text{ cm} - 8.00\text{ cm} = 13.0\text{ cm}.$$

(b) We solve $(1/p) + (1/i) = (1/f_{ob})$ for p. The image distance is

$$i = f_{ob} + s = 4.00\text{ cm} + 13.0\text{ cm} = 17.0\text{ cm},$$

so

$$p = \frac{if_{ob}}{i - f_{ob}} = \frac{(17.0\text{ cm})(4.00\text{ cm})}{17.0\text{ cm} - 4.00\text{ cm}} = 5.23\text{ cm}.$$

(c) The magnification of the objective is $m = -\dfrac{i}{p} = -\dfrac{17.0\text{ cm}}{5.23\text{ cm}} = -3.25.$

(d) The angular magnification of the eyepiece is $m_\theta = \dfrac{25\text{ cm}}{f_{ey}} = \dfrac{25\text{ cm}}{8.00\text{ cm}} = 3.13.$

(e) The overall magnification of the microscope is

$$M = mm_\theta = (-3.25)(3.13) = -10.2.$$

LEARN The objective produces a real image I of the object inside the focal point of the eyepiece ($i > f_{ey}$). Image I then serves as the object for the eyepiece, which produces a virtual image I' seen by the observer.

90. (a) Now, the lens-film distance is $i = \left(\dfrac{1}{f} - \dfrac{1}{p}\right)^{-1} = \left(\dfrac{1}{5.0\text{ cm}} - \dfrac{1}{100\text{ cm}}\right)^{-1} = 5.3\text{ cm}.$

(b) The change in the lens-film distance is 5.3 cm – 5.0 cm = 0.30 cm.

91. **THINK** This problem is about human eyes. We model the cornea and eye lens as a single effective thin lens, with image formed at the retina.

EXPRESS When the eye is relaxed, its lens focuses far-away objects on the retina, a distance i behind the lens. We set $p = \infty$ in the thin lens equation to obtain $1/i = 1/f$, where f is the focal length of the relaxed effective lens. Thus, $i = f = 2.50$ cm. When the eye focuses on closer objects, the image distance i remains the same but the object distance and focal length change.

ANALYZE (a) If p is the new object distance and f' is the new focal length, then

$$\frac{1}{p}+\frac{1}{i}=\frac{1}{f'}.$$

We substitute $i = f$ and solve for f': $f' = \dfrac{pf}{f+p} = \dfrac{(40.0\text{ cm})(2.50\text{ cm})}{40.0\text{ cm}+2.50\text{ cm}} = 2.35\text{ cm}.$

(b) Consider the lens maker's equation

$$\frac{1}{f}=(n-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$$

where r_1 and r_2 are the radii of curvature of the two surfaces of the lens and n is the index of refraction of the lens material. For the lens pictured in Fig. 34-46, r_1 and r_2 have about the same magnitude, r_1 is positive, and r_2 is negative. Since the focal length decreases, the combination $(1/r_1) - (1/r_2)$ must increase. This can be accomplished by decreasing the magnitudes of both radii.

LEARN When focusing on an object near the eye, the lens bulges a bit (smaller radius of curvature), and its focal length decreases.

92. We refer to Fig. 34-20. For the intermediate image, $p = 10$ mm and

$$i = (f_{ob} + s + f_{ey}) - f_{ey} = 300\text{ m} - 50\text{ mm} = 250\text{ mm},$$

so

$$\frac{1}{f_{ob}}=\frac{1}{i}+\frac{1}{p}=\frac{1}{250\text{ mm}}+\frac{1}{10\text{ mm}} \Rightarrow f_{ob}=9.62\text{ mm},$$

and

$$s = (f_{ob} + s + f_{ey}) - f_{ob} - f_{ey} = 300\text{ mm} - 9.62\text{ mm} - 50\text{ mm} = 240\text{ mm}.$$

Then from Eq. 34-14,

$$M=-\frac{s}{f_{ob}}\frac{25\text{ cm}}{f_{ey}}=-\left(\frac{240\text{ mm}}{9.62\text{ mm}}\right)\left(\frac{150\text{ mm}}{50\text{ mm}}\right)=-125.$$

93. (a) Without the magnifier, $\theta = h/P_n$ (see Fig. 34-19). With the magnifier, letting

$$i = -|i| = -P_n,$$

we obtain

$$\frac{1}{p} = \frac{1}{f} - \frac{1}{i} = \frac{1}{f} + \frac{1}{|i|} = \frac{1}{f} + \frac{1}{P_n}.$$

Consequently,

$$m_\theta = \frac{\theta'}{\theta} = \frac{h/p}{h/P_n} = \frac{1/f + 1/P_n}{1/P_n} = 1 + \frac{P_n}{f} = 1 + \frac{25\,\text{cm}}{f}.$$

With $f = 10$ cm, $m_\theta = 1 + \dfrac{25\text{ cm}}{10\text{ cm}} = 3.5$.

(b) In the case where the image appears at infinity, let $i = -|i| \to -\infty$, so that $1/p + 1/i = 1/p = 1/f$, we have

$$m_\theta = \frac{\theta'}{\theta} = \frac{h/p}{h/P_n} = \frac{1/f}{1/P_n} = \frac{P_n}{f} = \frac{25\,\text{cm}}{f}.$$

With $f = 10$ cm, $m_\theta = \dfrac{25\text{ cm}}{10\text{ cm}} = 2.5.$

94. By Eq. 34-9, $1/i + 1/p$ is equal to constant ($1/f$). Thus,

$$1/(-10) + 1/(15) = 1/i_{\text{new}} + 1/(70).$$

This leads to $i_{\text{new}} = -21$ cm.

95. A converging lens has a positive-valued focal length, so $f_1 = +8$ cm, $f_2 = +6$ cm, and $f_3 = +6$ cm. We use Eq. 34-9 for each lens separately, "bridging the gap" between the results of one calculation and the next with $p_2 = d_{12} - i_1$ and $p_3 = d_{23} - i_2$. We also use Eq. 34-7 for each magnification (m_1, etc.), and $m = m_1 m_2 m_3$ (a generalized version of Eq. 34-11) for the net magnification of the system. Our intermediate results for image distances are $i_1 = 24$ cm and $i_2 = -12$ cm. Our final results are as follows:

(a) $i_3 = +8.6$ cm.

(b) $m = +2.6$.

(c) The image is real (R).

(d) The image is not inverted (NI).

(e) It is on the opposite side of lens 3 from the object (which is expected for a real final image).

96. A converging lens has a positive-valued focal length, and a diverging lens has a negative-valued focal length. Therefore, $f_1 = -6.0$ cm, $f_2 = +6.0$ cm, and $f_3 = +4.0$ cm. We use Eq. 34-9 for each lens separately, "bridging the gap" between the results of one calculation and the next with $p_2 = d_{12} - i_1$ and $p_3 = d_{23} - i_2$. We also use Eq. 34-7 for each magnification (m_1, etc.), and $m = m_1 m_2 m_3$ (a generalized version of Eq. 34-11) for the net magnification of the system. Our intermediate results for image distances are $i_1 = -2.4$ cm and $i_2 = 12$ cm. Our final results are as follows:

(a) $i_3 = -4.0$ cm.

(b) $m = -1.2$.

(c) The image is virtual (V).

(d) The image is inverted (I).

(e) It is on the same side as the object (relative to lens 3) as expected for a virtual image.

97. A converging lens has a positive-valued focal length, so $f_1 = +6.0$ cm, $f_2 = +3.0$ cm, and $f_3 = +3.0$ cm. We use Eq. 34-9 for each lens separately, "bridging the gap" between the results of one calculation and the next with $p_2 = d_{12} - i_1$ and $p_3 = d_{23} - i_2$. We also use Eq. 34-7 for each magnification (m_1, etc.), and $m = m_1 m_2 m_3$ (a generalized version of Eq. 34-11) for the net magnification of the system. Our intermediate results for image distances are $i_1 = 9.0$ cm and $i_2 = 6.0$ cm. Our final results are as follows:

(a) $i_3 = +7.5$ cm.

(b) $m = -0.75$.

(c) The image is real (R).

(d) The image is inverted (I).

(e) It is on the opposite side of lens 3 from the object (which is expected for a real final image).

98. A converging lens has a positive-valued focal length, so $f_1 = +6.0$ cm, $f_2 = +6.0$ cm, and $f_3 = +5.0$ cm. We use Eq. 34-9 for each lens separately, "bridging the gap" between the results of one calculation and the next with $p_2 = d_{12} - i_1$ and $p_3 = d_{23} - i_2$. We also use Eq. 34-7 for each magnification (m_1, etc.), and $m = m_1 m_2 m_3$ (a generalized version of Eq. 34-11) for the net magnification of the system. Our intermediate results for image distances are $i_1 = -3.0$ cm and $i_2 = 9.0$ cm. Our final results are as follows:

(a) $i_3 = +10$ cm.

(b) $m = +0.75$.

(c) The image is real (R).

(d) The image is not inverted (NI).

(e) It is on the opposite side of lens 3 from the object (which is expected for a real final image).

99. A converging lens has a positive-valued focal length, and a diverging lens has a negative-valued focal length. Therefore, $f_1 = -8.0$ cm, $f_2 = -16$ cm, and $f_3 = +8.0$ cm. We use Eq. 34-9 for each lens separately, "bridging the gap" between the results of one calculation and the next with $p_2 = d_{12} - i_1$ and $p_3 = d_{23} - i_2$. We also use Eq. 34-7 for each magnification (m_1, etc.), and $m = m_1 m_2 m_3$ (a generalized version of Eq. 34-11) for the net magnification of the system. Our intermediate results for image distances are $i_1 = -4.0$ cm and $i_2 = -6.86$ cm. Our final results are as follows:

(a) $i_3 = +24.2$ cm.

(b) $m = -0.58$.

(c) The image is real (R).

(d) The image is inverted (I).

(e) It is on the opposite side of lens 3 from the object (as expected for a real image).

100. A converging lens has a positive-valued focal length, and a diverging lens has a negative-valued focal length. Therefore, $f_1 = +6.0$ cm, $f_2 = -4.0$ cm, and $f_3 = -12$ cm. We use Eq. 34-9 for each lens separately, "bridging the gap" between the results of one calculation and the next with $p_2 = d_{12} - i_1$ and $p_3 = d_{23} - i_2$. We also use Eq. 34-7 for each magnification (m_1, etc.), and $m = m_1 m_2 m_3$ (a generalized version of Eq. 34-11) for the net magnification of the system. Our intermediate results for image distances are $i_1 = -12$ cm and $i_2 = -3.33$ cm. Our final results are as follows:

(a) $i_3 = -5.15 \text{ cm} \approx -5.2 \text{ cm}$.

(b) $m = +0.285 \approx +0.29$.

(c) The image is virtual (V).

(d) The image is not inverted (NI).

(e) It is on the same side as the object (relative to lens 3) as expected for a virtual image.

101. **THINK** In this problem we convert the Gaussian form of the thin-lens formula to the Newtonian form.

EXPRESS For a thin lens, the Gaussian form of the thin-lens formula gives $(1/p) + (1/i) = (1/f)$, where p is the object distance, i is the image distance, and f is the focal length. To convert the formula to the Newtonian form, let $p = f + x$, where x is positive if the object is outside the focal point and negative if it is inside. In addition, let $i = f + x'$, where x' is positive if the image is outside the focal point and negative if it is inside.

ANALYZE From the Gaussian form, we solve for I and obtain:

$$i = \frac{fp}{p-f}.$$

Substituting $p = f + x$ gives

$$i = \frac{f(f+x)}{x}.$$

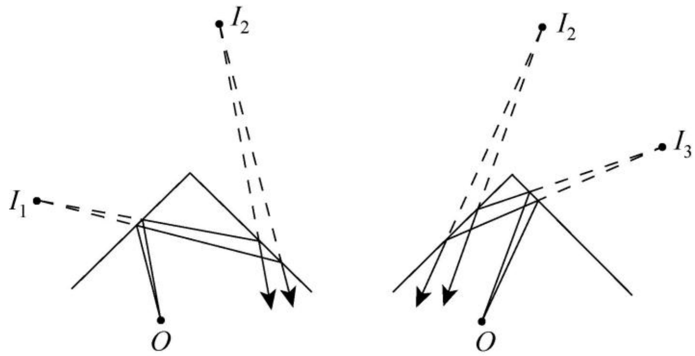
With $i = f + x'$, we have

$$x' = i - f = \frac{f(f+x)}{x} - f = \frac{f^2}{x}$$

which leads to $xx' = f^2$.

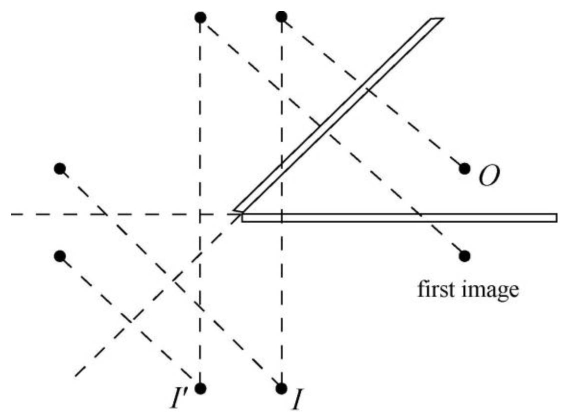
LEARN The Newtonain form is equivalent to the Gaussian form, and it provides another convenient way to analyze problems involving thin lenses.

102. (a) There are three images. Two are formed by single reflections from each of the mirrors and the third is formed by successive reflections from both mirrors. The positions of the images are shown on the two diagrams that follow. The diagram on the left shows the image I_1, formed by reflections from the left-hand mirror. It is the same distance behind the mirror as the object O is in front, and lies on the line perpendicular to the mirror and through the object. Image I_2 is formed by light that is reflected from both mirrors.



We may consider I_2 to be the image of I_1 formed by the right-hand mirror, extended. I_2 is the same distance behind the line of the right-hand mirror as I_1 is in front, and it is on the line that is perpendicular to the line of the mirror. The diagram on the right shows image I_3, formed by reflections from the right-hand mirror. It is the same distance behind the mirror as the object is in front, and lies on the line perpendicular to the mirror and through the object. As the diagram shows, light that is first reflected from the right-hand mirror and then from the left-hand mirror forms an image at I_2.

(b) For $\theta = 45°$, we have two images in the second mirror caused by the object and its "first" image, and from these one can construct two new images I and I' behind the first mirror plane. Extending the second mirror plane, we can find two further images of I and I' that are on equal sides of the extension of the first mirror plane. This circumstance implies there are no further images, since these final images are each other's "twins." We show this construction in the figure below. Summarizing, we find 1 + 2 + 2 + 2 = 7 images in this case.



(c) For $\theta = 60°$, we have two images in the second mirror caused by the object and its "first" image, and from these one can construct two new images I and I' behind the first mirror plane. The images I and I' are each other's "twins" in the sense that they are each other's reflections about the extension of the second mirror plane; there are no further images. Summarizing, we find 1 + 2 + 2 = 5 images in this case.

For $\theta = 120°$, we have two images I'_1 and I_2 behind the extension of the second mirror plane, caused by the object and its "first" image (which we refer to here as I_1). No further images can be constructed from I'_1 and I_2, since the method indicated above would place any further possibilities in front of the mirrors. This construction has the disadvantage of deemphasizing the actual ray-tracing, and thus any dependence on where the observer of these images is actually placing his or her eyes. It turns out in this case that the number of images that can be seen ranges from 1 to 3, depending on the locations of both the object and the observer.

(d) Thus, the smallest number of images that can be seen is 1. For example, if the observer's eye is collinear with I_1 and I'_1, then the observer can only see one image (I_1 and not the one behind it). Note that an observer who stands close to the second mirror would probably be able to see two images, I_1 and I_2.

(e) Similarly, the largest number would be 3. This happens if the observer moves further back from the vertex of the two mirrors. He or she should also be able to see the third image, I'_1, which is essentially the "twin" image formed from I_1 relative to the extension of the second mirror plane.

103. **THINK** Two lenses in contact can be treated as one single lens with an effective focal length.

EXPRESS We place an object far away from the composite lens and find the image distance i. Since the image is at a focal point, $i = f$, where f equals the effective focal length of the composite. The final image is produced by two lenses, with the image of the first lens being the object for the second. For the first lens, $(1/p_1) + (1/i_1) = (1/f_1)$, where f_1 is the focal length of this lens and i_1 is the image distance for the image it forms. Since $p_1 = \infty$, $i_1 = f_1$. The thin lens equation, applied to the second lens, is $(1/p_2) + (1/i_2) = (1/f_2)$, where p_2 is the object distance, i_2 is the image distance, and f_2 is the focal length. If the thickness of the lenses can be ignored, the object distance for the second lens is $p_2 = -i_1$. The negative sign must be used since the image formed by the first lens is beyond the second lens if i_1 is positive. This means the object for the second lens is virtual and the object distance is negative. If i_1 is negative, the image formed by the first lens is in front of the second lens and p_2 is positive.

ANALYZE In the thin lens equation, we replace p_2 with $-f_1$ and i_2 with f to obtain

$$-\frac{1}{f_1}+\frac{1}{f}=\frac{1}{f_2}$$

or

$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}=\frac{f_1+f_2}{f_1 f_2}.$$

Thus, the effective focal length of the system is $f=\dfrac{f_1 f_2}{f_1+f_2}$.

LEARN The reciprocal of the focal length, $1/f$, is known as the power of the lens, a quantity used by the optometrists to specify the strength of eyeglasses. From the derivation above, we see that when two lenses are in contact, the power of the effective lens is the sum of the two powers.

104. (a) In the closest mirror M_1, the "first" image I_1 is 10 cm behind M_1 and therefore 20 cm from the object O. This is the smallest distance between the object and an image of the object.

(b) There are images from both O and I_1 in the more distant mirror, M_2: an image I_2 located at 30 cm behind M_2. Since O is 30 cm in front of it, I_2 is 60 cm from O. This is the second smallest distance between the object and an image of the object.

(c) There is also an image I_3 that is 50 cm behind M_2 (since I_1 is 50 cm in front of it). Thus, I_3 is 80 cm from O. In addition, we have another image I_4 that is 70 cm behind M_1 (since I_2 is 70 cm in front of it). The distance from I_4 to O for is 80 cm.

(d) Returning to the closer mirror M_1, there is an image I_5 that is 90 cm behind the mirror (since I_3 is 90 cm in front of it). The distances (measured from O) for I_5 is 100 cm = 1.0 m.

105. (a) The "object" for the mirror that results in that box image is equally in front of the mirror (4 cm). This object is actually the first image formed by the system (produced by the first transmission through the lens); in those terms, it corresponds to i_1 = 10 – 4 = 6 cm. Thus, with f_1 = 2 cm, Eq. 34-9 leads to

$$\frac{1}{p_1}+\frac{1}{i_1}=\frac{1}{f_1}\Rightarrow p_1=3.00\text{ cm}.$$

(b) The previously mentioned box image (4 cm behind the mirror) serves as an "object" (at p_3 = 14 cm) for the return trip of light through the lens ($f_3 = f_1$ = 2 cm). This time, Eq. 34-9 leads to

$$\frac{1}{p_3}+\frac{1}{i_3}=\frac{1}{f_3}\Rightarrow i_3=2.33\text{ cm}.$$

106. (a) First, the lens forms a real image of the object located at a distance

$$i_1=\left(\frac{1}{f_1}-\frac{1}{p_1}\right)^{-1}=\left(\frac{1}{f_1}-\frac{1}{2f_1}\right)^{-1}=2f_1$$

to the right of the lens, or at

$$p_2 = 2(f_1 + f_2) - 2f_1 = 2f_2$$

in front of the mirror. The subsequent image formed by the mirror is located at a distance

$$i_2=\left(\frac{1}{f_2}-\frac{1}{p_2}\right)^{-1}=\left(\frac{1}{f_2}-\frac{1}{2f_2}\right)^{-1}=2f_2$$

to the left of the mirror, or at

$$p'_1 = 2(f_1 + f_2) - 2f_2 = 2f_1$$

to the right of the lens. The final image formed by the lens is at a distance i'_1 to the left of the lens, where

$$i'_1=\left(\frac{1}{f_1}-\frac{1}{p'_1}\right)^{-1}=\left(\frac{1}{f_1}-\frac{1}{2f_1}\right)^{-1}=2f_1.$$

This turns out to be the same as the location of the original object.

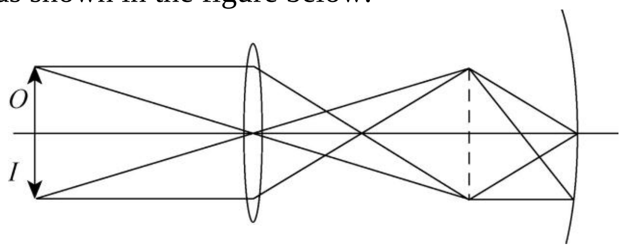
(b) The lateral magnification is

$$m=\left(-\frac{i_1}{p_1}\right)\left(-\frac{i_2}{p_2}\right)\left(-\frac{i'_1}{p'_1}\right)=\left(-\frac{2f_1}{2f_1}\right)\left(-\frac{2f_2}{2f_2}\right)\left(-\frac{2f_1}{2f_1}\right)=-1.0.$$

(c) The final image is real (R).

(d) It is at a distance i'_1 to the left of the lens,

(e) and inverted (I), as shown in the figure below.



107. **THINK** The nature of the lenses, whether converging or diverging, can be determined from the magnification and orientation of the images they produce.

EXPRESS By examining the ray diagrams shown in Fig. 34-16(a) – (c), we see that only a converging lens can produce an enlarged, upright image, while the image produced by a diverging lens is always virtual, reduced in size, and not inverted.

ANALYZE (a) In this case $m > +1$ and we know that lens 1 is converging (producing a virtual image), so that our result for focal length should be positive. Since $|P + i_1| = 20$ cm and $i_1 = -2p_1$, we find $p_1 = 20$ cm and $i_1 = -40$ cm. Substituting these into Eq. 34-9,

$$\frac{1}{p_1}+\frac{1}{i_1}=\frac{1}{f_1}$$

leads to

$$f_1=\frac{p_1 i_1}{p_1+i_1}=\frac{(20\text{ cm})(-40\text{ cm})}{20\text{ cm}+(-40\text{ cm})}=+40\text{ cm},$$

which is positive as we expected.

(b) The object distance is $p_1 = 20$ cm, as shown in part (a).

(c) In this case $0 < m < 1$ and we know that lens 2 is diverging (producing a virtual image), so that our result for focal length should be negative. Since $|p + i_2| = 20$ cm and $i_2 = -p_2/2$, we find $p_2 = 40$ cm and $i_2 = -20$ cm. Substituting these into Eq. 34-9 leads to

$$f_2=\frac{p_2 i_2}{p_2+i_2}=\frac{(40\text{ cm})(-20\text{ cm})}{40\text{ cm}+(-20\text{ cm})}=-40\text{ cm},$$

which is negative as we expected.

(d) The object distance is $p_2 = 40$ cm, as shown in part (c).

LEARN The ray diagram for lens 1 is similar to the one shown in Fig. 34-16(b). The lens is converging. With the fly inside the focal point ($p_1 < f_1$), we have a virtual image with the same orientation, and on the same side as the object. On the other hand, the ray diagram for lens 2 is similar to the one shown in Fig. 34-16(c). The lens is diverging, forming a virtual image with the same orientation but smaller in size as the object, and on the same side as the object.

108. We use Eq. 34-10, with the conventions for signs discussed in the text.

(a) For lens 1, the biconvex (or double convex) case, we have

$$f=\left[(n-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)\right]^{-1}=\left[(1.5-1)\left(\frac{1}{40\text{ cm}}-\frac{1}{-40\text{ cm}}\right)\right]^{-1}=40\text{ cm}.$$

(b) Since $f > 0$ the lens forms a real image of the Sun.

(c) For lens 2, of the planar convex type, we find

$$f=\left[(1.5-1)\left(\frac{1}{\infty}-\frac{1}{-40\text{cm}}\right)\right]^{-1}=80\text{cm}.$$

(d) The image formed is real (since $f > 0$).

(e) Now for lens 3, of the meniscus convex type, we have

$$f=\left[(1.5-1)\left(\frac{1}{40\text{cm}}-\frac{1}{60\text{cm}}\right)\right]^{-1}=240\text{cm}=2.4\text{ m}.$$

(f) The image formed is real (since $f > 0$).

(g) For lens 4, of the biconcave type, the focal length is

$$f=\left[(1.5-1)\left(\frac{1}{-40\text{cm}}-\frac{1}{40\text{cm}}\right)\right]^{-1}=-40\text{cm}.$$

(h) The image formed is virtual (since $f < 0$).

(i) For lens 5 (plane-concave), we have $f=\left[(1.5-1)\left(\frac{1}{\infty}-\frac{1}{40\text{cm}}\right)\right]^{-1}=-80\text{cm}.$

(j) The image formed is virtual (since $f < 0$).

(k) For lens 6 (meniscus concave), $f=\left[(1.5-1)\left(\dfrac{1}{60\,\text{cm}}-\dfrac{1}{40\,\text{cm}}\right)\right]^{-1}=-240\,\text{cm}=-2.4$ m.

(l) The image formed is virtual (since $f < 0$).

109. (a) The first image is figured using Eq. 34-8, with $n_1 = 1$ (using the rounded-off value for air) and $n_2 = 8/5$.

$$\frac{1}{p}+\frac{8}{5i}=\frac{1.6-1}{r}$$

For a "flat lens" $r = \infty$, so we obtain

$$i = -\,8p/5 = -\,64/5$$

(with the unit cm understood) for that object at $p = 10$ cm. Relative to the second surface, this image is at a distance of 3 + 64/5 = 79/5. This serves as an object in order to find the final image, using Eq. 34-8 again (and $r = \infty$) but with $n_1 = 8/5$ and $n_2 = 4/3$.

$$\frac{8}{5p'}+\frac{4}{3i'}=0$$

which produces (for $p' = 79/5$)

$$i' = -\,5p/6 = -\,79/6 \approx -\,13.2.$$

This means the observer appears 13.2 + 6.8 = 20 cm from the fish.

(b) It is straightforward to "reverse" the above reasoning, the result being that the final fish image is 7.0 cm to the right of the air-wall interface, and thus 15 cm from the observer.

110. Setting $n_{\text{air}} = 1$, $n_{\text{water}} = n$, and $p = r/2$ in Eq. 34-8 (and being careful with the sign convention for r in that equation), we obtain $i = -r/(1 + n)$, or $|i| = r/(1 + n)$. Then we use similar triangles (where h is the size of the fish and h' is that of the "virtual fish") to set up the ratio

$$\frac{h'}{r-|i|}=\frac{h}{r/2}\ .$$

Using our previous result for $|i|$, this gives $h'/h = 2(1 - 1/(1 + n)) = 1.14$.

111. (a) Parallel rays are bent by positive-f lenses to their focal points F_1, and rays that come from the focal point positions F_2 in front of positive-f lenses are made to emerge parallel. The key, then, to this type of beam expander is to have the rear focal point F_1 of the first lens coincide with the front focal point F_2 of the second lens. Since the triangles that meet at the coincident focal point are similar (they share the same angle; they are

vertex angles), then $W_f/f_2 = W_i/f_1$ follows immediately. Substituting the values given, we have

$$W_f = \frac{f_2}{f_1} W_i = \frac{30.0 \text{ cm}}{12.5 \text{ cm}} (2.5 \text{ mm}) = 6.0 \text{ mm}.$$

(b) The area is proportional to W^2. Since intensity is defined as power P divided by area, we have

$$\frac{I_f}{I_i} = \frac{P/W_f^2}{P/W_i^2} = \frac{W_i^2}{W_f^2} = \frac{f_1^2}{f_2^2} \Rightarrow I_f = \left(\frac{f_1}{f_2}\right)^2 I_i = 1.6 \text{ kW/m}^2.$$

(c) The previous argument can be adapted to the first lens in the expanding pair being of the diverging type, by ensuring that the front focal point of the first lens coincides with the front focal point of the second lens. The distance between the lenses in this case is

$$f_2 - |f_1| = 30.0 \text{ cm} - 26.0 \text{ cm} = 4.0 \text{ cm}.$$

112. The water is medium 1, so $n_1 = n_w$, which we simply write as n. The air is medium 2, for which $n_2 \approx 1$. We refer to points where the light rays strike the water surface as A (on the left side of Fig. 34-56) and B (on the right side of the picture). The point midway between A and B (the center point in the picture) is C. The penny P is directly below C, and the location of the "apparent" or virtual penny is V. We note that the angle $\angle CVB$ (the same as $\angle CVA$) is equal to θ_2, and the angle $\angle CPB$ (the same as $\angle CPA$) is equal to θ_1. The triangles CVB and CPB share a common side, the horizontal distance from C to B (which we refer to as x). Therefore,

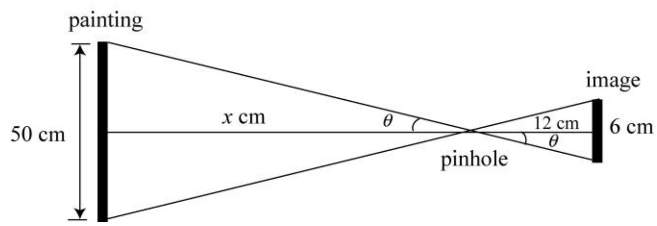
$$\tan\theta_2 = \frac{x}{d_a} \qquad \text{and} \qquad \tan\theta_1 = \frac{x}{d}.$$

Using the small angle approximation (so a ratio of tangents is nearly equal to a ratio of sines) and the law of refraction, we obtain

$$\frac{\tan\theta_2}{\tan\theta_1} \approx \frac{\sin\theta_2}{\sin\theta_1} \Rightarrow \frac{\frac{x}{d_a}}{\frac{x}{d}} \approx \frac{n_1}{n_2} \Rightarrow \frac{d}{d_a} \approx n$$

which yields the desired relation: $d_a = d/n$.

113. The top view of the arrangement is depicted in the figure below.

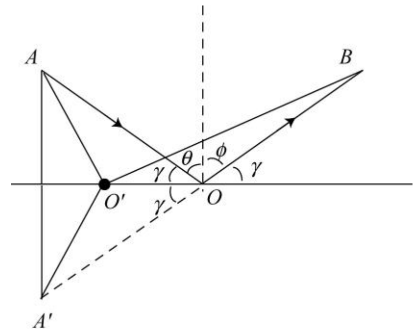


From the figure, we obtain

$$\tan\theta = \frac{25}{x} = \frac{3}{12}$$

which gives $x = 100$ cm.

114. Consider the ray diagram below.



Since $\theta + \gamma = \phi + \gamma = \pi/2$, we readily see that $\theta = \phi$, i.e., the angle of incidence is equal to the angle of reflection. To show that AOB is the shortest path, consider an incident ray AO' with a reflected ray $O'B$, where the angle of incidence is not equal to the angle of reflection. From the figure, we have

$$AO'B = AO' + O'B = A'O' + O'B > A'B = A'O + OB = AO + OB = AOB$$

The inequality comes from the fact that the sum of the two sides of a triangle is always greater than the hypotenuse.

115. We refer to Fig. 34-2 in the textbook. Consider the two light rays, r and r', which are closest to and on either side of the normal ray (the ray that reverses when it reflects). Each of these rays has an angle of incidence equal to θ when they reach the mirror. Consider that these two rays reach the top and bottom edges of the pupil after they have reflected. If ray r strikes the mirror at point A and ray r' strikes the mirror at B, the distance between A and B (call it x) is

$$x = 2d_o \tan\theta$$

where d_o is the distance from the mirror to the object. We can construct a right triangle starting with the image point of the object (a distance d_o behind the mirror; see I in Fig. 34-2). One side of the triangle follows the extended normal axis (which would reach from I to the middle of the pupil), and the hypotenuse is along the extension of ray r (after reflection). The distance from the pupil to I is $d_{ey} + d_o$, and the small angle in this triangle is again θ. Thus,

$$\tan\theta = \frac{R}{d_{ey} + d_o}$$

where R is the pupil radius (2.5 mm). Combining these relations, we find

$$x = 2d_o \frac{R}{d_{\text{ey}} + d_o} = 2(100\,\text{mm})\frac{2.5\,\text{mm}}{300\,\text{mm} + 100\,\text{mm}}$$

which yields x = 1.67 mm. Now, x serves as the diameter of a circular area A on the mirror, in which all rays that reflect will reach the eye. Therefore,

$$A = \frac{1}{4}\pi x^2 = \frac{\pi}{4}(1.67\,\text{mm})^2 = 2.2\,\text{mm}^2 \;.$$

116. For an object in front of a thin lens, the object distance p and the image distance i are related by $(1/p) + (1/i) = (1/f\,)$, where f is the focal length of the lens. For the situation described by the problem, all quantities are positive, so the distance x between the object and image is $x = p + i$. We substitute $i = x - p$ into the thin lens equation and solve for x:

$$x = \frac{p^2}{p - f}.$$

To find the minimum value of x, we set $dx/dp = 0$ and solve for p. Since

$$\frac{dx}{dp} = \frac{p(p - 2f\,)}{(p - f\,)^2},$$

the result is $p = 2f$. The minimum distance is

$$x_{\min} = \frac{p^2}{p - f} = \frac{(2f\,)^2}{2f - f} = 4f.$$

This is a minimum, rather than a maximum, since the image distance i becomes large without bound as the object approaches the focal point.

117. (a) If the object distance is x, then the image distance is $D - x$ and the thin lens equation becomes

$$\frac{1}{x} + \frac{1}{D - x} = \frac{1}{f}.$$

We multiply each term in the equation by $fx(D - x)$ and obtain $x^2 - Dx + Df = 0$. Solving for x, we find that the two object distances for which images are formed on the screen are

$$x_1 = \frac{D - \sqrt{D(D - 4f\,)}}{2} \quad \text{and} \quad x_2 = \frac{D + \sqrt{D(D - 4f\,)}}{2}.$$

The distance between the two object positions is

$$d = x_2 - x_1 = \sqrt{D(D-4f)}.$$

(b) The ratio of the image sizes is the same as the ratio of the lateral magnifications. If the object is at $p = x_1$, the magnitude of the lateral magnification is

$$|m_1| = \frac{i_1}{p_1} = \frac{D - x_1}{x_1}.$$

Now $x_1 = \frac{1}{2}(D-d)$, where $d = \sqrt{D(D-f)}$, so

$$|m_1| = \frac{D-(D-d)/2}{(D-d)/2} = \frac{D+d}{D-d}.$$

Similarly, when the object is at x_2, the magnitude of the lateral magnification is

$$|m_2| = \frac{I_2}{p_2} = \frac{D-x_2}{x_2} = \frac{D-(D+d)/2}{(D+d)/2} = \frac{D-d}{D+d}.$$

The ratio of the magnifications is

$$\frac{m_2}{m_1} = \frac{(D-d)/(D+d)}{(D+d)/(D-d)} = \left(\frac{D-d}{D+d}\right)^2.$$

118. (a) Our first step is to form the image from the first lens. With p_1 = 10 cm and $f_1 = -15$ cm, Eq. 34-9 leads to

$$\frac{1}{p_1} + \frac{1}{i_1} = \frac{1}{f_1} \quad \Rightarrow \quad i_1 = -6.0\,\text{cm}.$$

The corresponding magnification is $m_1 = -i_1/p_1 = 0.60$. This image serves the role of "object" for the second lens, with p_2 = 12 + 6.0 = 18 cm, and f_2 = 12 cm. Now, Eq. 34-9 leads to

$$\frac{1}{p_2} + \frac{1}{i_2} = \frac{1}{f_2} \quad \Rightarrow \quad i_2 = 36\text{ cm}.$$

(b) The corresponding magnification is $m_2 = -i_2/p_2 = -2.0$, which results in a net magnification of $m = m_1 m_2 = -1.2$. The height of the final image is (in absolute value) (1.2)(1.0 cm) = 1.2 cm.

(c) The fact that i_2 is positive means that the final image is real.

(d) The fact that m is negative means that the orientation of the final image is inverted with respect to the (original) object.

119. (a) Without the diverging lens (lens 2), the real image formed by the converging lens (lens 1) is located at a distance

$$i_1 = \left(\frac{1}{f_1} - \frac{1}{p_1}\right)^{-1} = \left(\frac{1}{20\text{ cm}} - \frac{1}{40\text{ cm}}\right)^{-1} = 40\text{ cm}$$

to the right of lens 1. This image now serves as an object for lens 2, with $p_2 = -(40\text{ cm} - 10\text{ cm}) = -30\text{ cm}$. So

$$i_2 = \left(\frac{1}{f_2} - \frac{1}{p_2}\right)^{-1} = \left(\frac{1}{-15\text{cm}} - \frac{1}{-30\text{cm}}\right)^{-1} = -30\text{cm}.$$

Thus, the image formed by lens 2 is located 30 cm to the left of lens 2.

(b) The magnification is $m = (-i_1/p_1) \times (-i_2/p_2) = +1.0 > 0$, so the image is not inverted.

(c) The image is virtual since $i_2 < 0$.

(d) The magnification is $m = (-i_1/p_1) \times (-i_2/p_2) = +1.0$, so the image has the same size as the object.

120. (a) For the image formed by the first lens

$$i_1 = \left(\frac{1}{f_1} - \frac{1}{p_1}\right)^{-1} = \left(\frac{1}{10\text{cm}} - \frac{1}{20\text{cm}}\right)^{-1} = 20\text{cm}.$$

For the subsequent image formed by the second lens $p_2 = 30\text{ cm} - 20\text{ cm} = 10\text{ cm}$, so

$$i_2 = \left(\frac{1}{f_2} - \frac{1}{p_2}\right)^{-1} = \left(\frac{1}{12.5\text{cm}} - \frac{1}{10\text{cm}}\right)^{-1} = -50\text{cm}.$$

Thus, the final image is 50 cm to the left of the second lens, which means that it coincides with the object.

(b) The magnification is

$$m = \left(\frac{i_1}{p_1}\right)\left(\frac{i_2}{p_2}\right) = \left(\frac{20\text{cm}}{20\text{cm}}\right)\left(\frac{-50\text{cm}}{10\text{cm}}\right) = -5.0,$$

which means that the final image is five times larger than the original object.

(c) The image is virtual since $i_2 < 0$.

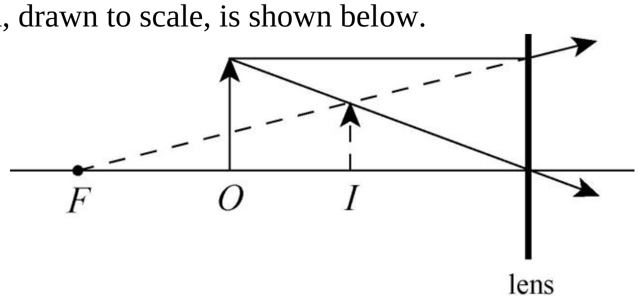
(d) The image is inverted since $m < 0$.

121. (a) We solve Eq. 34-9 for the image distance i: $i = pf/(p - f)$. The lens is diverging, so its focal length is $f = -30$ cm. The object distance is $p = 20$ cm. Thus,

$$i = \frac{(20\text{ cm})(-30\text{ cm})}{(20\text{ cm}) - (-30\text{ cm})} = -12\text{ cm}.$$

The negative sign indicates that the image is virtual and is on the same side of the lens as the object.

(b) The ray diagram, drawn to scale, is shown below.



122. (a) Suppose that the lens is placed to the left of the mirror. The image formed by the converging lens is located at a distance

$$i = \left(\frac{1}{f} - \frac{1}{p}\right)^{-1} = \left(\frac{1}{0.50\text{ m}} - \frac{1}{1.0\text{ m}}\right)^{-1} = 1.0\text{ m}$$

to the right of the lens, or 2.0 m – 1.0 m = 1.0 m in front of the mirror. The image formed by the mirror for this real image is then at 1.0 m to the right of the mirror, or 2.0 m + 1.0 m = 3.0 m to the right of the lens. This image then results in another image formed by the lens, located at a distance

$$i' = \left(\frac{1}{f} - \frac{1}{p'}\right)^{-1} = \left(\frac{1}{0.50\text{ m}} - \frac{1}{3.0\text{ m}}\right)^{-1} = 0.60\text{ m}$$

to the left of the lens (that is, 2.6 cm from the mirror).

(b) The lateral magnification is

$$m = \left(-\frac{i}{p}\right)\left(-\frac{i'}{p'}\right) = \left(-\frac{1.0\text{ m}}{1.0\text{ m}}\right)\left(-\frac{0.60\text{ m}}{3.0\text{ m}}\right) = +0.20\ .$$

(c) The final image is real since $i' > 0$.

(d) The image is to the left of the lens.

(e) It also has the same orientation as the object since $m > 0$. Therefore, the image is not inverted.

123. (a) We use Eq. 34-8 (and Fig. 34-12(b) is useful), with $n_1 = 1$ (using the rounded-off value for air) and $n_2 = 1.5$.

$$\frac{1}{p}+\frac{1.5}{i}=\frac{1.5-1}{r}$$

Using the sign convention for r stated in the paragraph following Eq. 34-8 (so that $r = +6.0$ cm), we obtain $i = -90$ cm for objects at $p = 10$ cm. Thus, the object and image are 80 cm apart.

(b) The image distance i is negative with increasing magnitude as p increases from very small values to some value p_0 at which point $i \to -\infty$. Since $1/(-\infty) = 0$, the above equation yields

$$\frac{1}{p_0}=\frac{1.5-1}{r} \quad\Rightarrow\quad p_0 = 2r.$$

Thus, the range for producing virtual images is $0 < p \le 12$ cm.

124. (a) Suppose one end of the object is a distance p from the mirror and the other end is a distance $p + L$. The position i_1 of the image of the first end is given by

$$\frac{1}{p}+\frac{1}{i_1}=\frac{1}{f}$$

where f is the focal length of the mirror. Thus, $i_1 = \dfrac{f_p}{p-f}$. The image of the other end is located at

$$i_2 = \frac{f(p+L)}{p+L-f},$$

so the length of the image is

$$L' = i_1 - i_2 = \frac{fp}{p-f} - \frac{f(p+L)}{p+L-f} = \frac{f^2L}{(p-f)(p+L-f)}.$$

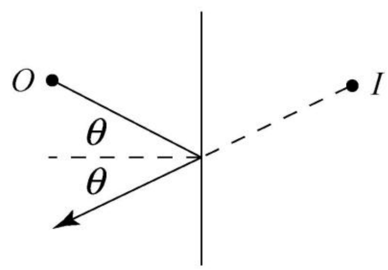
Since the object is short compared to $p - f$, we may neglect the L in the denominator and write

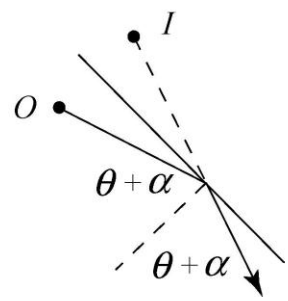
$$L' = L\left(\frac{f}{p-f}\right)^2.$$

(b) The lateral magnification is $m = -i/p$ and since $i = fp/(p - f)$, this can be written $m = -f/(p - f)$. The longitudinal magnification is

$$m' = \frac{L'}{L} = \left(\frac{f}{p-f}\right)^2 = m^2.$$

125. Consider a single ray from the source to the mirror and let θ be the angle of incidence. The angle of reflection is also θ and the reflected ray makes an angle of 2θ with the incident ray.





Now we rotate the mirror through the angle α so that the angle of incidence increases to $\theta + \alpha$. The reflected ray now makes an angle of $2(\theta + \alpha)$ with the incident ray. The reflected ray has been rotated through an angle of 2α. If the mirror is rotated so the angle of incidence is decreased by α, then the reflected ray makes an angle of $2(\theta - \alpha)$ with the incident ray. Again it has been rotated through 2α. The diagrams below show the situation for $\alpha = 45°$. The ray from the object to the mirror is the same in both cases and the reflected rays are 90° apart.

126. The fact that it is inverted implies $m < 0$. Therefore, with $m = -1/2$, we have $i = p/2$, which we substitute into Eq. 34-4:

$$\frac{1}{p} + \frac{1}{i} = \frac{1}{f} \quad \Rightarrow \quad \frac{1}{p} + \frac{2}{p} = \frac{1}{f}$$

or

$$\frac{3}{30.0\text{ cm}} = \frac{1}{f}.$$

Consequently, we find $f = (30.0\text{ cm})/3 = 10.0$ cm. The fact that $f > 0$ implies the mirror is concave.

127. (a) The mirror has focal length $f = 12.0$ cm. With $m = +3$, we have $i = -3p$. We substitute this into Eq. 34-4:

$$\frac{1}{p} + \frac{1}{i} = \frac{1}{f} \quad \Rightarrow \quad \frac{1}{p} + \frac{1}{-3p} = \frac{1}{12\text{ cm}}$$

or

$$\frac{2}{3p} = \frac{1}{12 \text{ cm}} .$$

Consequently, we find p = 2(12 cm)/3 = 8.0 cm.

(b) With m = –3, we have i = +3p, which we substitute into Eq. 34-4:

$$\frac{1}{p} + \frac{1}{i} = \frac{1}{f} \quad \Rightarrow \quad \frac{1}{p} + \frac{1}{3p} = \frac{1}{12}$$

or

$$\frac{4}{3p} = \frac{1}{12 \text{ cm}} .$$

Consequently, we find p = 4(12 cm)/3 = 16 cm.

(c) With m = –1/3, we have i = p/3. Thus, Eq. 34-4 leads to

$$\frac{1}{p} + \frac{1}{i} = \frac{1}{f} \quad \Rightarrow \quad \frac{1}{p} + \frac{3}{p} = \frac{1}{12 \text{ cm}}$$

or

$$\frac{4}{p} = \frac{1}{12 \text{ cm}} .$$

Consequently, we find p = 4(12 cm) = 48 cm.

128. Since 0 < m < 1, we conclude the lens is of the diverging type (so f = –40 cm). Thus, substituting i = –3p/10 into Eq. 34-9 produces

$$\frac{1}{p} - \frac{10}{3p} = -\frac{7}{3p} = \frac{1}{f} .$$

Therefore, we find p = 93.3 cm and i = –28.0 cm, or | i | = 28.0 cm.

129. (a) We show the α = 0.500 rad, r =12 cm, p = 20 cm calculation in detail. The understood length unit is the centimeter:

The distance from the object to point x:

$$d = p - r + x = 8 + x$$
$$y = d \tan\alpha = 4.3704 + 0.54630x$$

From the solution of $x^2 + y^2 = r^2$ we get x = 8.1398.

$$\beta = \tan^{-1}(y/x) = 0.8253 \text{ rad}$$

$$\gamma = 2\beta - \alpha = 1.151 \text{ rad}$$

From the solution of $\tan(\gamma) = y/(x + i - r)$ we get $i = 7.799$. The other results are shown without the intermediate steps:

For $\alpha = 0.100$ rad, we get $i = 8.544$ cm; for $\alpha = 0.0100$ rad, we get $i = 8.571$ cm. Eq. 34-3 and Eq. 34-4 (the mirror equation) yield $i = 8.571$ cm.

(b) Here the results are: ($\alpha = 0.500$ rad, $i = -13.56$ cm), ($\alpha = 0.100$ rad, $i = -12.05$ cm), ($\alpha = 0.0100$ rad, $i = -12.00$ cm). The mirror equation gives $i = -12.00$ cm.

130. (a) Since $m = +0.250$, we have $i = -0.25p$ which indicates that the image is virtual (as well as being diminished in size). We conclude from this that the mirror is convex and that $f < 0$; in fact, $f = -2.00$ cm. Substituting $i = -p/4$ into Eq. 34-4 produces

$$\frac{1}{p} - \frac{4}{p} = -\frac{3}{p} = \frac{1}{f}$$

Therefore, we find $p = 6.00$ cm and $i = -1.50$ cm, or $|i| = 1.50$ cm.

(b) The focal length is negative.

(c) As shown in (a), the image is virtual.

131. First, we note that — *relative to the water* — the index of refraction of the carbon tetrachloride should be thought of as $n = 1.46/1.33 = 1.1$ (this notation is chosen to be consistent with Problem 34-122). Now, if the observer were in the water, directly above the 40 mm deep carbon tetrachloride layer, then the apparent depth of the penny as measured below the surface of the carbon tetrachloride is $d_a = 40$ mm/1.1 = 36.4 mm. This "apparent penny" serves as an "object" for the rays propagating upward through the 20 mm layer of water, where this "object" should be thought of as being 20 mm + 36.4 mm = 56.4 mm from the top surface. Using the result of Problem 34-122 again, we find the perceived location of the penny, for a person at the normal viewing position above the water, to be 56.4 mm/1.33 = 42 mm below the water surface.

132. The sphere (of radius 0.35 m) is a convex mirror with focal length $f = -0.175$ m. We adopt the approximation that the rays are close enough to the central axis for Eq. 34-4 to be applicable.

(a) With $p = 1.0$ m, the equation $1/p + 1/i = 1/f$ yields $i = -0.15$ m, which means the image is 0.15 m from the front surface, appearing to be *inside* the sphere.

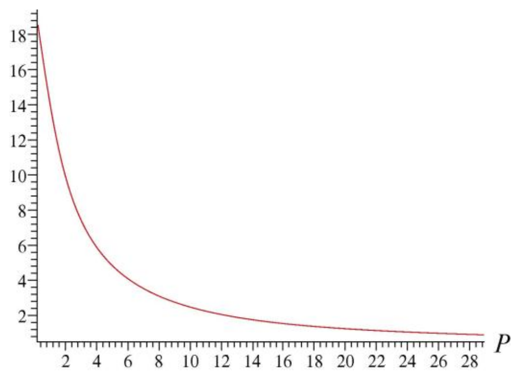
(b) The lateral magnification is $m = -i/p$ which yields $m = 0.15$. Therefore, the image distance is (0.15)(2.0 m) = 0.30 m.

(c) Since $m > 0$, the image is upright, or not inverted (NI).

133. (a) In this case $i < 0$ so $i = -|i|$, and Eq. 34-9 becomes $1/f = 1/p - 1/|i|$. We differentiate this with respect to time (t) to obtain

$$\frac{d|i|}{dt} = \left(\frac{|i|}{p}\right)^2 \frac{dp}{dt} \quad .$$

As the object is moved toward the lens, p is decreasing, so $dp/dt < 0$. Consequently, the above expression shows that $d|i|/dt < 0$; that is, the image moves in from infinity. The angular magnification $m_\theta = \theta'/\theta$ also increases as the following graph shows ("read" the graph from left to right since we are considering decreasing p from near the focal length to near 0). To obtain this graph of m_θ, we chose $f = 30$ cm and $h = 2$ cm.



(b) When the image appears to be at the near point (that is, $|i| = P_n$), m_θ is at its maximum usable value. Since one generally takes P_n to be equal to 25 cm (this value, too, was used in making the above graph).

(c) In this case,

$$p = \frac{if}{i-f} = \frac{|i|\,f}{|i|+f} = \frac{P_n f}{P_n + f} \quad .$$

If we use the small angle approximation, we have $\theta' \approx h'/|i|$ and $\theta \approx h/P_n$ (note: this approximation was not used in obtaining the graph, above). We therefore find

$$m_\theta \approx (h'/|i|)/(h/P_n)$$

which (using Eq. 34-7 relating the ratio of heights to the ratio of distances) becomes

$$m_\theta \approx \frac{h'}{h} \cdot \frac{P_n}{|i|} = \frac{|i|}{p} \cdot \frac{P_n}{|i|} = \frac{P_n}{p} = \frac{P_n}{P_n f/(P_n + f)} = \frac{P_n + f}{f}$$

which readily simplifies to the desired result.

(d) The linear magnification (Eq. 34-7) is given by $(h'/h) \approx m_\theta\,(|i|/P_n)$ (see the first in the chain of equalities, above). Once we set $|i| = P_n$ (see part (b)) then this shows the equality in the magnifications.

134. (a) The discussion in the textbook of the refracting telescope applies to the Newtonian arrangement if we replace the objective lens of Fig. 34-21 with an objective mirror (with the light incident on it from the right). This might suggest that the incident light would be blocked by the person's head in Fig. 34-21, which is why Newton added the mirror M' in his design (to move the head and eyepiece out of the way of the incoming light). The beauty of the idea of characterizing both lenses and mirrors by focal lengths is that it is easy, in a case like this, to simply carry over the results of the objective-lens telescope to the objective-mirror telescope, so long as we replace a positive f device with another positive f device. Thus, the converging lens serving as the objective of Fig. 34-21 must be replaced (as Newton has done in Fig. 34-58) with a concave mirror. With this change of language, the discussion in the textbook leading up to Eq. 34-15 applies equally as well to the Newtonian telescope: $m_\theta = -f_{ob}/f_{ey}$.

(b) A meter stick (held perpendicular to the line of sight) at a distance of 2000 m subtends an angle of

$$\theta_{\text{stick}} \approx \frac{1\text{ m}}{2000\text{ m}} = 0.0005\text{ rad.}$$

multiplying this by the mirror focal length gives (16.8 m) (0.0005) = 8.4 mm for the size of the image.

(c) With $r = 10$ m, Eq. 34-3 gives $f_{ob} = 5$ m. Plugging this into (the absolute value of) Eq. 34-15 leads to $f_{ey} = 5/200 = 2.5$ cm.

135. (a) If we let $p \to \infty$ in Eq. 34-8, we get $i = n_2\, r/(n_2 - n_1)$. If we set $n_1 = 1$ (for air) and restrict n_2 so that $1 < n_2 < 2$, then this suggests that $i > 2r$ (so this image does form before the rays strike the opposite side of the sphere). We can still consider this as a sort of "virtual" object for the second imaging event, where this "virtual" object distance is

$$2r - i = (n-2)\, r/(n-1),$$

where we have simplified the notation by writing $n_2 = n$. Putting this in for p in Eq. 34-8 and being careful with the sign convention for r in that equation, we arrive at the final image location: $i' = (0.5)(2-n)r/(n-1)$.

(b) The image is to the right of the right side of the sphere.

136. We set up an xyz coordinate system where the individual planes (xy, yz, xz) serve as the mirror surfaces. Suppose an incident ray of light A first strikes the mirror in the xy plane. If the unit vector denoting the direction of A is given by

$$\cos(\alpha)\hat{\text{i}} + \cos(\beta)\hat{\text{j}} + \cos(\gamma)\hat{\text{k}}$$

where α, β, γ are the angles A makes with the axes, then after reflection off the xy plane the unit vector becomes $\cos(\alpha)\hat{\mathrm{i}} + \cos(\beta)\hat{\mathrm{j}} - \cos(\gamma)\hat{\mathrm{k}}$ (one way to rationalize this is to think of the reflection as causing the angle γ to become $\pi - \gamma$). Next suppose it strikes the mirror in the xz plane. The unit vector of the reflected ray is now $\cos(\alpha)\hat{\mathrm{i}} - \cos(\beta)\hat{\mathrm{j}} - \cos(\gamma)\hat{\mathrm{k}}$. Finally as it reflects off the mirror in the yz plane α becomes $\pi - \alpha$, so the unit vector in the direction of the reflected ray is given by $-\cos(\alpha)\hat{\mathrm{i}} - \cos(\beta)\hat{\mathrm{j}} - \cos(\gamma)\hat{\mathrm{k}}$, exactly reversed from A's original direction. A further observation may be made: this argument would fail if the ray could strike any given surface twice and some consideration (perhaps an illustration) should convince the student that such an occurrence is not possible.

137. Since $m = -2$ and $p = 4.00$ cm, then $i = 8.00$ cm (and is real). Eq. 34-9 is

$$\frac{1}{p} + \frac{1}{i} = \frac{1}{f}$$

and leads to $f = 2.67$ cm (which is positive, as it must be for a converging lens).

138. (a) Since $m = +0.200$, we have $i = -0.2p$ which indicates that the image is virtual (as well as being diminished in size). We conclude from this that the mirror is convex (and that $f = -40.0$ cm).

(b) Substituting $i = -p/5$ into Eq. 34-4 produces

$$\frac{1}{p} - \frac{5}{p} = -\frac{4}{p} = \frac{1}{f}.$$

Therefore, we find $p = -4f = -4(-40.0\text{ cm}) = 160\text{ cm}$.

139. (a) Our first step is to form the image from the first lens. With $p_1 = 3.00$ cm and $f_1 = +4.00$ cm, Eq. 34-9 leads to

$$\frac{1}{p_1} + \frac{1}{i_1} = \frac{1}{f_1} \Rightarrow i_1 = \frac{f_1 p_1}{p_1 - f_1} = \frac{(4.00\text{ cm})(3.00\text{ cm})}{3.00\text{ cm} - 4.00\text{ cm}} = -12.0\text{cm}.$$

The corresponding magnification is $m_1 = -i_1/p_1 = 4$. This image serves the role of "object" for the second lens, with $p_2 = 8.00 + 12.0 = 20.0$ cm, and $f_2 = -4.00$ cm. Now, Eq. 34-9 leads to

$$\frac{1}{p_2} + \frac{1}{i_2} = \frac{1}{f_2} \Rightarrow i_2 = \frac{f_2 p_2}{p_2 - f_2} = \frac{(-4.00\text{ cm})(20.0\text{ cm})}{20.0\text{ cm} - (-4.00\text{ cm})} = -3.33\text{ cm},$$

or $|i_2| = 3.33\text{ cm}$.

(b) The fact that i_2 is negative means that the final image is virtual (and therefore to the left of the second lens).

(c) The image is virtual.

(d) With $m_2 = -i_2/p_2 = 1/6$, the net magnification is $m = m_1 m_2 = 2/3 > 0$. The fact that m is positive means that the orientation of the final image is the same as the (original) object. Therefore, the image is not inverted.

140. The far point of the person is 50 cm = 0.50 m from the eye. The object distance is taken to be at infinity, and the corrected lens will allow the image to be formed at the near point. Thus,

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{i} = \frac{1}{\infty} + \frac{1}{-0.50 \text{ m}}$$

and we find the focal length of the lens to $f = -0.50$ m.

(b) Since $f < 0$, the lens is diverging.

(c) The power of the lens is $P = \dfrac{1}{f} = \dfrac{1}{-0.50 \text{ m}} = -2.0 \text{ diopters}$.

141. (a) Without the magnifier, $\theta = h/P_n$. With the magnifier, letting $p = p_n$ and $i = -|i| = -P_n$, we obtain

$$\frac{1}{p} = \frac{1}{f} - \frac{1}{i} = \frac{1}{f} + \frac{1}{|i|} = \frac{1}{f} + \frac{1}{P_n}.$$

Consequently,

$$m_\theta = \frac{\theta'}{\theta} = \frac{h/p}{h/P_n} = \frac{1/f + 1/P_n}{1/P_n} = 1 + \frac{P_n}{f} = 1 + \frac{25 \text{ cm}}{f}.$$

(b) Now $i = -|i| \to -\infty$, so $1/p + 1/i = 1/p = 1/f$ and

$$m_\theta = \frac{\theta'}{\theta} = \frac{h/p}{h/P_n} = \frac{1/f}{1/P_n} = \frac{P_n}{f} = \frac{25 \text{ cm}}{f}.$$

(c) For $f = 10$ cm, we find the magnifications to be $m_\theta = 1 + \dfrac{25 \text{ cm}}{10 \text{ cm}} = 3.5$ for cases (a), and $m_\theta = \dfrac{25 \text{ cm}}{10 \text{ cm}} = 2.5$ for case (b).

Chapter 35

1. The fact that wave W_2 reflects two additional times has no substantive effect on the calculations, since two reflections amount to a $2(\lambda/2) = \lambda$ phase difference, which is effectively not a phase difference at all. The substantive difference between W_2 and W_1 is the extra distance $2L$ traveled by W_2.

(a) For wave W_2 to be a half-wavelength "behind" wave W_1, we require $2L = \lambda/2$, or $L = \lambda/4 = (620\text{ nm})/4 = 155$ nm using the wavelength value given in the problem.

(b) Destructive interference will again appear if W_2 is $\frac{3}{2}\lambda$ "behind" the other wave. In this case, $2L' = 3\lambda/2$, and the difference is

$$L' - L = \frac{3\lambda}{4} - \frac{\lambda}{4} = \frac{\lambda}{2} = \frac{620\text{ nm}}{2} = 310\,\text{nm} .$$

2. We consider waves W_2 and W_1 with an initial effective phase difference (in wavelengths) equal to $\frac{1}{2}$, and seek positions of the sliver that cause the wave to constructively interfere (which corresponds to an integer-valued phase difference in wavelengths). Thus, the extra distance $2L$ traveled by W_2 must amount to $\frac{1}{2}\lambda$, $\frac{3}{2}\lambda$, and so on. We may write this requirement succinctly as

$$L = \frac{2m+1}{4}\lambda \quad \text{where } m = 0, 1, 2, \ldots .$$

(a) Thus, the smallest value of L/λ that results in the final waves being exactly in phase is when $m = 0$, which gives $L/\lambda = 1/4 = 0.25$.

(b) The second smallest value of L/λ that results in the final waves being exactly in phase is when $m = 1$, which gives $L/\lambda = 3/4 = 0.75$.

(c) The third smallest value of L/λ that results in the final waves being exactly in phase is when $m = 2$, which gives $L/\lambda = 5/4 = 1.25$.

3. **THINK** The wavelength of light in a medium depends on the index of refraction of the medium. The nature of the interference, whether constructive or destructive, depends on the phase difference of the two waves.

EXPRESS We take the phases of both waves to be zero at the front surfaces of the layers. The phase of the first wave at the back surface of the glass is given by $\phi_1 = k_1L - \omega t$, where k_1 $(= 2\pi/\lambda_1)$ is the angular wave number and λ_1 is the wavelength in glass. Similarly, the phase of the second wave at the back surface of the plastic is given by $\phi_2 =$

$k_2L - \omega t$, where k_2 (= $2\pi/\lambda_2$) is the angular wave number and λ_2 is the wavelength in plastic. The angular frequencies are the same since the waves have the same wavelength in air and the frequency of a wave does not change when the wave enters another medium. The phase difference is

$$\phi_1 - \phi_2 = (k_1 - k_2)L = 2\pi\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right)L.$$

Now, $\lambda_1 = \lambda_{air}/n_1$, where λ_{air} is the wavelength in air and n_1 is the index of refraction of the glass. Similarly, $\lambda_2 = \lambda_{air}/n_2$, where n_2 is the index of refraction of the plastic. This means that the phase difference is

$$\phi_1 - \phi_2 = \frac{2\pi}{\lambda_{\text{air}}}(n_1 - n_2)L.$$

ANALYZE (a) The value of L that makes this 5.65 rad is

$$L = \frac{(\phi_1 - \phi_2)\lambda_{\text{air}}}{2\pi(n_1 - n_2)} = \frac{5.65(400 \times 10^{-9}\,\text{m})}{2\pi(1.60 - 1.50)} = 3.60 \times 10^{-6}\,\text{m}.$$

(b) A phase difference of 5.65 rad is less than 2π rad = 6.28 rad, the phase difference for completely constructive interference, but greater than π rad (= 3.14 rad), the phase difference for completely destructive interference. The interference is, therefore, intermediate, neither completely constructive nor completely destructive. It is, however, closer to completely constructive than to completely destructive.

LEARN The phase difference of two light waves can change when they travel through different materials having different indices of refraction.

4. Note that Snell's law (the law of refraction) leads to $\theta_1 = \theta_2$ when $n_1 = n_2$. The graph indicates that $\theta_2 = 30°$ (which is what the problem gives as the value of θ_1) occurs at n_2 = 1.5. Thus, n_1 = 1.5, and the speed with which light propagates in that medium is

$$v = \frac{c}{n_1} = \frac{2.998 \times 10^8\ \text{m/s}}{1.5} = 2.0 \times 10^8\ \text{m/s}.$$

5. Comparing the light speeds in sapphire and diamond, we obtain

$$\Delta v = v_s - v_d = c\left(\frac{1}{n_s} - \frac{1}{n_d}\right) = (2.998 \times 10^8\ \text{m/s})\left(\frac{1}{1.77} - \frac{1}{2.42}\right) = 4.55 \times 10^7\ \text{m/s}.$$

6. (a) The frequency of yellow sodium light is

$$f = \frac{c}{\lambda} = \frac{2.998 \times 10^8\ \text{m/s}}{589 \times 10^{-9}\,\text{m}} = 5.09 \times 10^{14}\,\text{Hz}.$$

(b) When traveling through the glass, its wavelength is

$$\lambda_n = \frac{\lambda}{n} = \frac{589\,\text{nm}}{1.52} = 388\text{ nm.}$$

(c) The light speed when traveling through the glass is

$$v = f\ \lambda_n = \left(5.09\times10^{14}\,\text{Hz}\right)\left(388\times10^{-9}\,\text{m}\right) = 1.97\times10^{8}\ \text{m/s}.$$

7. The index of refraction is found from Eq. 35-3:

$$n = \frac{c}{v} = \frac{2.998\times10^{8}\ \text{m/s}}{1.92\times10^{8}\ \text{m/s}} = 1.56.$$

8. (a) The time t_2 it takes for pulse 2 to travel through the plastic is

$$t_2 = \frac{L}{c/1.55} + \frac{L}{c/1.70} + \frac{L}{c/1.60} + \frac{L}{c/1.45} = \frac{6.30L}{c}.$$

Similarly for pulse 1:

$$t_1 = \frac{2L}{c/1.59} + \frac{L}{c/1.65} + \frac{L}{c/1.50} = \frac{6.33L}{c}.$$

Thus, pulse 2 travels through the plastic in less time.

(b) The time difference (as a multiple of L/c) is

$$\Delta t = t_2 - t_1 = \frac{6.33L}{c} - \frac{6.30L}{c} = \frac{0.03L}{c}.$$

Thus, the multiple is 0.03.

9. (a) We wish to set Eq. 35-11 equal to $1/2$, since a half-wavelength phase difference is equivalent to a π radians difference. Thus,

$$L_{\min} = \frac{\lambda}{2(n_2 - n_1)} = \frac{620\,\text{nm}}{2(1.65 - 1.45)} = 1550\,\text{nm} = 1.55\ \mu\text{m}.$$

(b) Since a phase difference of $\frac{3}{2}$ (wavelengths) is effectively the same as what we required in part (a), then

$$L = \frac{3\lambda}{2(n_2 - n_1)} = 3L_{\min} = 3(1.55\,\mu\text{m}) = 4.65\,\mu\text{m}.$$

10. (a) The exiting angle is 50º, the same as the incident angle, due to what one might call the "transitive" nature of Snell's law: $n_1 \sin\theta_1 = n_2 \sin\theta_2 = n_3 \sin\theta_3 = \ldots$

(b) Due to the fact that the speed (in a certain medium) is c/n (where n is that medium's index of refraction) and that speed is distance divided by time (while it's constant), we find

$$t = nL/c = (1.45)(25 \times 10^{-19}\text{ m})/(3.0 \times 10^{8}\text{ m/s}) = 1.4 \times 10^{-13}\text{ s} = 0.14\text{ ps}.$$

11. (a) Equation 35-11 (in absolute value) yields

$$\frac{L}{\lambda}|n_2 - n_1| = \frac{(8.50 \times 10^{-6}\text{ m})}{500 \times 10^{-9}\text{ m}}(1.60 - 1.50) = 1.70.$$

(b) Similarly,

$$\frac{L}{\lambda}|n_2 - n_1| = \frac{(8.50 \times 10^{-6}\text{ m})}{500 \times 10^{-9}\text{ m}}(1.72 - 1.62) = 1.70.$$

(c) In this case, we obtain

$$\frac{L}{\lambda}|n_2 - n_1| = \frac{(3.25 \times 10^{-6}\text{ m})}{500 \times 10^{-9}\text{ m}}(1.79 - 1.59) = 1.30.$$

(d) Since their phase differences were identical, the brightness should be the same for (a) and (b). Now, the phase difference in (c) differs from an integer by 0.30, which is also true for (a) and (b). Thus, their effective phase differences are equal, and the brightness in case (c) should be the same as that in (a) and (b).

12. (a) We note that ray 1 travels an extra distance $4L$ more than ray 2. To get the least possible L that will result in destructive interference, we set this extra distance equal to half of a wavelength:

$$4L = \frac{\lambda}{2} \quad \Rightarrow \quad L = \frac{\lambda}{8} = \frac{420.0\text{ nm}}{8} = 52.50\text{ nm}.$$

(b) The next case occurs when that extra distance is set equal to $\frac{3}{2}\lambda$. The result is

$$L = \frac{3\lambda}{8} = \frac{3(420.0\text{ nm})}{8} = 157.5\text{ nm}.$$

13. (a) We choose a horizontal x axis with its origin at the left edge of the plastic. Between $x = 0$ and $x = L_2$ the phase difference is that given by Eq. 35-11 (with L in that

equation replaced with L_2). Between $x = L_2$ and $x = L_1$ the phase difference is given by an expression similar to Eq. 35-11 but with L replaced with $L_1 - L_2$ and n_2 replaced with 1 (since the top ray in Fig. 35-36 is now traveling through air, which has index of refraction approximately equal to 1). Thus, combining these phase differences with $\lambda = 0.600\ \mu$m, we have

$$\frac{L_2}{\lambda}(n_2 - n_1) + \frac{L_1 - L_2}{\lambda}(1 - n_1) = \frac{3.50\ \mu\text{m}}{0.600\ \mu\text{m}}(1.60 - 1.40) + \frac{4.00\ \mu\text{m} - 3.50\ \mu\text{m}}{0.600\ \mu\text{m}}(1 - 1.40)$$
$$= 0.833.$$

(b) Since the answer in part (a) is closer to an integer than to a half-integer, the interference is more nearly constructive than destructive.

14. (a) For the maximum adjacent to the central one, we set $m = 1$ in Eq. 35-14 and obtain

$$\theta_1 = \sin^{-1}\left(\frac{m\lambda}{d}\right)\Bigg|_{m=1} = \sin^{-1}\left[\frac{(1)(\lambda)}{100\lambda}\right] = 0.010\,\text{rad}.$$

(b) Since $y_1 = D \tan \theta_1$ (see Fig. 35-10(a)), we obtain

$$y_1 = (500 \text{ mm}) \tan (0.010 \text{ rad}) = 5.0 \text{ mm}.$$

The separation is $\Delta y = y_1 - y_0 = y_1 - 0 = 5.0$ mm.

15. **THINK** The interference at a point depends on the path-length difference of the light rays reaching that point from the two slits.

EXPRESS The angular positions of the maxima of a two-slit interference pattern are given by $\Delta L = d\sin\theta = m\lambda$, where ΔL is the path-length difference, d is the slit separation, λ is the wavelength, and m is an integer. If θ is small, $\sin\theta$ may be approximated by θ in radians. Then, $\theta = m\lambda/d$ to good approximation. The angular separation of two adjacent maxima is $\Delta\theta = \lambda/d$.

ANALYZE Let λ' be the wavelength for which the angular separation is greater by10.0%. Then, $1.10\lambda/d = \lambda'/d$. or

$$\lambda' = 1.10\lambda = 1.10(589 \text{ nm}) = 648 \text{ nm}.$$

LEARN The angular separation $\Delta\theta$ is proportional to the wavelength of the light. For small θ, we have

$$\Delta\theta' = \left(\frac{\lambda'}{\lambda}\right)\Delta\theta.$$

16. The distance between adjacent maxima is given by $\Delta y = \lambda D/d$ (see Eqs. 35-17 and 35-18). Dividing both sides by D, this becomes $\Delta\theta = \lambda/d$ with θ in radians. In the steps that follow, however, we will end up with an expression where degrees may be directly used. Thus, in the present case,

$$\Delta\theta_n = \frac{\lambda_n}{d} = \frac{\lambda}{nd} = \frac{\Delta\theta}{n} = \frac{0.20°}{1.33} = 0.15°.$$

17. **THINK** Interference maxima occur at angles θ such that $d \sin \theta = m\lambda$, where m is an integer.

EXPRESS Since $d = 2.0$ m and $\lambda = 0.50$ m, this means that $\sin\theta = 0.25m$. We want all values of m (positive and negative) for which $|0.25m| \le 1$. These are –4, –3, –2, –1, 0, +1, +2, +3, and +4.

ANALYZE For each of these except –4 and +4, there are two different values for θ. A single value of θ (–90°) is associated with $m = -4$ and a single value (+90°) is associated with $m = +4$. There are sixteen different angles in all and, therefore, sixteen maxima.

LEARN The angles at which the maxima occur are given by

$$\theta = \sin^{-1}\left(\frac{m\lambda}{d}\right) = \sin^{-1}(0.25m)$$

Similarly, the condition for interference minima (destructive interference) is

$$d \sin\theta = \left(m + \frac{1}{2}\right)\lambda, \quad m = 0, 1, 2, \ldots$$

18. (a) The phase difference (in wavelengths) is

$$\phi = d \sin\theta/\lambda = (4.24\ \mu\text{m})\sin(20°)/(0.500\ \mu\text{m}) = 2.90\ .$$

(b) Multiplying this by 2π gives $\phi = 18.2$ rad.

(c) The result from part (a) is greater than $\frac{5}{2}$ (which would indicate the third minimum) and is less than 3 (which would correspond to the third side maximum).

19. **THINK** The condition for a maximum in the two-slit interference pattern is $d \sin \theta = m\lambda$, where d is the slit separation, λ is the wavelength, m is an integer, and θ is the angle made by the interfering rays with the forward direction.

EXPRESS If θ is small, $\sin \theta$ may be approximated by θ in radians. Then, $\theta = m\lambda/d$, and the angular separation of adjacent maxima, one associated with the integer m and the other associated with the integer $m + 1$, is given by $\Delta\theta = \lambda/d$. The separation on a screen a distance D away is given by

$$\Delta y = D\,\Delta\theta = \lambda D/d.$$

ANALYZE Thus,

$$\Delta y = \frac{(500\times10^{-9}\,\text{m})(5.40\,\text{m})}{1.20\times10^{-3}\,\text{m}} = 2.25\times10^{-3}\,\text{m} = 2.25\ \text{mm}.$$

LEARN For small θ, the spacing is nearly uniform. However, away from the center of the pattern, θ increases and the spacing gets larger.

20. (a) We use Eq. 35-14 with $m = 3$:

$$\theta = \sin^{-1}\left(\frac{m\lambda}{d}\right) = \sin^{-1}\left[\frac{2(550\times10^{-9}\,\text{m})}{7.70\times10^{-6}\,\text{m}}\right] = 0.216\,\text{rad}.$$

(b) $\theta = (0.216)\,(180°/\pi) = 12.4°$.

21. The maxima of a two-slit interference pattern are at angles θ given by $d \sin\theta = m\lambda$, where d is the slit separation, λ is the wavelength, and m is an integer. If θ is small, $\sin\theta$ may be replaced by θ in radians. Then, $d\theta = m\lambda$. The angular separation of two maxima associated with different wavelengths but the same value of m is

$$\Delta\theta = (m/d)(\lambda_2 - \lambda_1),$$

and their separation on a screen a distance D away is

$$\begin{aligned}\Delta y &= D\tan\Delta\theta \approx D\,\Delta\theta = \left[\frac{mD}{d}\right](\lambda_2 - \lambda_1)\\ &= \left[\frac{3(1.0\,\text{m})}{5.0\times10^{-3}\,\text{m}}\right](600\times10^{-9}\,\text{m} - 480\times10^{-9}\,\text{m}) = 7.2\times10^{-5}\,\text{m}.\end{aligned}$$

The small angle approximation $\tan\Delta\theta \approx \Delta\theta$ (in radians) is made.

22. Imagine a y axis midway between the two sources in the figure. Thirty points of destructive interference (to be considered in the xy plane of the figure) implies there are $7+1+7=15$ on each side of the y axis. There is no point of destructive interference on the y axis itself since the sources are in phase and any point on the y axis must therefore correspond to a zero phase difference (and corresponds to $\theta = 0$ in Eq. 35-14). In other words, there are 7 "dark" points in the first quadrant, one along the $+x$ axis, and 7 in the fourth quadrant, constituting the 15 dark points on the right-hand side of the y axis. Since the y axis corresponds to a minimum phase difference, we can count (say, in the first quadrant) the m values for the destructive interference (in the sense of Eq. 35-16)

beginning with the one closest to the y axis and going clockwise until we reach the x axis (at any point beyond S_2). This leads us to assign m = 7 (in the sense of Eq. 35-16) to the point on the x axis itself (where the path difference for waves coming from the sources is simply equal to the separation of the sources, d); this would correspond to $\theta = 90°$ in Eq. 35-16. Thus,

$$d = (7 + \tfrac{1}{2})\lambda = 7.5\,\lambda \;\Rightarrow\; \frac{d}{\lambda} = 7.5\,.$$

23. Initially, source A leads source B by 90°, which is equivalent to $1/4$ wavelength. However, source A also lags behind source B since r_A is longer than r_B by 100 m, which is $100\,\text{m}/400\,\text{m} = 1/4$ wavelength. So the net phase difference between A and B at the detector is zero.

24. (a) We note that, just as in the usual discussion of the double slit pattern, the $x = 0$ point on the screen (where that vertical line of length D in the picture intersects the screen) is a bright spot with phase difference equal to zero (it would be the middle fringe in the usual double slit pattern). We are not considering $x < 0$ values here, so that negative phase differences are not relevant (and if we did wish to consider $x < 0$ values, we could limit our discussion to absolute values of the phase difference, so that, again, negative phase differences do not enter it). Thus, the $x = 0$ point is the one with the minimum phase difference.

(b) As noted in part (a), the phase difference $\phi = 0$ at $x = 0$.

(c) The path length difference is greatest at the rightmost "edge" of the screen (which is assumed to go on forever), so ϕ is maximum at $x = \infty$.

(d) In considering $x = \infty$, we can treat the rays from the sources as if they are essentially horizontal. In this way, we see that the difference between the path lengths is simply the distance ($2d$) between the sources. The problem specifies $2d = 6.00\,\lambda$, or $2d/\lambda = 6.00$.

(e) Using the Pythagorean theorem, we have

$$\phi = \frac{\sqrt{D^2 + (x+d)^2}}{\lambda} - \frac{\sqrt{D^2 + (x-d)^2}}{\lambda} = 1.71$$

where we have plugged in $D = 20\lambda$, $d = 3\lambda$ and $x = 6\lambda$. Thus, the phase difference at that point is 1.71 wavelengths.

(f) We note that the answer to part (e) is closer to $\frac{3}{2}$ (destructive interference) than to 2 (constructive interference), so that the point is "intermediate" but closer to a minimum than to a maximum.

25. Let the distance in question be x. The path difference (between rays originating from S_1 and S_2 and arriving at points on the $x > 0$ axis) is

$$\sqrt{d^2 + x^2} - x = \left(m + \frac{1}{2}\right)\lambda,$$

where we are requiring destructive interference (half-integer wavelength phase differences) and $m = 0, 1, 2, \ldots$. After some algebraic steps, we solve for the distance in terms of m:

$$x = \frac{d^2}{(2m+1)\lambda} - \frac{(2m+1)\lambda}{4}.$$

To obtain the largest value of x, we set $m = 0$:

$$x_0 = \frac{d^2}{\lambda} - \frac{\lambda}{4} = \frac{(3.00\lambda)^2}{\lambda} - \frac{\lambda}{4} = 8.75\lambda = 8.75(900 \text{ nm}) = 7.88\times10^3 \text{ nm} = 7.88\,\mu\text{m}.$$

26. (a) We use Eq. 35-14 to find d:

$$d\sin\theta = m\lambda \quad \Rightarrow \quad d = (4)(450 \text{ nm})/\sin(90°) = 1800 \text{ nm}.$$

For the third-order spectrum, the wavelength that corresponds to $\theta = 90°$ is

$$\lambda = d\sin(90°)/3 = 600 \text{ nm}.$$

Any wavelength greater than this will not be seen. Thus, 600 nm < θ ≤ 700 nm are absent.

(b) The slit separation d needs to be decreased.

(c) In this case, the 400 nm wavelength in the $m = 4$ diffraction is to occur at 90°. Thus

$$d_{\text{new}}\sin\theta = m\lambda \quad \Rightarrow \quad d_{\text{new}} = (4)(400 \text{ nm})/\sin(90°) = 1600 \text{ nm}.$$

This represents a change of

$$|\Delta d| = d - d_{\text{new}} = 200 \text{ nm} = 0.20\,\mu\text{m}.$$

27. Consider the two waves, one from each slit, that produce the seventh bright fringe in the absence of the mica. They are in phase at the slits and travel different distances to the seventh bright fringe, where they have a phase difference of $2\pi m = 14\pi$. Now a piece of mica with thickness x is placed in front of one of the slits, and an additional phase difference between the waves develops. Specifically, their phases at the slits differ by

$$\frac{2\pi x}{\lambda_m} - \frac{2\pi x}{\lambda} = \frac{2\pi x}{\lambda}(n-1)$$

where λ_m is the wavelength in the mica and n is the index of refraction of the mica. The relationship $\lambda_m = \lambda/n$ is used to substitute for λ_m. Since the waves are now in phase at the screen,

$$\frac{2\pi x}{\lambda}(n-1) = 14\pi$$

or

$$x = \frac{7\lambda}{n-1} = \frac{7(550\times10^{-9}\,\text{m})}{1.58-1} = 6.64\times10^{-6}\,\text{m}.$$

28. The problem asks for "the greatest value of x… exactly out of phase," which is to be interpreted as the value of x where the curve shown in the figure passes through a phase value of π radians. This happens as some point P on the x axis, which is, of course, a distance x from the top source and (using Pythagoras' theorem) a distance $\sqrt{d^2 + x^2}$ from the bottom source. The difference (in normal length units) is therefore $\sqrt{d^2 + x^2} - x$, or (expressed in radians) is

$$\frac{2\pi}{\lambda}(\sqrt{d^2 + x^2} - x) .$$

We note (looking at the leftmost point in the graph) that at $x = 0$, this latter quantity equals 6π, which means $d = 3\lambda$. Using this value for d, we now must solve the condition

$$\frac{2\pi}{\lambda}\left(\sqrt{d^2 + x^2} - x\right) = \pi .$$

Straightforward algebra then leads to $x = (35/4)\lambda$, and using $\lambda = 400$ nm we find $x = 3500$ nm, or 3.5 μm.

29. **THINK** The intensity is proportional to the square of the resultant field amplitude.

EXPRESS Let the electric field components of the two waves be written as

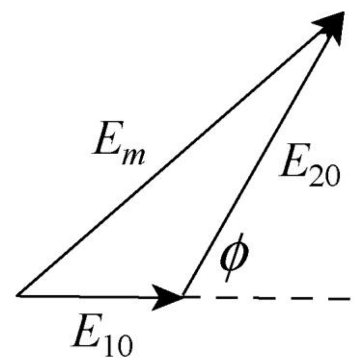
$$E_1 = E_{10}\sin\omega t$$
$$E_2 = E_{20}\sin(\omega t + \phi),$$

where $E_{10} = 1.00$, $E_{20} = 2.00$, and $\phi = 60°$. The resultant field is $E = E_1 + E_2$. We use phasor diagram to calculate the amplitude of E.

ANALYZE The phasor diagram is shown next.

The resultant amplitude E_m is given by the trigonometric law of cosines:

$$E_m^2 = E_{10}^2 + E_{20}^2 - 2E_{10}E_{20}\cos(180° - \phi) .$$



Thus,

$$E_m = \sqrt{(1.00)^2 + (2.00)^2 - 2(1.00)(2.00)\cos 120°} = 2.65 .$$

LEARN Summing over the horizontal components of the two fields gives

$$\sum E_h = E_{10}\cos 0 + E_{20}\cos 60° = 1.00 + (2.00)\cos 60° = 2.00$$

Similarly, the sum over the vertical components is

$$\sum E_v = E_{10}\sin 0 + E_{20}\sin 60° = 1.00\sin 0° + (2.00)\sin 60° = 1.732 .$$

The resultant amplitude is

$$E_m = \sqrt{(2.00)^2 + (1.732)^2} = 2.65 ,$$

which agrees with what we found above. The phase angle relative to the phasor representing E_1 is

$$\beta = \tan^{-1}\left(\frac{1.732}{2.00}\right) = 40.9°$$

Thus, the resultant field can be written as $E = (2.65)\sin(\omega t + 40.9°)$.

30. In adding these with the phasor method (as opposed to, say, trig identities), we may set $t = 0$ and add them as vectors:

$$y_h = 10\cos 0° + 8.0\cos 30° = 16.9$$
$$y_v = 10\sin 0° + 8.0\sin 30° = 4.0$$

so that

$$y_R = \sqrt{y_h^2 + y_v^2} = 17.4$$
$$\beta = \tan^{-1}\left(\frac{y_v}{y_h}\right) = 13.3° .$$

Thus,

$$y = y_1 + y_2 = y_R\sin(\omega t + \beta) = 17.4\sin(\omega t + 13.3°) .$$

Quoting the answer to two significant figures, we have $y \approx 17\sin\left(\omega t + 13°\right)$.

31. In adding these with the phasor method (as opposed to, say, trig identities), we may set t = 0 and add them as vectors:

$$y_h = 10\cos 0° + 15\cos 30° + 5.0\cos(-45°) = 26.5$$
$$y_v = 10\sin 0° + 15\sin 30° + 5.0\sin(-45°) = 4.0$$

so that

$$y_R = \sqrt{y_h^2 + y_v^2} = 26.8 \approx 27$$
$$\beta = \tan^{-1}\left(\frac{y_v}{y_h}\right) = 8.5°.$$

Thus, $y = y_1 + y_2 + y_3 = y_R\sin\left(\omega t + \beta\right) = 27\sin\left(\omega t + 8.5°\right)$.

32. (a) We can use phasor techniques or use trig identities. Here we show the latter approach. Since

$$\sin a + \sin(a + b) = 2\cos(b/2)\sin(a + b/2),$$

we find

$$E_1 + E_2 = 2E_0\cos(\phi/2)\sin(\omega t + \phi/2)$$

where E_0 = 2.00 μV/m, ω = 1.26 × 10^{15} rad/s, and ϕ = 39.6 rad. This shows that the electric field amplitude of the resultant wave is

$$E = 2E_0\cos(\phi/2) = 2(2.00\ \mu\text{V/m})\cos(19.2\text{ rad}) = 2.33\ \mu\text{V/m}.$$

(b) Equation 35-22 leads to

$$I = 4I_0\cos^2(\phi/2) = 1.35\, I_0$$

at point P, and

$$I_{\text{center}} = 4I_0\cos^2(0) = 4\, I_0$$

at the center. Thus, $I / I_{\text{center}} = 1.35/4 = 0.338$.

(c) The phase difference ϕ (in wavelengths) is gotten from ϕ in radians by dividing by 2π. Thus, $\phi = 39.6/2\pi = 6.3$ wavelengths. Thus, point P is between the sixth side maximum (at which ϕ = 6 wavelengths) and the seventh minimum (at which $\phi = 6\frac{1}{2}$ wavelengths).

(d) The rate is given by ω = 1.26 × 10^{15} rad/s.

(e) The angle between the phasors is $\phi = 39.6$ rad = 2270° (which would look like about 110° when drawn in the usual way).

33. With phasor techniques, this amounts to a vector addition problem $\vec{R} = \vec{A} + \vec{B} + \vec{C}$ where (in magnitude-angle notation) $\vec{A} = (10\angle 0°), \vec{B} = (5\angle 45°)$, and $\vec{C} = (5\angle -45°)$, where the magnitudes are understood to be in μV/m. We obtain the resultant (especially efficient on a vector-capable calculator in polar mode):

$$\vec{R} = (10\angle 0°) + (5\angle 45°) + (5\angle -45°) = (17.1\angle 0°)$$

which leads to

$$E_R = (17.1\,\mu\text{V/m})\sin(\omega t)$$

where $\omega = 2.0 \times 10^{14}$ rad/s.

34. (a) Referring to Figure 35-10(a) makes clear that

$$\theta = \tan^{-1}(y/D) = \tan^{-1}(0.205/4) = 2.93°.$$

Thus, the phase difference at point P is $\phi = d\sin\theta/\lambda = 0.397$ wavelengths, which means it is between the central maximum (zero wavelength difference) and the first minimum ($\frac{1}{2}$ wavelength difference). Note that the above computation could have been simplified somewhat by avoiding the explicit use of the tangent and sine functions and making use of the small-angle approximation ($\tan\theta \approx \sin\theta$).

(b) From Eq. 35-22, we get (with $\phi = (0.397)(2\pi) = 2.495$ rad)

$$I = 4I_0\cos^2(\phi/2) = 0.404\, I_0$$

at point P and

$$I_{\text{center}} = 4I_0\cos^2(0) = 4\, I_0$$

at the center. Thus, $I/I_{\text{center}} = 0.404/4 = 0.101$.

35. **THINK** For complete destructive interference, we want the waves reflected from the front and back of the coating to differ in phase by an odd multiple of π rad.

EXPRESS Each wave is incident on a medium of higher index of refraction from a medium of lower index, so both suffer phase changes of π rad on reflection. If L is the thickness of the coating, the wave reflected from the back surface travels a distance $2L$ farther than the wave reflected from the front. The phase difference is $2L(2\pi/\lambda_c)$, where λ_c is the wavelength in the coating. If n is the index of refraction of the coating, $\lambda_c = \lambda/n$, where λ is the wavelength in vacuum, and the phase difference is $2nL(2\pi/\lambda)$. We solve

$$2nL\left(\frac{2\pi}{\lambda}\right)=(2m+1)\pi$$

for L. Here m is an integer. The result is $L=\dfrac{(2m+1)\lambda}{4n}$.

ANALYZE To find the least thickness for which destructive interference occurs, we take $m = 0$. Then,

$$L=\frac{\lambda}{4n}=\frac{600\times10^{-9}\,\text{m}}{4(1.25)}=1.20\times10^{-7}\,\text{m}.$$

LEARN A light ray reflected by a material changes phase by π rad (or 180°) if the refractive index of the material is greater than that of the medium in which the light is traveling.

36. (a) On both sides of the soap is a medium with lower index (air) and we are examining the reflected light, so the condition for strong reflection is Eq. 35-36. With lengths in nm,

$$\lambda=\frac{2n_2L}{m+\frac{1}{2}}=\begin{cases}3360 & \text{for } m=0\\ 1120 & \text{for } m=1\\ 672 & \text{for } m=2\\ 480 & \text{for } m=3\\ 373 & \text{for } m=4\\ 305 & \text{for } m=5\end{cases}$$

from which we see the latter *four* values are in the given range.

(b) We now turn to Eq. 35-37 and obtain

$$\lambda=\frac{2n_2L}{m}=\begin{cases}1680 & \text{for } m=1\\ 840 & \text{for } m=2\\ 560 & \text{for } m=3\\ 420 & \text{for } m=4\\ 336 & \text{for } m=5\end{cases}$$

from which we see the latter *three* values are in the given range.

37. Light reflected from the front surface of the coating suffers a phase change of π rad while light reflected from the back surface does not change phase. If L is the thickness of the coating, light reflected from the back surface travels a distance $2L$ farther than light reflected from the front surface. The difference in phase of the two waves is $2L(2\pi/\lambda_c)-\pi$, where λ_c is the wavelength in the coating. If λ is the wavelength in vacuum, then $\lambda_c = \lambda/n$, where n is the index of refraction of the coating. Thus, the phase difference is

$2nL(2\pi/\lambda) - \pi$. For fully constructive interference, this should be a multiple of 2π. We solve

$$2nL\left(\frac{2\pi}{\lambda}\right) - \pi = 2m\pi$$

for L. Here m is an integer. The solution is

$$L = \frac{(2m+1)\lambda}{4n} .$$

To find the smallest coating thickness, we take $m = 0$. Then,

$$L = \frac{\lambda}{4n} = \frac{560\times10^{-9}\text{ m}}{4(2.00)} = 7.00\times10^{-8}\text{ m} .$$

38. (a) We are dealing with a thin film (material 2) in a situation where $n_1 > n_2 > n_3$, looking for strong *reflections*; the appropriate condition is the one expressed by Eq. 35-37. Therefore, with lengths in nm and $L = 500$ and $n_2 = 1.7$, we have

$$\lambda = \frac{2n_2L}{m} = \begin{cases} 1700 & \text{for } m = 1 \\ 850 & \text{for } m = 2 \\ 567 & \text{for } m = 3 \\ 425 & \text{for } m = 4 \end{cases}$$

from which we see the latter *two* values are in the given range. The longer wavelength (m=3) is $\lambda = 567$ nm.

(b) The shorter wavelength ($m = 4$) is $\lambda = 425$ nm.

(c) We assume the temperature dependence of the refraction index is negligible. From the proportionality evident in the part (a) equation, longer L means longer λ.

39. For constructive interference, we use Eq. 35-36:

$$2n_2L = (m + 1/2)\lambda .$$

For the smallest value of L, let $m = 0$:

$$L_0 = \frac{\lambda/2}{2n_2} = \frac{624\text{nm}}{4(1.33)} = 117\text{nm} = 0.117\,\mu\text{m}.$$

(b) For the second smallest value, we set $m = 1$ and obtain

$$L_1 = \frac{(1+1/2)\lambda}{2n_2} = \frac{3\lambda}{2n_2} = 3L_0 = 3(0.1173\,\mu\text{m}) = 0.352\,\mu\text{m}.$$

40. The incident light is in a low index medium, the thin film of acetone has somewhat higher $n = n_2$, and the last layer (the glass plate) has the highest refractive index. To see very little or no reflection, the condition

$$2L = \left(m + \tfrac{1}{2}\right)\frac{\lambda}{n_2} \quad \text{where} \quad m = 0, 1, 2, \ldots$$

must hold. This is the same as Eq. 35-36, which was developed for the opposite situation (constructive interference) regarding a thin film surrounded on both sides by air (a very different context from the one in this problem). By analogy, we expect Eq. 35-37 to apply in this problem to reflection *maxima*. Thus, using Eq. 35-37 with n_2 = 1.25 and λ = 700 nm yields

$$L = 0,\ 280\,\text{nm},\ 560\,\text{nm},\ 840\,\text{nm},\ 1120\,\text{nm}, \ldots$$

for the first several m values. And the equation shown above (equivalent to Eq. 35-36) gives, with λ = 600 nm,

$$L = 120\,\text{nm}, 360\,\text{nm}, 600\,\text{nm}, 840\,\text{nm}, 1080\,\text{nm}, \ldots$$

for the first several m values. The lowest number these lists have in common is $L = 840$ nm.

41. In this setup, we have $n_2 < n_1$ and $n_2 > n_3$, and the condition for destructive interference is

$$2L = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_2} \quad \Rightarrow L = \left(m + \frac{1}{2}\right)\frac{\lambda}{2n_2}\ , \quad m = 0, 1, 2, \ldots$$

The second least thickness is (m = 1)

$$L = \left(1 + \frac{1}{2}\right)\frac{342\text{ nm}}{2(1.59)} = 161\text{ nm}\ .$$

42. In this setup, we have $n_2 > n_1$ and $n_2 > n_3$, and the condition for constructive interference is

$$2L = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_2} \quad \Rightarrow \lambda = \frac{4Ln_2}{2m+1}, \quad m = 0, 1, 2, \ldots$$

Thus, we get

$$\lambda = \begin{cases} 4Ln_2 = 4(285\text{ nm})(1.60) = 1824\text{ nm} & (m=0) \\ 4Ln_2/3 = 4(285\text{ nm})(1.60)/3 = 608\text{ nm} & (m=1) \end{cases}.$$

For the wavelength to be in the visible range, we choose m = 1 with $\lambda = 608$ nm.

43. When a thin film of thickness L and index of refraction n_2 is placed between materials 1 and 3 such that $n_1 > n_2$ and $n_3 > n_2$ where n_1 and n_3 are the indexes of refraction of the materials, the general condition for destructive interference for a thin film is

$$2L = m\frac{\lambda}{n_2} \quad \Rightarrow \quad \lambda = \frac{2Ln_2}{m}, \quad m = 0,1,2,\ldots$$

where λ is the wavelength of light as measured in air. Thus, we have, for $m = 1$

$$\lambda = 2Ln_2 = 2(200\text{ nm})(1.40) = 560\text{ nm}.$$

44. In this setup, we have $n_2 < n_1$ and $n_2 < n_3$, and the condition for constructive interference is

$$2L = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_2} \quad \Rightarrow L = \left(m + \frac{1}{2}\right)\frac{\lambda}{2n_2}, \quad m = 0, 1, 2, \ldots$$

The second least thickness is (m = 1)

$$L = \left(1 + \frac{1}{2}\right)\frac{587\text{ nm}}{2(1.34)} = 329\text{ nm}.$$

45. In this setup, we have $n_2 > n_1$ and $n_2 > n_3$, and the condition for constructive interference is

$$2L = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_2} \quad \Rightarrow L = \left(m + \frac{1}{2}\right)\frac{\lambda}{2n_2}, \quad m = 0,1,2,\ldots$$

The third least thickness is (m = 2)

$$L = \left(2 + \frac{1}{2}\right)\frac{612\text{ nm}}{2(1.60)} = 478\text{ nm}.$$

46. In this setup, we have $n_2 < n_1$ and $n_2 > n_3$, and the condition for destructive interference is

$$2L = \left(m+\frac{1}{2}\right)\frac{\lambda}{n_2} \quad \Rightarrow \lambda = \frac{4Ln_2}{2m+1}, \quad m = 0,1,2,\ldots$$

Therefore,

$$\lambda = \begin{cases} 4Ln_2 = 4(415 \text{ nm})(1.59) = 2639 \text{ nm} \ \ (m=0) \\ 4Ln_2/3 = 4(415 \text{ nm})(1.59)/3 = 880 \text{ nm} \ \ (m=1) \\ 4Ln_2/5 = 4(415 \text{ nm})(1.59)/5 = 528 \text{ nm} \ \ (m=2) \end{cases}.$$

For the wavelength to be in the visible range, we choose m = 3 with $\lambda = 528$ nm.

47. **THINK** For a complete destructive interference, we want the waves reflected from the front and back of material 2 of refractive index n_2 to differ in phase by an odd multiple of π rad.

EXPRESS In this setup, we have $n_2 < n_1$, so there is no phase change from the first surface. On the other hand $n_2 < n_3$, so there is a phase change of π rad from the second surface. Since the second wave travels an extra distance of 2L, the phase difference is

$$\phi = \frac{2\pi}{\lambda_2}(2L) + \pi$$

where $\lambda_2 = \lambda / n_2$ is the wavelength in medium 2. The condition for destructive interference is

$$\frac{2\pi}{\lambda_2}(2L) + \pi = (2m+1)\pi,$$

or

$$2L = m\frac{\lambda}{n_2} \quad \Rightarrow \lambda = \frac{2Ln_2}{m}, \quad m = 0, 1, 2,\ldots$$

ANALYZE Thus, we have

$$\lambda = \begin{cases} 2Ln_2 = 2(380 \text{ nm})(1.34) = 1018 \text{ nm} \ \ (m=1) \\ Ln_2 = (380 \text{ nm})(1.34) = 509 \text{ nm} \ \ (m=2) \end{cases}$$

For the wavelength to be in the visible range, we choose m = 2 with $\lambda = 509$ nm.

LEARN In this setup, the condition for *constructive* interference is

$$\frac{2\pi}{\lambda_2}(2L) + \pi = 2m\pi,$$

or

$$2L = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_2}, \quad m = 0, 1, 2, \ldots$$

48. In this setup, we have $n_2 < n_1$ and $n_2 < n_3$, and the condition for constructive interference is

$$2L = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_2} \quad \Rightarrow L = \left(m + \frac{1}{2}\right)\frac{\lambda}{2n_2}, \quad m = 0, 1, 2, \ldots$$

The second least thickness is (m = 1)

$$L = \left(1 + \frac{1}{2}\right)\frac{632 \text{ nm}}{2(1.40)} = 339 \text{ nm}.$$

49. In this setup, we have $n_2 > n_1$ and $n_2 > n_3$, and the condition for constructive interference is

$$2L = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_2} \quad \Rightarrow L = \left(m + \frac{1}{2}\right)\frac{\lambda}{2n_2}, \quad m = 0, 1, 2, \ldots$$

The third least thickness is (m = 2)

$$L = \left(2 + \frac{1}{2}\right)\frac{382 \text{ nm}}{2(1.75)} = 273 \text{ nm}.$$

50. In this setup, we have $n_2 > n_1$ and $n_2 < n_3$, and the condition for destructive interference is

$$2L = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_2} \quad \Rightarrow L = \left(m + \frac{1}{2}\right)\frac{\lambda}{2n_2}, \quad m = 0, 1, 2, \ldots$$

The second least thickness is (m = 1)

$$L = \left(1 + \frac{1}{2}\right)\frac{482 \text{ nm}}{2(1.46)} = 248 \text{ nm}.$$

51. **THINK** For a complete destructive interference, we want the waves reflected from the front and back of material 2 of refractive index n_2 to differ in phase by an odd multiple of π rad.

EXPRESS In this setup, we have $n_1 < n_2$ and $n_2 < n_3$, which means that both waves are incident on a medium of higher refractive index from a medium of lower refractive index.

Thus, in both cases, there is a phase change of π rad from both surfaces. Since the second wave travels an additional distance of 2*L*, the phase difference is

$$\phi = \frac{2\pi}{\lambda_2}(2L)$$

where $\lambda_2 = \lambda / n_2$ is the wavelength in medium 2. The condition for destructive interference is

$$\frac{2\pi}{\lambda_2}(2L) = (2m+1)\pi,$$

or

$$2L = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_2} \quad \Rightarrow \lambda = \frac{4Ln_2}{2m+1}, \quad m = 0, 1, 2, \ldots$$

ANALYZE Thus,

$$\lambda = \begin{cases} 4Ln_2 = 4(210 \text{ nm})(1.46) = 1226 \text{ nm} \ \ (m=0) \\ 4Ln_2/3 = 4(210 \text{ nm})(1.46)/3 = 409 \text{ nm} \ \ (m=1) \end{cases}$$

For the wavelength to be in the visible range, we choose *m* = 1 with $\lambda = 409$ nm.

LEARN In this setup, the condition for *constructive* interference is

$$\frac{2\pi}{\lambda_2}(2L) = 2m\pi,$$

or

$$2L = m\frac{\lambda}{n_2}, \quad m = 0, 1, 2, \ldots$$

52. In this setup, we have $n_2 > n_1$ and $n_2 > n_3$, and the condition for constructive interference is

$$2L = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_2} \quad \Rightarrow \lambda = \frac{4Ln_2}{2m+1}, \quad m = 0, 1, 2, \ldots$$

Thus, we have

$$\lambda = \begin{cases} 4Ln_2 = 4(325 \text{ nm})(1.75) = 2275 \text{ nm} \ \ (m=0) \\ 4Ln_2/3 = 4(325 \text{ nm})(1.75)/3 = 758 \text{ nm} \ \ (m=1) \\ 4Ln_2/5 = 4(325 \text{ nm})(1.75)/5 = 455 \text{ nm} \ \ (m=2) \end{cases}.$$

For the wavelength to be in the visible range, we choose *m* = 2 with $\lambda = 455$ nm.

53. We solve Eq. 35-36 with n_2 = 1.33 and λ = 600 nm for *m* = 1, 2, 3,…:

$$L = 113\text{ nm}, 338\text{nm}, 564\text{nm}, 789\text{nm}, \ldots$$

And, we similarly solve Eq. 35-37 with the same n_2 and λ = 450 nm:

$$L = 0, 169\text{nm}, 338\text{nm}, 508\text{nm}, 677\text{nm}, \ldots$$

The lowest number these lists have in common is L = 338 nm.

54. The situation is analogous to that treated in Sample Problem — "Thin-film interference of a coating on a glass lens," in the sense that the incident light is in a low index medium, the thin film of oil has somewhat higher $n = n_2$, and the last layer (the glass plate) has the highest refractive index. To see very little or no reflection, according to the Sample Problem, the condition

$$2L = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_2} \quad \text{where} \quad m = 0, 1, 2, \ldots$$

must hold. With λ = 500 nm and n_2 = 1.30, the possible answers for L are

$$L = 96\text{nm}, 288\text{nm}, 481\text{nm}, 673\text{nm}, 865\text{nm}, \ldots$$

And, with λ = 700 nm and the same value of n_2, the possible answers for L are

$$L = 135\text{nm}, 404\text{nm}, 673\text{nm}, 942\text{nm}, \ldots$$

The lowest number these lists have in common is L = 673 nm.

55. **THINK** The index of refraction of oil is greater than that of the air, but smaller than that of the water.

EXPRESS Let the indices of refraction of the air, oil and water be n_1, n_2, and n_3, respectively. Since $n_1 < n_2$ and $n_2 < n_3$, there is a phase change of π rad from both surfaces. Since the second wave travels an additional distance of $2L$, the phase difference is

$$\phi = \frac{2\pi}{\lambda_2}(2L)$$

where $\lambda_2 = \lambda / n_2$ is the wavelength in the oil. The condition for constructive interference is

$$\frac{2\pi}{\lambda_2}(2L) = 2m\pi,$$

or

$$2L = m\frac{\lambda}{n_2}, \quad m = 0, 1, 2, \ldots$$

ANALYZE (a) For $m = 1, 2, \ldots,$ maximum reflection occurs for wavelengths

$$\lambda = \frac{2n_2L}{m} = \frac{2(1.20)(460\,\text{nm})}{m} = 1104\,\text{nm}\ ,\ 552\,\text{nm},\ 368\,\text{nm}\ldots$$

We note that only the 552 nm wavelength falls within the visible light range.

(b) Maximum transmission into the water occurs for wavelengths for which reflection is a minimum. The condition for such destructive interference is given by

$$2L = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_2} \Rightarrow \lambda = \frac{4n_2L}{2m+1}$$

which yields λ = 2208 nm, 736 nm, 442 nm … for the different values of m. We note that only the 442 nm wavelength (blue) is in the visible range, though we might expect some red contribution since the 736 nm is very close to the visible range.

LEARN A light ray reflected by a material changes phase by π rad (or 180°) if the refractive index of the material is greater than that of the medium in which the light is traveling. Otherwise, there is no phase change. Note that refraction at an interface does not cause a phase shift.

56. For constructive interference (which is obtained for λ = 600 nm) in this circumstance, we require

$$2L = \frac{k}{2}\lambda_n = \frac{k\lambda}{2n}$$

where k = some positive <u>odd</u> integer and n is the index of refraction of the thin film. Rearranging and plugging in L = 272.7 nm and the wavelength value, this gives

$$n = \frac{k\lambda}{4L} = \frac{k(600\text{ nm})}{4(272.7\text{ nm})} = \frac{k}{1.818} = 0.55k\,.$$

Since we expect $n > 1$, then $k = 1$ is ruled out. However, $k = 3$ seems reasonable, since it leads to $n = 1.65$, which is close to the "typical" values found in Table 34-1. Taking this to be the correct index of refraction for the thin film, we now consider the destructive interference part of the question. Now we have $2L$ = (integer)λ_{dest}/n. Thus,

$$\lambda_{\text{dest}} = (900\text{ nm})/(\text{integer}).$$

We note that setting the integer equal to 1 yields a λ_{dest} value outside the range of the visible spectrum. A similar remark holds for setting the integer equal to 3. Thus, we set it equal to 2 and obtain λ_{dest} = 450 nm.

57. In this setup, we have $n_2 > n_1$ and $n_2 > n_3$, and the condition for minimum transmission (maximum reflection) or destructive interference is

$$2L = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_2} \quad \Rightarrow \; \lambda = \frac{4Ln_2}{2m+1}\ , \quad m = 0, 1, 2, \ldots$$

Therefore,

$$\lambda = \begin{cases} 4Ln_2 = 4(285\ \text{nm})(1.60) = 1824\ \text{nm} \ \ (m=0) \\ 4Ln_2/3 = 4(415\ \text{nm})(1.59)/3 = 608\ \text{nm} \ \ (m=1) \end{cases}$$

For the wavelength to be in the visible range, we choose m = 1 with $\lambda = 608$ nm.

58. In this setup, we have $n_2 > n_1$ and $n_2 > n_3$, and the condition for minimum transmission (maximum reflection) or destructive interference is

$$2L = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_2} \quad \Rightarrow \; L = \left(m + \frac{1}{2}\right)\frac{\lambda}{2n_2}\ , \quad m = 0, 1, 2, \ldots$$

The third least thickness is (m = 2)

$$L = \left(2 + \frac{1}{2}\right)\frac{382\ \text{nm}}{2(1.75)} = 273\ \text{nm}\,.$$

59. **THINK** Maximum transmission means constructive interference.

EXPRESS As shown in Fig. 35-43, one wave travels a distance of 2L further than the other. This wave is reflected twice, once from the back surface (between materials 2 and 3), and once from the front surface (between materials 1 and 2). Since $n_2 > n_3$, there is no phase change at the back-surface reflection. On the other hand, since $n_2 < n_1$, there is a phase change of π rad due to the front-surface reflection. The phase difference of the two waves as they leave material 2 is

$$\phi = \frac{2\pi}{\lambda_2}(2L) + \pi$$

where $\lambda_2 = \lambda / n_2$ is the wavelength in material 2. The condition for constructive interference is

$$\frac{2\pi}{\lambda_2}(2L)+\pi=2m\pi,$$

or

$$2L=\left(m+\frac{1}{2}\right)\frac{\lambda}{n_2} \quad \Rightarrow \lambda=\frac{4Ln_2}{2m+1}, \quad m=0,1,2,\ldots$$

ANALYZE Thus, we have

$$\lambda=\begin{cases} 4Ln_2=4(415 \text{ nm})(1.59)=2639 \text{ nm} \ \ (m=0) \\ 4Ln_2/3=4(415 \text{ nm})(1.59)/3=880 \text{ nm} \ \ (m=1) \\ 4Ln_2/5=4(415 \text{ nm})(1.59)/5=528 \text{ nm} \ \ (m=2) \end{cases}.$$

For the wavelength to be in the visible range, we choose m = 2 with $\lambda=528$ nm.

LEARN similarly, the condition for destructive interference is

$$\frac{2\pi}{\lambda_2}(2L)+\pi=(2m+1)\pi,$$

or

$$2L=m\frac{\lambda}{n_2} \quad \Rightarrow \lambda=\frac{2Ln_2}{m}, \quad m=0,1,2,\ldots$$

60. In this setup, we have $n_2<n_1$ and $n_2<n_3$, and the condition for maximum transmission (minimum reflection) or constructive interference is

$$2L=m\frac{\lambda}{n_2} \quad \Rightarrow \lambda=\frac{2Ln_2}{m}, \quad m=0,1,2,\ldots$$

Thus, we obtain

$$\lambda=\begin{cases} 2Ln_2=2(380 \text{ nm})(1.34)=1018 \text{ nm} \ \ (m=1) \\ Ln_2=(380 \text{ nm})(1.34)=509 \text{ nm} \ \ (m=2) \end{cases}.$$

For the wavelength to be in the visible range, we choose m = 2 with $\lambda=509$ nm.

61. In this setup, we have $n_2>n_1$ and $n_2>n_3$, and the condition for minimum transmission (maximum reflection) or destructive interference is

$$2L=\left(m+\frac{1}{2}\right)\frac{\lambda}{n_2} \quad \Rightarrow \lambda=\frac{4Ln_2}{2m+1}, \quad m=0,1,2,\ldots$$

Therefore,

$$\lambda = \begin{cases} 4Ln_2 = 4(325\text{ nm})(1.75) = 2275\text{ nm} \quad (m=0) \\ 4Ln_2/3 = 4(415\text{ nm})(1.59)/3 = 758\text{ nm} \quad (m=1) \\ 4Ln_2/5 = 4(415\text{ nm})(1.59)/5 = 455\text{ nm} \quad (m=2) \end{cases}$$

For the wavelength to be in the visible range, we choose m = 2 with $\lambda = 455$ nm.

62. In this setup, we have $n_2 < n_1$ and $n_2 > n_3$, and the condition for maximum transmission (minimum reflection) or constructive interference is

$$2L = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_2} \quad \Rightarrow \; L = \left(m + \frac{1}{2}\right)\frac{\lambda}{2n_2}\;, \quad m = 0,1,2,\ldots$$

The second least thickness is (m = 1)

$$L = \left(1 + \frac{1}{2}\right)\frac{342\text{ nm}}{2(1.59)} = 161\text{ nm}\,.$$

63. In this setup, we have $n_2 > n_1$ and $n_2 < n_3$, and the condition for maximum transmission (minimum reflection) or constructive interference is

$$2L = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_2} \quad \Rightarrow \; L = \left(m + \frac{1}{2}\right)\frac{\lambda}{2n_2}\;, \quad m = 0,1,2,\ldots$$

The second least thickness is (m = 1)

$$L = \left(1 + \frac{1}{2}\right)\frac{482\text{ nm}}{2(1.46)} = 248\text{ nm}\,.$$

64. In this setup, we have $n_2 > n_1$ and $n_2 < n_3$, and the condition for maximum transmission (minimum reflection) or constructive interference is

$$2L = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_2} \quad \Rightarrow \; \lambda = \frac{4Ln_2}{2m+1}, \quad m = 0,1,2,\ldots$$

Thus, we have

$$\lambda = \begin{cases} 4Ln_2 = 4(210\text{ nm})(1.46) = 1226\text{ nm} \quad (m=0) \\ 4Ln_2/3 = 4(210\text{ nm})(1.46)/3 = 409\text{ nm} \quad (m=1) \end{cases}$$

For the wavelength to be in the visible range, we choose m = 1 with $\lambda = 409$ nm.

65. In this setup, we have $n_2 < n_1$ and $n_2 < n_3$, and the condition for minimum transmission (maximum reflection) or destructive interference is

$$2L = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_2} \quad \Rightarrow L = \left(m + \frac{1}{2}\right)\frac{\lambda}{2n_2} \;, \quad m = 0,1,2,\ldots$$

The second least thickness is (m = 1)

$$L = \left(1 + \frac{1}{2}\right)\frac{632\text{ nm}}{2(1.40)} = 339\text{ nm}.$$

66. In this setup, we have $n_2 < n_1$ and $n_2 < n_3$, and the condition for maximum transmission (minimum reflection) or constructive interference is

$$2L = m\frac{\lambda}{n_2} \quad \Rightarrow \lambda = \frac{2Ln_2}{m}, \quad m = 0,1,2,\ldots$$

Thus, we have (with m =1)

$$\lambda = 2Ln_2 = 2(200\text{ nm})(1.40) = 560\text{ nm}.$$

67. In this setup, we have $n_2 < n_1$ and $n_2 < n_3$, and the condition for minimum transmission (maximum reflection) or destructive interference is

$$2L = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_2} \quad \Rightarrow L = \left(m + \frac{1}{2}\right)\frac{\lambda}{2n_2} \;, \quad m = 0,1,2,\ldots$$

The second least thickness is (m = 1)

$$L = \left(1 + \frac{1}{2}\right)\frac{587\text{ nm}}{2(1.34)} = 329\text{ nm}.$$

68. In this setup, we have $n_2 > n_1$ and $n_2 > n_3$, and the condition for minimum transmission (maximum reflection) or destructive interference is

$$2L = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_2} \quad \Rightarrow L = \left(m + \frac{1}{2}\right)\frac{\lambda}{2n_2} \;, \quad m = 0,1,2,\ldots$$

The third least thickness is (m = 2)

$$L = \left(2 + \frac{1}{2}\right)\frac{612\text{ nm}}{2(1.60)} = 478\text{ nm}.$$

69. Assume the wedge-shaped film is in air, so the wave reflected from one surface undergoes a phase change of π rad while the wave reflected from the other surface does not. At a place where the film thickness is L, the condition for fully constructive interference is $2nL = \left(m + \frac{1}{2}\right)\lambda$, where n is the index of refraction of the film, λ is the wavelength in vacuum, and m is an integer. The ends of the film are bright. Suppose the end where the film is narrow has thickness L_1 and the bright fringe there corresponds to $m = m_1$. Suppose the end where the film is thick has thickness L_2 and the bright fringe there corresponds to $m = m_2$. Since there are ten bright fringes, $m_2 = m_1 + 9$. Subtract $2nL_1 = \left(m_1 + \frac{1}{2}\right)\lambda$ from $2nL_2 = \left(m_1 + 9 + \frac{1}{2}\right)\lambda$ to obtain $2n\,\Delta L = 9\lambda$, where $\Delta L = L_2 - L_1$ is the change in the film thickness over its length. Thus,

$$\Delta L = \frac{9\lambda}{2n} = \frac{9\left(630\times10^{-9}\,\text{m}\right)}{2(1.50)} = 1.89\times10^{-6}\,\text{m}.$$

70. (a) The third sentence of the problem implies $m_o = 9.5$ in $2\,d_o = m_o\lambda$ initially. Then, $\Delta t = 15$ s later, we have $m' = 9.0$ in $2d' = m'\lambda$. This means

$$|\Delta d| = d_o - d' = \tfrac{1}{2}(m_o\lambda - m'\lambda) = 155\text{ nm}.$$

Thus, $|\Delta d|$ divided by Δt gives 10.3 nm/s.

(b) In this case, $m_f = 6$ so that

$$d_o - d_f = \tfrac{1}{2}(m_o\lambda - m_f\lambda) = \tfrac{7}{4}\lambda = 1085\text{ nm} = 1.09\ \mu\text{m}.$$

71. The (vertical) change between the center of one dark band and the next is

$$\Delta y = \frac{\lambda}{2} = \frac{500\text{ nm}}{2} = 250\text{ nm} = 2.50\times10^{-4}\,\text{mm}.$$

Thus, with the (horizontal) separation of dark bands given by $\Delta x = 1.2$ mm, we have

$$\theta \approx \tan\theta = \frac{\Delta y}{\Delta x} = 2.08\times10^{-4}\,\text{rad}.$$

Converting this angle into degrees, we arrive at $\theta = 0.012°$.

72. We apply Eq. 35-27 to both scenarios: $m = 4001$ and $n_2 = n_{\text{air}}$, and $m = 4000$ and $n_2 = n_{\text{vacuum}} = 1.00000$:

$$2L = (4001)\frac{\lambda}{n_{\text{air}}} \quad\text{and}\quad 2L = (4000)\frac{\lambda}{1.00000}.$$

Since the $2L$ factor is the same in both cases, we set the right-hand sides of these expressions equal to each other and cancel the wavelength. Finally, we obtain

$$n_{\text{air}} = (1.00000)\frac{4001}{4000} = 1.00025.$$

We remark that this same result can be obtained starting with Eq. 35-43 (which is developed in the textbook for a somewhat different situation) and using Eq. 35-42 to eliminate the $2L/\lambda$ term.

73. **THINK** A light ray reflected by a material changes phase by π rad (or 180°) if the refractive index of the material is greater than that of the medium in which the light is traveling.

EXPRESS Consider the interference of waves reflected from the top and bottom surfaces of the air film. The wave reflected from the upper surface does not change phase on reflection but the wave reflected from the bottom surface changes phase by π rad. At a place where the thickness of the air film is L, the condition for fully constructive interference is $2L = \left(m + \frac{1}{2}\right)\lambda$ where λ (= 683 nm) is the wavelength and m is an integer.

ANALYZE For L = 48 μm, we find the value of m to be

$$m = \frac{2L}{\lambda} - \frac{1}{2} = \frac{2(4.80\times10^{-5}\,\text{m})}{683\times10^{-9}\,\text{m}} - \frac{1}{2} = 140.$$

At the thin end of the air film, there is a bright fringe. It is associated with m = 0. There are, therefore, 140 bright fringes in all.

LEARN The number of bright fringes increases with L, but decreases with λ.

74. By the condition $m\lambda = 2y$ where y is the thickness of the air film between the plates directly underneath the middle of a dark band), the edges of the plates (the edges where they are not touching) are $y = 8\lambda/2$ = 2400 nm apart (where we have assumed that the *middle* of the ninth dark band is at the edge). Increasing that to y' = 3000 nm would correspond to $m' = 2y'/\lambda$ = 10 (counted as the eleventh dark band, since the first one corresponds to m = 0). There are thus 11 dark fringes along the top plate.

75. **THINK** The formation of Newton's rings is due to the interference between the rays reflected from the flat glass plate and the curved lens surface.

EXPRESS Consider the interference pattern formed by waves reflected from the upper and lower surfaces of the air wedge. The wave reflected from the lower surface undergoes a π rad phase change while the wave reflected from the upper surface does not.

At a place where the thickness of the wedge is d, the condition for a maximum in intensity is $2d = \left(m + \tfrac{1}{2}\right)\lambda,$ where λ is the wavelength in air and m is an integer. Therefore,

$$d = (2m + 1)\lambda/4.$$

ANALYZE As the geometry of Fig. 35-46 shows, $d = R - \sqrt{R^2 - r^2}$, where R is the radius of curvature of the lens and r is the radius of a Newton's ring. Thus, $(2m+1)\lambda/4 = R - \sqrt{R^2 - r^2}$. First, we rearrange the terms so the equation becomes

$$\sqrt{R^2 - r^2} = R - \frac{(2m+1)\lambda}{4}.$$

Next, we square both sides, rearrange to solve for r^2, then take the square root. We get

$$r = \sqrt{\frac{(2m+1)R\lambda}{2} - \frac{(2m+1)^2\lambda^2}{16}}.$$

If R is much larger than a wavelength, the first term dominates the second and

$$r = \sqrt{\frac{(2m+1)R\lambda}{2}}.$$

LEARN Similarly, the radii of the dark fringes are given by

$$r = \sqrt{mR\lambda}.$$

76. (a) We find m from the last formula obtained in Problem 35-75:

$$m = \frac{r^2}{R\lambda} - \frac{1}{2} = \frac{\left(10\times10^{-3}\,\text{m}\right)^2}{(5.0\,\text{m})\left(589\times10^{-9}\,\text{m}\right)} - \frac{1}{2}$$

which (rounding down) yields $m = 33$. Since the first bright fringe corresponds to $m = 0$, $m = 33$ corresponds to the thirty-fourth bright fringe.

(b) We now replace λ by $\lambda_n = \lambda/n_w$. Thus,

$$m_n = \frac{r^2}{R\lambda_n} - \frac{1}{2} = \frac{n_w r^2}{R\lambda} - \frac{1}{2} = \frac{(1.33)\left(10\times10^{-3}\,\text{m}\right)^2}{(5.0\,\text{m})\left(589\times10^{-9}\,\text{m}\right)} - \frac{1}{2} = 45.$$

This corresponds to the forty-sixth bright fringe (see the remark at the end of our solution in part (a)).

77. We solve for m using the formula $r=\sqrt{(2m+1)R\lambda/2}$ obtained in Problem 35-75 and find $m = r^2/R\lambda - 1/2$. Now, when m is changed to $m + 20$, r becomes r', so

$$m + 20 = r'^2/R\lambda - 1/2.$$

Taking the difference between the two equations above, we eliminate m and find

$$R=\frac{r'^2-r^2}{20\lambda}=\frac{(0.368\,\text{cm})^2-(0.162\,\text{cm})^2}{20(546\times10^{-7}\,\text{cm})}=100\,\text{cm}.$$

78. The time to change from one minimum to the next is Δt = 12 s. This involves a change in thickness $\Delta L = \lambda/2n_2$ (see Eq. 35-37), and thus a change of volume

$$\Delta V=\pi r^2\Delta L=\frac{\pi r^2\lambda}{2n_2}\quad\Rightarrow\quad\frac{dV}{dt}=\frac{\pi r^2\lambda}{2n_2\Delta t}=\frac{\pi(0.0180)^2\,(550\times10^{-9})}{2(1.40)\,(12)}$$

using SI units. Thus, the rate of change of volume is $1.67\times10^{-11}\ \text{m}^3/\text{s}$.

79. A shift of one fringe corresponds to a change in the optical path length of one wavelength. When the mirror moves a distance d, the path length changes by $2d$ since the light traverses the mirror arm twice. Let N be the number of fringes shifted. Then, $2d = N\lambda$ and

$$\lambda=\frac{2d}{N}=\frac{2(0.233\times10^{-3}\,\text{m})}{792}=5.88\times10^{-7}\ \text{m}=588\,\text{nm}\ .$$

80. According to Eq. 35-43, the number of fringes shifted (ΔN) due to the insertion of the film of thickness L is $\Delta N = (2L / \lambda)(n - 1)$. Therefore,

$$L=\frac{\lambda\Delta N}{2(n-1)}=\frac{(589\,\text{nm})(7.0)}{2(1.40-1)}=5.2\,\mu\text{m}\ .$$

81. **THINK** The wavelength in air is different from the wavelength in vacuum.

EXPRESS Let ϕ_1 be the phase difference of the waves in the two arms when the tube has air in it, and let ϕ_2 be the phase difference when the tube is evacuated. If λ is the wavelength in vacuum, then the wavelength in air is λ/n, where n is the index of refraction of air. This means

$$\phi_1-\phi_2=2L\left[\frac{2\pi n}{\lambda}-\frac{2\pi}{\lambda}\right]=\frac{4\pi(n-1)L}{\lambda}$$

where L is the length of the tube. The factor 2 arises because the light traverses the tube twice, once on the way to a mirror and once after reflection from the mirror. Each shift by one fringe corresponds to a change in phase of 2π rad, so if the interference pattern shifts by N fringes as the tube is evacuated, then

$$\frac{4\pi(n-1)L}{\lambda} = 2N\pi\,.$$

ANALYZE Solving for n, we obtain

$$n = 1 + \frac{N\lambda}{2L} = 1 + \frac{60(500\times10^{-9}\text{ m})}{2(5.0\times10^{-2}\text{ m})} = 1.00030\,.$$

LEARN The interferometer provides an accurate way to measure the refractive index of the air (and other gases as well).

82. We apply Eq. 35-42 to both wavelengths and take the difference:

$$N_1 - N_2 = \frac{2L}{\lambda_1} - \frac{2L}{\lambda_2} = 2L\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right).$$

We now require $N_1 - N_2 = 1$ and solve for L:

$$L = \frac{1}{2}\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right)^{-1} = \frac{1}{2}\left(\frac{1}{588.9950\text{ nm}} - \frac{1}{589.5924\text{ nm}}\right)^{-1} = 2.91\times10^{5}\text{ nm} = 291\ \mu\text{m}.$$

83. (a) The path length difference between rays 1 and 2 is $7d - 2d = 5d$. For this to correspond to a half-wavelength requires $5d = \lambda/2$, so that $d = 50.0$ nm.

(b) The above requirement becomes $5d = \lambda/2n$ in the presence of the solution, with $n = 1.38$. Therefore, $d = 36.2$ nm.

84. (a) The minimum path length difference occurs when both rays are nearly vertical. This would correspond to a point as far up in the picture as possible. Treating the screen as if it extended forever, then the point is at $y = \infty$.

(b) When both rays are nearly vertical, there is no path length difference between them. Thus at $y = \infty$, the phase difference is $\phi = 0$.

(c) At $y = 0$ (where the screen crosses the x axis) both rays are horizontal, with the ray from S_1 being longer than the one from S_2 by distance d.

(d) Since the problem specifies d = 6.00λ, then the phase difference here is ϕ = 6.00 wavelengths and is at its maximum value.

(e) With $D = 20\lambda$, use of the Pythagorean theorem leads to

$$\phi = \frac{L_1 - L_2}{\lambda} = \frac{\sqrt{d^2 + (d + D)^2} - \sqrt{d^2 + D^2}}{\lambda} = 5.80$$

which means the rays reaching the point $y = d$ have a phase difference of roughly 5.8 wavelengths.

(f) The result of the previous part is "intermediate" — closer to 6 (constructive interference) than to $5\frac{1}{2}$ (destructive interference).

85. **THINK** The angle between adjacent fringes depends the wavelength of the light and the distance between the slits.

EXPRESS The angular positions of the maxima of a two-slit interference pattern are given by $\Delta L = d\sin\theta = m\lambda$, where ΔL is the path-length difference, d is the slit separation, λ is the wavelength, and m is an integer. If θ is small, sin θ may be approximated by θ in radians. Then, $\theta = m\lambda/d$ to good approximation. The angular separation of two adjacent maxima is $\Delta\theta = \lambda/d$. When the arrangement is immersed in water, the wavelength changes to $\lambda' = \lambda/n$, and the equation above becomes

$$\Delta\theta' = \frac{\lambda'}{d}.$$

ANALYZE Dividing the equation by $\Delta\theta = \lambda/d$, we obtain

$$\frac{\Delta\theta'}{\Delta\theta} = \frac{\lambda'}{\lambda} = \frac{1}{n}.$$

Therefore, with n = 1.33 and $\Delta\theta$ = 0.30°, we find $\Delta\theta'$ = 0.23°.

LEARN The angular separation decreases with increasing index of refraction; the greater the value of n, the smaller the value of $\Delta\theta$.

86. (a) The graph shows part of a periodic pattern of half-cycle "length" Δn = 0.4. Thus if we set $n = 1.0 + 2\Delta n = 1.8$ then the maximum at n = 1.0 should repeat itself there.

(b) Continuing the reasoning of part (a), adding another half-cycle "length" we get $1.8 + \Delta n = 2.2$ for the answer.

(c) Since Δn = 0.4 represents a half-cycle, then $\Delta n/2$ represents a quarter-cycle. To accumulate a total change of 2.0 – 1.0 = 1.0 (see problem statement), then we need $2\Delta n + \Delta n/2 = 5/4^{\text{th}}$ of a cycle, which corresponds to 1.25 wavelengths.

87. **THINK** For a completely destructive interference, the intensity produced by the two waves is zero.

EXPRESS When the interference between two waves is completely destructive, their phase difference is given by

$$\phi = (2m+1)\pi, \;\; m = 0, 1, 2, ...$$

The equivalent condition is that their path-length difference is an odd multiple of $\lambda/2$, where λ is the wavelength of the light.

ANALYZE (a) Looking at Fig. 35-52, we see that half of the periodic pattern is of length ΔL = 750 nm (judging from the maximum at x = 0 to the minimum at x = 750 nm); this suggests that $\Delta L = \lambda/2$, and the wavelength (the full length of the periodic pattern) is $\lambda = 2\Delta L$ = 1500 nm. Thus, a maximum should be reached again at x = 1500 nm (and at x = 3000nm, x = 4500 nm, …).

(b) From our discussion in part (a), we expect a minimum to be reached at odd multiple of $\lambda/2$, or x = 750 nm + n(1500 nm), where n = 1, 2, 3 … . For instance, for n = 1 we would find the minimum at x = 2250 nm.

(c) With λ = 1500 nm (found in part (a)), we can express x = 1200 nm as x = 1200/1500 = 0.80 wavelength.

LEARN For a completely destructive interference, the phase difference between two light sources is an odd multiple of π; however, for a completely constructive interference, the phase difference is a multiple of 2π.

88. (a) The difference in wavelengths, with and without the n = 1.4 material, is found using Eq. 35-9:

$$\Delta N = (n-1)\frac{L}{\lambda} = 1.143 .$$

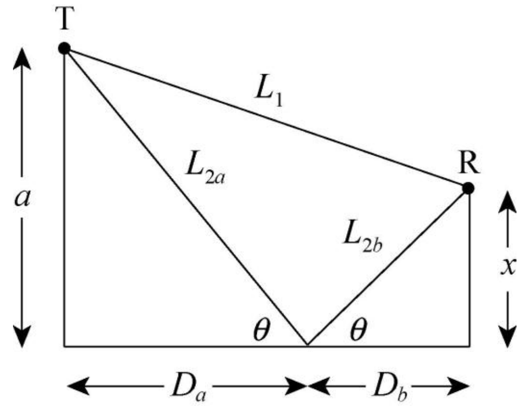
The result is equal to a phase shift of (1.143)(360°) = 411.4°, or

(b) more meaningfully, a shift of 411.4° – 360° = 51.4°.

89. **THINK** Since the index of refraction of water is greater than that of air, the wave that is reflected from the water surface suffers a phase change of π rad on reflection.

EXPRESS Suppose the wave that goes directly to the receiver travels a distance L_1 and the reflected wave travels a distance L_2. The last wave suffers a phase change on reflection of half a wavelength since water has higher refractive index than air. To obtain constructive interference at the receiver, the difference $L_2 - L_1$ must be an odd multiple of a half wavelength.

ANALYZE Consider the diagram below.



The right triangle on the left, formed by the vertical line from the water to the transmitter T, the ray incident on the water, and the water line, gives $D_a = a/\tan\theta$. The right triangle on the right, formed by the vertical line from the water to the receiver R, the reflected ray, and the water line leads to $D_b = x/\tan\theta$. Since $D_a + D_b = D$,

$$\tan\theta = \frac{a+x}{D}\ .$$

We use the identity $\sin^2\theta = \tan^2\theta/(1+\tan^2\theta)$ to show that

$$\sin\theta = (a+x)/\sqrt{D^2+(a+x)^2}\ .$$

This means

$$L_{2a} = \frac{a}{\sin\theta} = \frac{a\sqrt{D^2+(a+x)^2}}{a+x}$$

and

$$L_{2b} = \frac{x}{\sin\theta} = \frac{x\sqrt{D^2+(a+x)^2}}{a+x}.$$

Therefore,

$$L_2 = L_{2a} + L_{2b} = \frac{(a+x)\sqrt{D^2+(a+x)^2}}{a+x} = \sqrt{D^2+(a+x)^2}\ .$$

Using the binomial theorem, with D^2 large and $a^2 + x^2$ small, we approximate this expression: $L_2 \approx D + (a+x)^2/2D$. The distance traveled by the direct wave is

$L_1=\sqrt{D^2+(a-x)^2}$. Using the binomial theorem, we approximate this expression: $L_1 \approx D+(a-x)^2/2D.$ Thus,

$$L_2-L_1 \approx D+\frac{a^2+2ax+x^2}{2D}-D-\frac{a^2-2ax+x^2}{2D}=\frac{2ax}{D} .$$

Setting this equal to $(m+\frac{1}{2})\lambda$, where m is zero or a positive integer, we find $x=(m+\frac{1}{2})(\lambda D/2a)$.

LEARN Similarly, the condition for destructive interference is

$$L_2-L_1 \approx \frac{2ax}{D}=m\lambda,$$

or

$$x=m\frac{\lambda D}{2a}, \quad m=0,1,2,...$$

90. (a) Since P_1 is equidistant from S_1 and S_2 we conclude the sources are not in phase with each other. Their phase difference is $\Delta\phi_{\text{source}}$ = 0.60 π rad, which may be expressed in terms of "wavelengths" (thinking of the $\lambda \Leftrightarrow 2\pi$ correspondence in discussing a full cycle) as

$$\Delta\phi_{\text{source}} = (0.60\ \pi / 2\pi)\ \lambda = 0.3\ \lambda$$

(with S_2 "leading" as the problem states). Now S_1 is closer to P_2 than S_2 is. Source S_1 is 80 nm ($\Leftrightarrow$ 80/400 λ = 0.2 λ) from P_2 while source S_2 is 1360 nm ($\Leftrightarrow$ 1360/400 λ = 3.4 λ) from P_2. Here we find a difference of $\Delta\phi_{\text{path}}$ = 3.2 λ (with S_1 "leading" since it is closer). Thus, the net difference is

$$\Delta\phi_{\text{net}} = \Delta\phi_{\text{path}} - \Delta\phi_{\text{source}} = 2.90\ \lambda,$$

or 2.90 wavelengths.

(b) A whole number (like 3 wavelengths) would mean fully constructive, so our result is of the following nature: intermediate, but close to fully constructive.

91. (a) Applying the law of refraction, we obtain sin θ_2 / sin θ_1 = sin θ_2 / sin 30° = v_s/v_d. Consequently,

$$\theta_2=\sin^{-1}\left(\frac{v_s\sin 30°}{v_d}\right)=\sin^{-1}\left[\frac{(3.0\,\text{m/s})\sin 30°}{4.0\,\text{m/s}}\right]=22°.$$

(b) The angle of incidence is gradually reduced due to refraction, such as shown in the calculation above (from 30° to 22°). Eventually after several refractions, θ_2 will be virtually zero. This is why most waves come in normal to a shore.

92. When the depth of the liquid (L_{liq}) is zero, the phase difference ϕ is 60 wavelengths; this must equal the difference between the number of wavelengths in length L = 40 μm (since the liquid initially fills the hole) of the plastic (for ray r_1) and the number in that same length of the air (for ray r_2). That is,

$$\frac{Ln_{\text{plastic}}}{\lambda} - \frac{Ln_{\text{air}}}{\lambda} = 60 .$$

(a) Since $\lambda = 400 \times 10^{-9}$ m and $n_{\text{air}} = 1$ (to good approximation), we find $n_{\text{plastic}} = 1.6$.

(b) The slope of the graph can be used to determine n_{liq}, but we show an approach more closely based on the above equation:

$$\frac{Ln_{\text{plastic}}}{\lambda} - \frac{Ln_{\text{liq}}}{\lambda} = 20$$

which makes use of the leftmost point of the graph. This readily yields $n_{\text{liq}} = 1.4$.

93. **THINK** Knowing the slit separation and the distance between interference fringes allows us to calculate the wavelength of the light used.

EXPRESS The condition for a minimum in the two-slit interference pattern is $d \sin\theta = (m + ½)\lambda$, where d is the slit separation, λ is the wavelength, m is an integer, and θ is the angle made by the interfering rays with the forward direction. If θ is small, $\sin\theta$ may be approximated by θ in radians. Then, $\theta = (m + ½)\lambda/d$, and the distance from the minimum to the central fringe is

$$y = D\tan\theta \approx D\sin\theta \approx D\theta = \left(m + \frac{1}{2}\right)\frac{D\lambda}{d} ,$$

where D is the distance from the slits to the screen. For the first minimum $m = 0$ and for the tenth one, $m = 9$. The separation is

$$\Delta y = \left(9 + \frac{1}{2}\right)\frac{D\lambda}{d} - \frac{1}{2}\frac{D\lambda}{d} = \frac{9D\lambda}{d} .$$

ANALYZE We solve for the wavelength:

$$\lambda = \frac{d\Delta y}{9D} = \frac{(0.15\times10^{-3}\,\text{m})(18\times10^{-3}\,\text{m})}{9(50\times10^{-2}\,\text{m})} = 6.0\times10^{-7}\,\text{m} = 600\ \text{nm}.$$

LEARN The distance between two adjacent dark fringes, one associated with the integer *m* and the other associated with the integer *m* + 1, is

$$\Delta y = D\theta = D\lambda/d.$$

94. A light ray traveling directly along the central axis reaches the end in time

$$t_{\text{direct}} = \frac{L}{v_1} = \frac{n_1 L}{c}.$$

For the ray taking the critical zig-zag path, only its velocity component along the core axis direction contributes to reaching the other end of the fiber. That component is $v_1 \cos\theta'$, so the time of travel for this ray is

$$t_{\text{zig zag}} = \frac{L}{v_1\cos\theta'} = \frac{n_1 L}{c\sqrt{1-\left(\sin\theta/n_1\right)^2}}$$

using results from the previous solution. Plugging in $\sin\theta = \sqrt{n_1^2 - n_2^2}$ and simplifying, we obtain

$$t_{\text{zig zag}} = \frac{n_1 L}{c\left(n_2/n_1\right)} = \frac{n_1^2 L}{n_2 c}.$$

The difference is

$$\Delta t = t_{\text{zig zag}} - t_{\text{direct}} = \frac{n_1^2 L}{n_2 c} - \frac{n_1 L}{c} = \frac{n_1 L}{c}\left(\frac{n_1}{n_2} - 1\right).$$

With $n_1 = 1.58$, $n_2 = 1.53$, and $L = 300$ m, we obtain

$$\Delta t = \frac{n_1 L}{c}\left(\frac{n_1}{n_2} - 1\right) = \frac{(1.58)(300\text{ m})}{3.0\times10^8\text{ m/s}}\left(\frac{1.58}{1.53} - 1\right) = 5.16\times10^{-8}\text{ s} = 51.6\text{ ns}.$$

95. **THINK** The dark band corresponds to a completely destructive interference.

EXPRESS When the interference between two waves is completely destructive, their phase difference is given by

$$\phi = (2m+1)\pi,\ \ m = 0, 1, 2, \ldots$$

The equivalent condition is that their path-length difference is an odd multiple of $\lambda/2$, where λ is the wavelength of the light.

ANALYZE (a) A path length difference of $\lambda/2$ produces the first dark band, of $3\lambda/2$ produces the second dark band, and so on. Therefore, the fourth dark band corresponds to a path length difference of $7\lambda/2 = 1750$ nm = 1.75 μm.

(b) In the small angle approximation (which we assume holds here), the fringes are equally spaced, so that if Δy denotes the distance from one maximum to the next, then the distance from the middle of the pattern to the fourth dark band must be 16.8 mm = 3.5 Δy. Therefore, we obtain Δy = (16.8 mm)/3.5 = 4.8 mm.

LEARN The distance from the *m*th maximum to the central fringe is

$$y_{\text{bright}} = D\tan\theta \approx D\sin\theta \approx D\theta = m\frac{D\lambda}{d}.$$

Similarly, the distance from the *m*th minimum to the central fringe is

$$y_{\text{dark}} = \left(m+\frac{1}{2}\right)\frac{D\lambda}{d}.$$

96. We use the formula obtained in Sample Problem — "Thin-film interference of a coating on a glass lens:"

$$L_{\min} = \frac{\lambda}{4n_2} = \frac{\lambda}{4(1.25)} = 0.200\lambda \;\;\Rightarrow\;\; \frac{L_{\min}}{\lambda} = 0.200.$$

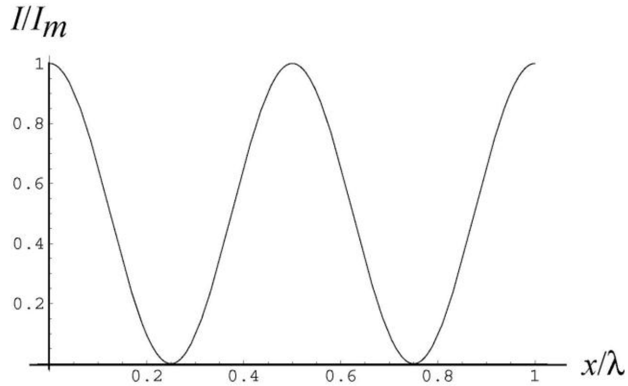
97. **THINK** The intensity of the light observed in the interferometer depends on the phase difference between the two waves.

EXPRESS Let the position of the mirror measured from the point at which $d_1 = d_2$ be x. We assume the beam-splitting mechanism is such that the two waves interfere constructively for $x = 0$ (with some beam-splitters, this would not be the case). We can adapt Eq. 35-23 to this situation by incorporating a factor of 2 (since the interferometer utilizes directly reflected light in contrast to the double-slit experiment) and eliminating the $\sin\theta$ factor. Thus, the path difference is $2x$, and the phase difference between the two light paths is $\Delta\phi = 2(2\pi x/\lambda) = 4\pi x/\lambda$.

ANALYZE From Eq. 35-22, we see that the intensity is proportional to $\cos^2(\Delta\phi/2)$. Thus, writing $4I_0$ as I_m, we find

$$I = I_m\cos^2\left(\frac{\Delta\phi}{2}\right) = I_m\cos^2\left(\frac{2\pi x}{\lambda}\right).$$

LEARN The intensity I/I_m as a function of x/λ is plotted below.



From the figure, we see that the intensity is at a maximum when

$$x = \frac{m}{2}\lambda, \quad m = 0, 1, 2, \ldots$$

Similarly, the condition for minima is

$$x = \frac{1}{4}(2m+1)\lambda, \quad m = 0, 1, 2, \ldots$$

98. We note that ray 1 travels an extra distance $4L$ more than ray 2. For constructive interference (which is obtained for λ = 620 nm) we require

$$4L = m\lambda \qquad \text{where } m = \text{some positive integer.}$$

For destructive interference (which is obtained for λ' = 4196 nm) we require

$$4L = \frac{k}{2}\lambda' \qquad \text{where } k = \text{some positive \underline{odd} integer.}$$

Equating these two equations (since their left-hand sides are equal) and rearranging, we obtain

$$k = 2\,m\,\frac{\lambda}{\lambda'} = 2\,m\,\frac{620}{496} = 2.5\,m\,.$$

We note that this condition is satisfied for k = 5 and m = 2. It is satisfied for some larger values, too, but recalling that we want the least possible value for L, we choose the solution set (k, m) = (5, 2). Plugging back into either of the equations above, we obtain the distance L:

$$4L = 2\lambda \quad \Rightarrow \quad L = \frac{\lambda}{2} = 310.0 \text{ nm}\,.$$

99. (a) Straightforward application of Eq. 35-3 $n = c/v$ and $v = \Delta x/\Delta t$ yields the result: pistol 1 with a time equal to $\Delta t = n\Delta x/c = 42.0 \times 10^{-12}$ s = 42.0 ps.

(b) For pistol 2, the travel time is equal to 42.3×10^{-12} s.

(c) For pistol 3, the travel time is equal to 43.2×10^{-12} s.

(d) For pistol 4, the travel time is equal to 41.8×10^{-12} s.

(e) We see that the blast from pistol 4 arrives first.

100. We use Eq. 35-36 for constructive interference: $2n_2L = (m + 1/2)\lambda$, or

$$\lambda = \frac{2n_2L}{m+1/2} = \frac{2(1.50)(410\,\text{nm})}{m+1/2} = \frac{1230\text{ nm}}{m+1/2},$$

where m = 0, 1, 2, …. The only value of m which, when substituted into the equation above, would yield a wavelength that falls within the visible light range is m = 1. Therefore,

$$\lambda = \frac{1230\,\text{nm}}{1+1/2} = 492\text{ nm}.$$

101. In the case of a distant screen the angle θ is close to zero so sin $\theta \approx \theta$. Thus from Eq. 35-14,

$$\Delta\theta \approx \Delta\sin\theta = \Delta\left(\frac{m\lambda}{d}\right) = \frac{\lambda}{d}\Delta m = \frac{\lambda}{d},$$

or $d \approx \lambda/\Delta\theta = 589 \times 10^{-9}$ m/0.018 rad = 3.3×10^{-5} m = 33 μm.

102. We note that $\Delta\phi = 60° = \frac{\pi}{3}$ rad. The phasors rotate with constant angular velocity

$$\omega = \frac{\Delta\phi}{\Delta t} = \frac{\pi/3\text{ rad}}{2.5\times10^{-16}\text{ s}} = 4.19\times10^{15}\text{ rad/s}.$$

Since we are working with light waves traveling in a medium (presumably air) where the wave speed is approximately c, then $kc = \omega$ (where $k = 2\pi/\lambda$), which leads to

$$\lambda = \frac{2\pi c}{\omega} = 450\text{ nm}.$$

103. (a) Each wave is incident on a medium of higher index of refraction from a medium of lower index (air to oil, and oil to water), so both suffer phase changes of π rad on reflection. If L is the thickness of the oil, the wave reflected from the back surface travels a distance $2L$ farther than the wave reflected from the front. The phase difference is $2L(2\pi/\lambda_o)$, where λ_o is the wavelength in oil. If n is the index of refraction of the oil, λ_o =

λ/n, where λ is the wavelength in vacuum, and the phase difference is $2nL(2\pi/\lambda)$. The conditions for constructive and destructive interferences are

$$\text{constructive}: 2nL\left(\frac{2\pi}{\lambda}\right) = 2m\pi \quad \Rightarrow \quad 2nL = m\lambda, \quad m = 0, 1, 2, \ldots$$

$$\text{destructive}: 2nL\left(\frac{2\pi}{\lambda}\right) = (2m+1)\pi \quad \Rightarrow 2nL = \left(m + \tfrac{1}{2}\right)\lambda, \quad m = 0, 1, 2, \ldots$$

Near the rim of the drop, $L < \lambda / 4n$, so only the condition for constructive interference with $m = 0$ can be met. So the outer (thinnest) region is bright.

(b) The third band from the rim corresponds to $2nL = 3\lambda / 2$. Thus, the film thickness there is

$$L = \frac{3\lambda}{2n} = \frac{3(475 \text{ nm})}{2(1.20)} = 594 \text{ nm}.$$

(c) The primary reason why colors gradually fade and then disappear as the oil thickness increases is because the colored bands begin to overlap too much to be distinguished. Also, the two reflecting surfaces would be too separated for the light reflecting from them to be coherent.

104. (a) The combination of the direct ray and the reflected ray from the mirror will produce an interference pattern on the screen, like the double-slit experiment. However, in this case, the reflected ray has a phase change of π, causing the locations of the dark and bright fringes to be interchanged. Thus, a zero path difference would correspond to a dark fringe.

(b) The condition for constructive interferences is

$$2h \sin\theta = (m + \tfrac{1}{2})\lambda, \quad m = 0, 1, 2, \ldots$$

(c) Similarly, the condition for destructive interference is

$$2h \sin\theta = m\lambda, \qquad m = 0, 1, 2, \ldots$$

105. The *Hint* essentially answers the question, but we put in some algebraic details and arrive at the familiar analytic-geometry expression for a hyperbola. The distance $d/2$ is denoted a and the constant value for the path length difference is denoted ϕ:

$$r_1 - r_2 = \phi$$

$$\sqrt{(a+x)^2 + y^2} - \sqrt{(a-x)^2 + y^2} = \phi$$

Rearranging and squaring, we have

$$\left(\sqrt{(a+x)^2 + y^2}\right)^2 = \left(\sqrt{(a-x)^2 + y^2} + \phi\right)^2$$

$$a^2 + 2ax + x^2 + y^2 = a^2 - 2ax + x^2 + y^2 + \phi^2 + 2\,\phi\sqrt{(a-x)^2 + y^2}$$

Many terms on both sides are identical and may be eliminated. This leaves us with

$$-2\,\phi\sqrt{(a-x)^2 + y^2} = \phi^2 - 4ax$$

at which point we square both sides again:

$$4\,\phi^2 a^2 - 8\,\phi^2 ax + 4\,\phi^2 x^2 + 4\,\phi^2 y^2 = \phi^4 - 8\,\phi^2 ax + 16a^2x^2$$

We eliminate the $-8\,\phi^2 ax$ term from both sides and plug in $a = 2d$ to get back to the original notation used in the problem statement:

$$\phi^2 d^2 + 4\,\phi^2 x^2 + 4\,\phi^2 y^2 = \phi^4 + 4\,d^2 x^2$$

Then a simple rearrangement puts it in the familiar analytic format for a hyperbola:

$$\phi^2 d^2 - \phi^4 = 4(d^2 - \phi^2)x^2 - 4\,\phi^2 y^2$$

which can be further simplified by dividing through by $\phi^2 d^2 - \phi^4$:

$$1 = \left(\frac{4}{\phi^2}\right)x^2 - \left(\frac{4}{d^2 - \phi^2}\right)y^2 .$$

Chapter 36

1. (a) We use Eq. 36-3 to calculate the separation between the first (m_1 = 1) and fifth $(m_2 = 5)$ minima:

$$\Delta y = D\Delta \sin\theta = D\Delta\left(\frac{m\lambda}{a}\right) = \frac{D\lambda}{a}\Delta m = \frac{D\lambda}{a}\left(m_2 - m_1\right) .$$

Solving for the slit width, we obtain

$$a = \frac{D\lambda(m_2 - m_1)}{\Delta y} = \frac{(400\,\text{mm})(550\times10^{-6}\,\text{mm})(5-1)}{0.35\,\text{mm}} = 2.5\,\text{mm} .$$

(b) For m = 1,

$$\sin\theta = \frac{m\lambda}{a} = \frac{(1)(550\times10^{-6}\,\text{mm})}{2.5\,\text{mm}} = 2.2\times10^{-4} .$$

The angle is $\theta = \sin^{-1}(2.2 \times 10^{-4}) = 2.2 \times 10^{-4}$ rad.

2. From Eq. 36-3,

$$\frac{a}{\lambda} = \frac{m}{\sin\theta} = \frac{1}{\sin 45.0^\circ} = 1.41.$$

3. (a) A plane wave is incident on the lens so it is brought to focus in the focal plane of the lens, a distance of 70 cm from the lens.

(b) Waves leaving the lens at an angle θ to the forward direction interfere to produce an intensity minimum if $a \sin \theta = m\lambda$, where a is the slit width, λ is the wavelength, and m is an integer. The distance on the screen from the center of the pattern to the minimum is given by $y = D \tan \theta$, where D is the distance from the lens to the screen. For the conditions of this problem,

$$\sin\theta = \frac{m\lambda}{a} = \frac{(1)(590\times10^{-9}\,\text{m})}{0.40\times10^{-3}\,\text{m}} = 1.475\times10^{-3} .$$

This means $\theta = 1.475 \times 10^{-3}$ rad and

$$y = (0.70\text{ m})\tan(1.475 \times 10^{-3}\text{ rad}) = 1.0 \times 10^{-3}\text{ m}.$$

4. (a) Equations 36-3 and 36-12 imply smaller angles for diffraction for smaller wavelengths. This suggests that diffraction effects in general would decrease.

(b) Using Eq. 36-3 with m = 1 and solving for 2θ (the angular width of the central diffraction maximum), we find

$$2\theta = 2\sin^{-1}\left(\frac{\lambda}{a}\right) = 2\sin^{-1}\left(\frac{0.50\,\text{m}}{5.0\,\text{m}}\right) = 11°.$$

(c) A similar calculation yields 0.23° for λ = 0.010 m.

5. (a) The condition for a minimum in a single-slit diffraction pattern is given by

$$a \sin\theta = m\lambda,$$

where a is the slit width, λ is the wavelength, and m is an integer. For $\lambda = \lambda_a$ and $m = 1$, the angle θ is the same as for $\lambda = \lambda_b$ and $m = 2$. Thus,

$$\lambda_a = 2\lambda_b = 2(350 \text{ nm}) = 700 \text{ nm}.$$

(b) Let m_a be the integer associated with a minimum in the pattern produced by light with wavelength λ_a, and let m_b be the integer associated with a minimum in the pattern produced by light with wavelength λ_b. A minimum in one pattern coincides with a minimum in the other if they occur at the same angle. This means $m_a\lambda_a = m_b\lambda_b$. Since $\lambda_a = 2\lambda_b$, the minima coincide if $2m_a = m_b$. Consequently, every other minimum of the λ_b pattern coincides with a minimum of the λ_a pattern. With m_a =2, we have m_b = 4.

(c) With m_a =3, we have m_b = 6.

6. (a) $\theta = \sin^{-1}$ (1.50 cm/2.00 m) = 0.430°.

(b) For the mth diffraction minimum, $a \sin\theta = m\lambda$. We solve for the slit width:

$$a = \frac{m\lambda}{\sin\theta} = \frac{2(441\,\text{nm})}{\sin 0.430°} = 0.118\,\text{mm}\ .$$

7. The condition for a minimum of a single-slit diffraction pattern is

$$a\sin\theta = m\lambda$$

where a is the slit width, λ is the wavelength, and m is an integer. The angle θ is measured from the forward direction, so for the situation described in the problem, it is 0.60° for m = 1. Thus,

$$a = \frac{m\lambda}{\sin\theta} = \frac{633\times10^{-9}\text{ m}}{\sin 0.60°} = 6.04\times10^{-5}\text{ m}\ .$$

8. Let the first minimum be a distance y from the central axis that is perpendicular to the speaker. Then

$$\sin\theta = y\big/\left(D^2+y^2\right)^{1/2} = m\lambda/a = \lambda/a \text{ (for } m = 1).$$

Therefore,

$$y=\frac{D}{\sqrt{(a/\lambda)^2-1}}=\frac{D}{\sqrt{(af/v_s)^2-1}}=\frac{100\,\text{m}}{\sqrt{\left[(0.300\,\text{m})(3000\,\text{Hz})/(343\,\text{m/s})\right]^2-1}}=41.2\,\text{m}\ .$$

9. **THINK** The condition for a minimum of intensity in a single-slit diffraction pattern is given by $a\sin\theta = m\lambda$, where a is the slit width, λ is the wavelength, and m is an integer.

EXPRESS To find the angular position of the first minimum to one side of the central maximum, we set $m = 1$:

$$\theta_1=\sin^{-1}\left(\frac{\lambda}{a}\right)=\sin^{-1}\left(\frac{589\times10^{-9}\text{ m}}{1.00\times10^{-3}\text{ m}}\right)=5.89\times10^{-4}\text{ rad}\ .$$

If D is the distance from the slit to the screen, the distance on the screen from the center of the pattern to the minimum is

$$y_1 = D\tan\theta_1 = (3.00\,\text{m})\tan\left(5.89\times10^{-4}\text{ rad}\right)=1.767\times10^{-3}\text{ m}\ .$$

To find the second minimum, we set $m = 2$:

$$\theta_2=\sin^{-1}\left(\frac{2\left(589\times10^{-9}\text{ m}\right)}{1.00\times10^{-3}\text{ m}}\right)=1.178\times10^{-3}\text{ rad}\ .$$

ANALYZE The distance from the center of the pattern to this second minimum is

$$y_2 = D\tan\theta_2 = (3.00\text{ m})\tan(1.178\times10^{-3}\text{ rad}) = 3.534\times10^{-3}\text{ m}.$$

The separation of the two minima is

$$\Delta y = y_2 - y_1 = 3.534\text{ mm} - 1.767\text{ mm} = 1.77\text{ mm}.$$

LEARN The angles θ_1 and θ_2 found above are quite small. In the small-angle approximation, $\sin\theta\approx\tan\theta\approx\theta$, and the separation between two adjacent diffraction minima can be approximated as

$$\Delta y = D(\tan\theta_{m+1}-\tan\theta_m)\approx D(\theta_{m+1}-\theta_m)=\frac{D\lambda}{a}\,.$$

10. From $y = m\lambda L/a$ we get

$$\Delta y = \Delta\left(\frac{m\lambda L}{a}\right) = \frac{\lambda L}{a}\Delta m = \frac{(632.8\,\text{nm})(2.60)}{1.37\,\text{mm}}[10-(-10)] = 24.0\,\text{mm}\ .$$

11. We note that 1 nm = 1 $\times 10^{-9}$ m = 1 $\times 10^{-6}$ mm. From Eq. 36-4,

$$\Delta\phi = \left(\frac{2\pi}{\lambda}\right)(\Delta x \sin\theta) = \left(\frac{2\pi}{589\times 10^{-6}\,\text{mm}}\right)\left(\frac{0.10\,\text{mm}}{2}\right)\sin 30° = 266.7\,\text{rad}\ .$$

This is equivalent to 266.7 rad – 84π = 2.8 rad = 160°.

12. (a) The slope of the plotted line is 12, and we see from Eq. 36-6 that this slope should correspond to

$$\frac{\pi a}{\lambda} = 12 \quad\Rightarrow\quad a = \frac{12\lambda}{\pi} = \frac{12(610\text{ nm})}{\pi} = 2330\text{ nm} \approx 2.33\,\mu\text{m}$$

(b) Consider Eq. 36-3 with "continuously variable" m (of course, m should be an integer for diffraction minima, but for the moment we will solve for it as if it could be any real number):

$$m_{\max} = \frac{a}{\lambda}(\sin\theta)_{\max} = \frac{a}{\lambda} = \frac{2330\text{ nm}}{610\text{ nm}} \approx 3.82$$

which suggests that, on each side of the central maximum (θ_{centr} = 0), there are three minima; considering both sides then implies there are six minima in the pattern.

(c) Setting m = 1 in Eq. 36-3 and solving for θ yields 15.2°.

(d) Setting m = 3 in Eq. 36-3 and solving for θ yields 51.8°.

13. (a) $\theta = \sin^{-1}$ (0.011 m/3.5 m) = 0.18°.

(b) We use Eq. 36-6:

$$\alpha = \left(\frac{\pi a}{\lambda}\right)\sin\theta = \frac{\pi(0.025\,\text{mm})\sin 0.18°}{538\times 10^{-6}\,\text{mm}} = 0.46\,\text{rad}\ .$$

(c) Making sure our calculator is in radian mode, Eq. 36-5 yields

$$\frac{I(\theta)}{I_m} = \left(\frac{\sin\alpha}{\alpha}\right)^2 = 0.93\ .$$

14. We will make use of arctangents and sines in our solution, even though they can be "shortcut" somewhat since the angles are small enough to justify the use of the small angle approximation.

(a) Given y/D = 15/300 (both expressed here in centimeters), then $\theta = \tan^{-1}(y/D) = 2.86°$. Use of Eq. 36-6 (with a = 6000 nm and λ = 500 nm) leads to

$$\alpha = \frac{\pi a \sin\theta}{\lambda} = \frac{\pi\left(6000\,\text{nm}\right)\sin 2.86°}{500\,\text{nm}} = 1.883\,\text{rad}.$$

Thus,

$$\frac{I_p}{I_m} = \left(\frac{\sin\alpha}{\alpha}\right)^2 = 0.256\ .$$

(b) Consider Eq. 36-3 with "continuously variable" m (of course, m should be an integer for diffraction minima, but for the moment we will solve for it as if it could be any real number):

$$m = \frac{a\sin\theta}{\lambda} = \frac{(6000\text{ nm})\sin 2.86°}{500\text{ nm}} \approx 0.60,$$

which suggests that the angle takes us to a point between the central maximum ($\theta_{\text{centr}} = 0$) and the first minimum (which corresponds to m = 1 in Eq. 36-3).

15. **THINK** The relative intensity in a single-slit diffraction depends on the ratio a/λ, where a is the slit width and λ is the wavelength.

EXPRESS The intensity for a single-slit diffraction pattern is given by

$$I = I_m \frac{\sin^2\alpha}{\alpha^2}$$

where I_m is the maximum intensity and $\alpha = (\pi a/\lambda)\sin\theta$. The angle θ is measured from the forward direction.

ANALYZE (a) We require $I = I_m/2$, so

$$\sin^2\alpha = \frac{1}{2}\alpha^2\ .$$

(b) We evaluate $\sin^2\alpha$ and $\alpha^2/2$ for α = 1.39 rad and compare the results. To be sure that 1.39 rad is closer to the correct value for α than any other value with three significant digits, we could also try 1.385 rad and 1.395 rad.

(c) Since $\alpha = (\pi a/\lambda)\sin\theta$,

$$\theta = \sin^{-1}\left(\frac{\alpha\lambda}{\pi a}\right).$$

Now $\alpha/\pi = 1.39/\pi = 0.442$, so

$$\theta = \sin^{-1}\left(\frac{0.442\lambda}{a}\right).$$

The angular separation of the two points of half intensity, one on either side of the center of the diffraction pattern, is

$$\Delta\theta = 2\theta = 2\sin^{-1}\left(\frac{0.442\lambda}{a}\right).$$

(d) For $a/\lambda = 1.0$,

$$\Delta\theta = 2\sin^{-1}(0.442/1.0) = 0.916\,\text{rad} = 52.5°.$$

(e) For $a/\lambda = 5.0$,

$$\Delta\theta = 2\sin^{-1}(0.442/5.0) = 0.177\,\text{rad} = 10.1°.$$

(f) For $a/\lambda = 10$,

$$\Delta\theta = 2\sin^{-1}(0.442/10) = 0.0884\,\text{rad} = 5.06°.$$

LEARN As shown in Fig. 36-8, the wider the slit is (relative to the wavelength), the narrower is the central diffraction maximum.

16. Consider Huygens' explanation of diffraction phenomena. When *A* is in place only the Huygens' wavelets that pass through the hole get to point *P*. Suppose they produce a resultant electric field E_A. When *B* is in place, the light that was blocked by *A* gets to *P* and the light that passed through the hole in *A* is blocked. Suppose the electric field at *P* is now $\vec{E}_B$. The sum $\vec{E}_A + \vec{E}_B$ is the resultant of all waves that get to *P* when neither *A* nor *B* are present. Since *P* is in the geometric shadow, this is zero. Thus $\vec{E}_A = -\vec{E}_B$, and since the intensity is proportional to the square of the electric field, the intensity at *P* is the same when *A* is present as when *B* is present.

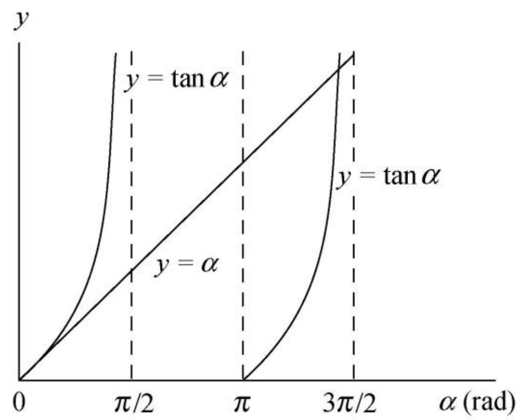
17. (a) The intensity for a single-slit diffraction pattern is given by

$$I = I_m \frac{\sin^2\alpha}{\alpha^2}$$

where α is described in the text (see Eq. 36-6). To locate the extrema, we set the derivative of *I* with respect to α equal to zero and solve for α. The derivative is

$$\frac{dI}{d\alpha} = 2I_m \frac{\sin\alpha}{\alpha^3}(\alpha\cos\alpha - \sin\alpha) .$$

The derivative vanishes if $\alpha \neq 0$ but sin α = 0. This yields $\alpha = m\pi$, where m is a nonzero integer. These are the intensity minima: I = 0 for $\alpha = m\pi$. The derivative also vanishes for α cos α – sin α = 0. This condition can be written tan α = α. These implicitly locate the maxima.



(b) The values of α that satisfy tan α = α can be found by trial and error on a pocket calculator or computer. Each of them is slightly less than one of the values $\left(m+\frac{1}{2}\right)\pi$ rad , so we start with these values. They can also be found graphically. As in the diagram that follows, we plot y = tan α and y = α on the same graph. The intersections of the line with the tan α curves are the solutions. The smallest α is $\alpha = 0$.

(c) We write $\alpha = \left(m+\frac{1}{2}\right)\pi$ for the maxima. For the central maximum, α = 0 and $m = -1/2 = -0.500$.

(d) The next one can be found to be α = 4.493 rad.

(e) For α = 4.4934, m = 0.930.

(f) The next one can be found to be α = 7.725 rad.

(g) For α = 7.7252, m = 1.96.

18. Using the notation of Sample Problem — "Pointillistic paintings use the diffraction of your eye," the maximum distance is

$$L = \frac{D}{\theta_R} = \frac{D}{1.22\lambda/d} = \frac{\left(5.0\times10^{-3}\text{ m}\right)\left(4.0\times10^{-3}\text{ m}\right)}{1.22\left(550\times10^{-9}\text{ m}\right)} = 30\text{ m} .$$

19. (a) Using the notation of Sample Problem — "Pointillistic paintings use the diffraction of your eye,"

$$L = \frac{D}{1.22\lambda/d} = \frac{2(50\times10^{-6}\text{ m})(1.5\times10^{-3}\text{ m})}{1.22(650\times10^{-9}\text{ m})} = 0.19\text{ m} .$$

(b) The wavelength of the blue light is shorter so $L_{\max} \propto \lambda^{-1}$ will be larger.

20. Using the notation of Sample Problem — "Pointillistic paintings use the diffraction of your eye," the minimum separation is

$$D = L\theta_R = L\left(\frac{1.22\lambda}{d}\right) = \left(6.2\times10^3\text{ m}\right)\frac{(1.22)\left(1.6\times10^{-2}\text{ m}\right)}{2.3\text{m}} = 53\text{m} .$$

21. **THINK** We apply the Rayleigh criterion to estimate the linear separation between the two objects.

EXPRESS If L is the distance from the observer to the objects, then the smallest separation D they can have and still be resolvable is $D = L\theta_R$, where θ_R is measured in radians.

ANALYZE (a) With small angle approximation, $\theta_R = 1.22\lambda/d$, where λ is the wavelength and d is the diameter of the aperture. Thus,

$$D = \frac{1.22\,L\lambda}{d} = \frac{1.22\left(8.0\times10^{10}\text{ m}\right)\left(550\times10^{-9}\text{ m}\right)}{5.0\times10^{-3}\text{ m}} = 1.1\times10^7\text{ m} = 1.1\times10^4\text{ km} .$$

This distance is greater than the diameter of Mars; therefore, one part of the planet's surface cannot be resolved from another part.

(b) Now d = 5.1 m and $D = \dfrac{1.22\left(8.0\times10^{10}\text{ m}\right)\left(550\times10^{-9}\text{ m}\right)}{5.1\text{m}} = 1.1\times10^4\text{ m} = 11\text{ km}$.

LEARN By the Rayleigh criterion for resolvability, two objects can be resolved only if their angular separation at the observer is greater than $\theta_R = 1.22\lambda/d$.

22. (a) Using the notation of Sample Problem — "Pointillistic paintings use the diffraction of your eye," the minimum separation is

$$D = L\theta_R = L\left(\frac{1.22\lambda}{d}\right) = \frac{\left(400\times10^3\text{m}\right)(1.22)\left(550\times10^{-9}\text{ m}\right)}{(0.005\text{m})} \approx 50\text{ m}.$$

(b) The Rayleigh criterion suggests that the astronaut will not be able to discern the Great Wall (see the result of part (a)).

(c) The signs of intelligent life would probably be, at most, ambiguous on the sunlit half of the planet. However, while passing over the half of the planet on the opposite side from the Sun, the astronaut would be able to notice the effects of artificial lighting.

23. **THINK** We apply the Rayleigh criterion to determine the conditions that allow the headlights to be resolved.

EXPRESS By the Rayleigh criteria, two point sources can be resolved if the central diffraction maximum of one source is centered on the first minimum of the diffraction pattern of the other. Thus, the angular separation (in radians) of the sources must be at least $\theta_R = 1.22\lambda/d$, where λ is the wavelength and d is the diameter of the aperture.

ANALYZE (a) For the headlights of this problem,

$$\theta_R = \frac{1.22\left(550\times10^{-9}\text{ m}\right)}{5.0\times10^{-3}\text{ m}} = 1.34\times10^{-4}\text{ rad},$$

or $1.3\times10^{-4}\text{ rad}$, in two significant figures.

(b) If L is the distance from the headlights to the eye when the headlights are just resolvable and D is the separation of the headlights, then $D = L\theta_R$, where the small angle approximation is made. This is valid for θ_R in radians. Thus,

$$L = \frac{D}{\theta_R} = \frac{1.4\text{ m}}{1.34\times10^{-4}\text{ rad}} = 1.0\times10^{4}\text{ m} = 10\text{ km} .$$

LEARN A distance of 10 km far exceeds what human eyes can resolve. In reality, our visual resolvability depends on other factors such as the relative brightness of the source and their surroundings, turbulence in the air between the lights and the eyes, the health of one's vision.

24. We use Eq. 36-12 with $\theta = 2.5°/2 = 1.25°$. Thus,

$$d = \frac{1.22\lambda}{\sin\theta} = \frac{1.22(550\text{ nm})}{\sin 1.25°} = 31\,\mu\text{m} .$$

25. Using the notation of Sample Problem — "Pointillistic paintings use the diffraction of your eye," the minimum separation is

$$D = L\theta_R = L\left(1.22\frac{\lambda}{d}\right) = \left(3.82\times10^{8}\text{ m}\right)\frac{(1.22)\left(550\times10^{-9}\text{ m}\right)}{5.1\text{m}} = 50\text{ m} .$$

26. Using the same notation found in Sample Problem — "Pointillistic paintings use the diffraction of your eye,"

$$\frac{D}{L} = \theta_R = 1.22\frac{\lambda}{d}$$

where we will assume a "typical" wavelength for visible light: $\lambda \approx 550 \times 10^{-9}$ m.

(a) With $L = 400 \times 10^3$ m and $D = 0.85$ m, the above relation leads to $d = 0.32$ m.

(b) Now with $D = 0.10$ m, the above relation leads to $d = 2.7$ m.

(c) The military satellites do not use Hubble Telescope-sized apertures. A great deal of very sophisticated optical filtering and digital signal processing techniques go into the final product, for which there is not space for us to describe here.

27. Using the notation of Sample Problem — "Pointillistic paintings use the diffraction of your eye,"

$$L = \frac{D}{\theta_R} = \frac{D}{1.22\lambda/d} = \frac{(5.0\times10^{-2}\text{ m})(4.0\times10^{-3}\text{m})}{1.22(0.10\times10^{-9}\text{ m})} = 1.6\times10^6\text{ m} = 1.6\times10^3\text{ km}.$$

28. Eq. 36-14 gives $\theta_R = 1.22\lambda/d$, where in our case $\theta_R \approx D/L$, with $D = 60$ μm being the size of the object your eyes must resolve, and L being the maximum viewing distance in question. If $d = 3.00$ mm $= 3000$ μm is the diameter of your pupil, then

$$L = \frac{Dd}{1.22\lambda} = \frac{(60\,\mu\text{m})(3000\,\mu\text{m})}{1.22(0.55\,\mu\text{m})} = 2.7\times10^5\ \mu\text{m} = 27\,\text{cm}.$$

29. (a) Using Eq. 36-14, the angular separation is

$$\theta_R = \frac{1.22\lambda}{d} = \frac{(1.22)(550\times10^{-9}\text{ m})}{0.76\,\text{m}} = 8.8\times10^{-7}\text{ rad}.$$

(b) Using the notation of Sample Problem — "Pointillistic paintings use the diffraction of your eye," the distance between the stars is

$$D = L\theta_R = \frac{(10\,\text{ly})(9.46\times10^{12}\text{ km/ly})(0.18)\pi}{(3600)(180)} = 8.4\times10^7\text{ km}.$$

(c) The diameter of the first dark ring is

$$d = 2\theta_R L = \frac{2(0.18)(\pi)(14\,\text{m})}{(3600)(180)} = 2.5\times10^{-5}\text{ m} = 0.025\,\text{mm}.$$

30. From Fig. 36-42(a), we find the diameter D' on the retina to be

$$D' = D\frac{L'}{L} = (2.00\text{ mm})\frac{2.00\text{ cm}}{45.0\text{ cm}} = 0.0889\text{ mm}.$$

Next, using Fig. 36-42(b), the angle from the axis is

$$\theta = \tan^{-1}\left(\frac{D'/2}{x}\right) = \tan^{-1}\left(\frac{0.0889\text{ mm}/2}{6.00\text{ mm}}\right) = 0.424° .$$

Since the angle corresponds to the first minimum in the diffraction pattern, we have $\sin\theta = 1.22\lambda/d$, where λ is the wavelength and d is the diameter of the defect. With $\lambda = 550\text{ nm},$ we obtain

$$d = \frac{1.22\lambda}{\sin\theta} = \frac{1.22(550\text{ nm})}{\sin(0.424°)} = 9.06\times10^{-5}\text{ m} \approx 91\mu\text{m} .$$

31. **THINK** We apply the Rayleigh criterion to calculate the angular width of the central maxima.

EXPRESS The first minimum in the diffraction pattern is at an angular position θ, measured from the center of the pattern, such that sin θ = 1.22λ/d, where λ is the wavelength and d is the diameter of the antenna. If f is the frequency, then the wavelength is

$$\lambda = \frac{c}{f} = \frac{3.00\times10^{8}\text{ m/s}}{220\times10^{9}\text{ Hz}} = 1.36\times10^{-3}\text{ m} .$$

ANALYZE (a) Thus, we have

$$\theta = \sin^{-1}\left(\frac{1.22\lambda}{d}\right) = \sin^{-1}\left(\frac{1.22\left(1.36\times10^{-3}\text{ m}\right)}{55.0\times10^{-2}\text{ m}}\right) = 3.02\times10^{-3}\text{ rad} .$$

The angular width of the central maximum is twice this, or 6.04 × 10^{-3} rad (0.346°).

(b) Now λ = 1.6 cm and d = 2.3 m, so

$$\theta = \sin^{-1}\left(\frac{1.22\left(1.6\times10^{-2}\text{ m}\right)}{2.3\text{m}}\right) = 8.5\times10^{-3}\text{ rad} .$$

The angular width of the central maximum is 1.7 × 10^{-2} rad (or 0.97°).

LEARN Using small angle approximation, we can write the angular width as

$$2\theta \approx 2\left(\frac{1.22\lambda}{d}\right) = \frac{2.44\lambda}{d} .$$

32. (a) We use Eq. 36-12:

$$\theta = \sin^{-1}\left(\frac{1.22\lambda}{d}\right) = \sin^{-1}\left[\frac{1.22(v_s/f)}{d}\right] = \sin^{-1}\left[\frac{(1.22)(1450\,\text{m/s})}{(25\times10^3\,\text{Hz})(0.60\,\text{m})}\right] = 6.8°.$$

(b) Now $f = 1.0 \times 10^3$ Hz so

$$\frac{1.22\lambda}{d} = \frac{(1.22)(1450\,\text{m/s})}{(1.0\times10^3\,\text{Hz})(0.60\,\text{m})} = 2.9 > 1.$$

Since sin θ cannot exceed 1 there is no minimum.

33. Equation 36-14 gives the Rayleigh angle (in radians):

$$\theta_R = \frac{1.22\lambda}{d} = \frac{D}{L}$$

where the rationale behind the second equality is given in Sample Problem — "Pointillistic paintings use the diffraction of your eye."

(a) We are asked to solve for D and are given $\lambda = 1.40 \times 10^{-9}$ m, $d = 0.200 \times 10^{-3}$ m, and $L = 2000\times10^3$ m. Consequently, we obtain D = 17.1 m.

(b) Intensity is power over area (with the area assumed spherical in this case, which means it is proportional to radius-squared), so the ratio of intensities is given by the square of a ratio of distances: $(d/D)^2 = 1.37 \times 10^{-10}$.

34. (a) Since $\theta = 1.22\lambda/d$, the larger the wavelength the larger the radius of the first minimum (and second maximum, etc). Therefore, the white pattern is outlined by red lights (with longer wavelength than blue lights).

(b) The diameter of a water drop is

$$d = \frac{1.22\lambda}{\theta} \approx \frac{1.22(7\times10^{-7}\,\text{m})}{1.5(0.50°)(\pi/180°)/2} = 1.3\times10^{-4}\,\text{m}\,.$$

35. Bright interference fringes occur at angles θ given by $d \sin\theta = m\lambda$, where m is an integer. For the slits of this problem, we have $d = 11a/2$, so

$$a \sin\theta = 2m\lambda/11\,.$$

The first minimum of the diffraction pattern occurs at the angle θ_1 given by $a \sin\theta_1 = \lambda$, and the second occurs at the angle θ_2 given by $a \sin\theta_2 = 2\lambda$, where a is the slit width. We should count the values of m for which $\theta_1 < \theta < \theta_2$, or, equivalently, the values of m for which $\sin\theta_1 < \sin\theta < \sin\theta_2$. This means $1 < (2m/11) < 2$. The values are m = 6, 7, 8, 9, and 10. There are five bright fringes in all.

36. Following the method of Sample Problem — "Double-slit experiment with diffraction of each slit included," we find

$$\frac{d}{a} = \frac{0.30\times10^{-3}\text{ m}}{46\times10^{-6}\text{ m}} = 6.52$$

which we interpret to mean that the first diffraction minimum occurs slightly farther "out" than the m = 6 interference maximum. This implies that the central diffraction envelope includes the central (m = 0) interference maximum as well as six interference maxima on each side of it. Therefore, there are 6 + 1 + 6 = 13 bright fringes (interference maxima) in the central diffraction envelope.

37. In a manner similar to that discussed in Sample Problem — "Double-slit experiment with diffraction of each slit included," we find the number is $2(d/a) - 1 = 2(2a/a) - 1 = 3$.

38. We note that the central diffraction envelope contains the central bright interference fringe (corresponding to m = 0 in Eq. 36-25) plus ten on either side of it. Since the eleventh order bright interference fringe is not seen in the central envelope, then we conclude the first diffraction minimum (satisfying $\sin\theta = \lambda/a$) coincides with the m = 11 instantiation of Eq. 36-25:

$$d = \frac{m\lambda}{\sin\theta} = \frac{11\,\lambda}{\lambda/a} = 11\,a\,.$$

Thus, the ratio d/a is equal to 11.

39. (a) The first minimum of the diffraction pattern is at 5.00°, so

$$a = \frac{\lambda}{\sin\theta} = \frac{0.440\,\mu\text{m}}{\sin 5.00°} = 5.05\,\mu\text{m}\,.$$

(b) Since the fourth bright fringe is missing, $d = 4a = 4(5.05\ \mu\text{m}) = 20.2\ \mu\text{m}$.

(c) For the m = 1 bright fringe,

$$\alpha = \frac{\pi a\sin\theta}{\lambda} = \frac{\pi\left(5.05\,\mu\text{m}\right)\sin 1.25°}{0.440\,\mu\text{m}} = 0.787\text{ rad}\,.$$

Consequently, the intensity of the m = 1 fringe is

$$I = I_m\left(\frac{\sin\alpha}{\alpha}\right)^2 = \left(7.0\text{ mW/cm}^2\right)\left(\frac{\sin 0.787\text{ rad}}{0.787}\right)^2 = 5.7\text{ mW/cm}^2\ ,$$

which agrees with Fig. 36-45. Similarly for m = 2, the intensity is I = 2.9 mW/cm^2, also in agreement with Fig. 36-45.

40. (a) We note that the slope of the graph is 80, and that Eq. 36-20 implies that the slope should correspond to

$$\frac{\pi d}{\lambda} = 80 \quad \Rightarrow \quad d = \frac{80\lambda}{\pi} = \frac{80(435 \text{ nm})}{\pi} = 11077 \text{ nm} \approx 11.1\,\mu\text{m}\,.$$

(b) Consider Eq. 36-25 with "continuously variable" m (of course, m should be an integer for interference maxima, but for the moment we will solve for it as if it could be any real number):

$$m_{\max} = \frac{d}{\lambda}(\sin\theta)_{\max} = \frac{d}{\lambda} = \frac{11077 \text{ nm}}{435 \text{ nm}} \approx 25.5$$

which indicates (on one side of the interference pattern) there are 25 bright fringes. Thus on the other side there are also 25 bright fringes. Including the one in the middle, then, means there are a total of 51 maxima in the interference pattern (assuming, as the problem remarks, that none of the interference maxima have been eliminated by diffraction minima).

(c) Clearly, the maximum closest to the axis is the middle fringe at $\theta = 0°$.

(d) If we set m = 25 in Eq. 36-25, we find

$$m\lambda = d\sin\theta \quad \Rightarrow \quad \theta = \sin^{-1}\left(\frac{m\lambda}{d}\right) = \sin^{-1}\left(\frac{(25)(435 \text{ nm})}{11077 \text{ nm}}\right) = 79.0°$$

41. We will make use of arctangents and sines in our solution, even though they can be "shortcut" somewhat since the angles are [almost] small enough to justify the use of the small angle approximation.

(a) Given y/D = (0.700 m)/(4.00 m), then

$$\theta = \tan^{-1}\left(\frac{y}{D}\right) = \tan^{-1}\left(\frac{0.700 \text{ m}}{4.00 \text{ m}}\right) = 9.93° = 0.173 \text{ rad}\,.$$

Equation 36-20 then gives

$$\beta = \frac{\pi d \sin\theta}{\lambda} = \frac{\pi(24.0\,\mu\text{m})\sin 9.93°}{0.600\,\mu\text{m}} = 21.66 \text{ rad.}$$

Thus, use of Eq. 36-21 (with a = 12 μm and λ = 0.60 μm) leads to

$$\alpha = \frac{\pi a \sin\theta}{\lambda} = \frac{\pi(12.0\,\mu\text{m})\sin 9.93°}{0.600\,\mu\text{m}} = 10.83 \text{ rad} .$$

Thus,

$$\frac{I}{I_m} = \left(\frac{\sin\alpha}{\alpha}\right)^2 (\cos\beta)^2 = \left(\frac{\sin 10.83\,\text{rad}}{10.83}\right)^2 (\cos 21.66 \text{ rad})^2 = 0.00743 .$$

(b) Consider Eq. 36-25 with "continuously variable" *m* (of course, *m* should be an integer for interference maxima, but for the moment we will solve for it as if it could be any real number):

$$m = \frac{d\sin\theta}{\lambda} = \frac{(24.0\,\mu\text{m})\sin 9.93°}{0.600\,\mu\text{m}} \approx 6.9$$

which suggests that the angle takes us to a point between the sixth minimum (which would have $m = 6.5$) and the seventh maximum (which corresponds to $m = 7$).

(c) Similarly, consider Eq. 36-3 with "continuously variable" *m* (of course, *m* should be an integer for diffraction minima, but for the moment we will solve for it as if it could be any real number):

$$m = \frac{a\sin\theta}{\lambda} = \frac{(12.0\,\mu\text{m})\sin 9.93°}{0.600\,\mu\text{m}} \approx 3.4$$

which suggests that the angle takes us to a point between the third diffraction minimum ($m = 3$) and the fourth one ($m = 4$). The maxima (in the smaller peaks of the diffraction pattern) are not exactly midway between the minima; their location would make use of mathematics not covered in the prerequisites of the usual sophomore-level physics course.

42. (a) In a manner similar to that discussed in Sample Problem — "Double-slit experiment with diffraction of each slit included," we find the ratio should be $d/a = 4$. Our reasoning is, briefly, as follows: we let the location of the fourth bright fringe coincide with the first minimum of diffraction pattern, and then set $\sin\theta = 4\lambda/d = \lambda/a$ (so $d = 4a$).

(b) Any bright fringe that happens to be at the same location with a diffraction minimum will vanish. Thus, if we let

$$\sin\theta = \frac{m_1\lambda}{d} = \frac{m_2\lambda}{a} = \frac{m_1\lambda}{4a} ,$$

or $m_1 = 4m_2$ where $m_2 = 1, 2, 3, \ldots$. The fringes missing are the 4th, 8th, 12th, and so on. Hence, every fourth fringe is missing.

43. **THINK** For relatively wide slits, the interference of light from two slits produces bright fringes that do not all have the same intensity; instead, the intensities are modified by diffraction of light passing through each slit.

EXPRESS The angular positions θ of the bright interference fringes are given by $d \sin\theta = m\lambda$, where d is the slit separation, λ is the wavelength, and m is an integer. The first diffraction minimum occurs at the angle θ_1 given by $a \sin\theta_1 = \lambda$, where a is the slit width. The diffraction peak extends from $-\theta_1$ to $+\theta_1$, so we should count the number of values of m for which $-\theta_1 < \theta < +\theta_1$, or, equivalently, the number of values of m for which

$$-\sin\theta_1 < \sin\theta < +\sin\theta_1.$$

The intensity at the screen is given by

$$I = I_m\left(\cos^2\beta\right)\left(\frac{\sin\alpha}{\alpha}\right)^2$$

where $\alpha = (\pi a/\lambda)\sin\theta$, $\beta = (\pi d/\lambda)\sin\theta$, and I_m is the intensity at the center of the pattern.

ANALYZE (a) The condition above means $-1/a < m/d < 1/a$, or $-d/a < m < +d/a$. Now

$$d/a = (0.150 \times 10^{-3}\text{ m})/(30.0 \times 10^{-6}\text{ m}) = 5.00,$$

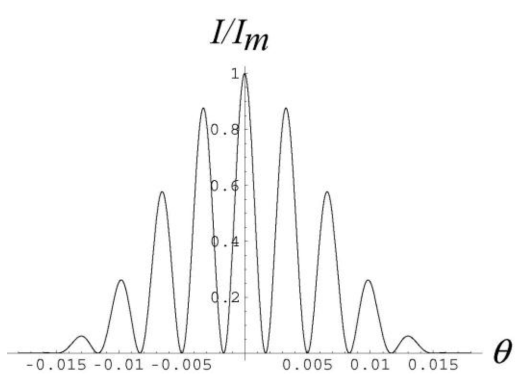
so the values of m are $m = -4, -3, -2, -1, 0, +1, +2, +3$, and $+4$. There are 9 fringes.

(b) For the third bright interference fringe, $d \sin\theta = 3\lambda$, so $\beta = 3\pi$ rad and $\cos^2\beta = 1$. Similarly, $\alpha = 3\pi a/d = 3\pi/5.00 = 0.600\pi$ rad and

$$\left(\frac{\sin\alpha}{\alpha}\right)^2 = \left(\frac{\sin 0.600\pi}{0.600\pi}\right)^2 = 0.255\,.$$

The intensity ratio is $I/I_m = 0.255$.

LEARN The expression for intensity contains two factors: (1) the interference factor $\cos^2\beta$ due to the interference between two slits with separation d, and (2) the diffraction factor $[(\sin\alpha)/\alpha]^2$ which arises due to diffraction by a single slit of width a. In the limit $a \to 0$, $(\sin\alpha)/\alpha \to 1$, and we recover Eq. 35-22 for the interference between two slits of vanishingly narrow slits separated by d. Similarly, setting $d = 0$ or equivalently, $\beta = 0$, we recover Eq. 36-5 for the diffraction of a single slit of width a. A plot of the relative intensity is shown to the right.



44. We use Eq. 36-25 for diffraction maxima: $d \sin\theta = m\lambda$. In our case, since the angle between the m = 1 and m = –1 maxima is 26°, the angle θ corresponding to m = 1 is θ = 26°/2 = 13°. We solve for the grating spacing:

$$d = \frac{m\lambda}{\sin\theta} = \frac{(1)(550\,\text{nm})}{\sin 13°} = 2.4\,\mu\text{m} \approx 2\,\mu\text{m}.$$

45. The distance between adjacent rulings is

$$d = 20.0\ \text{mm}/6000 = 0.00333\ \text{mm} = 3.33\ \mu\text{m}.$$

(a) Let $d\sin\theta = m\lambda\,(m = 0, \pm 1, \pm 2, \ldots)$. Since $|m|\lambda/d > 1$ for $|m| \geq 6$, the largest value of θ corresponds to $|m|$ = 5, which yields

$$\theta = \sin^{-1}(|m|\lambda/d) = \sin^{-1}\left(\frac{5(0.589\,\mu\text{m})}{3.33\,\mu\text{m}}\right) = 62.1°.$$

(b) The second largest value of θ corresponds to $|m|$ = 4, which yields

$$\theta = \sin^{-1}(|m|\lambda/d) = \sin^{-1}\left(\frac{4(0.589\,\mu\text{m})}{3.33\,\mu\text{m}}\right) = 45.0°.$$

(c) The third largest value of θ corresponds to $|m|$ = 3, which yields

$$\theta = \sin^{-1}(|m|\lambda/d) = \sin^{-1}\left(\frac{3(0.589\,\mu\text{m})}{3.33\,\mu\text{m}}\right) = 32.0°.$$

46. The angular location of the mth order diffraction maximum is given by $m\lambda = d\sin\theta$. To be able to observe the fifth-order maximum, we must let $\sin\theta|_{m=5} = 5\lambda/d < 1$, or

$$\lambda < \frac{d}{5} = \frac{1.00\,\text{nm}/315}{5} = 635\,\text{nm}.$$

Therefore, the longest wavelength that can be used is λ = 635 nm.

47. **THINK** Diffraction lines occur at angles θ such that $d\sin\theta = m\lambda$, where d is the grating spacing, λ is the wavelength and m is an integer.

EXPRESS The ruling separation is

$$d = 1/(400\ \text{mm}^{-1}) = 2.5 \times 10^{-3}\ \text{mm}.$$

Notice that for a given order, the line associated with a long wavelength is produced at a greater angle than the line associated with a shorter wavelength. We take λ to be the longest wavelength in the visible spectrum (700 nm) and find the greatest integer value of m such that θ is less than 90°. That is, find the greatest integer value of m for which $m\lambda < d$.

ANALYZE Since

$$\frac{d}{\lambda} = \frac{2.5\times10^{-6}\,\text{m}}{700\times10^{-9}\,\text{m}} \approx 3.57,$$

that value is $m = 3$. There are three complete orders on each side of the $m = 0$ order. The second and third orders overlap.

LEARN From $\theta = \sin^{-1}(m\lambda/d)$, the condition for maxima or lines, we see that for a given diffraction grating, the angle from the central axis to any line depends on the wavelength of the light being used.

48. (a) For the maximum with the greatest value of $m = M$ we have $M\lambda = a \sin\theta < d$, so $M < d/\lambda = 900\text{ nm}/600\text{ nm} = 1.5$, or $M = 1$. Thus three maxima can be seen, with $m = 0, \pm1$.

(b) From Eq. 36-28, we obtain

$$\begin{aligned}\Delta\theta_{\text{hw}} &= \frac{\lambda}{N\,d\cos\theta} = \frac{d\sin\theta}{N\,d\cos\theta} = \frac{\tan\theta}{N} = \frac{1}{N}\tan\left[\sin^{-1}\left(\frac{\lambda}{d}\right)\right] \\ &= \frac{1}{1000}\tan\left[\sin^{-1}\left(\frac{600\,\text{nm}}{900\,\text{nm}}\right)\right] = 0.051°.\end{aligned}$$

49. **THINK** Maxima of a diffraction grating pattern occur at angles θ given by $d \sin\theta = m\lambda$, where d is the slit separation, λ is the wavelength, and m is an integer.

EXPRESS If two lines are adjacent, then their order numbers differ by unity. Let m be the order number for the line with $\sin\theta = 0.2$ and $m + 1$ be the order number for the line with $\sin\theta = 0.3$. Then,

$$0.2d = m\lambda, \quad 0.3d = (m + 1)\lambda.$$

ANALYZE (a) We subtract the first equation from the second to obtain $0.1d = \lambda$, or

$$d = \lambda/0.1 = (600 \times 10^{-9}\text{m})/0.1 = 6.0 \times 10^{-6}\text{ m}.$$

(b) Minima of the single-slit diffraction pattern occur at angles θ given by $a \sin\theta = m\lambda$, where a is the slit width. Since the fourth-order interference maximum is missing, it must fall at one of these angles. If a is the smallest slit width for which this order is missing, the angle must be given by $a \sin \theta = \lambda$. It is also given by $d \sin \theta = 4\lambda$, so

$$a = d/4 = (6.0 \times 10^{-6}\ \text{m})/4 = 1.5 \times 10^{-6}\ \text{m}.$$

(c) First, we set $\theta = 90°$ and find the largest value of m for which $m\lambda < d \sin\theta$. This is the highest order that is diffracted toward the screen. The condition is the same as $m < d/\lambda$ and since

$$d/\lambda = (6.0 \times 10^{-6}\ \text{m})/(600 \times 10^{-9}\ \text{m}) = 10.0,$$

the highest order seen is the $m = 9$ order. The fourth and eighth orders are missing, so the observable orders are $m = 0, 1, 2, 3, 5, 6, 7,$ and 9. Thus, the largest value of the order number is $m = 9$.

(d) Using the result obtained in (c), the second largest value of the order number is $m = 7$.

(e) Similarly, the third largest value of the order number is $m = 6$.

LEARN Interference maxima occur when $d \sin \theta = m\lambda$, while the condition for diffraction minima is $a \sin \theta = m'\lambda$. Thus, a particular interference maximum with order m may coincide with the diffraction minimum of order m'. The value of m is given by

$$\frac{d \sin\theta}{a \sin\theta} = \frac{m\lambda}{m'\lambda} \quad \Rightarrow \quad m = \left(\frac{d}{a}\right)m'.$$

Since $m = 4$ when $m' = 1$, we conclude that $d/a = 4$. Thus, $m = 8$ would correspond to the second diffraction minimum ($m' = 2$).

50. We use Eq. 36-25. For $m = \pm 1$

$$\lambda = \frac{d \sin\theta}{m} = \frac{(1.73\,\mu\text{m})\sin(\pm 17.6°)}{\pm 1} = 523\ \text{nm},$$

and for $m = \pm 2$,

$$\lambda = \frac{(1.73\,\mu\text{m})\sin(\pm 37.3°)}{\pm 2} = 524\ \text{nm}.$$

Similarly, we may compute the values of λ corresponding to the angles for $m = \pm 3$. The average value of these λ's is 523 nm.

51. (a) Since $d = (1.00\ \text{mm})/180 = 0.0056\ \text{mm}$, we write Eq. 36-25 as

$$\theta = \sin^{-1}\left(\frac{m\lambda}{d}\right) = \sin^{-1}(180)(2)\lambda$$

where $\lambda_1 = 4\times10^{-4}$ mm and $\lambda_2 = 5\times10^{-4}$ mm. Thus, $\Delta\theta = \theta_2 - \theta_1 = 2.1°$.

(b) Use of Eq. 36-25 for each wavelength leads to the condition

$$m_1\lambda_1 = m_2\lambda_2$$

for which the smallest possible choices are m_1 = 5 and m_2 = 4. Returning to Eq. 36-25, then, we find

$$\theta = \sin^{-1}\left(\frac{m_1\lambda_1}{d}\right) = \sin^{-1}\left(\frac{5(4.0\times10^{-4}\ \text{mm})}{0.0056\ \text{mm}}\right) = \sin^{-1}(0.36) = 21°.$$

(c) There are no refraction angles greater than 90°, so we can solve for "m_{max}" (realizing it might not be an integer):

$$m_{\text{max}} = \frac{d\sin 90°}{\lambda_2} = \frac{d}{\lambda_2} = \frac{0.0056\ \text{mm}}{5.0\times10^{-4}\ \text{mm}} \approx 11$$

where we have rounded down. There are no values of m (for light of wavelength λ_2) greater than m = 11.

52. We are given the "number of lines per millimeter" (which is a common way to express $1/d$ for diffraction gratings); thus,

$$\frac{1}{d} = 160\ \text{lines/mm} \quad\Rightarrow\quad d = 6.25\times10^{-6}\ \text{m}.$$

(a) We solve Eq. 36-25 for θ with various values of m and λ. We show here the m = 2 and λ = 460 nm calculation:

$$\theta = \sin^{-1}\left(\frac{m\lambda}{d}\right) = \sin^{-1}\left(\frac{2(460\times10^{-9}\ \text{m})}{6.25\times10^{-6}\ \text{m}}\right) = \sin^{-1}(0.1472) = 8.46°.$$

Similarly, we get 11.81° for m = 2 and λ = 640 nm, 12.75° for m = 3 and λ = 460 nm, and 17.89° for m = 3 and λ = 640 nm. The first indication of overlap occurs when we compute the angle for m = 4 and λ = 460 nm; the result is 17.12° which clearly shows overlap with the large-wavelength portion of the m = 3 spectrum.

(b) We solve Eq. 36-25 for m with θ = 90° and λ = 640 nm. In this case, we obtain m = 9.8 which means that the largest order in which the full range (which must include that largest wavelength) is seen is ninth order.

(c) Now with m = 9, Eq. 36-25 gives θ = 41.5° for λ = 460 nm.

(d) It similarly gives $\theta = 67.2°$ for $\lambda = 640$ nm.

(e) We solve Eq. 36-25 for m with $\theta = 90°$ and $\lambda = 460$ nm. In this case, we obtain $m = 13.6$ which means that the largest order in which the wavelength is seen is the thirteenth order. Now with $m = 13$, Eq. 36-25 gives $\theta = 73.1°$ for $\lambda = 460$ nm.

53. At the point on the screen where we find the inner edge of the hole, we have $\tan\theta = 5.0$ cm/30 cm, which gives $\theta = 9.46°$. We note that d for the grating is equal to 1.0 mm/350 = 1.0×10^6 nm/350.

(a) From $m\lambda = d \sin\theta$, we find

$$m = \frac{d\sin\theta}{\lambda} = \frac{(1.0\times10^6 \text{ nm}/350)(0.1644)}{\lambda} = \frac{470\,\text{nm}}{\lambda}.$$

Since for white light $\lambda > 400$ nm, the only integer m allowed here is $m = 1$. Thus, at one edge of the hole, $\lambda = 470$ nm. This is the shortest wavelength of the light that passes through the hole.

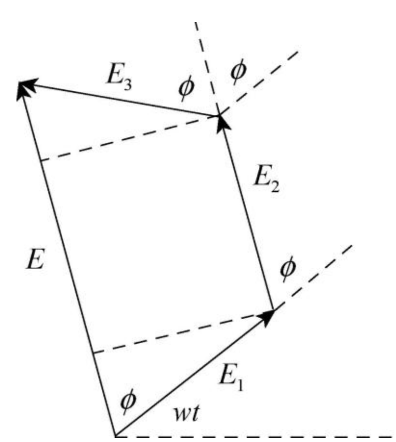
(b) At the other edge, we have $\tan\theta' = 6.0$ cm/30 cm, which gives $\theta' = 11.31°$. This leads to

$$\lambda' = d\sin\theta' = \left(\frac{1.0\times10^6\text{ nm}}{350}\right)\sin(11.31°) = 560\text{ nm}.$$

This corresponds to the longest wavelength of the light that passes through the hole.

54. Since the slit width is much less than the wavelength of the light, the central peak of the single-slit diffraction pattern is spread across the screen and the diffraction envelope can be ignored. Consider three waves, one from each slit. Since the slits are evenly spaced, the phase difference for waves from the first and second slits is the same as the phase difference for waves from the second and third slits. The electric fields of the waves at the screen can be written as

$$E_1 = E_0\sin(\omega t)$$
$$E_2 = E_0\sin(\omega t + \phi)$$
$$E_3 = E_0\sin(\omega t + 2\phi)$$



where $\phi = (2\pi d/\lambda)\sin\theta$. Here d is the separation of adjacent slits and λ is the wavelength. The phasor diagram is shown on the right. It yields

$$E = E_0\cos\phi + E_0\cos\phi = E_0(1 + 2\cos\phi).$$

for the amplitude of the resultant wave. Since the intensity of a wave is proportional to the square of the electric field, we may write $I = AE_0^2(1+2\cos\phi)^2$, where A is a constant of proportionality. If I_m is the intensity at the center of the pattern, for which $\phi = 0$, then $I_m = 9AE_0^2$. We take A to be $I_m/9E_0^2$ and obtain

$$I = \frac{I_m}{9}(1+2\cos\phi)^2 = \frac{I_m}{9}\left(1+4\cos\phi+4\cos^2\phi\right).$$

55. **THINK** If a grating just resolves two wavelengths whose average is λ_{avg} and whose separation is $\Delta\lambda$, then its resolving power is defined by $R = \lambda_{avg}/\Delta\lambda$.

EXPRESS As shown in Eq. 36-32, the resolving power can also be written as Nm, where N is the number of rulings in the grating and m is the order of the lines.

ANALYZE Thus $\lambda_{avg}/\Delta\lambda = Nm$ and

$$N = \frac{\lambda_{avg}}{m\Delta\lambda} = \frac{656.3\,\text{nm}}{(1)(0.18\,\text{nm})} = 3.65\times10^3 \text{ rulings.}$$

LEARN A large N (more rulings) means greater resolving power.

56. (a) From $R = \lambda/\Delta\lambda = Nm$ we find

$$N = \frac{\lambda}{m\Delta\lambda} = \frac{(415.496 \text{ nm} + 415.487 \text{ nm})/2}{2(415.96 \text{ nm} - 415.487 \text{ nm})} = 23100.$$

(b) We note that $d = (4.0 \times 10^7 \text{ nm})/23100 = 1732$ nm. The maxima are found at

$$\theta = \sin^{-1}\left(\frac{m\lambda}{d}\right) = \sin^{-1}\left[\frac{(2)(415.5\,\text{nm})}{1732\,\text{nm}}\right] = 28.7°.$$

57. (a) We note that $d = (76 \times 10^6 \text{ nm})/40000 = 1900$ nm. For the first order maxima $\lambda = d\sin\theta$, which leads to

$$\theta = \sin^{-1}\left(\frac{\lambda}{d}\right) = \sin^{-1}\left(\frac{589 \text{ nm}}{1900 \text{ nm}}\right) = 18°.$$

Now, substituting $m = d\sin\theta/\lambda$ into Eq. 36-30 leads to

$$D = \tan\theta/\lambda = \tan 18°/589 \text{ nm} = 5.5 \times 10^{-4} \text{ rad/nm} = 0.032°/\text{nm}.$$

(b) For $m = 1$, the resolving power is $R = Nm = 40000\,m = 40000 = 4.0 \times 10^4$.

(c) For $m = 2$ we have $\theta = 38°$, and the corresponding value of dispersion is 0.076°/nm.

(d) For $m = 2$, the resolving power is $R = Nm = 40000\, m = (40000)2 = 8.0 \times 10^4$.

(e) Similarly for $m = 3$, we have $\theta = 68°$, and the corresponding value of dispersion is 0.24°/nm.

(f) For $m = 3$, the resolving power is $R = Nm = 40000\, m = (40000)3 = 1.2 \times 10^5$.

58. (a) We find $\Delta\lambda$ from $R = \lambda/\Delta\lambda = Nm$:

$$\Delta\lambda = \frac{\lambda}{Nm} = \frac{500\,\text{nm}}{(600/\text{mm})(5.0\,\text{mm})(3)} = 0.056\,\text{nm} = 56\,\text{pm}.$$

(b) Since $\sin\theta = m_{\max}\lambda/d < 1$,

$$m_{\max} < \frac{d}{\lambda} = \frac{1}{(600/\text{mm})(500\times10^{-6}\,\text{mm})} = 3.3.$$

Therefore, $m_{\max} = 3$. No higher orders of maxima can be seen.

59. Assuming all $N = 2000$ lines are uniformly illuminated, we have

$$\frac{\lambda_{\text{av}}}{\Delta\lambda} = Nm$$

from Eq. 36-31 and Eq. 36-32. With $\lambda_{\text{av}} = 600$ nm and $m = 2$, we find $\Delta\lambda = 0.15$ nm.

60. Letting $R = \lambda/\Delta\lambda = Nm$, we solve for N:

$$N = \frac{\lambda}{m\Delta\lambda} = \frac{(589.6\,\text{nm} + 589.0\,\text{nm})/2}{2(589.6\,\text{nm} - 589.0\,\text{nm})} = 491.$$

61. (a) From $d\sin\theta = m\lambda$ we find

$$d = \frac{m\lambda_{\text{avg}}}{\sin\theta} = \frac{3(589.3\,\text{nm})}{\sin 10°} = 1.0\times10^4\,\text{nm} = 10\,\mu\text{m}.$$

(b) The total width of the ruling is

$$L = Nd = \left(\frac{R}{m}\right)d = \frac{\lambda_{\text{avg}}d}{m\Delta\lambda} = \frac{(589.3\,\text{nm})(10\,\mu\text{m})}{3(589.59\,\text{nm} - 589.00\,\text{nm})} = 3.3\times10^3\,\mu\text{m} = 3.3\,\text{mm}.$$

62. (a) From the expression for the half-width $\Delta\theta_{\text{hw}}$ (given by Eq. 36-28) and that for the resolving power R (given by Eq. 36-32), we find the product of $\Delta\theta_{\text{hw}}$ and R to be

$$\Delta\theta_{\text{hw}} R = \left(\frac{\lambda}{N d\cos\theta}\right) Nm = \frac{m\lambda}{d\cos\theta} = \frac{d\sin\theta}{d\cos\theta} = \tan\theta,$$

where we used $m\lambda = d\sin\theta$ (see Eq. 36-25).

(b) For first order $m = 1$, so the corresponding angle θ_1 satisfies $d\sin\theta_1 = m\lambda = \lambda$. Thus the product in question is given by

$$\tan\theta_1 = \frac{\sin\theta_1}{\cos\theta_1} = \frac{\sin\theta_1}{\sqrt{1-\sin^2\theta_1}} = \frac{1}{\sqrt{(1/\sin\theta_1)^2-1}} = \frac{1}{\sqrt{(d/\lambda)^2-1}}$$
$$= \frac{1}{\sqrt{(900\text{nm}/600\text{nm})^2-1}} = 0.89.$$

63. The angular positions of the first-order diffraction lines are given by $d\sin\theta = \lambda$. Let λ_1 be the shorter wavelength (430 nm) and θ be the angular position of the line associated with it. Let λ_2 be the longer wavelength (680 nm), and let $\theta + \Delta\theta$ be the angular position of the line associated with it. Here $\Delta\theta = 20°$. Then,

$$\lambda_1 = d\sin\theta, \quad \lambda_2 = d\sin(\theta+\Delta\theta)\,.$$

We write

$$\sin(\theta+\Delta\theta) \text{ as } \sin\theta\cos\Delta\theta + \cos\theta\sin\Delta\theta,$$

then use the equation for the first line to replace $\sin\theta$ with λ_1/d, and $\cos\theta$ with $\sqrt{1-\lambda_1^2/d^2}$. After multiplying by d, we obtain

$$\lambda_1\cos\Delta\theta + \sqrt{d^2-\lambda_1^2}\,\sin\Delta\theta = \lambda_2.$$

Solving for d, we find

$$d = \sqrt{\frac{(\lambda_2-\lambda_1\cos\Delta\theta)^2+(\lambda_1\sin\Delta\theta)^2}{\sin^2\Delta\theta}}$$
$$= \sqrt{\frac{[(680\text{ nm})-(430\text{ nm})\cos 20°]^2+[(430\text{ nm})\sin 20°]^2}{\sin^2 20°}}$$
$$= 914\text{ nm} = 9.14\times10^{-4}\text{ mm}.$$

There are $1/d = 1/(9.14\times10^{-4}\text{ mm}) = 1.09\times10^{3}$ rulings per mm.

64. We use Eq. 36-34. For smallest value of θ, we let m = 1. Thus,

$$\theta_{\min} = \sin^{-1}\left(\frac{m\lambda}{2d}\right) = \sin^{-1}\left[\frac{(1)(30\,\text{pm})}{2(0.30\times10^{3}\,\text{pm})}\right] = 2.9°.$$

65. (a) For the first beam $2d \sin \theta_1 = \lambda_A$ and for the second one $2d \sin \theta_2 = 3\lambda_B$. The values of d and λ_A can then be determined:

$$d = \frac{3\lambda_B}{2\sin\theta_2} = \frac{3(97\,\text{pm})}{2\sin 60°} = 1.7\times10^{2}\,\text{pm}.$$

(b) $\lambda_A = 2d\sin\theta_1 = 2\left(1.7\times10^{2}\ \text{pm}\right)\left(\sin 23°\right) = 1.3\times10^{2}\,\text{pm}.$

66. The x-ray wavelength is $\lambda = 2d \sin \theta = 2(39.8\text{ pm}) \sin 30.0° = 39.8$ pm.

67. We use Eq. 36-34.

(a) From the peak on the left at angle 0.75° (estimated from Fig. 36-46), we have

$$\lambda_1 = 2d\sin\theta_1 = 2\left(0.94\ \text{nm}\right)\sin\left(0.75°\right) = 0.025\,\text{nm} = 25\ \text{pm}.$$

This is the shorter wavelength of the beam. Notice that the estimation should be viewed as reliable to within ±2 pm.

(b) We now consider the next peak:

$$\lambda_2 = 2d\sin\theta_2 = 2(0.94\ \text{nm})\sin 1.15° = 0.038\,\text{nm} = 38\,\text{pm}.$$

This is the longer wavelength of the beam. One can check that the third peak from the left is the second-order one for λ_1.

68. For x-ray ("Bragg") scattering, we have $2d \sin \theta_m = m\,\lambda$. This leads to

$$\frac{2d\sin\theta_2}{2d\sin\theta_1} = \frac{2\,\lambda}{1\,\lambda} \quad\Rightarrow\quad \sin\theta_2 = 2\sin\theta_1\,.$$

Thus, with θ_1= 3.4°, this yields θ_2 = 6.8°. The fact that θ_2 is very nearly twice the value of θ_1 is due to the small angles involved (when angles are small, $\sin\theta_2 / \sin\theta_1 = \theta_2/\theta_1$).

69. Bragg's law gives the condition for diffraction maximum:

$$2d\sin\theta = m\lambda$$

where d is the spacing of the crystal planes and λ is the wavelength. The angle θ is measured from the surfaces of the planes. For a second-order reflection $m = 2$, so

$$d = \frac{m\lambda}{2\sin\theta} = \frac{2\left(0.12\times10^{-9}\,\mathrm{m}\right)}{2\sin 28^\circ} = 2.56\times10^{-10}\,\mathrm{m} \approx 0.26\,\mathrm{nm}.$$

70. The angle of incidence on the reflection planes is $\theta = 63.8° - 45.0° = 18.8°$, and the plane-plane separation is $d = a_0/\sqrt{2}$. Thus, using $2d \sin\theta = \lambda$, we get

$$a_0 = \sqrt{2}d = \frac{\sqrt{2}\lambda}{2\sin\theta} = \frac{0.260\,\mathrm{nm}}{\sqrt{2}\sin 18.8^\circ} = 0.570\,\mathrm{nm}.$$

71. **THINK** The criterion for diffraction maxima is given by the Bragg's law.

EXPRESS We want the reflections to obey the Bragg condition: $2d \sin\theta = m\lambda$, where θ is the angle between the incoming rays and the reflecting planes, λ is the wavelength, and m is an integer. We solve for θ:

$$\theta = \sin^{-1}\left(\frac{m\lambda}{2d}\right) = \sin^{-1}\left(\frac{\left(0.125\times10^{-9}\,\mathrm{m}\right)m}{2\left(0.252\times10^{-9}\,\mathrm{m}\right)}\right) = 0.2480m.$$

ANALYZE (a) For $m = 2$ the above equation gives $\theta = 29.7°$. The crystal should be turned $\phi = 45° - 29.7° = 15.3°$ clockwise.

(b) For $m = 1$ the above equation gives $\theta = 14.4°$. The crystal should be turned $\phi = 45° - 14.4° = 30.6°$ clockwise.

(c) For $m = 3$ the above equation gives $\theta = 48.1°$. The crystal should be turned $\phi = 48.1° - 45° = 3.1°$ counterclockwise.

(d) For $m = 4$ the above equation gives $\theta = 82.8°$. The crystal should be turned $\phi = 82.8° - 45° = 37.8°$ counterclockwise.

LEARN Note that there are no intensity maxima for $m > 4$ as one can verify by noting that $m\lambda/2d$ is greater than 1 for m greater than 4.

72. The wavelengths satisfy

$$m\lambda = 2d \sin\theta = 2(275\text{ pm})(\sin 45°) = 389\text{ pm}.$$

In the range of wavelengths given, the allowed values of m are $m = 3, 4$.

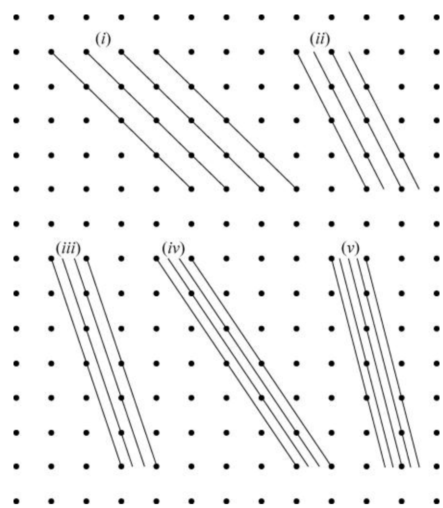
(a) The longest wavelength is 389 pm/3 = 130 pm.

(b) The associated order number is m = 3.

(c) The shortest wavelength is 389 pm/4 = 97.2 pm.

(d) The associated order number is m = 4.

73. The sets of planes with the next five smaller interplanar spacings (after a_0) are shown in the diagram that follows.



(a) In terms of a_0, the second largest interplanar spacing is $a_0/\sqrt{2} = 0.7071a_0$.

(b) The third largest interplanar spacing is $a_0/\sqrt{5} = 0.4472a_0$.

(c) The fourth largest interplanar spacing is $a_0/\sqrt{10} = 0.3162a_0$.

(d) The fifth largest interplanar spacing is $a_0/\sqrt{13} = 0.2774a_0$.

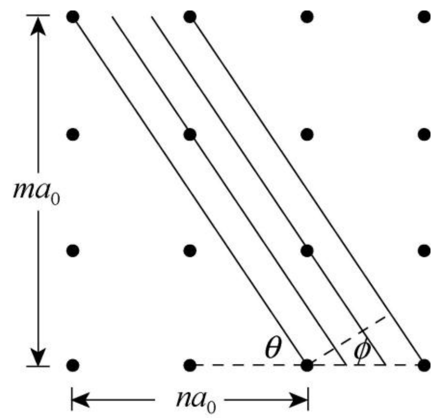
(e) The sixth largest interplanar spacing is $a_0/\sqrt{17} = 0.2425a_0$.

(f) Since a crystal plane passes through lattice points, its slope can be written as the ratio of two integers. Consider a set of planes with slope m/n, as shown in the diagram that follows. The first and last planes shown pass through adjacent lattice points along a horizontal line and there are $m - 1$ planes between. If h is the separation of the first and last planes, then the interplanar spacing is $d = h/m$. If the planes make the angle θ with the horizontal, then the normal to the planes (shown dashed) makes the angle $\phi = 90° - \theta$. The distance h is given by $h = a_0 \cos \phi$ and the interplanar spacing is $d = h/m = (a_0/m) \cos \phi$. Since $\tan \theta = m/n$, $\tan \phi = n/m$ and

$$\cos\phi = 1\Big/\sqrt{1+\tan^2\phi} = m\Big/\sqrt{n^2+m^2}\,.$$

Thus,

$$d = \frac{h}{m} = \frac{a_0\cos\phi}{m} = \frac{a_0}{\sqrt{n^2+m^2}}\,.$$



74. (a) We use Eq. 36-14:

$$\theta_R = 1.22\frac{\lambda}{d} = \frac{(1.22)\left(540\times10^{-6}\text{ mm}\right)}{5.0\,\text{mm}} = 1.3\times10^{-4}\text{ rad}\,.$$

(b) The linear separation is $D = L\theta_R = (160 \times 10^3 \text{ m})\,(1.3 \times 10^{-4} \text{ rad}) = 21$ m.

75. **THINK** Maxima of a diffraction grating pattern occur at angles θ given by $d \sin\theta = m\lambda$, where d is the slit separation, λ is the wavelength, and m is an integer.

EXPRESS The ruling separation is given by

$$d = \frac{1}{200\text{ mm}^{-1}} = 5.00\times10^{-3}\text{ mm} = 5.00\times10^{-6}\text{ m} = 5000\text{ nm}\,.$$

Letting $d \sin\theta = m\lambda$, we solve for λ:

$$\lambda = \frac{d\sin\theta}{m} = \frac{(5000\text{ nm})(\sin 30°)}{m} = \frac{2500\,\text{nm}}{m}$$

where $m = 1, 2, 3 \ldots$. In the visible light range m can assume the following values: $m_1 = 4$, $m_2 = 5$ and $m_3 = 6$.

(a) The longest wavelength corresponds to $m_1 = 4$ with $\lambda_1 = 2500$ nm/4 = 625 nm.

(b) The second longest wavelength corresponds to $m_2 = 5$ with $\lambda_2 = 2500$ nm/5 = 500 nm.

(c) The third longest wavelength corresponds to $m_3 = 6$ with $\lambda_3 = 2500$ nm/6 = 416 nm.

LEARN As shown above, only three values of *m* give wavelengths that are in the visible spectrum. Note that if the light incident on the diffraction grating is not monochromatic, a *spectrum* would be observed since the grating spreads out light into its component wavelength,

76. We combine Eq. 36-31 ($R = \lambda_{avg}/\Delta\lambda$) with Eq. 36-32 ($R = Nm$) and solve for *N*:

$$N = \frac{\lambda_{avg}}{m\,\Delta\lambda} = \frac{590.2\text{ nm}}{2\,(0.061\text{ nm})} = 4.84 \times 10^3 .$$

77. **THINK** The condition for a minimum of intensity in a single-slit diffraction pattern is given by $a \sin\theta = m\lambda$, where *a* is the slit width, λ is the wavelength, and *m* is an integer.

EXPRESS As a slit is narrowed, the pattern spreads outward, so the question about "minimum width" suggests that we are looking at the lowest possible values of *m* (the label for the minimum produced by light $\lambda = 600$ nm) and m' (the label for the minimum produced by light $\lambda' = 500$ nm). Since the angles are the same, then Eq. 36-3 leads to

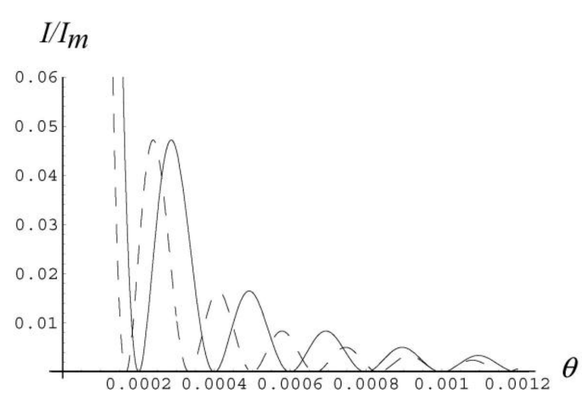
$$m\lambda = m'\lambda'$$

which leads to the choices $m = 5$ and $m' = 6$.

ANALYZE We find the slit width from Eq. 36-3:

$$a = \frac{m\lambda}{\sin\theta} = \frac{5(600\times10^{-9}\text{m})}{\sin(1.00\times10^{-9}\text{rad})} = 3.00\times10^{-3}\text{ m}.$$

LEARN The intensities of the diffraction are shown next (solid line for orange light, and dashed line for blue-green light). The angle $\theta = 0.001$ rad corresponds to $m = 5$ for the orange light, but $m' = 6$ for the blue-green light.



78. The central diffraction envelope spans the range $-\theta_1 < \theta < +\theta_1$ where $\theta_1 = \sin^{-1}(\lambda/a)$. The maxima in the double-slit pattern are located at

$$\theta_m = \sin^{-1}\frac{m\lambda}{d},$$

so that our range specification becomes

$$-\sin^{-1}\left(\frac{\lambda}{a}\right) < \sin^{-1}\left(\frac{m\lambda}{d}\right) < +\sin^{-1}\left(\frac{\lambda}{a}\right),$$

which we change (since sine is a monotonically increasing function in the fourth and first quadrants, where all these angles lie) to

$$-\frac{\lambda}{a} < \frac{m\lambda}{d} < +\frac{\lambda}{a}.$$

Rewriting this as $-d/a < m < +d/a$, we find $-6 < m < +6$, or, since m is an integer, $-5 \le m \le +5$. Thus, we find eleven values of m that satisfy this requirement.

79. **THINK** We relate the resolving power of a diffraction grating to the frequency range.

EXPRESS Since the resolving power of a grating is given by $R = \lambda/\Delta\lambda$ and by Nm, the range of wavelengths that can just be resolved in order m is $\Delta\lambda = \lambda/Nm$. Here N is the number of rulings in the grating and λ is the average wavelength. The frequency f is related to the wavelength by $f\lambda = c$, where c is the speed of light. This means $f\,\Delta\lambda + \lambda\Delta f = 0$, so

$$\Delta\lambda = -\frac{\lambda}{f}\Delta f = -\frac{\lambda^2}{c}\Delta f$$

where $f = c/\lambda$ is used. The negative sign means that an increase in frequency corresponds to a decrease in wavelength.

ANALYZE (a) Equating the two expressions for $\Delta\lambda$, we have

$$\frac{\lambda^2}{c}\Delta f = \frac{\lambda}{Nm}$$

and

$$\Delta f = \frac{c}{Nm\lambda}.$$

(b) The difference in travel time for waves traveling along the two extreme rays is $\Delta t = \Delta L/c$, where ΔL is the difference in path length. The waves originate at slits that are separated by $(N-1)d$, where d is the slit separation and N is the number of slits, so the path difference is $\Delta L = (N-1)d \sin\theta$ and the time difference is

$$\Delta t = \frac{(N-1)d\sin\theta}{c}.$$

If N is large, this may be approximated by $\Delta t = (Nd/c)\sin\theta$. The lens does not affect the travel time.

(c) Substituting the expressions we derived for Δt and Δf, we obtain

$$\Delta f\ \Delta t = \left(\frac{c}{Nm\lambda}\right)\left(\frac{N\,d\sin\theta}{c}\right) = \frac{d\sin\theta}{m\lambda} = 1.$$

The condition $d\sin\theta = m\lambda$ for a diffraction line is used to obtain the last result.

LEARN We take Δf to be positive and interpret it as the range of frequencies that can be resolved.

80. Eq. 36-14 gives the Rayleigh angle (in radians):

$$\theta_R = \frac{1.22\lambda}{d} = \frac{D}{L}$$

where the rationale behind the second equality is given in Sample Problem — "Pointillistic paintings use the diffraction of your eye." We are asked to solve for D and are given $\lambda = 500 \times 10^{-9}$ m, $d = 5.00 \times 10^{-3}$ m, and $L = 0.250$ m. Consequently, $D = 3.05 \times 10^{-5}$ m.

81. Consider two of the rays shown in Fig. 36-49, one just above the other. The extra distance traveled by the lower one may be found by drawing perpendiculars from where the top ray changes direction (point P) to the incident and diffracted paths of the lower one. Where these perpendiculars intersect the lower ray's paths are here referred to as points A and C. Where the bottom ray changes direction is point B. We note that angle $\angle APB$ is the same as ψ, and angle BPC is the same as θ (see Fig. 36-49). The difference in path lengths between the two adjacent light rays is

$$\Delta x = |AB| + |BC| = d\sin\psi + d\sin\theta.$$

The condition for bright fringes to occur is therefore

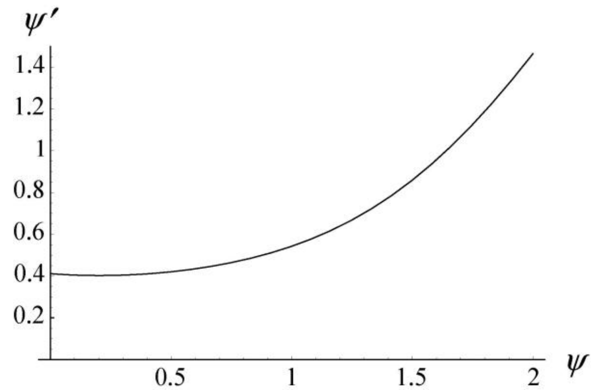
$$\Delta x = d(\sin\psi + \sin\theta) = m\lambda$$

where $m = 0, 1, 2, \ldots$. If we set $\psi = 0$ then this reduces to Eq. 36-25.

82. The angular deviation of a diffracted ray (the angle between the forward extrapolation of the incident ray and its diffracted ray) is $\psi' = \psi + \theta$. For m = 1, this becomes

$$\psi' = \psi + \theta = \psi + \sin^{-1}\left(\frac{\lambda}{d} - \sin\psi\right)$$

where the ratio λ/d = 0.40 using the values given in the problem statement. The graph of this is shown next (with radians used along both axes).



83. **THINK** For relatively wide slits, we consider both the interference of light from two slits, as well as the diffraction of light passing through each slit.

EXPRESS The central diffraction envelope spans the range $-\theta_1 < \theta < +\theta_1$ where $\theta_1 = \sin^{-1}(\lambda/a)$ is the angle that corresponds to the first diffraction minimum. The maxima in the double-slit pattern are at

$$\theta_m = \sin^{-1}\frac{m\lambda}{d},$$

so that our range specification becomes

$$-\sin^{-1}\left(\frac{\lambda}{a}\right) < \sin^{-1}\left(\frac{m\lambda}{d}\right) < +\sin^{-1}\left(\frac{\lambda}{a}\right),$$

which we change (since sine is a monotonically increasing function in the fourth and first quadrants, where all these angles lie) to

$$-\frac{\lambda}{a} < \frac{m\lambda}{d} < +\frac{\lambda}{a}.$$

The equation above sets the range of allowable values of m.

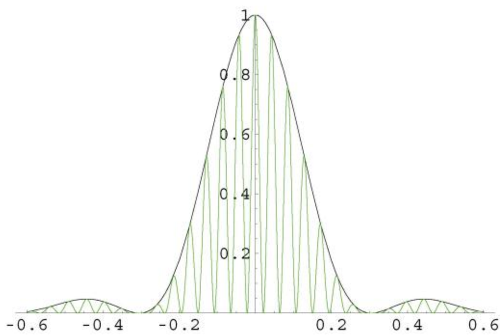
ANALYZE (a) Rewriting the equation as $-d/a < m < +d/a$, noting that d/a = (14 μm)/(2.0 μm) = 7, we arrive at the result $-7 < m < +7$, or (since m must be an integer) $-6 \le m \le +6$, which amounts to 13 distinct values for *m*. Thus, thirteen maxima are within the central envelope.

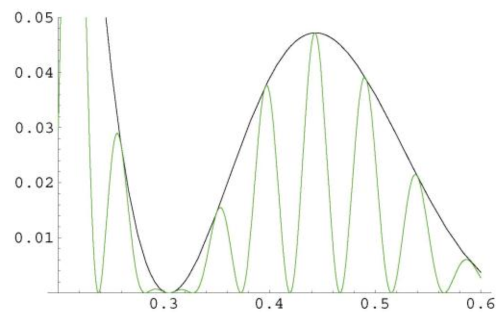
(b) The range (within *one* of the first-order envelopes) is now

$$-\sin^{-1}\left(\frac{\lambda}{a}\right) < \sin^{-1}\left(\frac{m\lambda}{d}\right) < +\sin^{-1}\left(\frac{2\lambda}{a}\right),$$

which leads to $d/a < m < 2d/a$ or $7 < m < 14$. Since *m* is an integer, this means $8 \leq m \leq 13$ which includes 6 distinct values for *m* in that one envelope. If we were to include the total from both first-order envelopes, the result would be twelve, but the wording of the problem implies six should be the answer (just one envelope).

LEARN The intensity of the double-slit interference experiment is plotted below. The central diffraction envelope contains 13 maxima, and the first-order envelope has 6 on each side.





84. The central diffraction envelope spans the range $-\theta_1 < \theta < +\theta_1$ where $\theta_1 = \sin^{-1}(\lambda/a)$. The maxima in the double-slit pattern are at

$$\theta_m = \sin^{-1}\frac{m\lambda}{d},$$

so that our range specification becomes

$$-\sin^{-1}\left(\frac{\lambda}{a}\right) < \sin^{-1}\left(\frac{m\lambda}{d}\right) < +\sin^{-1}\left(\frac{\lambda}{a}\right),$$

which we change (since sine is a monotonically increasing function in the fourth and first quadrants, where all these angles lie) to

$$-\frac{\lambda}{a} < \frac{m\lambda}{d} < +\frac{\lambda}{a}.$$

Rewriting this as $-d/a < m < +d/a$ we arrive at the result $m_{\max} < d/a \le m_{\max} + 1$. Due to the symmetry of the pattern, the multiplicity of the m values is $2m_{\max} + 1 = 17$ so that $m_{\max} = 8$, and the result becomes

$$8 < \frac{d}{a} \le 9$$

where these numbers are as accurate as the experiment allows (that is, "9" means "9.000" if our measurements are that good).

85. We see that the total number of lines on the grating is (1.8 cm)(1400/cm) = 2520 = N. Combining Eq. 36-31 and Eq. 36-32, we find

$$\Delta\lambda = \frac{\lambda_{\text{avg}}}{Nm} = \frac{450 \text{ nm}}{(2520)(3)} = 0.0595 \text{ nm} = 59.5 \text{ pm}.$$

86. Use of Eq. 36-21 leads to $D = \dfrac{1.22\lambda L}{d} = 6.1$ mm.

87. Following the method of Sample Problem — "Pointillistic paintings use the diffraction of your eye," we have

$$\frac{1.22\lambda}{d} = \frac{D}{L}$$

where $\lambda = 550 \times 10^{-9}$ m, $D = 0.60$ m, and $d = 0.0055$ m. Thus we get $L = 4.9 \times 10^3$ m.

88. We use Eq. 36-3 for $m = 2$: $m\lambda = a\sin\theta \;\Rightarrow\; \dfrac{a}{\lambda} = \dfrac{m}{\sin\theta} = \dfrac{2}{\sin 37°} = 3.3$.

89. We solve Eq. 36-25 for d:

$$d = \frac{m\lambda}{\sin\theta} = \frac{2(600\times10^{-9}\text{ m})}{\sin 33°} = 2.203\times10^{-6}\text{ m} = 2.203\times10^{-4}\text{ cm}$$

which is typically expressed in reciprocal form as the "number of lines per centimeter" (or per millimeter, or per inch):

$$\frac{1}{d} = 4539 \text{ lines/cm} .$$

The full width is 3.00 cm, so the number of lines is $(4539\,/\text{cm})(3.00\text{ cm}) = 1.36 \times 10^4$.

90. Although the angles in this problem are not particularly big (so that the small angle approximation could be used with little error), we show the solution appropriate for large as well as small angles (that is, we do not use the small angle approximation here). Equation 36-3 gives

$$m\lambda = a\sin\theta \;\Rightarrow\; \theta = \sin^{-1}(m\lambda/a) = \sin^{-1}[2(0.42\ \mu\text{m})/(5.1\ \mu\text{m})] = 9.48°.$$

The geometry of Figure 35-10(a) is a useful reference (even though it shows a double slit instead of the single slit that we are concerned with here). We see in that figure the relation between y, D, and θ:

$$y = D\tan\theta = (3.2\text{ m})\tan(9.48°) = 0.534\text{ m}.$$

91. The problem specifies d = 12/8900 using the mm unit, and we note there are no refraction angles greater than 90°. We convert λ = 500 nm to 5×10^{-4} mm and solve Eq. 36-25 for "$m_{\max}$" (realizing it might not be an integer):

$$m_{\max} = \frac{d\sin 90°}{\lambda} = \frac{12}{(8900)(5\times10^{-4})} \approx 2$$

where we have rounded down. There are no values of m (for light of wavelength λ) greater than m = 2.

92. We denote the Earth-Moon separation as L. The energy of the beam of light that is projected onto the Moon is concentrated in a circular spot of diameter d_1, where $d_1/L = 2\theta_R = 2(1.22\lambda/d_0)$, with d_0 the diameter of the mirror on Earth. The fraction of energy picked up by the reflector of diameter d_2 on the Moon is then $\eta' = (d_2/d_1)^2$. This reflected light, upon reaching the Earth, has a circular cross section of diameter d_3 satisfying

$$d_3/L = 2\theta_R = 2(1.22\lambda/d_2).$$

The fraction of the reflected energy that is picked up by the telescope is then $\eta'' = (d_0/d_3)^2$. Consequently, the fraction of the original energy picked up by the detector is

$$\eta = \eta'\eta'' = \left(\frac{d_0}{d_3}\right)^2\left(\frac{d_2}{d_1}\right)^2 = \left[\frac{d_0 d_2}{(2.44\lambda d_{em}/d_0)(2.44\lambda d_{em}/d_2)}\right]^2 = \left(\frac{d_0 d_2}{2.44\lambda d_{em}}\right)^4$$
$$= \left[\frac{(2.6\text{ m})(0.10\text{ m})}{2.44(0.69\times10^{-6}\text{ m})(3.82\times10^{8}\text{ m})}\right]^4 \approx 4\times10^{-13}.$$

93. Since we are considering the *diameter* of the central diffraction maximum, then we are working with *twice* the Rayleigh angle. Using notation similar to that in Sample Problem — "Pointillistic paintings use the diffraction of your eye," we have $2(1.22\lambda/d) = D/L$. Therefore,

$$d = 2\frac{1.22\lambda L}{D} = 2\frac{(1.22)(500\times10^{-9}\text{ m})(3.54\times10^{5}\text{ m})}{9.1\text{ m}} = 0.047\text{ m}.$$

94. Letting $d\sin\theta = (L/N)\sin\theta = m\lambda$, we get

$$\lambda = \frac{(L/N)\sin\theta}{m} = \frac{(1.0\times10^{7}\ \text{nm})(\sin\ 30°)}{(1)(10000)} = 500\ \text{nm}\ .$$

95. **THINK** We use phasors to explore how doubling slit width changes the intensity of the central maximum of diffraction and the energy passing through the slit.

EXPRESS We imagine dividing the original slit into N strips and represent the light from each strip, when it reaches the screen, by a phasor. Then, at the central maximum in the diffraction pattern, we would add the N phasors, all in the same direction and each with the same amplitude. We would find that the intensity there is proportional to N^2.

ANALYZE If we double the slit width, we need $2N$ phasors if they are each to have the amplitude of the phasors we used for the narrow slit. The intensity at the central maximum is proportional to $(2N)^2$ and is, therefore, four times the intensity for the narrow slit. The energy reaching the screen per unit time, however, is only twice the energy reaching it per unit time when the narrow slit is in place. The energy is simply redistributed. For example, the central peak is now half as wide and the integral of the intensity over the peak is only twice the analogous integral for the narrow slit.

LEARN From the discussion above, we see that the intensity of the central maximum increases as N^2. The dependence arises from the following two considerations: (1) The total power reaching the screen is proportional to N, and (2) the width of each maximum (distance between two adjacent minima) is proportional to $1/N$.

96. The condition for a minimum in a single-slit diffraction pattern is given by Eq. 36-3, which we solve for the wavelength:

$$\lambda = \frac{a\sin\theta}{m} = \frac{(0.022\,\text{mm})\sin\ 1.8°}{1} = 6.91\times10^{-4}\ \text{mm} = 691\ \text{nm}\ .$$

97. Equation 36-14 gives the Rayleigh angle (in radians):

$$\theta_R = \frac{1.22\lambda}{d} = \frac{D}{L}$$

where the rationale behind the second equality is given in Sample Problem — "Pointillistic paintings use the diffraction of your eye." We are asked to solve for d and are given $\lambda = 550\times10^{-9}$ m, $D = 30\times10^{-2}$ m, and $L = 160\times10^{3}$ m. Consequently, we obtain d = 0.358 m $\approx 36\ \text{cm}$.

98. Following Sample Problem — "Pointillistic paintings use the diffraction of your eye," we use Eq. 36-17 and obtain $L = \dfrac{Dd}{1.22\lambda} = 164\,\text{m}$.

99. (a) Use of Eq. 36-25 for the limit-wavelengths (λ_1 = 700 nm and λ_2 = 550 nm) leads to the condition

$$m_1\lambda_1 \geq m_2\lambda_2$$

for $m_1 + 1 = m_2$ (the low end of a high-order spectrum is what is overlapping with the high end of the next-lower-order spectrum). Assuming equality in the above equation, we can solve for "m_1" (realizing it might not be an integer) and obtain $m_1 \approx 4$ where we have rounded *up*. It is the fourth-order spectrum that is the lowest-order spectrum to overlap with the next higher spectrum.

(b) The problem specifies d = (1/200) mm, and we note there are no refraction angles greater than 90°. We concentrate on the largest wavelength λ = 700 nm = 7×10^{-4} mm and solve Eq. 36-25 for "m_{max}" (realizing it might not be an integer):

$$m_{\max} = \frac{d\sin 90°}{\lambda} = \frac{(1/200)\text{ mm}}{7\times10^{-4}\text{ mm}} \approx 7$$

where we have rounded down. There are no values of m (for the appearance of the full spectrum) greater than m = 7.

100. (a) Maxima of a diffraction grating pattern occur at angles θ given by $d \sin\theta = m\lambda$, where d is the slit separation, λ is the wavelength, and m is an integer. With $\theta = 30°$, and $d = (1\text{ mm})/200 = 5.0\times10^{-6}\text{m}$, the wavelengths for the mth order maxima are given by

$$\lambda = \frac{d\sin\theta}{m} = \frac{(5.0\times10^{-6}\text{m})\sin 30°}{m} = \frac{2.5\times10^{-6}\text{m}}{m} = \frac{2500\text{ nm}}{m}$$

For the light to be in the visible spectrum (400 – 750 nm), the values of m are m = 4, 5, and 6. The wavelengths are: $\lambda_4 = (2500\text{ nm})/4 = 625\text{ nm}$, $\lambda_5 = (2500\text{ nm})/5 = 500\text{ nm}$, and $\lambda_6 = (2500\text{ nm})/6 = 417\text{ nm}$.

(c) The three wavelengths correspond to orange, blue-green, and violet, respectively.

101. The dispersion of a grating is given by $D = d\theta/d\lambda$, where θ is the angular position of a line associated with wavelength λ. The angular position and wavelength are related by $\mathbf{d}\sin\theta = m\lambda$, where $\mathbf{d}$ is the slit separation (which we made boldfaced in order not to confuse it with the d used in the derivative, below) and m is an integer. We differentiate this expression with respect to θ to obtain

$$\frac{d\theta}{d\lambda}\mathbf{d}\cos\theta = m,$$

or

$$D = \frac{d\theta}{d\lambda} = \frac{m}{\mathbf{d}\cos\theta}.$$

Now $m = (\mathbf{d}/\lambda)\sin\theta$, so $D = \dfrac{\mathbf{d}\sin\theta}{\mathbf{d}\lambda\cos\theta} = \dfrac{\tan\theta}{\lambda}$.

102. (a) Employing Eq. 36-3 with the small angle approximation ($\sin\theta \approx \tan\theta = y/D$ where y locates the minimum relative to the middle of the pattern), we find (with $m = 1$)

$$D = \frac{ya}{m\lambda} = \frac{(0.90 \text{ mm})(0.40 \text{ mm})}{4.50\times10^{-4} \text{ mm}} = 800 \text{ mm} = 80 \text{ cm}$$

which places the screen 80 cm away from the slit.

(b) The above equation gives for the value of y (for $m = 3$)

$$y = \frac{(3)\lambda D}{a} = \frac{(3)(4.50\times10^{-4} \text{ mm})(800 \text{ mm})}{(0.40 \text{ mm})} = 2.7 \text{ mm} .$$

Subtracting this from the first minimum position y = 0.9 mm, we find the result $\Delta y = 1.8$ mm.

103. (a) We require that $\sin\theta = m\lambda_{1,2}/d \le \sin 30°$, where m = 1, 2 and λ_1 = 500 nm. This gives

$$d \ge \frac{2\lambda_s}{\sin 30°} = \frac{2(600\text{nm})}{\sin 30°} = 2400\text{nm} = 2.4\mu\text{m}.$$

For a grating of given total width L we have $N = L/d \propto d^{-1}$, so we need to minimize d to maximize $R = mN \propto d^{-1}$. Thus we choose d = 2400 nm = 2.4 μm.

(b) Let the third-order maximum for λ_2 = 600 nm be the first minimum for the single-slit diffraction profile. This requires that $d\sin\theta = 3\lambda_2 = a\sin\theta$, or

$$a = d/3 = 2400 \text{ nm}/3 = 800 \text{ nm} = 0.80 \ \mu\text{m}.$$

(c) Letting $\sin\theta = m_{\max}\lambda_2/d \le 1$, we obtain

$$m_{\max} \le \frac{d}{\lambda_2} = \frac{2400\text{ nm}}{800\text{ nm}} = 3 .$$

Since the third order is missing the only maxima present are the ones with m = 0, 1 and 2. Thus, the largest order of maxima produced by the grating is m = 2.

104. For λ = 0.10 nm, we have scattering for order m, and for λ' = 0.075 nm, we have scattering for order m'. From Eq. 36-34, we see that we must require $m\lambda = m'\lambda'$, which suggests (looking for the smallest integer solutions) that m = 3 and m' = 4. Returning with this result and with d = 0.25 nm to Eq. 36-34, we obtain

$$\theta = \sin^{-1}\frac{m\lambda}{2d} = 37° .$$

Studying Figure 36-30, we conclude that the angle between incident and scattered beams is 180° – 2θ = 106°.

105. The key trigonometric identity used in this proof is $\sin(2\theta) = 2\sin\theta\cos\theta$. Now, we wish to show that Eq. 36-19 becomes (when $d = a$) the pattern for a single slit of width $2a$ (see Eq. 36-5 and Eq. 36-6):

$$I(\theta) = I_m\left(\frac{\sin(2\pi a\sin\theta/\lambda)}{2\pi a\sin\theta/\lambda}\right)^2 .$$

We note from Eq. 36-20 and Eq. 36-21, that the parameters β and α are identical in this case (when $d = a$), so that Eq. 36-19 becomes

$$I(\theta) = I_m\left(\frac{\cos(\pi a\sin\theta/\lambda)\sin(\pi a\sin\theta/\lambda)}{\pi a\sin\theta/\lambda}\right)^2 .$$

Multiplying numerator and denominator by 2 and using the trig identity mentioned above, we obtain

$$I(\theta) = I_m\left(\frac{2\cos(\pi a\sin\theta/\lambda)\sin(\pi a\sin\theta/\lambda)}{2\pi a\sin\theta/\lambda}\right)^2 = I_m\left(\frac{\sin(2\pi a\sin\theta/\lambda)}{2\pi a\sin\theta/\lambda}\right)^2$$

which is what we set out to show.

106. Employing Eq. 36-3, we find (with m = 3 and all lengths in μm)

$$\theta = \sin^{-1}\frac{m\lambda}{a} = \sin^{-1}\frac{(3)(0.5)}{2}$$

which yields θ = 48.6°. Now, we use the experimental geometry ($\tan\theta = y/D$ where y locates the minimum relative to the middle of the pattern) to find

$$y = D\tan\theta = 2.27 \text{ m}.$$

107. (a) The central diffraction envelope spans the range $-\theta_1 < \theta < +\theta_1$ where

$$\theta_1 = \sin^{-1}\frac{\lambda}{a},$$

which could be further simplified *if* the small-angle approximation were justified (which it is *not*, since a is so small). The maxima in the double-slit pattern are at

$$\theta_m = \sin^{-1}\frac{m\lambda}{d},$$

so that our range specification becomes

$$-\sin^{-1}\left(\frac{\lambda}{a}\right) < \sin^{-1}\left(\frac{m\lambda}{d}\right) < +\sin^{-1}\left(\frac{\lambda}{a}\right),$$

which we change (since sine is a monotonically increasing function in the fourth and first quadrants, where all these angles lie) to

$$-\frac{\lambda}{a} < \frac{m\lambda}{d} < +\frac{\lambda}{a}.$$

Rewriting this as $-d/a < m < +d/a$ we arrive at the result $m_{\max} < d/a \le m_{\max} + 1$. Due to the symmetry of the pattern, the multiplicity of the m values is $2m_{\max} + 1 = 17$ so that $m_{\max} = 8$, and the result becomes

$$8 < \frac{d}{a} \le 9$$

where these numbers are as accurate as the experiment allows (that is, "9" means "9.000" if our measurements are that good).

108. We refer (somewhat sloppily) to the 400 nm wavelength as “blue” and the 700 nm wavelength as “red.” Consider Eq. 36-25 ($m\lambda = d\sin\theta$), for the 3$^{\text{rd}}$ order blue, and also for the 2$^{\text{nd}}$ order red:

$$(3)\,\lambda_{\text{blue}} = 1200\text{ nm} = d\sin(\theta_{\text{blue}})$$

$$(2)\,\lambda_{\text{red}} = 1400\text{ nm} = d\sin(\theta_{\text{red}})\,.$$

Since sine is an increasing function of angle (in the first quadrant) then the above set of values make clear that $\theta_{\text{red (second order)}} > \theta_{\text{blue (third order)}}$ which shows that the spectrums overlap (regardless of the value of d).

109. One strategy is to divide Eq. 36-25 by Eq. 36-3, assuming the same angle (a point we’ll come back to, later) and the same light wavelength for both:

$$\frac{m}{m'} = \frac{m\lambda}{m'\lambda} = \frac{d\sin\theta}{a\sin\theta} = \frac{d}{a}\,.$$

We recall that d is measured from middle of transparent strip to the middle of the next transparent strip, which in this particular setup means $d = 2a$. Thus, $m/m' = 2$, or $m = 2m'$.

Now we interpret our result. First, the division of the equations is not valid when $m = 0$ (which corresponds to $\theta = 0$), so our remarks do not apply to the $m = 0$ maximum. Second, Eq. 36-25 gives the "bright" interference results, and Eq. 36-3 gives the "dark" diffraction results (where the latter overrules the former in places where they coincide – see Figure 36-17 in the textbook). For $m' =$ any nonzero integer, the relation $m = 2m'$ implies that $m =$ any nonzero *even* integer. As mentioned above, these are occurring at the same angle, so the even integer interference maxima are eliminated by the diffraction minima.

110. The derivation is similar to that used to obtain Eq. 36-27. At the first minimum beyond the mth principal maximum, two waves from adjacent slits have a phase difference of $\Delta\phi = 2\pi m + (2\pi/N)$, where N is the number of slits. This implies a difference in path length of

$$\Delta L = (\Delta\phi/2\pi)\lambda = m\lambda + (\lambda/N).$$

If θ_m is the angular position of the mth maximum, then the difference in path length is also given by $\Delta L = d\sin(\theta_m + \Delta\theta)$. Thus

$$d\sin(\theta_m + \Delta\theta) = m\lambda + (\lambda/N).$$

We use the trigonometric identity

$$\sin(\theta_m + \Delta\theta) = \sin\theta_m\cos\Delta\theta + \cos\theta_m\sin\Delta\theta.$$

Since $\Delta\theta$ is small, we may approximate $\sin\Delta\theta$ by $\Delta\theta$ in radians and $\cos\Delta\theta$ by unity. Thus,

$$d\sin\theta_m + d\,\Delta\theta\cos\theta_m = m\lambda + (\lambda/N).$$

We use the condition $d\sin\theta_m = m\lambda$ to obtain $d\,\Delta\theta\cos\theta_m = \lambda/N$ and

$$\Delta\theta = \frac{\lambda}{N\,d\cos\theta_m}.$$

111. There are two unknowns, the x-ray wavelength λ and the plane separation d, so data for scattering at two angles from the same planes should suffice. The observations obey Bragg's law, so

$$2d\sin\theta_1 = m_1\lambda,\ 2d\sin\theta_2 = m_2\lambda.$$

However, these cannot be solved for the unknowns. For example, we can use the first equation to eliminate λ from the second. We obtain

$$m_2\sin\theta_1 = m_1\sin\theta_2,$$

an equation that does not contain either of the unknowns.

112. The problem specifies d = (1 mm)/500 = 2.00 μm unit, and we note there are no refraction angles greater than 90°. We concentrate on the largest wavelength λ = 700 nm = 0.700 μm and solve Eq. 36-25 for "m_{max}" (realizing it might not be an integer):

$$m_{\max} = \frac{d \sin 90°}{\lambda} = \frac{d}{\lambda} = \frac{2.00\ \mu\text{m}}{0.700\ \mu\text{m}} \approx 2$$

where we have rounded down. There are no values of m (for appearance of the full spectrum) greater than m = 2.

113. When the speaker phase difference is π rad (180°), we expect to see the "reverse" of Fig. 36-15 [translated into the acoustic context, so that "bright" becomes "loud" and "dark" becomes "quiet"]. That is, with 180° phase difference, all the peaks in Fig. 36-15 become valleys and all the valleys become peaks. As the phase changes from zero to 180° (and similarly for the change from 180° back to 360° = original pattern), the peaks should shift (and change height) in a continuous fashion – with the most dramatic feature being a large "dip" in the center diffraction envelope which deepens until it seems to split the central maximum into smaller diffraction maxima which (once the phase difference reaches π rad) will be located at angles given by a sinθ = ± λ. How many interference fringes would actually "be inside" each of these smaller diffraction maxima would, of course, depend on the particular values of a, λ and d.

114. From d sin θ = mλ, where d is the slit separation, λ is the wavelength, and m is an integer, we write

$$d \sin(\theta + \Delta\theta) = m(\lambda + \Delta\lambda)$$

Subtracting the first equation from the second gives

$$d\left[\sin(\theta + \Delta\theta) - \sin\theta\right] = m(\lambda + \Delta\lambda) - m\lambda = m\Delta\lambda\,.$$

Noting that

$$\lim_{\Delta\theta \to 0} \frac{\sin(\theta + \Delta\theta) - \sin(\theta + \Delta\theta)}{\Delta\theta} = \cos\theta\,,$$

the above expression simplifies to

$$\cos\theta = \frac{m\Delta\lambda}{d\Delta\theta}\,.$$

Thus,

$$\Delta\theta = \frac{m\Delta\lambda}{d\cos\theta} = \frac{m\Delta\lambda}{d\sqrt{1-\sin^2\theta}} = \frac{m\Delta\lambda}{d\sqrt{1-(m\lambda/d)^2}} = \frac{m\Delta\lambda}{\sqrt{d^2-(m\lambda)^2}} = \frac{\Delta\lambda}{\sqrt{(d/m)^2-\lambda^2}}\,.$$

Chapter 37

1. From the time dilation equation $\Delta t = \gamma\Delta t_0$ (where Δt_0 is the proper time interval, $\gamma = 1/\sqrt{1-\beta^2}$, and $\beta = v/c$), we obtain

$$\beta = \sqrt{1-\left(\frac{\Delta t_0}{\Delta t}\right)^2}.$$

The proper time interval is measured by a clock at rest relative to the muon. Specifically, $\Delta t_0 = 2.2000\ \mu$s. We are also told that Earth observers (measuring the decays of moving muons) find $\Delta t = 16.000\ \mu$s. Therefore,

$$\beta = \sqrt{1-\left(\frac{2.2000\ \mu\text{s}}{16.000\ \mu\text{s}}\right)^2} = 0.99050.$$

2. (a) We find β from $\gamma = 1/\sqrt{1-\beta^2}$:

$$\beta = \sqrt{1-\frac{1}{\gamma^2}} = \sqrt{1-\frac{1}{(1.0100000)^2}} = 0.14037076.$$

(b) Similarly, $\beta = \sqrt{1-(10.000000)^{-2}} = 0.99498744.$

(c) In this case, $\beta = \sqrt{1-(100.00000)^{-2}} = 0.99995000.$

(d) The result is $\beta = \sqrt{1-(1000.0000)^{-2}} = 0.99999950.$

3. (a) The round-trip (discounting the time needed to "turn around") should be one year according to the clock you are carrying (this is your proper time interval Δt_0) and 1000 years according to the clocks on Earth, which measure Δt. We solve Eq. 37-7 for β:

$$\beta = \sqrt{1-\left(\frac{\Delta t_0}{\Delta t}\right)^2} = \sqrt{1-\left(\frac{1\text{y}}{1000\text{y}}\right)^2} = 0.99999950.$$

(b) The equations do not show a dependence on acceleration (or on the direction of the velocity vector), which suggests that a circular journey (with its constant magnitude centripetal acceleration) would give the same result (if the speed is the same) as the one described in the problem. A more careful argument can be given to support this, but it should be admitted that this is a fairly subtle question that has occasionally precipitated debates among professional physicists.

4. Due to the time-dilation effect, the time between initial and final ages for the daughter is longer than the four years experienced by her father:

$$t_{f\,\text{daughter}} - t_{i\,\text{daughter}} = \gamma(4.000\text{ y})$$

where γ is the Lorentz factor (Eq. 37-8). Letting T denote the age of the father, then the conditions of the problem require

$$T_i = t_{i\,\text{daughter}} + 20.00\text{ y}\,,\quad T_f = t_{f\,\text{daughter}} - 20.00\text{ y}\ .$$

Since $T_f - T_i = 4.000$ y, then these three equations combine to give a single condition from which γ can be determined (and consequently v):

$$44 = 4\gamma \quad\Rightarrow\quad \gamma = 11 \quad\Rightarrow\quad \beta = \frac{2\sqrt{30}}{11} = 0.9959.$$

5. In the laboratory, it travels a distance $d = 0.00105$ m $= vt$, where $v = 0.992c$ and t is the time measured on the laboratory clocks. We can use Eq. 37-7 to relate t to the proper lifetime of the particle t_0:

$$t = \frac{t_0}{\sqrt{1-(v/c)^2}} \quad\Rightarrow\quad t_0 = t\sqrt{1-\left(\frac{v}{c}\right)^2} = \frac{d}{0.992c}\sqrt{1-0.992^2}$$

which yields $t_0 = 4.46 \times 10^{-13}$ s $= 0.446$ ps.

6. From the value of Δt in the graph when $\beta = 0$, we infer than Δt_o in Eq. 37-9 is 8.0 s. Thus, that equation (which describes the curve in Fig. 37-22) becomes

$$\Delta t = \frac{\Delta t_0}{\sqrt{1-(v/c)^2}} = \frac{8.0\text{ s}}{\sqrt{1-\beta^2}}\,.$$

If we set $\beta = 0.98$ in this expression, we obtain approximately 40 s for Δt.

7. We solve the time dilation equation for the time elapsed (as measured by Earth observers):

$$\Delta t = \frac{\Delta t_0}{\sqrt{1-(0.9990)^2}}$$

where Δt_0 = 120 y. This yields Δt = 2684 y $\approx 2.68\times10^3$ y.

8. The contracted length of the tube would be

$$L = L_0\sqrt{1-\beta^2} = (3.00\,\text{m})\sqrt{1-(0.999987)^2} = 0.0153\,\text{m}.$$

9. **THINK** The length of the moving spaceship is measured to be shorter by a stationary observer

EXPRESS Let the rest length of the spaceship be L_0. The length measured by the timing station is

$$L = L_0\sqrt{1-(v/c)^2}.$$

ANALYZE (a) The rest length is L_0 = 130 m. With v = 0.740c, we obtain

$$L = L_0\sqrt{1-(v/c)^2} = (130\,\text{m})\sqrt{1-(0.740)^2} = 87.4\,\text{m}.$$

(b) The time interval for the passage of the spaceship is

$$\Delta t = \frac{L}{v} = \frac{87.4\,\text{m}}{(0.740)(3.00\times10^8\,\text{m/s})} = 3.94\times10^{-7}\,\text{s}.$$

LEARN The length of the spaceship appears to be contracted by a factor of

$$\gamma = \frac{1}{\sqrt{1-(v/c)^2}} = \frac{1}{\sqrt{1-(0.740)^2}} = 1.487.$$

10. Only the "component" of the length in the x direction contracts, so its y component stays

$$\ell'_y = \ell_y = \ell\sin 30^\circ = (1.0\text{ m})(0.50) = 0.50\,\text{m}$$

while its x component becomes

$$\ell'_x = \ell_x\sqrt{1-\beta^2} = (1.0\text{ m})(\cos 30^\circ)\sqrt{1-(0.90)^2} = 0.38\,\text{m}.$$

Therefore, using the Pythagorean theorem, the length measured from S' is

$$\ell' = \sqrt{(\ell'_x)^2 + (\ell'_y)^2} = \sqrt{(0.38\text{ m})^2 + (0.50\text{ m})^2} = 0.63\,\text{m}.$$

11. The length L of the rod, as measured in a frame in which it is moving with speed v parallel to its length, is related to its rest length L_0 by $L = L_0/\gamma$, where $\gamma = 1/\sqrt{1-\beta^2}$ and $\beta = v/c$. Since γ must be greater than 1, L is less than L_0. For this problem, L_0 = 1.70 m and β = 0.630, so

$$L = L_0\sqrt{1-\beta^2} = (1.70\,\text{m})\sqrt{1-(0.630)^2} = 1.32\,\text{m}.$$

12. (a) We solve Eq. 37-13 for v and then plug in:

$$\beta = \sqrt{1-\left(\frac{L}{L_0}\right)^2} = \sqrt{1-\left(\frac{1}{2}\right)^2} = 0.866.$$

(b) The Lorentz factor in this case is $\gamma = \dfrac{1}{\sqrt{1-(v/c)^2}} = 2.00$.

13. (a) The speed of the traveler is $v = 0.99c$, which may be equivalently expressed as 0.99 ly/y. Let d be the distance traveled. Then, the time for the trip as measured in the frame of Earth is

$$\Delta t = d/v = (26 \text{ ly})/(0.99 \text{ ly/y}) = 26.26 \text{ y}.$$

(b) The signal, presumed to be a radio wave, travels with speed c and so takes 26.0 y to reach Earth. The total time elapsed, in the frame of Earth, is

$$26.26 \text{ y} + 26.0 \text{ y} = 52.26 \text{ y}.$$

(c) The proper time interval is measured by a clock in the spaceship, so $\Delta t_0 = \Delta t/\gamma$. Now

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-(0.99)^2}} = 7.09.$$

Thus, Δt_0 = (26.26 y)/(7.09) = 3.705 y.

14. From the value of L in the graph when $\beta = 0$, we infer that L_0 in Eq. 37-13 is 0.80 m. Thus, that equation (which describes the curve in Fig. 37-23) with SI units understood becomes

$$L = L_0\sqrt{1-(v/c)^2} = (0.80\,\text{m})\sqrt{1-\beta^2}\ .$$

If we set $\beta = 0.95$ in this expression, we obtain approximately 0.25 m for L.

15. (a) Let d = 23000 ly = 23000 c y, which would give the distance in meters if we included a conversion factor for years $\rightarrow$ seconds. With Δt_0 = 30 y and $\Delta t = d/v$ (see Eq. 37-10), we wish to solve for v from Eq. 37-7. Our first step is as follows:

$$\Delta t = \frac{d}{v} = \frac{\Delta t_0}{\sqrt{1-\beta^2}} \quad \Rightarrow \quad \frac{23000\ \text{y}}{\beta} = \frac{30\ \text{y}}{\sqrt{1-\beta^2}},$$

at which point we can cancel the unit year and manipulate the equation to solve for the speed parameter β. This yields

$$\beta = \frac{1}{\sqrt{1+(30/23000)^2}} = 0.99999915.$$

(b) The Lorentz factor is $\gamma = 1/\sqrt{1-\beta^2} = 766.6680752$. Thus, the length of the galaxy measured in the traveler's frame is

$$L = \frac{L_0}{\gamma} = \frac{23000\ \text{ly}}{766.6680752} = 29.99999\ \text{ly} \approx 30\ \text{ly}.$$

16. The "coincidence" of $x = x' = 0$ at $t = t' = 0$ is important for Eq. 37-21 to apply without additional terms. In part (a), we apply these equations directly with

$$v = +0.400c = 1.199 \times 10^8\ \text{m/s},$$

and in part (c) we simply change $v \to -v$ and recalculate the primed values.

(a) The position coordinate measured in the S' frame is

$$x' = \gamma(x - vt) = \frac{x - vt}{\sqrt{1-\beta^2}} = \frac{3.00\times10^8\ \text{m} - (1.199\times10^8\ \text{m/s})(2.50\ \text{s})}{\sqrt{1-(0.400)^2}} = 2.7\times10^5\ \text{m} \approx 0,$$

where we conclude that the numerical result (2.7×10^5 m or 2.3×10^5 m depending on how precise a value of v is used) is not meaningful (in the significant figures sense) and should be set equal to zero (that is, it is "consistent with zero" in view of the statistical uncertainties involved).

(b) The time coordinate measured in the S' frame is

$$t' = \gamma\left(t - \frac{vx}{c^2}\right) = \frac{t - \beta x/c}{\sqrt{1-\beta^2}} = \frac{2.50\ \text{s} - (0.400)(3.00\times10^8\ \text{m})/2.998\times10^8\ \text{m/s}}{\sqrt{1-(0.400)^2}} = 2.29\ \text{s}.$$

(c) Now, we obtain

$$x' = \frac{x+vt}{\sqrt{1-\beta^2}} = \frac{3.00\times10^8\ \text{m} + \left(1.199\times10^8\ \text{m/s}\right)\left(2.50\ \text{s}\right)}{\sqrt{1-\left(0.400\right)^2}} = 6.54\times10^8\ \text{m}.$$

(d) Similarly,

$$t' = \gamma\left(t+\frac{vx}{c^2}\right) = \frac{2.50\text{s} + \left(0.400\right)\left(3.00\times10^8\ \text{m}\right)/\,2.998\times10^8\,\text{m/s}}{\sqrt{1-\left(0.400\right)^2}} = 3.16\text{s}.$$

17. **THINK** We apply Lorentz transformation to calculate x' and t' according to an observer in S'.

EXPRESS The proper time is not measured by clocks in either frame S or frame S' since a single clock at rest in either frame cannot be present at the origin and at the event. The full Lorentz transformation must be used:

$$x' = \gamma(x-vt),\ \ t' = \gamma(t-\beta x/c)$$

where $\beta = v/c = 0.950$ and

$$\gamma = 1\sqrt{1-\beta^2} = 1/\sqrt{1-(0.950)^2} = 3.20256\,.$$

ANALYZE (a) Thus, the spatial coordinate in S' is

$$\begin{aligned} x' &= \gamma(x-vt) = (3.20256)\left(100\times10^3\,\text{m} - (0.950)(2.998\times10^8\ \text{m/s})(200\times10^{-6}\text{s})\right) \\ &= 1.38\times10^5\ \text{m} = 138\ \text{km}. \end{aligned}$$

(b) The temporal coordinate in S' is

$$\begin{aligned} t' &= \gamma(t-\beta x/c) = (3.20256)\left[200\times10^{-6}\,\text{s} - \frac{(0.950)(100\times10^3\ \text{m})}{2.998\times10^8\ \text{m/s}}\right] \\ &= -3.74\times10^{-4}\ \text{s} = -374\,\mu\text{s}\ . \end{aligned}$$

LEARN The time and the location of the collision recorded by an observer S' are different than that by another observer in S.

18. The "coincidence" of $x = x' = 0$ at $t = t' = 0$ is important for Eq. 37-21 to apply without additional terms. We label the event coordinates with subscripts: $(x_1, t_1) = (0, 0)$ and $(x_2, t_2) = (3000\ \text{m}, 4.0\times10^{-6}\ \text{s})$.

(a) We expect $(x'_1, t'_1) = (0, 0)$, and this may be verified using Eq. 37-21.

(b) We now compute (x'_2, t'_2), assuming v = +0.60c = +1.799 × 10^8 m/s (the sign of v is not made clear in the problem statement, but the figure referred to, Fig. 37-9, shows the motion in the positive x direction).

$$x'_2 = \frac{x - vt}{\sqrt{1-\beta^2}} = \frac{3000 \text{ m} - (1.799\times10^8 \text{ m/s})(4.0\times10^{-6} \text{ s})}{\sqrt{1-(0.60)^2}} = 2.85\times10^3 \text{ m}$$

$$t'_2 = \frac{t - \beta x/c}{\sqrt{1-\beta^2}} = \frac{4.0\times10^{-6} \text{ s} - (0.60)(3000 \text{ m})/(2.998\times10^8 \text{ m/s})}{\sqrt{1-(0.60)^2}} = -2.5\times10^{-6} \text{ s}$$

(c) The two events in frame S occur in the order: first 1, then 2. However, in frame S' where $t'_2 < 0$, they occur in the reverse order: first 2, then 1. So the two observers see the two events in the reverse sequence.

We note that the distances $x_2 - x_1$ and $x'_2 - x'_1$ are larger than how far light can travel during the respective times ($c(t_2 - t_1) = 1.2$ km and $c\,|t'_2 - t'_1| \approx 750$ m), so that no inconsistencies arise as a result of the order reversal (that is, no signal from event 1 could arrive at event 2 or vice versa).

19. (a) We take the flashbulbs to be at rest in frame S, and let frame S' be the rest frame of the second observer. Clocks in neither frame measure the proper time interval between the flashes, so the full Lorentz transformation (Eq. 37-21) must be used. Let t_s be the time and x_s be the coordinate of the small flash, as measured in frame S. Then, the time of the small flash, as measured in frame S', is

$$t'_s = \gamma\left(t_s - \frac{\beta x_s}{c}\right)$$

where $\beta = v/c = 0.250$ and

$$\gamma = 1/\sqrt{1-\beta^2} = 1/\sqrt{1-(0.250)^2} = 1.0328\,.$$

Similarly, let t_b be the time and x_b be the coordinate of the big flash, as measured in frame S. Then, the time of the big flash, as measured in frame S', is

$$t'_b = \gamma\left(t_b - \frac{\beta x_b}{c}\right).$$

Subtracting the second Lorentz transformation equation from the first and recognizing that $t_s = t_b$ (since the flashes are simultaneous in S), we find

$$\Delta t' = \frac{\gamma\beta(x_s - x_b)}{c} = \frac{(1.0328)(0.250)(30\times10^3\ \text{m})}{3.00\times10^8\ \text{m/s}} = 2.58\times10^{-5}\ \text{s}$$

where $\Delta t' = t_b' - t_s'$.

(b) Since $\Delta t'$ is negative, t_b' is greater than t_s' . The small flash occurs first in S'.

20. From Eq. 2 in Table 37-2, we have

$$\Delta t = v\,\gamma\,\Delta x'/c^2 + \gamma\,\Delta t'.$$

The coefficient of $\Delta x'$ is the slope (4.0 μs/400 m) of the graph, and the last term involving $\Delta t'$ is the "y-intercept" of the graph. From the first observation, we can solve for $\beta = v/c = 0.949$ and consequently $\gamma = 3.16$. Then, from the second observation, we find

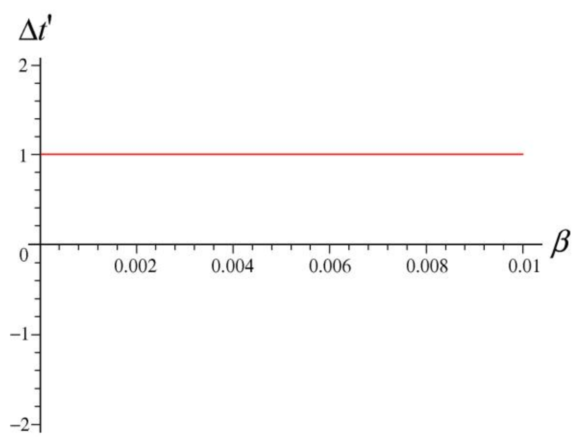
$$\Delta t' = \frac{\Delta t}{\gamma} = \frac{2.00\times10^{-6}\text{s}}{3.16} = 6.3\times10^{-7}\text{s} .$$

21. (a) Using Eq. 2′ of Table 37-2, we have

$$\Delta t' = \gamma\left(\Delta t - \frac{v\Delta x}{c^2}\right) = \gamma\left(\Delta t - \frac{\beta\Delta x}{c}\right) = \gamma\left(1.00\times10^{-6}\text{s} - \frac{\beta(400\,\text{m})}{2.998\times10^8\ \text{m/s}}\right)$$

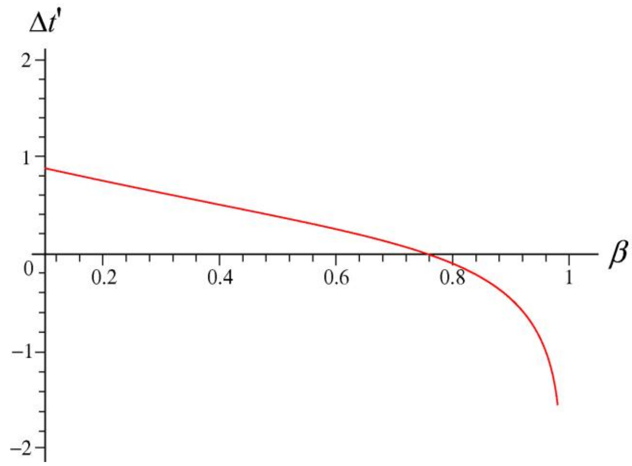
where the Lorentz factor is itself a function of β (see Eq. 37-8).

(b) A plot of $\Delta t'$ as a function of β in the range $0 < \beta < 0.01$ is shown below:



Note the limits of the vertical axis are +2 μs and –2 μs. We note how "flat" the curve is in this graph; the reason is that for low values of β, Bullwinkle's measure of the temporal separation between the two events is approximately our measure, namely +1.0 μs. There are no nonintuitive relativistic effects in this case.

(c) A plot of $\Delta t'$ as a function of β in the range $0.1 < \beta < 1$ is shown below:



(d) Setting

$$\Delta t' = \gamma\left(\Delta t - \frac{\beta \Delta x}{c}\right) = \gamma\left(1.00\times10^{-6}\,\text{s} - \frac{\beta(400\,\text{m})}{2.998\times10^{8}\,\text{m/s}}\right) = 0$$

leads to

$$\beta = \frac{c\Delta t}{\Delta x} = \frac{(2.998\times10^{8}\,\text{m/s})(1.00\times10^{-6}\,\text{s})}{400\,\text{m}} = 0.7495 \approx 0.750\,.$$

(e) For the graph shown in part (c), as we increase the speed, the temporal separation according to Bullwinkle is positive for the lower values and then goes to zero and finally (as the speed approaches that of light) becomes progressively more negative. For the lower speeds with

$$\Delta t' > 0 \Rightarrow t_A' < t_B' \Rightarrow \quad 0 < \beta < 0.750,$$

according to Bullwinkle event A occurs before event B just as we observe.

(f) For the higher speeds with

$$\Delta t' < 0 \;\Rightarrow\; \quad t_A' > t_B' \Rightarrow 0.750 < \beta < 1,$$

according to Bullwinkle event B occurs before event A (the opposite of what we observe).

(g) No, event A cannot cause event B or vice versa. We note that

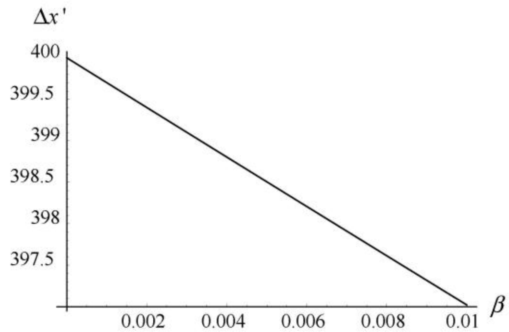
$$\Delta x/\Delta t = (400\text{ m})/(1.00\ \mu\text{s}) = 4.00\times10^{8}\text{ m/s} > c.$$

A signal cannot travel from event *A* to event *B* without exceeding *c*, so causal influences cannot originate at *A* and thus affect what happens at *B*, or vice versa.

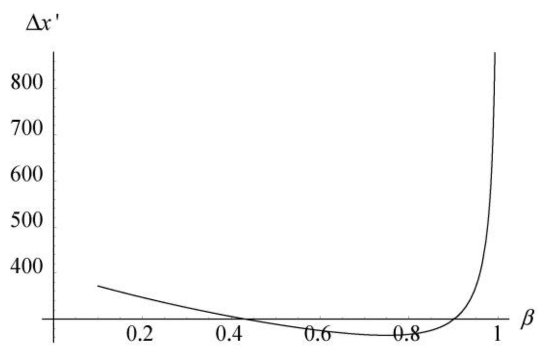
22. (a) From Table 37-2, we find

$$\Delta x' = \gamma\left(\Delta x - v\Delta t\right) = \gamma\left(\Delta x - \beta c\Delta t\right) = \gamma[400\text{ m} - \beta c(1.00\ \mu\text{s})] = \frac{400\text{ m} - (299.8\text{ m})\beta}{\sqrt{1-\beta^2}}$$

(b) A plot of $\Delta x'$ as a function of β with $0 < \beta < 0.01$ is shown below:



(c) A plot of $\Delta x'$ as a function of β with $0.1 < \beta < 1$ is shown below:



(d) To find the minimum, we can take a derivative of $\Delta x'$ with respect to β, simplify, and then set equal to zero:

$$\frac{d\Delta x'}{d\beta} = \frac{d}{d\beta}\left(\frac{\Delta x - \beta c\Delta t}{\sqrt{1-\beta^2}}\right) = \frac{\beta\Delta x - c\Delta t}{(1-\beta^2)^{3/2}} = 0$$

This yields

$$\beta = \frac{c\Delta t}{\Delta x} = \frac{(2.998\times10^8\text{ m/s})(1.00\times10^{-6}\text{s})}{400\text{ m}} = 0.7495 \approx 0.750$$

(e) Substituting this value of β into the part (a) expression yields $\Delta x'$ = 264.8 m ≈ 265 m for its minimum value.

23. (a) The Lorentz factor is

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-(0.600)^2}} = 1.25 .$$

(b) In the unprimed frame, the time for the clock to travel from the origin to x = 180 m is

$$t = \frac{x}{v} = \frac{180\,\text{m}}{(0.600)(3.00\times10^8\ \text{m/s})} = 1.00\times10^{-6}\ \text{s} .$$

The proper time interval between the two events (at the origin and at x = 180 m) is measured by the clock itself. The reading on the clock at the beginning of the interval is zero, so the reading at the end is

$$t' = \frac{t}{\gamma} = \frac{1.00\times10^{-6}\,\text{s}}{1.25} = 8.00\times10^{-7}\,\text{s} .$$

24. The time-dilation information in the problem (particularly, the 15 s on "his wristwatch… which takes 30.0 s according to you") reveals that the Lorentz factor is γ = 2.00 (see Eq. 37-9), which implies his speed is v = 0.866c.

(a) With γ = 2.00, Eq. 37-13 implies the contracted length is 0.500 m.

(b) There is no contraction along the direction perpendicular to the direction of motion (or "boost" direction), so meter stick 2 still measures 1.00 m long.

(c) As in part (b), the answer is 1.00 m.

(d) Equation 1′ in Table 37-2 gives

$$\begin{aligned}\Delta x' &= x_2' - x_1' = \gamma\left(\Delta x - v\Delta t\right) = (2.00)\left[20.0\ \text{m} - (0.866)(2.998\times10^8\,\text{m/s})(40.0\times10^{-9}\ \text{s})\right]\\ &= 19.2\ \text{m}\end{aligned}$$

(e) Equation 2′ in Table 37-2 gives

$$\begin{aligned}\Delta t' &= t_2' - t_1' = \gamma\left(\Delta t - v\Delta x / c^2\right) = \gamma\left(\Delta t - \beta\Delta x / c\right)\\ &= (2.00)\left[40.0\times10^{-9}\ \text{s} - (0.866)(20.0\ \text{m})/(2.998\times10^8\,\text{m/s})\right]\\ &= -35.5\ \text{ns} .\end{aligned}$$

In absolute value, the two events are separated by 35.5 ns.

(f) The negative sign obtained in part (e) implies event 2 occurred before event 1.

25. (a) In frame *S*, our coordinates are such that x_1 = +1200 m for the big flash, and x_2 = 1200 – 720 = 480 m for the small flash (which occurred later). Thus,

$$\Delta x = x_2 - x_1 = -720 \text{ m}.$$

If we set $\Delta x' = 0$ in Eq. 37-25, we find

$$0 = \gamma(\Delta x - v\Delta t) = \gamma\left(-720\,\text{m} - v(5.00\times10^{-6}\,\text{s})\right)$$

which yields $v = -1.44 \times 10^8$ m/s, or $\beta = v/c = 0.480$.

(b) The negative sign in part (a) implies that frame *S′* must be moving in the –*x* direction.

(c) Equation 37-28 leads to

$$\Delta t' = \gamma\left(\Delta t - \frac{v\Delta x}{c^2}\right) = \gamma\left(5.00\times10^{-6}\,\text{s} - \frac{(-1.44\times10^8\,\text{m/s})(-720\,\text{m})}{(2.998\times10^8\,\text{m/s})^2}\right),$$

which turns out to be positive (regardless of the specific value of γ). Thus, the order of the flashes is the same in the *S′* frame as it is in the *S* frame (where Δt is also positive). Thus, the big flash occurs first, and the small flash occurs later.

(d) Finishing the computation begun in part (c), we obtain

$$\Delta t' = \frac{5.00\times10^{-6}\,\text{s} - (-1.44\times10^8\,\text{m/s})(-720\,\text{m})/(2.998\times10^8\,\text{m/s})^2}{\sqrt{1-0.480^2}} = 4.39\times10^{-6}\,\text{s}.$$

26. We wish to adjust Δt so that

$$0 = \Delta x' = \gamma\left(\Delta x - v\Delta t\right) = \gamma(-720\,\text{m} - v\Delta t)$$

in the limiting case of $|v| \to c$. Thus,

$$\Delta t = \frac{\Delta x}{v} = \frac{\Delta x}{c} = \frac{720\,\text{m}}{2.998\times10^8\,\text{m/s}} = 2.40\times10^{-6}\,\text{s}.$$

27. **THINK** We apply relativistic velocity transformation to calculate the velocity of the particle with respect to frame *S*.

EXPRESS We assume *S′* is moving in the +*x* direction. Let *u′* be the velocity of the particle as measured in *S′* and *v* be the velocity of *S′* relative to *S*, the velocity of the particle as measured in *S* is given by Eq. 37-29:

$$u = \frac{u' + v}{1 + u'v/c^2}.$$

ANALYZE With $u' = +0.40c$ and $v = +0.60c$, we obtain

$$u = \frac{u' + v}{1 + u'v/c^2} = \frac{0.40c + 0.60c}{1 + (0.40c)(+0.60c)/c^2} = 0.81c\ .$$

LEARN The classical Galilean transformation would have given

$$u = u' + v = 0.40c + 0.60c = 1.0c.$$

28. (a) We use Eq. 37-29:

$$v = \frac{v' + u}{1 + uv'/c^2} = \frac{0.47c + 0.62c}{1 + (0.47)(0.62)} = 0.84c\ ,$$

in the direction of increasing x (since $v > 0$). In unit-vector notation, we have $\vec{v} = (0.84c)\hat{\mathrm{i}}$.

(b) The classical theory predicts that $v = 0.47c + 0.62c = 1.1c$, or $\vec{v} = (1.1c)\hat{\mathrm{i}}$.

(c) Now $v' = -0.47c\,\hat{\mathrm{i}}$ so

$$v = \frac{v' + u}{1 + uv'/c^2} = \frac{-0.47c + 0.62c}{1 + (-0.47)(0.62)} = 0.21c\ ,$$

or $\vec{v} = (0.21c)\hat{\mathrm{i}}$

(d) By contrast, the classical prediction is $v = 0.62c - 0.47c = 0.15c$, or $\vec{v} = (0.15c)\hat{\mathrm{i}}$.

29. (a) One thing Einstein's relativity has in common with the more familiar (Galilean) relativity is the reciprocity of relative velocity. If Joe sees Fred moving at 20 m/s eastward away from him (Joe), then Fred should see Joe moving at 20 m/s westward away from him (Fred). Similarly, if we see Galaxy A moving away from us at $0.35c$ then an observer in Galaxy A should see our galaxy move away from him at $0.35c$, or 0.35 in multiple of c.

(b) We take the positive axis to be in the direction of motion of Galaxy A, as seen by us. Using the notation of Eq. 37-29, the problem indicates $v = +0.35c$ (velocity of Galaxy A relative to Earth) and $u = -0.35c$ (velocity of Galaxy B relative to Earth). We solve for the velocity of B relative to A:

$$\frac{u'}{c}=\frac{u/c-v/c}{1-uv/c^2}=\frac{(-0.35)-0.35}{1-(-0.35)(0.35)}=-0.62\,,$$

or $|u'/c|=0.62.$

30. Using the notation of Eq. 37-29 and taking "away" (from us) as the positive direction, the problem indicates v = +0.4c and u = +0.8c (with 3 significant figures understood). We solve for the velocity of Q_2 relative to Q_1 (in multiple of c):

$$\frac{u'}{c}=\frac{u/c-v/c}{1-uv/c^2}=\frac{0.8-0.4}{1-(0.8)(0.4)}=0.588$$

in a direction away from Earth.

31. **THINK** Both the spaceship and the micrometeorite are moving relativistically, and we apply relativistic speed transformation to calculate the velocity of the micrometeorite relative to the spaceship.

EXPRESS Let S be the reference frame of the micrometeorite, and S' be the reference frame of the spaceship. We assume S to be moving in the $+x$ direction. Let u be the velocity of the micrometeorite as measured in S and v be the velocity of S' relative to S, the velocity of the micrometeorite as measured in S' can be solved by using Eq. 37-29:

$$u=\frac{u'+v}{1+u'v/c^2}\;\;\Rightarrow\; u'=\frac{u-v}{1-uv/c^2}.$$

ANALYZE The problem indicates that v = –0.82c (spaceship velocity) and u = +0.82c (micrometeorite velocity). We solve for the velocity of the micrometeorite relative to the spaceship:

$$u'=\frac{u-v}{1-uv/c^2}=\frac{0.82c-(-0.82c)}{1-(0.82)(-0.82)}=0.98c$$

or 2.94×10^8 m/s. Using Eq. 37-10, we conclude that observers on the ship measure a transit time for the micrometeorite (as it passes along the length of the ship) equal to

$$\Delta t=\frac{d}{u'}=\frac{350\,\text{m}}{2.94\times10^8\ \text{m/s}}=1.2\times10^{-6}\ \text{s}\,.$$

LEARN The classical Galilean transformation would have given

$$u' = u - v = 0.82c - (-0.82c) = 1.64c,$$

which exceeds c and therefore, is physically impossible.

32. The figure shows that $u' = 0.80c$ when $v = 0$. We therefore infer (using the notation of Eq. 37-29) that $u = 0.80c$. Now, u is a fixed value and v is variable, so u' as a function of v is given by

$$u' = \frac{u - v}{1 - uv/c^2} = \frac{0.80c - v}{1 - (0.80)v/c}$$

which is Eq. 37-29 rearranged so that u' is isolated on the left-hand side. We use this expression to answer parts (a) and (b).

(a) Substituting $v = 0.90c$ in the expression above leads to $u' = -0.357c \approx -0.36c$.

(b) Substituting $v = c$ in the expression above leads to $u' = -c$ (regardless of the value of u).

33. (a) In the messenger's rest system (called S_m), the velocity of the armada is

$$v' = \frac{v - v_m}{1 - vv_m/c^2} = \frac{0.80c - 0.95c}{1 - (0.80c)(0.95c)/c^2} = -0.625c\ .$$

The length of the armada as measured in S_m is

$$L_1 = \frac{L_0}{\gamma v'} = (1.0\,\text{ly})\sqrt{1 - (-0.625)^2} = 0.781\ \text{ly}\ .$$

Thus, the length of the trip is

$$t' = \frac{L'}{|v'|} = \frac{0.781\text{ly}}{0.625\text{c}} = 1.25\ \text{y}\ .$$

(b) In the armada's rest frame (called S_a), the velocity of the messenger is

$$v' = \frac{v - v_a}{1 - vv_a/c^2} = \frac{0.95c - 0.80c}{1 - (0.95c)(0.80c)/c^2} = 0.625c\ .$$

Now, the length of the trip is

$$t' = \frac{L_0}{v'} = \frac{1.0\,\text{ly}}{0.625c} = 1.60\ \text{y}\ .$$

(c) Measured in system S, the length of the armada is

$$L = \frac{L_0}{\gamma} = 1.0\,\text{ly}\sqrt{1 - (0.80)^2} = 0.60\ \text{ly}\ ,$$

so the length of the trip is

$$t = \frac{L}{v_m - v_a} = \frac{0.60\,\text{ly}}{0.95c - 0.80c} = 4.00\ \text{y}\ .$$

34. We use the transverse Doppler shift formula, Eq. 37-37: $f = f_0\sqrt{1-\beta^2}$, or

$$\frac{1}{\lambda} = \frac{1}{\lambda_0}\sqrt{1-\beta^2}.$$

We solve for $\lambda - \lambda_0$:

$$\lambda - \lambda_0 = \lambda_0\left(\frac{1}{\sqrt{1-\beta^2}} - 1\right) = (589.00\,\text{mm})\left[\frac{1}{\sqrt{1-(0.100)^2}} - 1\right] = +2.97\,\text{nm}\ .$$

35. **THINK** This problem deals with the Doppler effect of light. The source is the spaceship that is moving away from the Earth, where the detector is located.

EXPRESS With the source and the detector separating, the frequency received is given directly by Eq. 37-31:

$$f = f_0\sqrt{\frac{1-\beta}{1+\beta}}$$

where f_0 is the frequency in the frames of the spaceship, $\beta = v/c$, and v is the speed of the spaceship relative to the Earth.

ANALYZE With $\beta = 0.90$ and $f_0 = 100$ MHz, we obtain

$$f = f_0\sqrt{\frac{1-\beta}{1+\beta}} = (100\,\text{MHz})\sqrt{\frac{1-0.9000}{1+0.9000}} = 22.9\,\text{MHz}\ .$$

LEARN Since the source is moving away from the detector, $f < f_0$. Note that in the low speed limit, $\beta \ll 1$, Eq. 37-31 can be approximated as

$$f \approx f_0\left(1 - \beta + \frac{1}{2}\beta^2\right).$$

36. (a) Equation 37-36 leads to a speed of

$$v = \frac{\Delta\lambda}{\lambda}c = (0.004)(3.0\times10^8\,\text{m/s}) = 1.2\times10^6\ \text{m/s} \approx 1\times10^6\,\text{m/s}.$$

(b) The galaxy is receding.

37. We obtain

$$v = \frac{\Delta\lambda}{\lambda}c = \left(\frac{620\text{ nm} - 540\text{ nm}}{620\text{ nm}}\right)c = 0.13c.$$

38. (a) Equation 37-36 leads to

$$v = \frac{\Delta\lambda}{\lambda}c = \frac{12.00\,\text{nm}}{513.0\,\text{nm}}(2.998\times10^8\,\text{m/s}) = 7.000\times10^6\,\text{m/s}.$$

(b) The line is shifted to a larger wavelength, which means shorter frequency. Recalling Eq. 37-31 and the discussion that follows it, this means galaxy NGC is moving away from Earth.

39. **THINK** This problem deals with the Doppler effect of light. The source is the spaceship that is moving away from the Earth, where the detector is located.

EXPRESS With the source and the detector separating, the frequency received is given directly by Eq. 37-31:

$$f = f_0\sqrt{\frac{1-\beta}{1+\beta}}$$

where f_0 is the frequency in the frames of the spaceship, $\beta = v/c$, and v is the speed of the spaceship relative to the Earth. The frequency and the wavelength are related by $f\lambda = c$. Thus, if λ_0 is the wavelength of the light as seen on the spaceship, using $c = f_0\lambda_0 = f\lambda$, then the wavelength detected on Earth would be

$$\lambda = \lambda_0\left(\frac{f_0}{f}\right) = \lambda_0\sqrt{\frac{1+\beta}{1-\beta}}\,.$$

ANALYZE (a) With λ_0 = 450 nm and β = 0.20, we obtain

$$\lambda = (450\,\text{nm})\sqrt{\frac{1+0.20}{1-0.20}} = 550\text{ nm}.$$

(b) This is in the green-yellow portion of the visible spectrum.

LEARN Since λ_0 = 450 nm, the color of the light as seen on the spaceship is violet-blue. With $\lambda > \lambda_0$, this Doppler shift is red shift.

40. (a) The work-kinetic energy theorem applies as well to relativistic physics as to Newtonian; the only difference is the specific formula for kinetic energy. Thus, we use Eq. 37-52

$$W = \Delta K = m_e c^2(\gamma - 1)$$

and $m_e c^2$ = 511 keV = 0.511 MeV (Table 37-3), and obtain

$$W = m_e c^2\left(\frac{1}{\sqrt{1-\beta^2}}-1\right) = (511\,\text{keV})\left[\frac{1}{\sqrt{1-(0.500)^2}}-1\right] = 79.1\ \text{keV}\ .$$

(b) $W = (0.511\,\text{MeV})\left(\frac{1}{\sqrt{1-(0.990)^2}}-1\right) = 3.11\ \text{MeV}.$

(c) $W = (0.511\,\text{MeV})\left(\frac{1}{\sqrt{1-(0.990)^2}}-1\right) = 10.9\,\text{MeV}.$

41. **THINK** The electron is moving at a relativistic speed since its kinetic energy greatly exceeds its rest energy.

EXPRESS The kinetic energy of the electron is given by Eq. 37-52:

$$K = E - mc^2 = \gamma mc^2 - mc^2 = mc^2(\gamma - 1)\ .$$

Thus, $\gamma = (K/mc^2) + 1$. Similarly, by inverting the Lorentz factor $\gamma = 1/\sqrt{1-\beta^2}$, we obtain $\beta = \sqrt{1-(1/\gamma)^2}$.

ANALYZE (a) Table 37-3 gives mc^2 = 511 keV = 0.511 MeV for the electron rest energy, so the Lorentz factor is

$$\gamma = \frac{K}{mc^2} + 1 = \frac{100\,\text{MeV}}{0.511\,\text{MeV}} + 1 = 196.695.$$

(b) The speed parameter is

$$\beta = \sqrt{1 - \frac{1}{(196.695)^2}} = 0.999987.$$

Thus, the speed of the electron is 0.999987c, or 99.9987% of the speed of light.

LEARN The classical expression $K = mv^2/2$, for kinetic energy, is adequate only when the speed of the object is well below the speed of light.

42. From Eq. 28-37, we have

$$\begin{aligned} Q &= -\Delta Mc^2 = -\left[3(4.00151\text{u}) - 11.99671\text{u}\right]c^2 = -(0.00782\text{u})(931.5\text{MeV/u}) \\ &= -7.28\text{Mev}. \end{aligned}$$

Thus, it takes a minimum of 7.28 MeV supplied to the system to cause this reaction. We note that the masses given in this problem are strictly for the nuclei involved; they are not the "atomic" masses that are quoted in several of the other problems in this chapter.

43. (a) The work-kinetic energy theorem applies as well to relativistic physics as to Newtonian; the only difference is the specific formula for kinetic energy. Thus, we use $W = \Delta K$ where $K = m_e c^2(\gamma - 1)$ (Eq. 37-52), and $m_e c^2$ = 511 keV = 0.511 MeV (Table 37-3). Noting that

$$\Delta K = m_e c^2(\gamma_f - \gamma_i),$$

we obtain

$$W = \Delta K = m_e c^2 \left(\frac{1}{\sqrt{1-\beta_f^2}} - \frac{1}{\sqrt{1-\beta_i^2}} \right) = (511\,\text{keV}) \left(\frac{1}{\sqrt{1-(0.19)^2}} - \frac{1}{\sqrt{1-(0.18)^2}} \right)$$
$$= 0.996 \text{ keV} \approx 1.0 \text{ keV}.$$

(b) Similarly,

$$W = (511\,\text{keV}) \left(\frac{1}{\sqrt{1-(0.99)^2}} - \frac{1}{\sqrt{1-(0.98)^2}} \right) = 1055\,\text{keV} \approx 1.1 \text{ MeV}.$$

We see the dramatic increase in difficulty in trying to accelerate a particle when its initial speed is very close to the speed of light.

44. The mass change is

$$\Delta M = (4.002603\,\text{u} + 15.994915\,\text{u}) - (1.007825\,\text{u} + 18.998405\,\text{u}) = -0.008712\,\text{u}.$$

Using Eq. 37-50 and Eq. 37-46, this leads to

$$Q = -\Delta M\, c^2 = -(-0.008712\,\text{u})(931.5\,\text{MeV/u}) = 8.12\,\text{MeV}.$$

45. The distance traveled by the pion in the frame of Earth is (using Eq. 37-12) $d = v\Delta t$. The proper lifetime Δt_0 is related to Δt by the time-dilation formula: $\Delta t = \gamma\Delta t_0$. To use this equation, we must first find the Lorentz factor γ (using Eq. 37-48). Since the total energy of the pion is given by $E = 1.35 \times 10^5$ MeV and its mc^2 value is 139.6 MeV, then

$$\gamma = \frac{E}{mc^2} = \frac{1.35 \times 10^5\,\text{MeV}}{139.6\,\text{MeV}} = 967.05.$$

Therefore, the lifetime of the moving pion as measured by Earth observers is

$$\Delta t = \gamma \Delta t_0 = (967.1)(35.0 \times 10^{-9}\,\text{s}) = 3.385 \times 10^{-5}\,\text{s},$$

and the distance it travels is

$$d \approx c\Delta t = \left(2.998\times10^{8}\ \text{m/s}\right)\left(3.385\times10^{-5}\ \text{s}\right) = 1.015\times10^{4}\ \text{m} = 10.15\,\text{km}$$

where we have approximated its speed as c (note: its speed can be found by solving Eq. 37-8, which gives v = 0.9999995c; this more precise value for v would not significantly alter our final result). Thus, the altitude at which the pion decays is 120 km – 10.15 km = 110 km.

46. (a) Squaring Eq. 37-47 gives

$$E^2 = \left(mc^2\right)^2 + 2mc^2K + K^2$$

which we set equal to Eq. 37-55. Thus,

$$\left(mc^2\right)^2 + 2mc^2K + K^2 = \left(pc\right)^2 + \left(mc^2\right)^2 \quad\Rightarrow\quad m = \frac{(pc)^2 - K^2}{2Kc^2}.$$

(b) At low speeds, the pre-Einsteinian expressions $p = mv$ and $K = \frac{1}{2}mv^2$ apply. We note that $pc \gg K$ at low speeds since $c \gg v$ in this regime. Thus,

$$m \rightarrow \frac{(mvc)^2 - \left(\frac{1}{2}mv^2\right)^2}{2\left(\frac{1}{2}mv^2\right)c^2} \approx \frac{(mvc)^2}{2\left(\frac{1}{2}mv^2\right)c^2} = m.$$

(c) Here, pc = 121 MeV, so

$$m = \frac{121^2 - 55^2}{2(55)c^2} = 105.6\,\text{MeV}/\text{c}^2.$$

Now, the mass of the electron (see Table 37-3) is m_e = 0.511 MeV/c^2, so our result is roughly 207 times bigger than an electron mass, i.e., $m/m_e \approx 207$. The particle is a muon.

47. **THINK** As a consequence of the theory of relativity, mass can be considered as another form of energy.

EXPRESS The mass of an object and its equivalent energy is given by

$$E_0 = mc^2.$$

ANALYZE The energy equivalent of one tablet is

$$E_0 = mc^2 = (320 \times 10^{-6} \text{ kg})\,(3.00 \times 10^8 \text{ m/s})^2 = 2.88 \times 10^{13} \text{ J}.$$

This provides the same energy as

$$(2.88 \times 10^{13} \text{ J})/(3.65 \times 10^7 \text{ J/L}) = 7.89 \times 10^5 \text{ L}$$

of gasoline. The distance the car can go is

$$d = (7.89 \times 10^5 \text{ L})\,(12.75 \text{ km/L}) = 1.01 \times 10^7 \text{ km}.$$

LEARN The distance is roughly 250 times larger than the circumference of Earth (see Appendix C). However, this is possible only if the mass-energy conversion were perfect.

48. (a) The proper lifetime Δt_0 is 2.20 μs, and the lifetime measured by clocks in the laboratory (through which the muon is moving at high speed) is Δt = 6.90 μs. We use Eq. 37-7 to solve for the speed parameter:

$$\beta = \sqrt{1-\left(\frac{\Delta t_0}{\Delta t}\right)^2} = \sqrt{1-\left(\frac{2.20\ \mu\text{s}}{6.90\ \mu\text{s}}\right)^2} = 0.948\,.$$

(b) From the answer to part (a), we find γ = 3.136. Thus, with (see Table 37-3)

$$m_\mu c^2 = 207 m_e c^2 = 105.8 \text{ MeV},$$

Eq. 37-52 yields

$$K = m_\mu c^2\left(\gamma - 1\right) = (105.8\,\text{MeV})(3.136-1) = 226\,\text{MeV}.$$

(c) We write $m_\mu c$ = 105.8 MeV/c and apply Eq. 37-41:

$$p = \gamma m_\mu v = \gamma m_\mu c\beta = (3.136)(105.8\,\text{MeV}/c)(0.9478) = 314\ \text{MeV}/c$$

which can also be expressed in SI units ($p = 1.7 \times 10^{-19}$ kg·m/s).

49. (a) The strategy is to find the γ factor from $E = 14.24 \times 10^{-9}$ J and $m_p c^2 = 1.5033 \times 10^{-10}$ J and from that find the contracted length. From the energy relation (Eq. 37-48), we obtain

$$\gamma = \frac{E}{m_p c^2} = \frac{14.24\times10^{-9}\ \text{J}}{1.5033\times10^{-10}\ \text{J}} = 94.73.$$

Consequently, Eq. 37-13 yields

$$L = \frac{L_0}{\gamma} = \frac{21\text{ cm}}{94.73} = 0.222\text{ cm} = 2.22\times10^{-3}\text{ m}.$$

(b) From the γ factor, we find the speed:

$$v = c\sqrt{1-\left(\frac{1}{\gamma}\right)^2} = 0.99994c.$$

Therefore, in our reference frame the time elapsed is

$$\Delta t = \frac{L_0}{v} = \frac{0.21\text{ m}}{(0.99994)(2.998\times10^8\text{ m/s})} = 7.01\times10^{-10}\text{ s}.$$

(c) The time dilation formula (Eq. 37-7) leads to

$$\Delta t = \gamma\Delta t_0 = 7.01\times10^{-10}\text{ s}$$

Therefore, according to the proton, the trip took

$$\Delta t_0 = 2.22\times10^{-3}/0.99994c = 7.40\times10^{-12}\text{ s}.$$

50. From Eq. 37-52, $\gamma = (K/mc^2) + 1$, and from Eq. 37-8, the speed parameter is $\beta = \sqrt{1-(1/\gamma)^2}$.

(a) Table 37-3 gives m_ec^2 = 511 keV = 0.511 MeV, so the Lorentz factor is

$$\gamma = \frac{10.00\,\text{MeV}}{0.5110\,\text{MeV}} + 1 = 20.57,$$

(b) and the speed parameter is

$$\beta = \sqrt{1-(1/\gamma)^2} = \sqrt{1-\frac{1}{(20.57)^2}} = 0.9988.$$

(c) Using m_pc^2 = 938.272 MeV, the Lorentz factor is

$$\gamma = 1 + 10.00\text{ MeV}/938.272\text{ MeV} = 1.01065 \approx 1.011.$$

(d) The speed parameter is

$$\beta = \sqrt{1-\gamma^{-2}} = 0.144844 \approx 0.1448.$$

(e) With $m_\alpha c^2$ = 3727.40 MeV, we obtain γ = 10.00/3727.4 + 1 = 1.00268 $\approx$ 1.003.

(f) The speed parameter is

$$\beta = \sqrt{1-\gamma^{-2}} = 0.0731037 \approx 0.07310 .$$

51. We set Eq. 37-55 equal to $(3.00mc^2)^2$, as required by the problem, and solve for the speed. Thus,

$$(pc)^2 + (mc^2)^2 = 9.00(mc^2)^2$$

leads to $p = mc\sqrt{8} \approx 2.83mc.$

52. (a) The binomial theorem tells us that, for x small,

$$(1 + x)^\nu \approx 1 + \nu x + \tfrac{1}{2}\nu(\nu - 1)x^2$$

if we ignore terms involving x^3 and higher powers (this is reasonable since if x is small, say x = 0.1, then x^3 is much smaller: x^3 = 0.001). The relativistic kinetic energy formula, when the speed v is much smaller than c, has a term that we can apply the binomial theorem to; identifying $-\beta^2$ as x and $-1/2$ as ν, we have

$$\gamma = (1-\beta^2)^{-1/2} \approx 1 + (-\tfrac{1}{2})(-\beta^2) + \tfrac{1}{2}(-\tfrac{1}{2})\big((-\tfrac{1}{2}) - 1\big)(-\beta^2)^2.$$

Substituting this into Eq. 37-52 leads to

$$K = mc^2(\gamma - 1) \approx mc^2\big[(-\tfrac{1}{2})(-\beta^2) + \tfrac{1}{2}(-\tfrac{1}{2})\big((-\tfrac{1}{2}) - 1\big)(-\beta^2)^2\big]$$

which simplifies to

$$K \approx \tfrac{1}{2}mc^2\beta^2 + \tfrac{3}{8}mc^2\beta^4 = \tfrac{1}{2}mv^2 + \tfrac{3}{8}mv^4/c^2 .$$

(b) If we use the mc^2 value for the electron found in Table 37-3, then for β = 1/20, the classical expression for kinetic energy gives

$$K_{\text{classical}} = \tfrac{1}{2}mv^2 = \tfrac{1}{2}mc^2\beta^2 = \tfrac{1}{2}(8.19\times10^{-14}\text{ J})(1/20)^2 = 1.0\times10^{-16}\text{ J} .$$

(c) The first-order correction becomes

$$K_{\text{first-order}} = \tfrac{3}{8}mv^4/c^2 = \tfrac{3}{8}mc^2\beta^4 = \tfrac{3}{8}(8.19\times10^{-14}\text{ J})(1/20)^4 = 1.9\times10^{-19}\text{ J}$$

which we note is much smaller than the classical result.

(d) In this case, $\beta = 0.80 = 4/5$, and the classical expression yields

$$K_{\text{classical}} = \tfrac{1}{2}mv^2 = \tfrac{1}{2}mc^2\beta^2 = \tfrac{1}{2}(8.19\times 10^{-14}\text{ J})(4/5)^2 = 2.6\times 10^{-14}\text{ J}.$$

(e) And the first-order correction is

$$K_{\text{first-order}} = \tfrac{3}{8}mv^4/c^2 = \tfrac{3}{8}mc^2\beta^4 = \tfrac{3}{8}(8.19\times 10^{-14}\text{ J})(4/5)^4 = 1.3\times 10^{-14}\text{ J}$$

which is comparable to the classical result. This is a signal that ignoring the higher order terms in the binomial expansion becomes less reliable the closer the speed gets to c.

(f) We set the first-order term equal to one-tenth of the classical term and solve for β:

$$\tfrac{3}{8}mc^2\beta^4 = \tfrac{1}{10}\left(\tfrac{1}{2}mc^2\beta^2\right)$$

and obtain $\beta = \sqrt{2/15} \approx 0.37$.

53. Using the classical orbital radius formula $r_0 = mv/|q|B$, the period is

$$T_0 = 2\pi r_0/v = 2\pi m/|q|B.$$

In the relativistic limit, we must use

$$r = \frac{p}{|q|B} = \frac{\gamma mv}{|q|B} = \gamma r_0$$

which yields

$$T = \frac{2\pi r}{v} = \gamma\frac{2\pi m}{|q|B} = \gamma T_0$$

(b) The period T is not independent of v.

(c) We interpret the given 10.0 MeV to be the kinetic energy of the electron. In order to make use of the mc^2 value for the electron given in Table 37-3 (511 keV = 0.511 MeV) we write the classical kinetic energy formula as

$$K_{\text{classical}} = \frac{1}{2}mv^2 = \frac{1}{2}\left(mc^2\right)\left(\frac{v^2}{c^2}\right) = \frac{1}{2}\left(mc^2\right)\beta^2.$$

If $K_{\text{classical}}$ = 10.0 MeV, then

$$\beta = \sqrt{\frac{2K_{\text{classical}}}{mc^2}} = \sqrt{\frac{2(10.0\,\text{MeV})}{0.511\,\text{MeV}}} = 6.256,$$

which, of course, is impossible since it exceeds 1. If we use this value anyway, then the classical orbital radius formula yields

$$r = \frac{mv}{|q|B} = \frac{m\beta c}{eB} = \frac{(9.11\times10^{-31}\,\text{kg})\,(6.256)\,(2.998\times10^{8}\,\text{m/s})}{(1.6\times10^{-19}\,\text{C})\,(2.20\text{T})} = 4.85\times10^{-3}\,\text{m}.$$

(d) Before using the relativistically correct orbital radius formula, we must compute β in a relativistically correct way:

$$K = mc^2(\gamma - 1) \;\Rightarrow\; \gamma = \frac{10.0\text{ MeV}}{0.511\text{ MeV}} + 1 = 20.57$$

which implies (from Eq. 37-8)

$$\beta = \sqrt{1 - \frac{1}{\gamma^2}} = \sqrt{1 - \frac{1}{(20.57)^2}} = 0.99882.$$

Therefore,

$$r = \frac{\gamma mv}{|q|B} = \frac{\gamma m\beta c}{eB} = \frac{(20.57)(9.11\times10^{-31}\,\text{kg})(0.99882)(2.998\times10^{8}\,\text{m/s})}{(1.6\times10^{-19}\,\text{C})(2.20\text{T})}$$
$$= 1.59\times10^{-2}\,\text{m}.$$

(e) The classical period is

$$T = \frac{2\pi r}{\beta c} = \frac{2\pi(4.85\times10^{-3}\text{ m})}{(6.256)(2.998\times10^{8}\text{ m/s})} = 1.63\times10^{-11}\text{ s}.$$

(f) The period obtained with relativistic correction is

$$T = \frac{2\pi r}{\beta c} = \frac{2\pi(0.0159\text{ m})}{(0.99882)(2.998\times10^{8}\text{ m/s})} = 3.34\times10^{-10}\text{ s}.$$

54. (a) We set Eq. 37-52 equal to $2mc^2$, as required by the problem, and solve for the speed. Thus,

$$mc^2\left(\frac{1}{\sqrt{1-\beta^2}} - 1\right) = 2mc^2$$

leads to $\beta = 2\sqrt{2}/3 \approx 0.943$.

(b) We now set Eq. 37-48 equal to $2mc^2$ and solve for the speed. In this case,

$$\frac{mc^2}{\sqrt{1-\beta^2}} = 2mc^2$$

leads to $\beta = \sqrt{3}/2 \approx 0.866.$

55. (a) We set Eq. 37-41 equal to mc, as required by the problem, and solve for the speed. Thus,

$$\frac{mv}{\sqrt{1-v^2/c^2}} = mc$$

leads to $\beta = 1/\sqrt{2} = 0.707.$

(b) Substituting $\beta = 1/\sqrt{2}$ into the definition of γ, we obtain

$$\gamma = \frac{1}{\sqrt{1-v^2/c^2}} = \frac{1}{\sqrt{1-(1/2)}} = \sqrt{2} \approx 1.41.$$

(c) The kinetic energy is

$$K = (\gamma-1)mc^2 = \left(\sqrt{2}-1\right)mc^2 = 0.414mc^2 = 0.414E_0.$$

which implies $K/E_0 = 0.414$.

56. (a) From the information in the problem, we see that each kilogram of TNT releases $(3.40 \times 10^6$ J/mol)/(0.227 kg/mol) = 1.50×10^7 J. Thus,

$$(1.80 \times 10^{14} \text{ J})/(1.50 \times 10^7 \text{ J/kg}) = 1.20 \times 10^7 \text{ kg}$$

of TNT are needed. This is equivalent to a weight of $\approx 1.2 \times 10^8$ N.

(b) This is certainly more than can be carried in a backpack. Presumably, a train would be required.

(c) We have $0.00080mc^2 = 1.80 \times 10^{14}$ J, and find $m = 2.50$ kg of fissionable material is needed. This is equivalent to a weight of about 25 N, or 5.5 pounds.

(d) This can be carried in a backpack.

57. Since the rest energy E_0 and the mass m of the quasar are related by $E_0 = mc^2$, the rate P of energy radiation and the rate of mass loss are related by

$$P = dE_0/dt = (dm/dt)c^2.$$

Thus,

$$\frac{dm}{dt} = \frac{P}{c^2} = \frac{1\times 10^{41}\ \text{W}}{\left(2.998\times 10^{8}\ \text{m/s}\right)^2} = 1.11\times 10^{24}\ \text{kg/s}.$$

Since a solar mass is 2.0×10^{30} kg and a year is 3.156×10^{7} s,

$$\frac{dm}{dt} = \left(1.11\times 10^{24}\ \text{kg/s}\right)\left(\frac{3.156\times 10^{7}\ \text{s/y}}{2.0\times 10^{30}\ \text{kg/smu}}\right) \approx 18\,\text{smu/y}.$$

58. (a) Using $K = m_e c^2 (\gamma - 1)$ (Eq. 37-52) and

$$m_e c^2 = 510.9989\ \text{keV} = 0.5109989\ \text{MeV},$$

we obtain

$$\gamma = \frac{K}{m_e c^2} + 1 = \frac{1.0000000\,\text{keV}}{510.9989\,\text{keV}} + 1 = 1.00195695 \approx 1.0019570.$$

(b) Therefore, the speed parameter is

$$\beta = \sqrt{1 - \frac{1}{\gamma^2}} = \sqrt{1 - \frac{1}{(1.0019570)^2}} = 0.062469542.$$

(c) For $K = 1.0000000$ MeV, we have

$$\gamma = \frac{K}{m_e c^2} + 1 = \frac{1.0000000\,\text{MeV}}{0.5109989\,\text{MeV}} + 1 = 2.956951375 \approx 2.9569514.$$

(d) The corresponding speed parameter is

$$\beta = \sqrt{1 - \gamma^{-2}} = 0.941079236 \approx 0.94107924.$$

(e) For K = 1.0000000 GeV, we have

$$\gamma = \frac{K}{m_e c^2} + 1 = \frac{1000.0000\,\text{MeV}}{0.5109989\,\text{MeV}} + 1 = 1957.951375 \approx 1957.9514.$$

(f) The corresponding speed parameter is

$$\beta = \sqrt{1 - \gamma^{-2}} = 0.99999987\,.$$

59. (a) Before looking at our solution to part (a) (which uses momentum conservation), it might be advisable to look at our solution (and accompanying remarks) for part (b) (where a very different approach is used). Since momentum is a vector, its conservation involves two equations (along the original direction of alpha particle motion, the *x* direction, as well as along the final proton direction of motion, the *y* direction). The problem states that all speeds are much less than the speed of light, which allows us to use the classical formulas for kinetic energy and momentum ($K = \frac{1}{2}mv^2$ and $\vec{p} = m\vec{v}$, respectively). Along the *x* and *y* axes, momentum conservation gives (for the components of $\vec{v}_{\text{oxy}}$):

$$m_\alpha v_\alpha = m_{\text{oxy}} v_{\text{oxy},x} \quad \Rightarrow \quad v_{\text{oxy},x} = \frac{m_\alpha}{m_{\text{oxy}}} v_\alpha \approx \frac{4}{17} v_\alpha$$

$$0 = m_{\text{oxy}} v_{\text{oxy},y} + m_p v_p \quad \Rightarrow \quad v_{\text{oxy},y} = -\frac{m_p}{m_{\text{oxy}}} v_p \approx -\frac{1}{17} v_p.$$

To complete these determinations, we need values (inferred from the kinetic energies given in the problem) for the initial speed of the alpha particle (v_α) and the final speed of the proton (v_p). One way to do this is to rewrite the classical kinetic energy expression as $K = \frac{1}{2}(mc^2)\beta^2$ and solve for β (using Table 37-3 and/or Eq. 37-46). Thus, for the proton, we obtain

$$\beta_p = \sqrt{\frac{2K_p}{m_p c^2}} = \sqrt{\frac{2(4.44 \text{ MeV})}{938 \text{ MeV}}} = 0.0973.$$

This is almost 10% the speed of light, so one might worry that the relativistic expression (Eq. 37-52) should be used. If one does so, one finds β_p = 0.969, which is reasonably close to our previous result based on the classical formula. For the alpha particle, we write

$$m_\alpha c^2 = (4.0026 \text{ u})(931.5 \text{ MeV/u}) = 3728 \text{ MeV}$$

(which is actually an overestimate due to the use of the "atomic mass" value in our calculation, but this does not cause significant error in our result), and obtain

$$\beta_\alpha = \sqrt{\frac{2K_\alpha}{m_\alpha c^2}} = \sqrt{\frac{2(7.70 \text{ MeV})}{3728 \text{ MeV}}} = 0.064.$$

Returning to our oxygen nucleus velocity components, we are now able to conclude:

$$v_{\text{oxy},x} \approx \frac{4}{17} v_\alpha \Rightarrow \beta_{\text{oxy},x} \approx \frac{4}{17}\beta_\alpha = \frac{4}{17}(0.064) = 0.015$$

$$|v_{\text{oxy},y}| \approx \frac{1}{17} v_p \Rightarrow \beta_{\text{oxy},y} \approx \frac{1}{17}\beta_p = \frac{1}{17}(0.097) = 0.0057$$

Consequently, with

$$m_{\text{oxy}}c^2 \approx (17\text{ u})(931.5\text{ MeV/u}) = 1.58 \times 10^4\text{ MeV},$$

we obtain

$$K_{\text{oxy}} = \frac{1}{2}(m_{\text{oxy}}c^2)\,(\beta^2_{\text{oxy},x} + \beta^2_{\text{oxy},y}) = \frac{1}{2}(1.58\times10^4\text{ MeV})\,(0.015^2 + 0.0057^2)$$
$$\approx 2.08\text{ MeV}.$$

(b) Using Eq. 37-50 and Eq. 37-46,

$$Q = -(1.007825\text{u} + 16.99914\text{u} - 4.00260\text{u} - 14.00307\text{u})c^2$$
$$= -(0.001295\text{u})(931.5\text{MeV/u})$$

which yields $Q = -1.206$ MeV ≈ -1.21 MeV. Incidentally, this provides an alternate way to obtain the answer (and a more accurate one at that!) to part (a). Equation 37-49 leads to

$$K_{\text{oxy}} = K_\alpha + Q - K_p = 7.70\text{MeV} - 1206\text{MeV} - 4.44\text{MeV} = 2.05\text{MeV}.$$

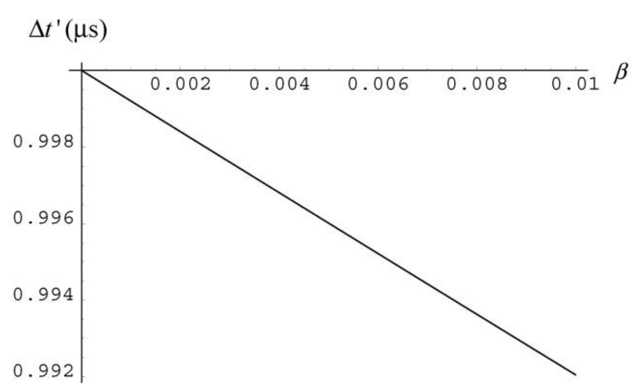
This approach to finding K_{oxy} avoids the many computational steps and approximations made in part (a).

60. (a) Equation 2′ of Table 37-2 becomes

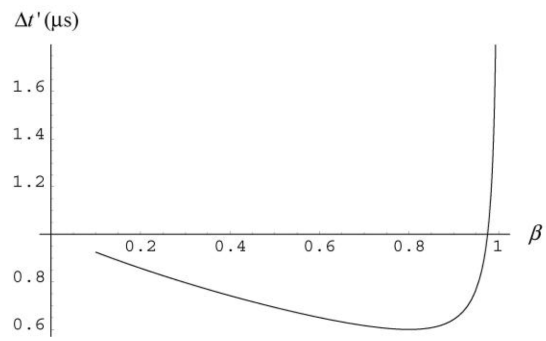
$$\Delta t' = \gamma(\Delta t - \beta\Delta x/c) = \gamma\,[1.00\ \mu\text{s} - \beta(240\text{ m})/(2.998 \times 10^2\text{ m}/\mu\text{s})]$$
$$= \gamma(1.00 - 0.800\beta)\ \mu\text{s}$$

where the Lorentz factor is itself a function of β (see Eq. 37-8).

(b) A plot of $\Delta t'$ is shown for the range $0 < \beta < 0.01$:



(c) A plot of $\Delta t'$ is shown for the range $0.1 < \beta < 1$:



(d) The minimum for the $\Delta t'$ curve can be found by taking the derivative and simplifying and then setting equal to zero:

$$\frac{d\Delta t'}{d\beta} = \gamma^3(\beta\Delta t - \Delta x/c) = 0 .$$

Thus, the value of β for which the curve is minimum is $\beta = \Delta x/c\Delta t = 240/299.8$, or $\beta = 0.801$.

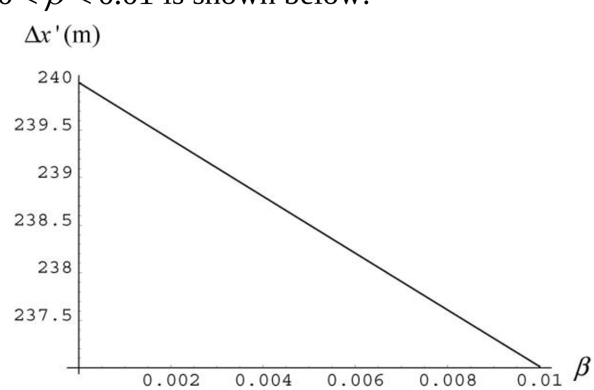
(e) Substituting the value of β from part (d) into the part (a) expression yields the minimum value $\Delta t' = 0.599\ \mu$s.

(f) Yes. We note that $\Delta x/\Delta t = 2.4 \times 10^8$ m/s < c. A signal can indeed travel from event A to event B without exceeding c, so causal influences can originate at A and thus affect what happens at B. Such events are often described as being "time-like separated" – and we see in this problem that it is (always) possible in such a situation for us to find a frame of reference (here with $\beta \approx 0.801$) where the two events will seem to be at the same location (though at different times).

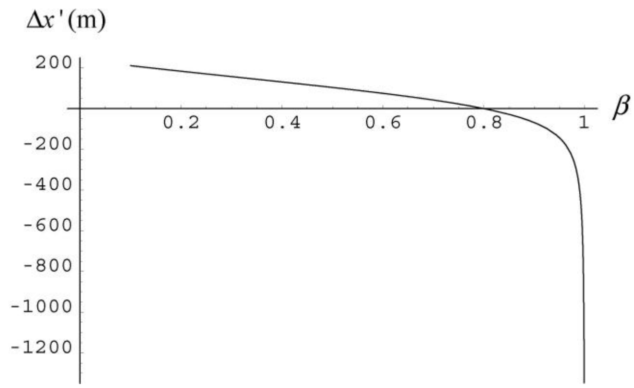
61. (a) Equation 1′ of Table 37-2 becomes

$$\Delta x' = \gamma(\Delta x - \beta c\Delta t) = \gamma[(240 \text{ m}) - \beta(299.8 \text{ m})] .$$

(b) A plot of $\Delta x'$ for $0 < \beta < 0.01$ is shown below:



(c) A plot of $\Delta x'$ for $0.1 < \beta < 1$ is shown below:



We see that $\Delta x'$ decreases from its $\beta = 0$ value (where it is equal to Δx = 240 m) to its zero value (at $\beta \approx 0.8$), and continues (without bound) downward in the graph (where it is negative, implying event B has a *smaller* value of x' than event A!).

(d) The zero value for $\Delta x'$ is easily seen (from the expression in part (b)) to come from the condition $\Delta x - \beta c\Delta t = 0$. Thus $\beta = 0.801$ provides the zero value of $\Delta x'$.

62. By examining the value of u' when v = 0 on the graph, we infer $u = -0.20c$. Solving Eq. 37-29 for u' and inserting this value for u, we obtain

$$u' = \frac{u - v}{1 - uv/c^2} = \frac{-0.20c - v}{1 + 0.20v/c}$$

for the equation of the curve shown in the figure.

(a) With $v = 0.80c$, the above expression yields $u' = -0.86c$.

(b) As expected, setting $v = c$ in this expression leads to $u' = -c$.

63. (a) The spatial separation between the two bursts is vt. We project this length onto the direction perpendicular to the light rays headed to Earth and obtain $D_{\text{app}} = vt \sin\theta$.

(b) Burst 1 is emitted a time t ahead of burst 2. Also, burst 1 has to travel an extra distance L more than burst 2 before reaching the Earth, where $L = vt\cos\theta$ (see Fig. 37-29); this requires an additional time $t' = L/c$. Thus, the apparent time is given by

$$T_{\text{app}} = t - t' = t - \frac{vt\cos\theta}{c} = t\left[1 - \left(\frac{v}{c}\right)\cos\theta\right].$$

(c) We obtain

$$V_{\text{app}} = \frac{D_{\text{app}}}{T_{\text{app}}} = \left[\frac{(v/c)\sin\theta}{1-(v/c)\cos\theta}\right]c = \left[\frac{(0.980)\sin 30.0°}{1-(0.980)\cos 30.0°}\right]c = 3.24\,c.$$

64. The line in the graph is described by Eq. 1 in Table 37-2:

$$\Delta x = v\gamma\Delta t' + \gamma\Delta x' = (\text{“slope”})\Delta t' \;+\; \text{“y-intercept”}$$

where the “slope” is 7.0×10^8 m/s. Setting this value equal to $v\gamma$ leads to $v = 2.8 \times 10^8$ m/s and $\gamma = 2.54$. Since the “y-intercept” is 2.0 m, we see that dividing this by γ leads to $\Delta x' = 0.79$ m.

65. Interpreting v_{AB} as the x-component of the velocity of A relative to B, and defining the corresponding speed parameter $\beta_{AB} = v_{AB}/c$, then the result of part (a) is a straightforward rewriting of Eq. 37-29 (after dividing both sides by c). To make the correspondence with Fig. 37-11 clear, the particle in that picture can be labeled A, frame S' (or an observer at rest in that frame) can be labeled B, and frame S (or an observer at rest in it) can be labeled C. The result of part (b) is less obvious, and we show here some of the algebra steps:

$$M_{AC} = M_{AB} \cdot M_{BC} \quad \Rightarrow \quad \frac{1-\beta_{AC}}{1+\beta_{AC}} = \frac{1-\beta_{AB}}{1+\beta_{AB}} \cdot \frac{1-\beta_{BC}}{1+\beta_{BC}}$$

We multiply both sides by factors to get rid of the denominators

$$(1-\beta_{AC})(1+\beta_{AB})(1+\beta_{BC}) = (1-\beta_{AB})(1-\beta_{BC})(1+\beta_{AC})$$

and expand:

$$1 - \beta_{AC} + \beta_{AB} + \beta_{BC} - \beta_{AC}\,\beta_{AB} - \beta_{AC}\,\beta_{BC} + \beta_{AB}\,\beta_{BC} - \beta_{AB}\,\beta_{BC}\,\beta_{AC} =$$
$$1 + \beta_{AC} - \beta_{AB} - \beta_{BC} - \beta_{AC}\,\beta_{AB} - \beta_{AC}\,\beta_{BC} + \beta_{AB}\,\beta_{BC} + \beta_{AB}\,\beta_{BC}\,\beta_{AC}$$

We note that several terms are identical on both sides of the equals sign, and thus cancel, which leaves us with

$$-\beta_{AC} + \beta_{AB} + \beta_{BC} - \beta_{AB}\,\beta_{BC}\,\beta_{AC} = \beta_{AC} - \beta_{AB} - \beta_{BC} + \beta_{AB}\,\beta_{BC}\,\beta_{AC}$$

which can be rearranged to produce

$$2\beta_{AB} + 2\beta_{BC} = 2\beta_{AC} + 2\beta_{AB}\beta_{BC}\beta_{AC} \;.$$

The left-hand side can be written as $2\beta_{AC}\,(1 + \beta_{AB}\,\beta_{BC})$ in which case it becomes clear how to obtain the result from part (a) [just divide both sides by $2(1 + \beta_{AB}\,\beta_{BC})$].

66. We note, because it is a pretty symmetry and because it makes the part (b) computation move along more quickly, that

$$M = \frac{1-\beta}{1+\beta} \;\Rightarrow\; \beta = \frac{1-M}{1+M}\,.$$

Here, with β_{AB} given as 1/2 (see the problem statement), then M_{AB} is seen to be 1/3 (which is (1 – 1/2) divided by (1 + 1/2)). Similarly for β_{BC} .

(a) Thus,

$$M_{AC} = M_{AB} \cdot M_{BC} = \frac{1}{3} \cdot \frac{1}{3} = \frac{1}{9}\;.$$

(b) Consequently,

$$\beta_{AC} = \frac{1 - M_{AC}}{1 + M_{AC}} = \frac{1-1/9}{1+1/9} = \frac{8}{10} = \frac{4}{5} = 0.80.$$

(c) By the definition of the speed parameter, we finally obtain $v_{AC} = 0.80c$.

67. We note, for use later in the problem, that

$$M = \frac{1-\beta}{1+\beta} \;\Rightarrow\; \beta = \frac{1-M}{1+M}$$

Now, with β_{AB} given as 1/5 (see problem statement), then M_{AB} is seen to be 2/3 (which is (1 – 1/5) divided by (1 + 1/5)). With $\beta_{BC} = -2/5$, we similarly find $M_{BC} = 7/3$, and for $\beta_{CD} = 3/5$ we get $M_{CD} = 1/4$. Thus,

$$M_{AD} = M_{AB} M_{BC} M_{CD} = \frac{2}{3} \cdot \frac{7}{3} \cdot \frac{1}{4} = \frac{7}{18}\;.$$

Consequently,

$$\beta_{AD} = \frac{1 - M_{AD}}{1 + M_{AD}} = \frac{1-7/18}{1+7/18} = \frac{11}{25} = 0.44.$$

By the definition of the speed parameter, we obtain $v_{AD} = 0.44c$.

68. (a) According to the ship observers, the duration of proton flight is $\Delta t' = (760\text{ m})/0.980c = 2.59\ \mu\text{s}$ (assuming it travels the entire length of the ship).

(b) To transform to our point of view, we use Eq. 2 in Table 37-2. Thus, with $\Delta x' = -750$ m, we have

$$\Delta t = \gamma\left(\Delta t' + (0.950c)\Delta x'/c^2\right) = 0.572\mu\text{s}.$$

(c) For the ship observers, firing the proton from back to front makes no difference, and $\Delta t' = 2.59\ \mu\text{s}$ as before.

(d) For us, the fact that now $\Delta x' = +750$ m is a significant change.

$$\Delta t = \gamma\left(\Delta t' + (0.950c)\Delta x'/c^2\right) = 16.0\mu\text{s}.$$

69. (a) From the length contraction equation, the length L_c' of the car according to Garageman is

$$L_c' = \frac{L_c}{\gamma} = L_c\sqrt{1-\beta^2} = (30.5\ \text{m})\sqrt{1-(0.9980)^2} = 1.93\ \text{m}.$$

(b) Since the x_g axis is fixed to the garage, $x_{g2} = L_g = 6.00$ m.

(c) As for t_{g2}, note from Fig. 37-32(b) that at $t_g = t_{g1} = 0$ the coordinate of the front bumper of the limo in the x_g frame is L_c', meaning that the front of the limo is still a distance $L_g - L_c'$ from the back door of the garage. Since the limo travels at a speed v, the time it takes for the front of the limo to reach the back door of the garage is given by

$$\Delta t_g = t_{g2} - t_{g1} = \frac{L_g - L_c'}{v} = \frac{6.00\ \text{m} - 1.93\ \text{m}}{0.9980(2.998\times10^8\ \text{m/s})} = 1.36\times10^{-8}\ \text{s}.$$

Thus $t_{g2} = t_{g1} + \Delta t_g = 0 + 1.36 \times 10^{-8}$ s $= 1.36 \times 10^{-8}$ s.

(d) The limo is inside the garage between times t_{g1} and t_{g2}, so the time duration is $t_{g2} - t_{g1} = 1.36 \times 10^{-8}$ s.

(e) Again from Eq. 37-13, the length L_g' of the garage according to Carman is

$$L_g' = \frac{L_g}{\gamma} = L_g\sqrt{1-\beta^2} = (6.00\ \text{m})\sqrt{1-(0.9980)^2} = 0.379\ \text{m}.$$

(f) Again, since the x_c axis is fixed to the limo, $x_{c2} = L_c = 30.5$ m.

(g) Now, from the two diagrams described in part (h) below, we know that at $t_c = t_{c2}$ (when event 2 takes place), the distance between the rear bumper of the limo and the back door of the garage is given by $L_c - L_g'$. Since the garage travels at a speed v, the front door of the garage will reach the rear bumper of the limo a time Δt_c later, where Δt_c satisfies

$$\Delta t_c = t_{c1} - t_{c2} = \frac{L_c - L_g'}{v} = \frac{30.5\ \text{m} - 0.379\ \text{m}}{0.9980(2.998\times10^8\ \text{m/s})} = 1.01\times10^{-7}\ \text{s}.$$

Thus $t_{c2} = t_{c1} - \Delta t_c = 0 - 1.01 \times 10^{-7}$ s $= -1.01 \times 10^{-7}$ s.

(h) From Carman's point of view, the answer is clearly no.

(i) Event 2 occurs first according to Carman, since $t_{c2} < t_{c1}$.

(j) We describe the essential features of the two pictures. For event 2, the front of the limo coincides with the back door, and the garage itself seems very short (perhaps failing to reach as far as the front window of the limo). For event 1, the rear of the car coincides with the front door and the front of the limo has traveled a significant distance beyond the back door. In this picture, as in the other, the garage seems very short compared to the limo.

(k) No, the limo cannot be in the garage with both doors shut.

(l) Both Carman and Garageman are correct in their respective reference frames. But, in a sense, Carman should lose the bet since he dropped his physics course before reaching the Theory of Special Relativity!

70. (a) The relative contraction is

$$\frac{|\Delta L|}{L_0} = \frac{L_0(1-\gamma^{-1})}{L_0} = 1-\sqrt{1-\beta^2} \approx 1-\left(1-\frac{1}{2}\beta^2\right) = \frac{1}{2}\beta^2 = \frac{1}{2}\left(\frac{630\,\text{m/s}}{3.00\times10^8\,\text{m/s}}\right)^2$$
$$= 2.21\times10^{-12}\,.$$

(b) Letting $|\Delta t - \Delta t_0| = \Delta t_0(\gamma - 1) = \tau = 1.00\,\mu\text{s}$, we solve for Δt_0:

$$\Delta t_0 = \frac{\tau}{\gamma-1} = \frac{\tau}{(1-\beta^2)^{-1/2}-1} \approx \frac{\tau}{1+\frac{1}{2}\beta^2-1} = \frac{2\tau}{\beta^2} = \frac{2(1.00\times10^{-6}\,\text{s})(1\,\text{d}/86400\,\text{s})}{[(630\,\text{m/s})/(2.998\times10^8\,\text{m/s})]^2}$$
$$= 5.25\ \text{d}\,.$$

71. **THINK** We calculate the relative speed of the satellites using both the Galilean transformation and the relativistic speed transformation.

EXPRESS Let v be the speed of the satellites relative to Earth. As they pass each other in opposite directions, their relative speed is given by $v_{\text{rel},\,c} = 2v$ according to the classical Galilean transformation. On the other hand, applying relativistic velocity transformation gives

$$v_{\text{rel}} = \frac{2v}{1+v^2/c^2}\,.$$

ANALYZE (a) With v = 27000 km/h, we obtain

$$v_{\text{rel},\,c} = 2v = 2(27000\ \text{km/h}) = 5.4\times10^4\ \text{km/h}.$$

(b) We can express c in these units by multiplying by 3.6: $c = 1.08\times10^9$ km/h. The fractional error is

$$\frac{v_{\text{rel},c}-v_{\text{rel}}}{v_{\text{rel},c}}=1-\frac{1}{1+v^2/c^2}=1-\frac{1}{1+[(27000\text{ km/h})/(1.08\times10^9\text{ km/h})]^2}=6.3\times10^{-10}.$$

LEARN Since the speeds of the satellites are well below the speed of light, calculating their relative speed using the classical Galilean transformation is adequate.

72. Using Eq. 37-10, we obtain $\beta=\frac{v}{c}=\frac{d/c}{t}=\frac{6.0\text{ y}}{2.0\text{ y}+6.0\text{ y}}=0.75.$

73. **THINK** The work done to the proton is equal to the change in kinetic energy.

EXPRESS The kinetic energy of the electron is given by Eq. 37-52:

$$K=E-mc^2=\gamma mc^2-mc^2=mc^2(\gamma-1)$$

where $\gamma=1/\sqrt{1-\beta^2}$ is the Lorentz factor.

Let v_1 be the initial speed and v_2 be the final speed of the proton. The work required is

$$W=\Delta K=mc^2(\gamma_2-1)-mc^2(\gamma_1-1)=mc^2(\gamma_2-\gamma_1)=mc^2\Delta\gamma\,.$$

ANALYZE When β_2 = 0.9860, we have γ_2 = 5.9972, and when β_1 = 0.9850, we have γ_1 = 5.7953. Thus, $\Delta\gamma$ = 0.202 and the change in kinetic energy (equal to the work) becomes (using Eq. 37-52)

$$W=\Delta K=(mc^2)\Delta\gamma=(938\text{ MeV})(5.9972-5.7953)=189\text{ MeV}$$

where mc^2 = 938 MeV has been used (see Table 37-3).

LEARN Using the classical expression $K_c=mv^2/2$ for kinetic energy, one would have obtain

$$\begin{aligned}W_c&=\Delta K_c=\frac{1}{2}m(v_2^2-v_1^2)=\frac{1}{2}mc^2(\beta_2^2-\beta_1^2)=\frac{1}{2}(938\text{ MeV})\left[(0.9860)^2-(0.9850)^2\right]\\&=0.924\text{ MeV}\end{aligned}$$

which is substantially lowered than that using relativistic formulation.

74. The mean lifetime of a pion measured by observers on the Earth is $\Delta t=\gamma\Delta t_0$, so the distance it can travel (using Eq. 37-12) is

$$d = v\Delta t = \gamma v \Delta t_0 = \frac{(0.99)(2.998\times 10^8 \text{ m/s})(26\times 10^{-9}\text{ s})}{\sqrt{1-(0.99)^2}} = 55\text{m} .$$

75. **THINK** The electron is moving toward the Earth at a relativistic speed since $E \gg mc^2$, where mc^2 is the rest energy of the electron.

EXPRESS The energy of the electron is given by

$$E = \gamma mc^2 = \frac{mc^2}{\sqrt{1-(v/c)^2}} .$$

With E = 1533 MeV and mc^2 = 0.511 MeV (see Table 37-3), we obtain

$$v = c\sqrt{1-\left(\frac{mc^2}{E}\right)^2} = c\sqrt{1-\left(\frac{0.511\text{ MeV}}{1533\text{ MeV}}\right)^2} = 0.99999994c \approx c .$$

Thus, in the rest frame of Earth, it took the electron 26 y to reach us. In order to transform to its own "clock" it's useful to compute γ directly from Eq. 37-48:

$$\gamma = \frac{E}{mc^2} = \frac{1533\text{ MeV}}{0.511\text{ MeV}} = 3000$$

though if one is careful one can also get this result from $\gamma = 1/\sqrt{1-(v/c)^2}$.

ANALYZE Then, Eq. 37-7 leads to

$$\Delta t_0 = \frac{\Delta t}{\gamma} = \frac{26\text{ y}}{3000} = 0.0087\text{ y}$$

so that the electron "concludes" the distance he traveled is only 0.0087 light-years.

LEARN In the rest frame of the electron, the Earth appears to be rushing toward the electron with a speed $0.99999994c$. Thus, the electron starts its journey from a distance of 0.0087 light-years away.

76. We are asked to solve Eq. 37-48 for the speed v. Algebraically, we find

$$\beta = \sqrt{1-\left(\frac{mc^2}{E}\right)^2} .$$

Using E = 10.611×10^{-9} J and the very accurate values for c and m (in SI units) found in Appendix B, we obtain β = 0.99990.

77. The speed of the spaceship after the first increment is v_1 = 0.5c. After the second one, it becomes

$$v_2 = \frac{v' + v_1}{1 + v'v_1/c^2} = \frac{0.50c + 0.50c}{1 + (0.50c)^2/c^2} = 0.80c,$$

and after the third one, the speed is

$$v_3 = \frac{v' + v_2}{1 + v'v_2/c^2} = \frac{0.50c + 0.50c}{1 + (0.50c)(0.80c)/c^2} = 0.929c.$$

Continuing with this process, we get v_4 = 0.976c, v_5 = 0.992c, v_6 = 0.997c, and v_7 = 0.999c. Thus, seven increments are needed.

78. (a) Equation 37-37 yields

$$\frac{\lambda_0}{\lambda} = \sqrt{\frac{1-\beta}{1+\beta}} \quad \Rightarrow \quad \beta = \frac{1-(\lambda_0/\lambda)^2}{1+(\lambda_0/\lambda)^2} \; .$$

With $\lambda_0/\lambda = 434/462$, we obtain $\beta = 0.062439$, or v = 1.87 × 10^7 m/s.

(b) Since it is shifted "toward the red" (toward longer wavelengths) then the galaxy is moving away from us (receding).

79. **THINK** The electron is moving at a relativistic speed since its total energy E is much great than mc^2, the rest energy of the electron.

EXPRESS To calculate the momentum of the electron, we use Eq. 37-54:

$$(pc)^2 = K^2 + 2Kmc^2 \; .$$

ANALYZE With K = 2.00 MeV and mc^2 = 0.511 MeV (see Table 37-3), we have

$$pc = \sqrt{K^2 + 2Kmc^2} = \sqrt{(2.00\text{ MeV})^2 + 2(2.00\text{ MeV})(0.511\text{ MeV})}$$

This readily yields p = 2.46 MeV/c.

LEARN Classically, the electron momentum is

$$p_c = \sqrt{2Km} = \frac{\sqrt{2Kmc^2}}{c} \frac{\sqrt{2(2.00\text{ MeV})(0.511\text{ MeV})}}{c} = 1.43\text{ MeV}/c$$

which is smaller than that obtained using relativistic formulation.

80. Using Appendix C, we find that the contraction is

$$|\Delta L| = L_0 - L = L_0\left(1-\frac{1}{\gamma}\right) = L_0\left(1-\sqrt{1-\beta^2}\right)$$
$$= 2(6.370\times10^6\text{ m})\left(1-\sqrt{1-\left(\frac{3.0\times10^4\text{ m/s}}{2.998\times10^8\text{ m/s}}\right)^2}\right)$$
$$= 0.064\text{ m}.$$

81. We refer to the particle in the first sentence of the problem statement as particle 2. Since the total momentum of the two particles is zero in S', it must be that the velocities of these two particles are equal in magnitude and opposite in direction in S'. Letting the velocity of the S' frame be v relative to S, then the particle that is at rest in S must have a velocity of $u'_1 = -v$ as measured in S', while the velocity of the other particle is given by solving Eq. 37-29 for u':

$$u'_2 = \frac{u_2 - v}{1-u_2 v/c^2} = \frac{(c/2)-v}{1-(c/2)(v/c^2)}\ .$$

Letting $u'_2 = -u'_1 = v$, we obtain

$$\frac{(c/2)-v}{1-(c/2)(v/c^2)} = v \quad\Rightarrow\quad v = c(2\pm\sqrt{3}) \approx 0.27c$$

where the quadratic formula has been used (with the smaller of the two roots chosen so that $v \le c$).

82. (a) Our lab-based measurement of its lifetime is figured simply from

$$t = L/v = 7.99\times10^{-13}\text{ s}.$$

Use of the time-dilation relation (Eq. 37-7) leads to

$$\Delta t_0 = (7.99\times10^{-13}\text{ s})\sqrt{1-(0.960)^2} = 2.24\times10^{-13}\text{ s}.$$

(b) The length contraction formula can be used, or we can use the simple speed-distance relation (from the point of view of the particle, who watches the lab and all its meter sticks rushing past him at $0.960c$ until he expires): $L = v\Delta t_0 = 6.44\times10^{-5}$ m.

83. (a) For a proton (using Table 37-3), we have

$$E=\gamma m_p c^2=\frac{938\,\text{MeV}}{\sqrt{1-(0.990)^2}}=6.65\,\text{GeV}$$

which gives $K=E-m_p c^2=6.65\,\text{GeV}-938\,\text{MeV}=5.71\,\text{GeV}$.

(b) From part (a), $E=6.65\,\text{GeV}$.

(c) Similarly, we have $p=\gamma m_p v=\gamma(m_p c^2)\beta/c=\dfrac{(938\,\text{MeV})(0.990)/c}{\sqrt{1-(0.990)^2}}=6.58\,\text{GeV}/c$.

(d) For an electron, we have

$$E=\gamma m_e c^2=\frac{0.511\,\text{MeV}}{\sqrt{1-(0.990)^2}}=3.62\,\text{MeV}$$

which yields $K=E-m_e c^2=3.625\,\text{MeV}-0.511\,\text{MeV}=3.11\,\text{MeV}$.

(e) From part (d), $E=3.62\,\text{MeV}$.

(f) $p=\gamma m_e v=\gamma(m_e c^2)\beta/c=\dfrac{(0.511\,\text{MeV})(0.990)/c}{\sqrt{1-(0.990)^2}}=3.59\,\text{MeV}/c$.

84. (a) Using Eq. 37-7, we expect the dilated time intervals to be

$$\tau=\gamma\tau_0=\frac{\tau_0}{\sqrt{1-(v/c)^2}}\ .$$

(b) We rewrite Eq. 37-31 using the fact that the period is the reciprocal of frequency ($f_R=\tau_R^{-1}$ and $f_0=\tau_0^{-1}$):

$$\tau_R=\frac{1}{f_R}=\left(f_0\sqrt{\frac{1-\beta}{1+\beta}}\right)^{-1}=\tau_0\sqrt{\frac{1+\beta}{1-\beta}}=\tau_0\sqrt{\frac{c+v}{c-v}}\ .$$

(c) The Doppler shift combines two physical effects: the time dilation of the moving source *and* the travel-time differences involved in periodic emission (like a sine wave or a series of pulses) from a traveling source to a "stationary" receiver). To isolate the purely time-dilation effect, it's useful to consider "local" measurements (say, comparing the readings on a moving clock to those of two of your clocks, spaced some distance apart, such that the moving clock and each of your clocks can make a close comparison of readings at the moment of passage).

85. Let the reference frame be S in which the particle (approaching the South Pole) is at rest, and let the frame that is fixed on Earth be S'. Then $v = 0.60c$ and $u' = 0.80c$ (calling

"downward" [in the sense of Fig. 37-34] positive). The relative speed is now the speed of the other particle as measured in *S*:

$$u = \frac{u' + v}{1 + u'v/c^2} = \frac{0.80c + 0.60c}{1 + (0.80c)(0.60c)/c^2} = 0.95c\ .$$

86. (a) $\Delta E = \Delta mc^2$ = (3.0 kg)(0.0010)(2.998 × 10^8 m/s)^2 = 2.7 × 10^{14} J.

(b) The mass of TNT is

$$m_{\text{TNT}} = \frac{\left(2.7\times10^{14}\ \text{J}\right)(0.227\ \ \text{kg/mol})}{3.4\times10^{6}\ \text{J}} = 1.8\times10^{7}\ \text{kg}.$$

(c) The fraction of mass converted in the TNT case is

$$\frac{\Delta m_{\text{TNT}}}{m_{\text{TNT}}} = \frac{(3.0\ \text{kg})(0.0010)}{1.8\times10^{7}\ \text{kg}} = 1.6\times10^{-9},$$

Therefore, the fraction is 0.0010/1.6 × 10^{-9} = 6.0 × 10^6.

87. (a) We assume the electron starts from rest. The classical formula for kinetic energy is Eq. 37-51, so if $v = c$ then this (for an electron) would be $\frac{1}{2}mc^2 = \frac{1}{2}(511\ \text{ke V}) = 255.5\ \text{ke V}$ (using Table 37-3). Setting this equal to the potential energy loss (which is responsible for its acceleration), we find (using Eq. 25-7)

$$V = \frac{255.5\ \text{keV}}{|q|} = \frac{255\ \text{keV}}{e} = 255.5\ \text{kV} \approx 256\ \text{kV}.$$

(b) Setting this amount of potential energy loss ($|\Delta U|$ = 255.5 keV) equal to the correct relativistic kinetic energy, we obtain (using Eq. 37-52)

$$mc^2\left(\frac{1}{\sqrt{1-(v/c)^2}} - 1\right) = |\Delta U| \;\;\Rightarrow\;\; v = c\sqrt{1 + \left(\frac{1}{1-\Delta U/mc^2}\right)^2}$$

which yields $v = 0.745c$ = 2.23 × 10^8 m/s.

88. We use the relative velocity formula (Eq. 37-29) with the primed measurements being those of the scout ship. We note that $v = -0.900c$ since the velocity of the scout ship relative to the cruiser is opposite to that of the cruiser relative to the scout ship.

$$u = \frac{u' + v}{1 + u'v/c^2} = \frac{0.980c - 0.900c}{1 - (0.980)(0.900)} = 0.678c\ .$$

89. (a) Since both spaceships A and C are approaching B at the same speed (relative to B), with $v_A > v_B > v_C$, using relativistic velocity addition formula, we have $v'_A = -v'_c$, or

$$\frac{v_A - v_B}{1 - v_A v_B / c^2} = \frac{v_B - v_C}{1 - v_B v_C / c^2} \quad \Rightarrow \quad \frac{\beta_A - \beta_B}{1 - \beta_A \beta_B} = \frac{\beta_B - \beta_C}{1 - \beta_B \beta_C}$$

We multiply both sides by factors to get rid of the denominators:

$$(\beta_A - \beta_B)(1 - \beta_B \beta_C) = (\beta_B - \beta_C)(1 - \beta_A \beta_B)$$

Expanding and simplifying gives

$$(\beta_A + \beta_C)\beta_B^2 - 2(1 + \beta_A \beta_C)\beta_B + (\beta_A + \beta_C) = 0$$

Solving the quadratic equation with $\beta_A = 0.90$ and $\beta_C = 0.80$ leads to $\beta_B = 0.858$, or $v_B = 0.858c$.

(b) The relative speed (say, A relative to B) is

$$v'_A = \frac{v_A - v_B}{1 - v_A v_B / c^2} = \frac{0.90c - 0.858c}{1 - (0.90)(0.858)} = 0.185c\,.$$

90. In the rest frame of Cruiser A, Cruiser B is moving at a speed of 0.900c, and has a length of 200 m. The proper length of Cruiser B, according to its pilot, is

$$L_{B0} = \gamma L_B = \frac{200 \text{ m}}{\sqrt{1 - (0.900)^2}} = 458.8 \text{ m},$$

and the length of Cruiser A is $L_A = L_{A0} / \gamma = \sqrt{1 - (0.900)^2}\,(200 \text{ m}) = 87.2 \text{ m}$. Therefore, according to pilot in Cruiser B, the time elapsed for the tails to align is

$$\Delta t = \frac{L_{B0} - L_A}{v_A} = \frac{458.8 \text{ m} - 87.2 \text{ m}}{(0.90)(3.0 \times 10^8 \text{ m/s})} = 1.38 \times 10^{-6} \text{ s}\,.$$

91. Let the speed of B relative to the station be v_B. We require the speed of A relative to B to be the same as v_B:

$$v'_A = \frac{v_A - v_B}{1 - v_A v_B / c^2} = v_B\,.$$

The above expression can be rewritten as $v_B^2 - (2c^2/v_A)v_B + c^2 = 0$. Solving the quadratic equation for v_B, with $v_A = 0.80c$, we obtain $v_B = 0.50c$.

92. (a) From the train view, the tunnel appears to be contracted by a factor of

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-(0.900)^2}} = 2.29.$$

Thus, the length is $L_{\text{tunnel}} = L_{\text{tunnel},0}/\gamma = (200 \text{ m})/2.29 = 87.2 \text{ m}$.

(b) From the train view, since the tunnel appears to be shorter than the train, event FF will occur first.

(c) According to an observer on the train, the time between the two events is

$$\Delta t = \frac{L_{\text{train},0} - L_{\text{tunnel}}}{v} = \frac{200 \text{ m} - 87.2 \text{ m}}{(0.900)(3.0\times10^8 \text{ m/s})} = 0.418\mu\text{s}.$$

(d) Since event FF occurs first, the paint will explode.

(e) From the tunnel view, the train appears to be contracted by a factor of

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-(0.900)^2}} = 2.29.$$

Thus, the length is $L_{\text{train}} = L_{\text{train},0}/\gamma = (200 \text{ m})/2.29 = 87.2 \text{ m}$.

(f) From the tunnel view, since the train appears to be shorter than the tunnel, event RN will occur first.

(g) According to an observer in the rest frame of the tunnel, the time between the two events is

$$\Delta t = \frac{L_{\text{tunnel},0} - L_{\text{train}}}{v} = \frac{200 \text{ m} - 87.2 \text{ m}}{(0.900)(3.0\times10^8 \text{ m/s})} = 0.418\mu\text{s}.$$

(h) The bomb will explode also. The reason is that one must take into consideration the time is takes for the deactivation signal to propagate from the rear of the train to the front, which is $\Delta t_{\text{signal}} = \dfrac{L_{\text{train},0}}{v} = \dfrac{200 \text{ m}}{(0.900)(3.0\times10^8 \text{ m/s})} = 0.741\mu\text{s}$. This is longer than the time elapsed between the two events. So the bomb does explode.

93. (a) The condition for energy conservation is $E_A = E_B + E_C$. Similarly, momentum conservation requires $p_B = p_C$ (same magnitude but opposite directions). Using $E = \gamma mc^2$ gives $m_A c^2 = \gamma_B m_B c^2 + \gamma_C m_C c^2$, or

$$200 = 100\gamma_B + 50\gamma_C \;\Rightarrow\; 4 = 2\gamma_B + \gamma_C$$

Now using $p = \gamma mv$, we have

$$\gamma_B m_B v_B = \gamma_C m_C v_C \;\Rightarrow\; \gamma_B m_B \beta_B = \gamma_C m_C \beta_C$$

Noting that $\gamma\beta = \gamma\sqrt{1-1/\gamma^2} = \sqrt{\gamma^2 - 1}$, the above expression can be rewritten as

$$\frac{\sqrt{\gamma_B^2 - 1}}{\sqrt{\gamma_C^2 - 1}} = \frac{m_C}{m_B} = \frac{50\text{ MeV}/c^2}{100\text{ MeV}/c^2} = \frac{1}{2}$$

which implies $4\gamma_B^2 = \gamma_C^2 + 3$. Solving the two simultaneous equations gives $\gamma_B = 19/16$ and $\gamma_C = 13/8$. The total energy of *B* is

$$E_B = \gamma_B m_B c^2 = \left(\frac{19}{16}\right)(100\text{ MeV}) = 119\text{ MeV}.$$

(b) Using $p = \gamma mv = \sqrt{\gamma^2 - 1}\,\dfrac{mc^2}{c}$, we find the momentum of *B* to be

$$p_B = \sqrt{\gamma_B^2 - 1}\,\frac{m_B c^2}{c} = \sqrt{(19/16)^2 - 1}\,(100\text{ MeV}/c) = 64.0\text{ MeV}/c.$$

(c) The total energy of *C* is $E_C = \gamma_C m_C c^2 = \left(\dfrac{13}{8}\right)(50\text{ MeV}) = 81.3\text{ MeV}.$

(d) The magnitude of momentum of *C* is the same as *B*: $p_C = 64.0\text{ MeV}/c$.

94. (a) The travel time for trip 1 measured by an Earth observer is $\Delta t_1 = 2D/c$.

(b) For trip 2, we have $\Delta t_2 = 4D/c$,

(c) and $\Delta t_3 = 6D/c$, for trip 3.

(d) In the rest frame of the starship, the distance appears to be shortened by the Lorentz factor γ. Thus, $\Delta t_1' = \dfrac{2D'}{c} = \dfrac{2D}{c\gamma_1} = \dfrac{D}{5c}$.

(e) Similarly, for trip 2, we have $\Delta t_2' = \frac{4D'}{c} = \frac{4D}{c\gamma_2} = \frac{4D}{c(24)} = \frac{D}{6c}$.

(f) For trip 3, the time is $\Delta t_3' = \frac{6D'}{c} = \frac{6D}{c\gamma_3} = \frac{6D}{c(30)} = \frac{D}{5c}$.

95. The radius r of the path is $r = \gamma mvqB$. Thus,

$$m = \frac{qBr\sqrt{1-\beta^2}}{v} = \frac{2(1.60\times10^{-19}\text{ C})(1.00\text{ T})(6.28\text{ m})\sqrt{1-(0.710)^2}}{(0.710)(3.00\times10^8\text{ m/s})} = 6.64\times10^{-27}\text{ kg}.$$

Since 1.00 u = 1.66×10^{-27} kg, the mass is m = 4.00 u. The nuclear particle contains four nucleons. Since there must be two protons to provide the charge $2e$, the nuclear particle is a helium nucleus (usually referred to as an alpha particle) with two protons and two neutrons.

96. We interpret the given 2.50 MeV = 2500 keV to be the kinetic energy of the electron. Using Table 37-3, we find

$$\gamma = \frac{K}{m_e c^2} + 1 = \frac{2500\text{ keV}}{511\text{ keV}} + 1 = 5.892,$$

and

$$\beta = \sqrt{1 - \frac{1}{\gamma^2}} = 0.9855.$$

Therefore, using the equation $r = \gamma mvqB$ (with "q" interpreted as $|q|$), we obtain

$$B = \frac{\gamma m_e v}{|q|r} = \frac{\gamma m_e \beta c}{er} = \frac{(5.892)(9.11\times10^{-31}\text{ kg})(0.9855)(2.998\times10^8\text{ m/s})}{(1.6\times10^{-19}\text{ C})(0.030\text{ m})}$$
$$= 0.33\text{ T}.$$

97. (a) Using Table 37-3 and Eq. 37-58, we find

$$\gamma = \frac{K}{m_p c^2} + 1 = \frac{500\times10^3\text{ MeV}}{938.3\text{ MeV}} + 1 = 533.88.$$

(b) From Eq. 37-8, we obtain

$$\beta = \sqrt{1 - \frac{1}{\gamma^2}} = 0.99999825.$$

(c) To make use of the precise $m_p c^2$ value given here, we rewrite the expression introduced in problem 46 (as applied to the proton) as follows:

$$r = \frac{\gamma mv}{qB} = \frac{\gamma(mc^2)\left(\frac{v}{c^2}\right)}{eB} = \frac{\gamma(mc^2)\beta}{ecB}.$$

Therefore, the magnitude of the magnetic field is

$$B = \frac{\gamma(mc^2)\beta}{ecr} = \frac{(533.88)(938.3\text{ MeV})(0.99999825)}{ec(750\text{ m})} = \frac{667.92\times10^6\text{ V/m}}{c}$$

where we note the cancellation of the "e" in MeV with the e in the denominator. After substituting $c = 2.998 \times 10^8$ m/s, we obtain $B = 2.23$ T.

98. (a) The pulse rate as measured by an observer at the station is

$$R = \frac{\Delta N}{\Delta t} = \frac{\Delta N}{\gamma\Delta t_0} = \frac{R_0}{\gamma} = (150/\text{min})\sqrt{1-(0.900)^2} = 65.4/\text{min}.$$

(b) According to the observer at the station, the stride length appears to be shortened, and the clock runs slower in the spaceship, the speed observed is

$$v = \frac{\Delta L}{\Delta t} = \frac{L_0/\gamma}{\gamma\Delta t_0} = \frac{v_0}{\gamma^2},$$

and the distance the astronaut walked is measured to be

$$d = v\Delta t = \frac{v_0}{\gamma^2}\gamma\Delta t_0 = \frac{v_0\Delta t_0}{\gamma} = \sqrt{1-(0.900)^2}(1.0\text{ m/s})(3600\text{ s}) = 1570\text{ m}.$$

99. The frequency received is given by

$$f = f_0\sqrt{\frac{1+\beta}{1-\beta}} \quad \Rightarrow \quad \frac{c}{\lambda} = \frac{c}{\lambda_0}\sqrt{\frac{1+\beta}{1-\beta}}$$

which implies

$$\lambda = \lambda_0\sqrt{\frac{1-\beta}{1+\beta}} = (650\text{ nm})\sqrt{\frac{1-0.42}{1+0.42}} = 415\text{ nm}.$$

This is in the blue portion of the visible spectrum.

100. (a) Using the classical Doppler equation $f' = \frac{v}{v+v_s} f$, we have

$$v_s = v\left(\frac{f}{f'}-1\right) = v\left(\frac{\lambda'}{\lambda}-1\right) = c\left(\frac{3\lambda}{\lambda}-1\right) = 2c > c.$$

(b) Using $f = f_0\sqrt{\dfrac{1-\beta}{1+\beta}}$, we solve for β and obtain

$$\beta = \frac{1-(f/f_0)^2}{1+(f/f_0)^2} = \frac{1-(1/3)^2}{1+(1/3)^2} = \frac{8/9}{10/9} = 0.80$$

or $v = 0.80c$.

101. Using $E = mc^2$, we find the required mass to be

$$m = \frac{E}{c^2} = \frac{(2.2\times10^{12}\ \text{kWh})(3.6\times10^{12}\ \text{J/kWh})}{(3\times10^8\ \text{m/s})^2} = 88\ \text{kg}.$$

(b) No, the energy consumed is still about 2.2×10^{12} kWh regardless of how it's generated (oil-burning, nuclear, or hydroelectric….).

102. (a) The time an electron with a horizontal component of velocity v takes to travel a horizontal distance L is

$$t = \frac{L}{v} = \frac{20\times10^{-2}\,\text{m}}{(0.992)(2.998\times10^8\ \text{m/s})} = 6.72\times10^{-10}\ \text{s}.$$

(b) During this time, it falls a vertical distance

$$y = \frac{1}{2}gt^2 = \frac{1}{2}(9.8\,\text{m/s}^2)(6.72\times10^{-10}\ \text{s})^2 = 2.2\times10^{-18}\ \text{m}.$$

This distance is much less than the radius of a proton.

(c) We can conclude that for particles traveling near the speed of light in a laboratory, Earth may be considered an approximately inertial frame.

103. (a) The speed parameter β is v/c. Thus,

$$\beta = \frac{(3\,\text{cm/y})(0.01\,\text{m/cm})(1\,\text{y}/3.15\times10^7\ \text{s})}{3.0\times10^8\ \text{m/s}} = 3\times10^{-18}.$$

(b) For the highway speed limit, we find

$$\beta = \frac{(90\,\text{km/h})(1000\,\text{m/km})(1\,\text{h}/3600\,\text{s})}{3.0\times 10^{8}\ \text{m/s}} = 8.3\times 10^{-8}.$$

(c) Mach 2.5 corresponds to

$$\beta = \frac{(1200\,\text{km/h})(1000\,\text{m/km})(1\,\text{h}/3600\,\text{s})}{3.0\times 10^{8}\ \text{m/s}} = 1.1\times 10^{-6}.$$

(d) We refer to Table 14-2:

$$\beta = \frac{(11.2\,\text{km/s})(1000\,\text{m/km})}{3.0\times 10^{8}\ \text{m/s}} = 3.7\times 10^{-5}.$$

(e) For the quasar recession speed, we obtain

$$\beta = \frac{(3.0\times 10^{4}\ \text{km/s})(1000\,\text{m/km})}{3.0\times 10^{8}\ \text{m/s}} = 0.10.$$

Chapter 38

1. (a) With $E = hc/\lambda_{\min} = 1240\ \text{eV}\cdot\text{nm}/\lambda_{\min} = 0.6$ eV, we obtain $\lambda = 2.1 \times 10^3$ nm = 2.1 μm.

(b) It is in the infrared region.

2. Let

$$\frac{1}{2}m_e v^2 = E_{\text{photon}} = \frac{hc}{\lambda}$$

and solve for v:

$$v = \sqrt{\frac{2hc}{\lambda m_e}} = \sqrt{\frac{2hc}{\lambda m_e c^2}c^2} = c\sqrt{\frac{2hc}{\lambda\left(m_e c^2\right)}}$$
$$= \left(2.998\times10^8\ \text{m/s}\right)\sqrt{\frac{2\left(1240\,\text{eV}\cdot\text{nm}\right)}{\left(590\,\text{nm}\right)\left(511\times10^3\ \text{eV}\right)}} = 8.6\times10^5\ \text{m/s}.$$

Since $v \ll c$, the nonrelativistic formula $K = \frac{1}{2}mv^2$ may be used. The $m_e c^2$ value of Table 37-3 and $hc = 1240\text{eV}\cdot\text{nm}$ are used in our calculation.

3. Let R be the rate of photon emission (number of photons emitted per unit time) of the Sun and let E be the energy of a single photon. Then the power output of the Sun is given by $P = RE$. Now

$$E = hf = hc/\lambda,$$

where $h = 6.626 \times 10^{-34}$ J·s is the Planck constant, f is the frequency of the light emitted, and λ is the wavelength. Thus $P = Rhc/\lambda$ and

$$R = \frac{\lambda P}{hc} = \frac{\left(550\,\text{nm}\right)\left(3.9\times10^{26}\ \text{W}\right)}{\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(2.998\times10^8\ \text{m/s}\right)} = 1.0\times10^{45}\ \text{photons/s}.$$

4. We denote the diameter of the laser beam as d. The cross-sectional area of the beam is $A = \pi d^{\,2}/4$. From the formula obtained in Problem 38-3, the rate is given by

$$\frac{R}{A} = \frac{\lambda P}{hc\left(\pi d^2/4\right)} = \frac{4\left(633\text{nm}\right)\left(5.0\times10^{-3}\,\text{W}\right)}{\pi\left(6.63\times10^{-34}\,\text{J}\cdot\text{s}\right)\left(2.998\times10^8\,\text{m/s}\right)\left(3.5\times10^{-3}\,\text{m}\right)^2}$$
$$= 1.7\times10^{21}\,\text{photons/m}^2\cdot\text{s}.$$

5. The energy of a photon is given by $E = hf$, where h is the Planck constant and f is the frequency. The wavelength λ is related to the frequency by $\lambda f = c$, so $E = hc/\lambda$. Since $h = 6.626 \times 10^{-34}$ J·s and $c = 2.998 \times 10^{8}$ m/s,

$$hc = \frac{(6.626\times10^{-34}\ \mathrm{J\cdot s})(2.998\times10^{8}\ \mathrm{m/s})}{(1.602\times10^{-19}\ \mathrm{J/eV})(10^{-9}\ \mathrm{m/nm})} = 1240\,\mathrm{eV\cdot nm}.$$

Thus,

$$E = \frac{1240\,\mathrm{eV\cdot nm}}{\lambda}.$$

With

$$\lambda = (1,\ 650,\ 763.73)^{-1}\ \mathrm{m} = 6.0578021 \times 10^{-7}\ \mathrm{m} = 605.78021\ \mathrm{nm},$$

we find the energy to be

$$E = \frac{hc}{\lambda} = \frac{1240\,\mathrm{eV\cdot nm}}{605.78021\,\mathrm{nm}} = 2.047\,\mathrm{eV}.$$

6. The energy of a photon is given by $E = hf$, where h is the Planck constant and f is the frequency. The wavelength λ is related to the frequency by $\lambda f = c$, so $E = hc/\lambda$. Since $h = 6.626 \times 10^{-34}$ J·s and $c = 2.998 \times 10^{8}$ m/s,

$$hc = \frac{(6.626\times10^{-34}\ \mathrm{J\cdot s})(2.998\times10^{8}\ \mathrm{m/s})}{(1.602\times10^{-19}\ \mathrm{J/eV})(10^{-9}\ \mathrm{m/nm})} = 1240\,\mathrm{eV\cdot nm}.$$

Thus,

$$E = \frac{1240\,\mathrm{eV\cdot nm}}{\lambda}.$$

With $\lambda = 589\ \mathrm{nm}$, we obtain

$$E = \frac{hc}{\lambda} = \frac{1240\,\mathrm{eV\cdot nm}}{589\,\mathrm{nm}} = 2.11\,\mathrm{eV}.$$

7. The rate at which photons are absorbed by the detector is related to the rate of photon emission by the light source via

$$R_{\mathrm{abs}} = (0.80)\frac{A_{\mathrm{abs}}}{4\pi r^2} R_{\mathrm{emit}}.$$

Given that $A_{\mathrm{abs}} = 2.00\times10^{-6}\ \mathrm{m}^2$ and $r = 3.00$ m, with $R_{\mathrm{abs}} = 4.000$ photons/s, we find the rate at which photons are emitted to be

$$R_{\mathrm{emit}} = \frac{4\pi r^2}{(0.80)A_{\mathrm{abs}}} R_{\mathrm{abs}} = \frac{4\pi(3.00\ \mathrm{m})^2}{(0.80)(2.00\times10^{-6}\ \mathrm{m}^2)}(4.000\ \text{photons/s}) = 2.83\times10^{8}\ \text{photons/s}.$$

Since the energy of each emitted photon is

$$E_{\text{ph}} = \frac{hc}{\lambda} = \frac{1240\,\text{eV}\cdot\text{nm}}{500\,\text{nm}} = 2.48\,\text{eV},$$

the power output of source is

$$P_{\text{emit}} = R_{\text{emit}}E_{\text{ph}} = \left(2.83\times10^{8}\,\text{photons/s}\right)(2.48\,\text{eV}) = 7.0\times10^{8}\ \text{eV/s} = 1.1\times10^{-10}\ \text{W}.$$

8. The rate at which photons are emitted from the argon laser source is given by $R = P/E_{\text{ph}}$, where P = 1.5 W is the power of the laser beam and $E_{\text{ph}} = hc/\lambda$ is the energy of each photon of wavelength λ. Since α = 84% of the energy of the laser beam falls within the central disk, the rate of photon absorption of the central disk is

$$R' = \alpha R = \frac{\alpha P}{hc/\lambda} = \frac{(0.84)(1.5\,\text{W})}{\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(2.998\times10^{8}\ \text{m/s}\right)/\left(515\times10^{-9}\ \text{m}\right)}$$
$$= 3.3\times10^{18}\ \text{photons/s}.$$

9. (a) We assume all the power results in photon production at the wavelength $\lambda = 589\ \text{nm}$. Let R be the rate of photon production and E be the energy of a single photon. Then,

$$P = RE = Rhc/\lambda,$$

where $E = hf$ and $f = c/\lambda$ are used. Here h is the Planck constant, f is the frequency of the emitted light, and λ is its wavelength. Thus,

$$R = \frac{\lambda P}{hc} = \frac{\left(589\times10^{-9}\ \text{m}\right)(100\ \text{W})}{\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00\times10^{8}\ \text{m/s}\right)} = 2.96\times10^{20}\ \text{photon/s}.$$

(b) Let I be the photon flux a distance r from the source. Since photons are emitted uniformly in all directions, $R = 4\pi r^2 I$ and

$$r = \sqrt{\frac{R}{4\pi I}} = \sqrt{\frac{2.96\times10^{20}\ \text{photon/s}}{4\pi\left(1.00\times10^{4}\ \text{photon/m}^2\cdot\text{s}\right)}} = 4.86\times10^{7}\ \text{m}.$$

(c) The photon flux is

$$I = \frac{R}{4\pi r^2} = \frac{2.96\times10^{20}\ \text{photon/s}}{4\pi\left(2.00\ \text{m}\right)^2} = 5.89\times10^{18}\ \frac{\text{photon}}{\text{m}^2\cdot\text{s}}.$$

10. (a) The rate at which solar energy strikes the panel is

$$P = (1.39 \text{ kW/m}^2)(2.60 \text{ m}^2) = 3.61 \text{ kW}.$$

(b) The rate at which solar photons are absorbed by the panel is

$$R = \frac{P}{E_{\text{ph}}} = \frac{3.61\times10^3 \text{ W}}{(6.63\times10^{-34} \text{ J}\cdot\text{s})(2.998\times10^8 \text{ m/s})/(550\times10^{-9} \text{ m})}$$
$$= 1.00\times10^{22} \text{ photons/s}.$$

(c) The time in question is given by

$$t = \frac{N_A}{R} = \frac{6.02\times10^{23}}{1.00\times10^{22}\text{ /s}} = 60.2 \text{ s}.$$

11. **THINK** The rate of photon emission is the number of photons emitted per unit time.

EXPRESS Let R be the photon emission rate and E be the energy of a single photon. The power output of a lamp is given by $P = RE$, where we assume that all the power goes into photon production. Now, $E = hf = hc/\lambda$, where h is the Planck constant, f is the frequency of the light emitted, and λ is the wavelength. Thus

$$P = \frac{Rhc}{\lambda} \quad \Rightarrow \quad R = \frac{\lambda P}{hc} \; .$$

ANALYZE (a) The fact that $R \sim \lambda$ means that the lamp that emits light with the longer wavelength (the 700 nm infrared lamp) emits more photons per unit time. The energy of each photon is less, so it must emit photons at a greater rate.

(b) Let R be the rate of photon production for the 700 nm lamp. Then,

$$R = \frac{\lambda P}{hc} = \frac{(700\,\text{nm})(400\,\text{J/s})}{(1.60\times10^{-19} \text{ J/eV})(1240 \text{ eV}\cdot\text{nm})} = 1.41\times10^{21} \text{ photon/s}.$$

LEARN With $P = Rhc/\lambda$, we readily see that when the rate of photon emission is held constant, the shorter the wavelength, the greater the power, or rate of energy emission.

12. Following Sample Problem — "Emission and absorption of light as photons," we have

$$P = \frac{Rhc}{\lambda} = \frac{(100/\text{s})(6.63\times10^{-34} \text{ J}\cdot\text{s})(2.998\times10^8 \text{ m/s})}{550\times10^{-9} \text{ m}} = 3.6\times10^{-17} \text{ W}.$$

13. The total energy emitted by the bulb is $E = 0.93Pt$, where $P = 60$ W and

$$t = 730 \text{ h} = (730 \text{ h})(3600 \text{ s/h}) = 2.628 \times 10^6 \text{ s}.$$

The energy of each photon emitted is $E_{ph} = hc/\lambda$. Therefore, the number of photons emitted is

$$N = \frac{E}{E_{ph}} = \frac{0.93Pt}{hc/\lambda} = \frac{(0.93)(60\,\text{W})(2.628\times10^6\text{ s})}{(6.63\times10^{-34}\text{ J}\cdot\text{s})(2.998\times10^8\text{ m/s})/(630\times10^{-9}\text{ m})} = 4.7\times10^{26}.$$

14. The average power output of the source is

$$P_{\text{emit}} = \frac{\Delta E}{\Delta t} = \frac{7.2\text{ nJ}}{2\text{ s}} = 3.6\text{ nJ/s} = 3.6\times10^{-9}\text{ J/s} = 2.25\times10^{10}\text{ eV/s}.$$

Since the energy of each photon emitted is

$$E_{ph} = \frac{hc}{\lambda} = \frac{1240\text{ eV}\cdot\text{nm}}{600\,\text{nm}} = 2.07\text{ eV},$$

the rate at which photons are emitted by the source is

$$R_{\text{emit}} = \frac{P_{\text{emit}}}{E_{ph}} = \frac{2.25\times10^{10}\text{ eV/s}}{2.07\text{ eV}} = 1.09\times10^{10}\,\text{photons/s}.$$

Given that the source is isotropic, and the detector (located 12.0 m away) has an absorbing area of $A_{\text{abs}} = 2.00\times10^{-6}\text{ m}^2$ and absorbs 50% of the incident light, the rate of photon absorption is

$$R_{\text{abs}} = (0.50)\frac{A_{\text{abs}}}{4\pi r^2}R_{\text{emit}} = (0.50)\frac{2.00\times10^{-6}\text{ m}^2}{4\pi(12.0\text{ m})^2}\left(1.09\times10^{10}\,\text{photons/s}\right) = 6.0\text{ photons/s}.$$

15. **THINK** The energy of an incident photon is $E = hf$, where h is the Planck constant, and f is the frequency of the electromagnetic radiation.

EXPRESS The kinetic energy of the most energetic electron emitted is

$$K_m = E - \Phi = (hc/\lambda) - \Phi,$$

where Φ is the work function for sodium, and $f = c/\lambda$, where λ is the wavelength of the photon.

The stopping potential V_{stop} is related to the maximum kinetic energy by $eV_{\text{stop}} = K_m$, so

$$eV_{\text{stop}} = (hc/\lambda) - \Phi$$

and

$$\lambda = \frac{hc}{eV_{\text{stop}} + \Phi} = \frac{1240\,\text{eV}\cdot\text{nm}}{5.0\,\text{eV} + 2.2\,\text{eV}} = 170\,\text{nm}.$$

Here eV_{stop} = 5.0 eV and hc = 1240 eV·nm are used.

LEARN The cutoff frequency for this problem is

$$f_0 = \frac{\Phi}{h} = \frac{(2.2\text{ eV})(1.6\times10^{-19}\text{ J/eV})}{6.626\times10^{-34}\,\text{J}\cdot\text{s}} = 5.3\times10^{14}\text{ Hz}.$$

16. We use Eq. 38-5 to find the maximum kinetic energy of the ejected electrons:

$$K_{\max} = hf - \Phi = \left(4.14\times10^{-15}\text{ eV}\cdot\text{s}\right)\left(3.0\times10^{15}\text{ Hz}\right) - 2.3\,\text{eV} = 10\,\text{eV}.$$

17. The speed v of the electron satisfies

$$K_{\max} = \tfrac{1}{2}m_e v^2 = \tfrac{1}{2}\left(m_e c^2\right)\left(v/c\right)^2 = E_{\text{photon}} - \Phi.$$

Using Table 37-3, we find

$$v = c\sqrt{\frac{2\left(E_{\text{photon}} - \Phi\right)}{m_e c^2}} = \left(2.998\times10^{8}\text{ m/s}\right)\sqrt{\frac{2\left(5.80\,\text{eV} - 4.50\,\text{eV}\right)}{511\times10^{3}\text{ eV}}} = 6.76\times10^{5}\text{ m/s}.$$

18. The energy of the most energetic photon in the visible light range (with wavelength of about 400 nm) is about E = (1240 eV·nm/400 nm) = 3.1 eV (using the value hc = 1240 eV·nm). Consequently, barium and lithium can be used, since their work functions are both lower than 3.1 eV.

19. (a) We use Eq. 38-6:

$$V_{\text{stop}} = \frac{hf - \Phi}{e} = \frac{hc/\lambda - \Phi}{e} = \frac{\left(1240\,\text{eV}\cdot\text{nm}/400\,\text{nm}\right) - 1.8\,\text{eV}}{e} = 1.3\,\text{V}.$$

(b) The speed v of the electron satisfies

$$K_{\max} = \tfrac{1}{2}m_e v^2 = \tfrac{1}{2}\left(m_e c^2\right)\left(v/c\right)^2 = E_{\text{photon}} - \Phi.$$

Using Table 37-3, we find

$$v=\sqrt{\frac{2\left(E_{\text{photon}}-\Phi\right)}{m_e}}=\sqrt{\frac{2eV_{\text{stop}}}{m_e}}=c\sqrt{\frac{2eV_{\text{stop}}}{m_ec^2}}=\left(2.998\times10^8\text{ m/s}\right)\sqrt{\frac{2e\left(1.3\text{V}\right)}{511\times10^3\text{eV}}}$$
$$=6.8\times10^5\text{ m/s}.$$

20. Using the value hc = 1240 eV·nm, the number of photons emitted from the laser per unit time is

$$R=\frac{P}{E_{\text{ph}}}=\frac{2.00\times10^{-3}\text{ W}}{(1240\text{ eV}\cdot\text{nm}/600\text{ nm})(1.60\times10^{-19}\text{ J}/\text{eV})}=6.05\times10^{15}/\text{s},$$

of which $(1.0\times10^{-16})(6.05\times10^{15}/\text{s}) = 0.605/\text{s}$ actually cause photoelectric emissions. Thus the current is

$$i = (0.605/\text{s})(1.60\times10^{-19}\text{ C}) = 9.68\times10^{-20}\text{ A}.$$

21. (a) From $r = m_ev/eB$, the speed of the electron is $v = rBe/m_e$. Thus,

$$K_{\max}=\frac{1}{2}m_ev^2=\frac{1}{2}m_e\left(\frac{rBe}{m_e}\right)^2=\frac{(rB)^2e^2}{2m_e}=\frac{(1.88\times10^{-4}\text{ T}\cdot\text{m})^2(1.60\times10^{-19}\text{ C})^2}{2(9.11\times10^{-31}\text{ kg})(1.60\times10^{-19}\text{ J/eV})}$$
$$=3.1\text{ keV}.$$

(b) Using the value hc = 1240 eV·nm, the work done is

$$W=E_{\text{photon}}-K_{\max}=\frac{1240\text{ eV}\cdot\text{nm}}{71\times10^{-3}\text{ nm}}-3.10\text{ keV}=14\text{ keV}.$$

22. We use Eq. 38-6 and the value hc = 1240 eV·nm:

$$K_{\max}=E_{\text{photon}}-\Phi=\frac{hc}{\lambda}-\frac{hc}{\lambda_{\max}}=\frac{1240\,\text{eV}\cdot\text{nm}}{254\,\text{nm}}-\frac{1240\,\text{eV}\cdot\text{nm}}{325\,\text{nm}}=1.07\,\text{eV}.$$

23. **THINK** The kinetic energy K_m of the fastest electron emitted is given by

$$K_m = hf - \Phi,$$

where Φ is the work function of aluminum, and f is the frequency of the incident radiation.

EXPRESS Since $f = c/\lambda$, where λ is the wavelength of the photon, the above expression can be rewritten as

$$K_m = (hc/\lambda) - \Phi.$$

ANALYZE (a) Thus, the kinetic energy of the fastest electron is

$$K_m = \frac{1240\,\text{eV}\cdot\text{nm}}{200\,\text{nm}} - 4.20\,\text{eV} = 2.00\ \text{eV}\,,$$

where we have used hc = 1240 eV·nm.

(b) The slowest electron just breaks free of the surface and so has zero kinetic energy.

(c) The stopping potential V_{stop} is given by $K_m = eV_{\text{stop}}$, so

$$V_{\text{stop}} = K_m/e = (2.00\ \text{eV})/e = 2.00\ \text{V}.$$

(d) The value of the cutoff wavelength is such that $K_m = 0$. Thus, $hc/\lambda_0 = \Phi$, or

$$\lambda_0 = hc/\Phi = (1240\ \text{eV}\cdot\text{nm})/(4.2\ \text{eV}) = 295\ \text{nm}.$$

LEARN If the wavelength is longer than λ_0, the photon energy is less than Φ and a photon does not have sufficient energy to knock even the most energetic electron out of the aluminum sample.

24. (a) For the first and second case (labeled 1 and 2) we have

$$eV_{01} = hc/\lambda_1 - \Phi\,, \qquad eV_{02} = hc/\lambda_2 - \Phi,$$

from which h and Φ can be determined. Thus,

$$h = \frac{e(V_1 - V_2)}{c(\lambda_1^{-1} - \lambda_2^{-1})} = \frac{1.85\text{eV} - 0.820\text{eV}}{(3.00\times10^{17}\,\text{nm/s})\left[(300\text{nm})^{-1} - (400\text{nm})^{-1}\right]} = 4.12\times10^{-15}\,\text{eV}\cdot\text{s}.$$

(b) The work function is

$$\Phi = \frac{3(V_2\lambda_2 - V_1\lambda_1)}{\lambda_1 - \lambda_2} = \frac{(0.820\ \text{eV})(400\ \text{nm}) - (1.85\ \text{eV})(300\ \text{nm})}{300\ \text{nm} - 400\ \text{nm}} = 2.27\ \text{eV}.$$

(c) Let $\Phi = hc/\lambda_{\text{max}}$ to obtain

$$\lambda_{\text{max}} = \frac{hc}{\Phi} = \frac{1240\ \text{eV}\cdot\text{nm}}{2.27\ \text{eV}} = 545\ \text{nm}.$$

25. (a) We use the photoelectric effect equation (Eq. 38-5) in the form $hc/\lambda = \Phi + K_m$. The work function depends only on the material and the condition of the surface, and not on the wavelength of the incident light. Let λ_1 be the first wavelength described and λ_2 be the second. Let K_{m1} = 0.710 eV be the maximum kinetic energy of electrons ejected by

light with the first wavelength, and K_{m2} = 1.43 eV be the maximum kinetic energy of electrons ejected by light with the second wavelength. Then,

$$\frac{hc}{\lambda_1} = \Phi + K_{m1}, \qquad \frac{hc}{\lambda_2} = \Phi + K_{m2}.$$

The first equation yields $\Phi = (hc/\lambda_1) - K_{m1}$. When this is used to substitute for Φ in the second equation, the result is

$$(hc/\lambda_2) = (hc/\lambda_1) - K_{m1} + K_{m2}.$$

The solution for λ_2 is

$$\lambda_2 = \frac{hc\lambda_1}{hc + \lambda_1(K_{m2} - K_{m1})} = \frac{(1240\,\text{V}\cdot\text{nm})(491\,\text{nm})}{1240\,\text{eV}\cdot\text{nm} + (491\,\text{nm})(1.43\,\text{eV} - 0.710\,\text{eV})}$$
$$= 382\,\text{nm}.$$

Here hc = 1240 eV·nm has been used.

(b) The first equation displayed above yields

$$\Phi = \frac{hc}{\lambda_1} - K_{m1} = \frac{1240\ \text{eV}\cdot\text{nm}}{491\ \text{nm}} - 0.710\ \text{eV} = 1.82\ \text{eV}.$$

26. To find the longest possible wavelength λ_{max} (corresponding to the lowest possible energy) of a photon that can produce a photoelectric effect in platinum, we set $K_{max} = 0$ in Eq. 38-5 and use $hf = hc/\lambda$. Thus $hc/\lambda_{max} = \Phi$. We solve for λ_{max}:

$$\lambda_{max} = \frac{hc}{\Phi} = \frac{1240\,\text{eV}\cdot\text{nm}}{5.32\,\text{nm}} = 233\,\text{nm}.$$

27. **THINK** The scattering between a photon and an electron initially at rest results in a change or photon's wavelength, or Compton shift.

EXPRESS When a photon scatters off from an electron initially at rest, the change in wavelength is given by

$$\Delta\lambda = (h/mc)(1 - \cos\phi),$$

where m is the mass of an electron and ϕ is the scattering angle.

ANALYZE (a) The Compton wavelength of the electron is $h/mc = 2.43 \times 10^{-12}$ m = 2.43 pm. Therefore, we find the shift to be

$$\Delta\lambda = (h/mc)(1 - \cos\phi) = (2.43\text{ pm})(1 - \cos 30°) = 0.326\text{ pm}.$$

The final wavelength is

$$\lambda' = \lambda + \Delta\lambda = 2.4 \text{ pm} + 0.326 \text{ pm} = 2.73 \text{ pm}.$$

(b) With $\phi = 120°$, $\Delta\lambda = (2.43 \text{ pm})(1 - \cos 120°) = 3.645$ pm and

$$\lambda' = 2.4 \text{ pm} + 3.645 \text{ pm} = 6.05 \text{ pm}.$$

LEARN The wavelength shift is greatest when $\phi = 180°$, where $\cos 180° = -1$. At this angle, the photon is scattered back along its initial direction of travel, and $\Delta\lambda = 2h/mc$.

28. (a) The rest energy of an electron is given by $E = m_e c^2$. Thus the momentum of the photon in question is given by

$$p = \frac{E}{c} = \frac{m_e c^2}{c} = m_e c = (9.11\times10^{-31}\text{ kg})(2.998\times10^{8}\text{ m/s}) = 2.73\times10^{-22}\text{ kg}\cdot\text{m/s}$$
$$= 0.511\text{ MeV}/c.$$

(b) From Eq. 38-7,

$$\lambda = \frac{h}{p} = \frac{6.63\times10^{-34}\text{ J}\cdot\text{s}}{2.73\times10^{-22}\text{ kg}\cdot\text{m/s}} = 2.43\times10^{-12}\text{ m} = 2.43\text{ pm}.$$

(c) Using Eq. 38-1,

$$f = \frac{c}{\lambda} = \frac{2.998\times10^{8}\text{ m/s}}{2.43\times10^{-12}\text{ m}} = 1.24\times10^{20}\text{ Hz}.$$

29. (a) The x-ray frequency is

$$f = \frac{c}{\lambda} = \frac{2.998\times10^{8}\text{ m/s}}{35.0\times10^{-12}\text{ m}} = 8.57\times10^{18}\text{ Hz}.$$

(b) The x-ray photon energy is

$$E = hf = (4.14\times10^{-15}\text{ eV}\cdot\text{s})(8.57\times10^{18}\text{ Hz}) = 3.55\times10^{4}\text{ eV}.$$

(c) From Eq. 38-7,

$$p = \frac{h}{\lambda} = \frac{6.63\times10^{-34}\text{ J}\cdot\text{s}}{35.0\times10^{-12}\text{ m}} = 1.89\times10^{-23}\text{ kg}\cdot\text{m/s} = 35.4\text{ keV}/c.$$

30. The $(1 - \cos\phi)$ factor in Eq. 38-11 is largest when $\phi = 180°$. Thus, using Table 37-3, we obtain

$$\Delta\lambda_{\max}=\frac{hc}{m_pc^2}(1-\cos180°)=\frac{1240\,\text{MeV}\cdot\text{fm}}{938\,\text{MeV}}(1-(-1))=2.64\ \text{fm}$$

where we have used the value hc = 1240 eV·nm =1240 MeV·fm.

31. If E is the original energy of the photon and E' is the energy after scattering, then the fractional energy loss is

$$\frac{\Delta E}{E}=\frac{E-E'}{E}=\frac{\Delta\lambda}{\lambda+\Delta\lambda}$$

using the result from Sample Problem – "Compton scattering of light by electrons." Thus

$$\frac{\Delta\lambda}{\lambda}=\frac{\Delta E/E}{1-\Delta E/E}=\frac{0.75}{1-0.75}=3=300\ \%.$$

A 300% increase in the wavelength leads to a 75% decrease in the energy of the photon.

32. (a) Equation 38-11 yields

$$\Delta\lambda=\frac{h}{m_ec}(1-\cos\phi)=(2.43\ \text{pm})(1-\cos180°)=+4.86\ \text{pm}.$$

(b) Using the value hc = 1240 eV·nm, the change in photon energy is

$$\Delta E=\frac{hc}{\lambda'}-\frac{hc}{\lambda}=(1240\ \text{eV·nm})\left(\frac{1}{0.01\ \text{nm}+4.86\ \text{pm}}-\frac{1}{0.01\ \text{nm}}\right)=-40.6\ \text{keV}.$$

(c) From conservation of energy, $\Delta K = -\Delta E$ = 40.6 keV.

(d) The electron will move straight ahead after the collision, since it has acquired some of the forward linear momentum from the photon. Thus, the angle between $+x$ and the direction of the electron's motion is zero.

33. (a) The fractional change is

$$\frac{\Delta E}{E}=\frac{\Delta(hc/\lambda)}{hc/\lambda}=\lambda\Delta\left(\frac{1}{\lambda}\right)=\lambda\left(\frac{1}{\lambda'}-\frac{1}{\lambda}\right)=\frac{\lambda}{\lambda'}-1=\frac{\lambda}{\lambda+\Delta\lambda}-1$$
$$=-\frac{1}{\lambda/\Delta\lambda+1}=-\frac{1}{(\lambda/\lambda_C)(1-\cos\phi)^{-1}+1}.$$

If λ = 3.0 cm = 3.0×10^{10} pm and ϕ = 90°, the result is

$$\frac{\Delta E}{E}=-\frac{1}{(3.0\times10^{10}\,\text{pm}/2.43\,\text{pm})(1-\cos 90°)^{-1}+1}=-8.1\times10^{-11}=-8.1\times10^{-9}\ \%.$$

(b) Now λ = 500 nm = 5.00 × 10^5 pm and ϕ = 90°, so

$$\frac{\Delta E}{E}=-\frac{1}{(5.00\times10^{5}\,\text{pm}/2.43\,\text{pm})(1-\cos 90°)^{-1}+1}=-4.9\times10^{-6}=-4.9\times10^{-4}\ \%.$$

(c) With λ = 25 pm and ϕ = 90°, we find

$$\frac{\Delta E}{E}=-\frac{1}{(25\,\text{pm}/2.43\,\text{pm})(1-\cos 90°)^{-1}+1}=-8.9\times10^{-2}=-8.9\ \%.$$

(d) In this case,

$$\lambda = hc/E = 1240\ \text{nm}\cdot\text{eV}/1.0\ \text{MeV} = 1.24\times10^{-3}\ \text{nm} = 1.24\ \text{pm},$$

so

$$\frac{\Delta E}{E}=-\frac{1}{(1.24\,\text{pm}/2.43\,\text{pm})(1-\cos 90°)^{-1}+1}=-0.66=-66\ \%.$$

(e) From the calculation above, we see that the shorter the wavelength the greater the fractional energy change for the photon as a result of the Compton scattering. Since $\Delta E/E$ is virtually zero for microwave and visible light, the Compton effect is significant only in the x-ray to gamma ray range of the electromagnetic spectrum.

34. The initial energy of the photon is (using hc = 1240 eV·nm)

$$E=\frac{hc}{\lambda}=\frac{1240\,\text{eV}\cdot\text{nm}}{0.00300\ \text{nm}}=4.13\times10^{5}\ \text{eV}.$$

Using Eq. 38-11 (applied to an electron), the Compton shift is given by

$$\Delta\lambda=\frac{h}{m_ec}(1-\cos\phi)=\frac{h}{m_ec}(1-\cos 90.0°)=\frac{hc}{m_ec^2}=\frac{1240\,\text{eV}\cdot\text{nm}}{511\times10^{3}\ \text{eV}}=2.43\ \text{pm}$$

Therefore, the new photon wavelength is

$$\lambda' = 3.00\ \text{pm} + 2.43\ \text{pm} = 5.43\ \text{pm}.$$

Consequently, the new photon energy is

$$E'=\frac{hc}{\lambda'}=\frac{1240\,\text{eV}\cdot\text{nm}}{0.00543\,\text{nm}}=2.28\times10^{5}\ \text{eV}$$

By energy conservation, then, the kinetic energy of the electron must be equal to

$$K_e = \Delta E = E - E' = 4.13\times10^5 - 2.28\times10^5 \text{ eV} = 1.85\times10^5 \text{ eV} \approx 3.0\times10^{-14} \text{ J}.$$

35. (a) Since the mass of an electron is $m = 9.109 \times 10^{-31}$ kg, its Compton wavelength is

$$\lambda_C = \frac{h}{mc} = \frac{6.626\times10^{-34} \text{ J}\cdot\text{s}}{(9.109\times10^{-31} \text{ kg})(2.998\times10^8 \text{ m/s})} = 2.426\times10^{-12} \text{ m} = 2.43 \text{ pm}.$$

(b) Since the mass of a proton is $m = 1.673 \times 10^{-27}$ kg, its Compton wavelength is

$$\lambda_C = \frac{6.626\times10^{-34} \text{ J}\cdot\text{s}}{(1.673\times10^{-27} \text{ kg})(2.998\times10^8 \text{ m/s})} = 1.321\times10^{-15} \text{ m} = 1.32 \text{ fm}.$$

(c) We note that hc = 1240 eV·nm, which gives E = (1240 eV·nm)/λ, where E is the energy and λ is the wavelength. Thus for the electron,

$$E = (1240 \text{ eV}\cdot\text{nm})/(2.426 \times 10^{-3} \text{ nm}) = 5.11 \times 10^5 \text{ eV} = 0.511 \text{ MeV}.$$

(d) For the proton,

$$E = (1240 \text{ eV}\cdot\text{nm})/(1.321 \times 10^{-6} \text{ nm}) = 9.39 \times 10^8 \text{ eV} = 939 \text{ MeV}.$$

36. (a) Using the value hc = 1240 eV·nm, we find

$$\lambda = \frac{hc}{E} = \frac{1240 \text{ nm}\cdot\text{eV}}{0.511 \text{ MeV}} = 2.43\times10^{-3} \text{ nm} = 2.43 \text{ pm}.$$

(b) Now, Eq. 38-11 leads to

$$\lambda' = \lambda + \Delta\lambda = \lambda + \frac{h}{m_e c}(1-\cos\phi) = 2.43\text{pm} + (2.43\text{pm})(1-\cos 90.0°)$$
$$= 4.86\text{pm}.$$

(c) The scattered photons have energy equal to

$$E' = E\left(\frac{\lambda}{\lambda'}\right) = (0.511 \text{ MeV})\left(\frac{2.43 \text{ pm}}{4.86 \text{ pm}}\right) = 0.255 \text{ MeV}.$$

37. (a) From Eq. 38-11,

$$\Delta\lambda = \frac{h}{m_e c}(1-\cos\theta).$$

In this case ϕ = 180° (so cos ϕ = –1), and the change in wavelength for the photon is given by $\Delta\lambda = 2h/m_e c$. The energy E' of the scattered photon (with initial energy $E = hc/\lambda$) is then

$$E' = \frac{hc}{\lambda+\Delta\lambda} = \frac{E}{1+\Delta\lambda/\lambda} = \frac{E}{1+(2h/m_e c)(E/hc)} = \frac{E}{1+2E/m_e c^2}$$
$$= \frac{50.0\,\text{keV}}{1+2(50.0\,\text{keV})/0.511\,\text{MeV}} = 41.8\,\text{keV} .$$

(b) From conservation of energy the kinetic energy K of the electron is given by

$$K = E - E' = 50.0 \text{ keV} - 41.8 \text{ keV} = 8.2 \text{ keV}.$$

38. Referring to Sample Problem — "Compton scattering of light by electrons," we see that the fractional change in photon energy is

$$\frac{E-E_\text{n}}{E} = \frac{\Delta\lambda}{\lambda+\Delta\lambda} = \frac{(h/mc)(1-\cos\phi)}{(hc/E)+(h/mc)(1-\cos\phi)} .$$

Energy conservation demands that $E - E' = K$, the kinetic energy of the electron. In the maximal case, ϕ = 180°, and we find

$$\frac{K}{E} = \frac{(h/mc)(1-\cos 180°)}{(hc/E)+(h/mc)(1-\cos 180°)} = \frac{2h/mc}{(hc/E)+(2h/mc)} .$$

Multiplying both sides by E and simplifying the fraction on the right-hand side leads to

$$K = E\left(\frac{2/mc}{c/E+2/mc}\right) = \frac{E^2}{mc^2/2+E} .$$

39. The magnitude of the fractional energy change for the photon is given by

$$\left|\frac{\Delta E_\text{ph}}{E_\text{ph}}\right| = \left|\frac{\Delta(hc/\lambda)}{hc/\lambda}\right| = \left|\lambda\Delta\left(\frac{1}{\lambda}\right)\right| = \lambda\left(\frac{1}{\lambda}-\frac{1}{\lambda+\Delta\lambda}\right) = \frac{\Delta\lambda}{\lambda+\Delta\lambda} = \beta$$

where β = 0.10. Thus $\Delta\lambda = \lambda\beta/(1-\beta)$. We substitute this expression for $\Delta\lambda$ in Eq. 38-11 and solve for cos ϕ:

$$\cos\phi = 1-\frac{mc}{h}\Delta\lambda = 1-\frac{mc\lambda\beta}{h(1-\beta)} = 1-\frac{\beta(mc^2)}{(1-\beta)E_\text{ph}}$$
$$= 1-\frac{(0.10)(511 \text{ keV})}{(1-0.10)(200\,\text{keV})} = 0.716 .$$

This leads to an angle of $\phi = 44°$.

40. The initial wavelength of the photon is (using hc = 1240 eV·nm)

$$\lambda = \frac{hc}{E} = \frac{1240\,\text{eV}\cdot\text{nm}}{17500\,\text{eV}} = 0.07086\text{ nm}$$

or 70.86 pm. The maximum Compton shift occurs for $\phi = 180°$, in which case Eq. 38-11 (applied to an electron) yields

$$\Delta\lambda = \left(\frac{hc}{m_e c^2}\right)(1-\cos 180°) = \left(\frac{1240\,\text{eV}\cdot\text{nm}}{511\times 10^3\,\text{eV}}\right)(1-(-1)) = 0.00485\text{ nm}$$

where Table 37-3 is used. Therefore, the new photon wavelength is

$$\lambda' = 0.07086\text{ nm} + 0.00485\text{ nm} = 0.0757\text{ nm}.$$

Consequently, the new photon energy is

$$E' = \frac{hc}{\lambda'} = \frac{1240\,\text{eV}\cdot\text{nm}}{0.0757\,\text{nm}} = 1.64\times 10^4\text{ eV} = 16.4\text{ keV}.$$

By energy conservation, then, the kinetic energy of the electron must equal

$$E' - E = 17.5\text{ keV} - 16.4\text{ keV} = 1.1\text{ keV}.$$

41. (a) From Eq. 38-11

$$\Delta\lambda = \frac{h}{m_e c}(1-\cos\phi) = (2.43\,\text{pm})(1-\cos 90°) = 2.43\,\text{pm}.$$

(b) The fractional shift should be interpreted as $\Delta\lambda$ divided by the original wavelength:

$$\frac{\Delta\lambda}{\lambda} = \frac{2.425\,\text{pm}}{590\,\text{nm}} = 4.11\times 10^{-6}.$$

(c) The change in energy for a photon with λ = 590 nm is given by

$$\Delta E_{\text{ph}} = \Delta\left(\frac{hc}{\lambda}\right) \approx -\frac{hc\Delta\lambda}{\lambda^2} = -\frac{(4.14\times 10^{-15}\,\text{eV}\cdot\text{s})(2.998\times 10^8\,\text{m/s})(2.43\,\text{pm})}{(590\,\text{nm})^2}$$
$$= -8.67\times 10^{-6}\text{ eV}.$$

(d) For an x-ray photon of energy E_{ph} = 50 keV, $\Delta\lambda$ remains the same (2.43 pm), since it is independent of E_{ph}.

(e) The fractional change in wavelength is now

$$\frac{\Delta\lambda}{\lambda}=\frac{\Delta\lambda}{hc/E_{\text{ph}}}=\frac{(50\times10^{3}\,\text{eV})(2.43\text{pm})}{(4.14\times10^{-15}\,\text{eV}\cdot\text{s})(2.998\times10^{8}\text{m/s})}=9.78\times10^{-2}\,.$$

(f) The change in photon energy is now

$$\Delta E_{\text{ph}}=hc\left(\frac{1}{\lambda+\Delta\lambda}-\frac{1}{\lambda}\right)=-\left(\frac{hc}{\lambda}\right)\frac{\Delta\lambda}{\lambda+\Delta\lambda}=-E_{\text{ph}}\left(\frac{\alpha}{1+\alpha}\right)$$

where $\alpha = \Delta\lambda/\lambda$. With E_{ph} = 50 keV and $\alpha = 9.78 \times 10^{-2}$, we obtain ΔE_{ph} = –4.45 keV. (Note that in this case $\alpha \approx 0.1$ is not close enough to zero so the approximation $\Delta E_{ph} \approx hc\Delta\lambda/\lambda^2$ is not as accurate as in the first case, in which $\alpha = 4.12 \times 10^{-6}$. In fact if one were to use this approximation here, one would get $\Delta E_{ph} \approx$ –4.89 keV, which does not amount to a satisfactory approximation.)

42. (a) Using Wien's law, $\lambda_{\max}T = 2898\ \mu\text{m}\cdot\text{K}$, we obtain

$$\lambda_{\max}=\frac{2898\ \mu\text{m}\cdot\text{K}}{T}=\frac{2898\ \mu\text{m}\cdot\text{K}}{5800\ \text{K}}=0.50\ \mu\text{m}=500\ \text{nm}\,.$$

(b) The electromagnetic wave is in the visible spectrum.

(c) If $\lambda_{\max}=1.06\ \text{mm}=1060\ \mu\text{m}$, then $T=\dfrac{2898\ \mu\text{m}\cdot\text{K}}{\lambda_{\max}}=\dfrac{2898\ \mu\text{m}\cdot\text{K}}{1060\ \mu\text{m}}=2.73\,\text{K}$.

43. (a) Using Wien's law, the wavelength that corresponds to thermal radiation maximum is

$$\lambda_{\max}=\frac{2898\ \mu\text{m}\cdot\text{K}}{T}=\frac{2898\ \mu\text{m}\cdot\text{K}}{1.0\times10^{7}\ \text{K}}=2.9\times10^{-4}\,\mu\text{m}=2.9\times10^{-10}\,\text{m}\,.$$

(b) The wave is in the x-ray region of the electromagnetic spectrum.

(c) Using Wien's law, the wavelength that corresponds to thermal radiation maximum is

$$\lambda_{\max}=\frac{2898\ \mu\text{m}\cdot\text{K}}{T}=\frac{2898\ \mu\text{m}\cdot\text{K}}{1.0\times10^{5}\ \text{K}}=2.9\times10^{-2}\,\mu\text{m}=2.9\times10^{-8}\,\text{m}$$

(d) The wave is in the ultraviolet region of the electromagnetic spectrum.

44. (a) The intensity per unit length according to the classical radiation law shown in Eq. 38-13 is

$$I_C = \frac{2\pi ckT}{\lambda^4}$$

On the other hand, Planck's radiation law (Eq. 38-14) gives

$$I_P = \frac{2\pi c^2 h}{\lambda^5}\frac{1}{e^{hc/\lambda kT}-1}.$$

The ratio of the two expressions can be written as

$$\frac{I_C}{I_P} = \frac{\lambda kT}{hc}\left(e^{hc/\lambda kT}-1\right) = \frac{1}{x}\left(e^x - 1\right)$$

where $x = hc/\lambda kT$. For T = 200 K, and $\lambda = 400$ nm,

$$x = \frac{hc}{\lambda kT} = \frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(2.998\times10^{8}\ \text{m/s})}{(400\times10^{-9}\,\text{m})(1.38\times10^{-23}\ \text{J/K})(2000\ \text{K})} \approx 17.98,$$

and the ratio of the intensities is $\dfrac{I_C}{I_P} \approx \dfrac{1}{17.98}\left(e^{17.98}-1\right) \approx 3.6\times10^{6}$.

(b) For $\lambda = 200\ \mu\text{m},$ we have

$$x = \frac{hc}{\lambda kT} = \frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(2.998\times10^{8}\ \text{m/s})}{(200\times10^{-6}\,\text{m})(1.38\times10^{-23}\ \text{J/K})(2000\ \text{K})} \approx 0.03596,$$

and the ratio of the intensities is

$$\frac{I_C}{I_P} \approx \frac{1}{0.03596}\left(e^{0.03596}-1\right) \approx 1.02.$$

(c) The agreement is better at longer wavelength, with $I_C / I_P \approx 1$.

45. (a) With $T = 98.6\ ^\circ\text{F} = 37\ ^\circ\text{C} = 310\ \text{K},$ we use Wien's law and find the wavelength that corresponds to spectral radiancy maximum to be

$$\lambda_{\max} = \frac{2898\ \mu\text{m}\cdot\text{K}}{T} = \frac{2898\ \mu\text{m}\cdot\text{K}}{310\ \text{K}} = 9.35\mu\text{m}.$$

(b) With $\lambda = 9.35\ \mu\text{m},$ and T = 310 K, the spectral radiancy is

$$S(\lambda)=\frac{2\pi c^2 h}{\lambda^5}\frac{1}{e^{hc/\lambda kT}-1}$$
$$=\frac{2\pi(2.998\times10^8\ \text{m/s})^2(6.626\times10^{-34}\ \text{J}\cdot\text{s})}{(9.35\times10^{-6}\,\text{m})^5}\left(\exp\left[\frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(2.998\times10^8\ \text{m/s})}{(9.35\times10^{-6}\,\text{m})(1.38\times10^{-23}\ \text{J/K})(310\ \text{K})}\right]\right)^{-1}$$
$$=3.688\times10^7\ \text{W/m}^3$$

For small range of wavelength, the radiated power may be approximated as

$$P=S(\lambda)A\Delta\lambda=(3.688\times10^7\ \text{W/m}^3)(4\times10^{-4}\,\text{m}^2)(10^{-9}\,\text{m})=1.475\times10^{-5}\ \text{W}.$$

(c) The energy carried by each photon is

$$\varepsilon=hf=\frac{hc}{\lambda}=\frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(2.998\times10^8\ \text{m/s})}{9.35\times10^{-6}\,\text{m}}=2.1246\times10^{-20}\ \text{J}$$

Writing $P=(dN/dt)\varepsilon,$ we find the rate to be

$$\frac{dN}{dt}=\frac{P}{\varepsilon}=\frac{1.475\times10^{-5}\ \text{W}}{2.1246\times10^{-20}\ \text{J}}=6.94\times10^{14}\ \text{photons/s}\,.$$

(d) If $\lambda=500\,\text{nm},$ and T = 310 K, the spectral radiancy is

$$S(\lambda)=\frac{2\pi c^2 h}{\lambda^5}\frac{1}{e^{hc/\lambda kT}-1}$$
$$=\frac{2\pi(2.998\times10^8\ \text{m/s})^2(6.626\times10^{-34}\ \text{J}\cdot\text{s})}{(500\times10^{-9}\,\text{m})^5}\left(\exp\left[\frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(2.998\times10^8\ \text{m/s})}{(500\times10^{-9}\,\text{m})(1.38\times10^{-23}\ \text{J/K})(310\ \text{K})}\right]\right)^{-1}$$
$$=5.95\times10^{-25}\ \text{W/m}^3$$

For small range of wavelength, the radiated power may be approximated as

$$P=S(\lambda)A\Delta\lambda=(5.95\times10^{-25}\ \text{W/m}^3)(4\times10^{-4}\,\text{m}^2)(10^{-9}\,\text{m})=2.38\times10^{-37}\ \text{W}.$$

(e) The energy carried by each photon is

$$\varepsilon=hf=\frac{hc}{\lambda}=\frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(2.998\times10^8\ \text{m/s})}{500\times10^{-9}\,\text{m}}=3.97\times10^{-19}\ \text{J}$$

The corresponding photon emission rate is

$$\frac{dN}{dt}=\frac{P}{\varepsilon}=\frac{2.38\times10^{-5}\ \text{W}}{3.97\times10^{-19}\ \text{J}}=5.9\times10^{-19}\ \text{photons/s}$$

46. (a) Using Table 37-3 and the value hc = 1240 eV·nm, we obtain

$$\lambda=\frac{h}{p}=\frac{h}{\sqrt{2m_eK}}=\frac{hc}{\sqrt{2m_ec^2K}}=\frac{1240\,\text{eV}\cdot\text{nm}}{\sqrt{2(511000\text{eV})(1000\text{eV})}}=0.0388\text{nm}.$$

(b) A photon's de Broglie wavelength is equal to its familiar wave-relationship value. Using the value hc = 1240 eV·nm,

$$\lambda=\frac{hc}{E}=\frac{1240\,\text{eV}\cdot\text{nm}}{1.00\,\text{keV}}=1.24\,\text{nm}\ .$$

(c) The neutron mass may be found in Appendix B. Using the conversion from electron-volts to Joules, we obtain

$$\lambda=\frac{h}{\sqrt{2m_nK}}=\frac{6.63\times10^{-34}\,\text{J}\cdot\text{s}}{\sqrt{2(1.675\times10^{-27}\,\text{kg})(1.6\times10^{-16}\,\text{J})}}=9.06\times10^{-13}\,\text{m}.$$

47. **THINK** The de Broglie wavelength of the electron is given by $\lambda = h/p$, where p is the momentum of the electron.

EXPRESS The momentum of the electron can be written as

$$p=m_ev=\sqrt{2m_eK}=\sqrt{2m_eeV},$$

where V is the accelerating potential and e is the fundamental charge. Thus,

$$\lambda=\frac{h}{p}=\frac{h}{\sqrt{2m_eeV}}.$$

ANALYZE With V = 25.0 kV, we obtain

$$\begin{aligned}\lambda&=\frac{h}{\sqrt{2m_eeV}}=\frac{6.626\times10^{-34}\,\text{J}\cdot\text{s}}{\sqrt{2(9.109\times10^{-31}\,\text{kg})(1.602\times10^{-19}\ \text{C})(25.0\times10^{3}\ \text{V})}}\\&=7.75\times10^{-12}\ \text{m}=7.75\,\text{pm}.\end{aligned}$$

LEARN The wavelength is of the same order as the Compton wavelength of the electron. Increasing the potential difference V would make the wavelength even smaller.

48. The same resolution requires the same wavelength, and since the wavelength and particle momentum are related by $p = h/\lambda$, we see that the same particle momentum is required. The momentum of a 100 keV photon is

$$p = E/c = (100 \times 10^3 \text{ eV})(1.60 \times 10^{-19} \text{ J/eV})/(3.00 \times 10^8 \text{ m/s}) = 5.33 \times 10^{-23} \text{ kg·m/s}.$$

This is also the magnitude of the momentum of the electron. The kinetic energy of the electron is

$$K = \frac{p^2}{2m} = \frac{(5.33\times10^{-23}\text{ kg}\cdot\text{m/s})^2}{2(9.11\times10^{-31}\text{ kg})} = 1.56\times10^{-15}\text{ J}.$$

The accelerating potential is

$$V = \frac{K}{e} = \frac{1.56\times10^{-15}\text{ J}}{1.60\times10^{-19}\text{ C}} = 9.76\times10^3\text{ V}.$$

49. **THINK** The de Broglie wavelength of the sodium ion is given by $\lambda = h/p$, where p is the momentum of the ion.

EXPRESS The kinetic energy acquired is $K = qV$, where q is the charge on an ion and V is the accelerating potential. Thus, the momentum of an ion is $p=\sqrt{2mK}$, and the corresponding de Broglie wavelength is $\lambda=\dfrac{h}{p}=\dfrac{h}{\sqrt{2mK}}$.

ANALYZE (a) The kinetic energy of the ion is

$$K = qV = (1.60 \times 10^{-19} \text{ C})(300 \text{ V}) = 4.80 \times 10^{-17} \text{ J}.$$

The mass of a single sodium atom is, from Appendix F,

$$m = (22.9898 \text{ g/mol})/(6.02 \times 10^{23} \text{ atom/mol}) = 3.819 \times 10^{-23} \text{ g} = 3.819 \times 10^{-26} \text{ kg}.$$

Thus, the momentum of a sodium ion is

$$p = \sqrt{2mK} = \sqrt{2(3.819\times10^{-26}\text{ kg})(4.80\times10^{-17}\text{ J})} = 1.91\times10^{-21}\text{ kg}\cdot\text{m/s}.$$

(b) The de Broglie wavelength is

$$\lambda = \frac{h}{p} = \frac{6.63\times10^{-34}\text{ J}\cdot\text{s}}{1.91\times10^{-21}\text{kg}\cdot\text{m/s}} = 3.46\times10^{-13}\text{ m}.$$

LEARN The greater the potential difference, the greater the kinetic energy and momentum, and hence, the smaller the de Broglie wavelength.

50. (a) We need to use the relativistic formula

$$p=\sqrt{(E/c)^2-m_e^2c^2}\approx E/c\approx K/c$$

$\left(\text{since } E\gg m_ec^2\right)$. So

$$\lambda=\frac{h}{p}\approx\frac{hc}{K}=\frac{1240\,\text{eV}\cdot\text{nm}}{50\times10^9\,\text{eV}}=2.5\times10^{-8}\,\text{nm}=0.025\ \text{fm}.$$

(b) With $R=5.0\ \text{fm}$, we obtain $R/\lambda=2.0\times10^2$.

51. **THINK** The de Broglie wavelength of a particle is given by $\lambda = h/p$, where p is the momentum of the particle.

EXPRESS Let K be the kinetic energy of the electron, in units of electron volts (eV). Since $K=p^2/2m$, the electron momentum is $p=\sqrt{2mK}$. Thus, the de Broglie wavelength is

$$\lambda=\frac{h}{p}=\frac{h}{\sqrt{2mK}}=\frac{6.626\times10^{-34}\,\text{J}\cdot\text{s}}{\sqrt{2(9.109\times10^{-31}\,\text{kg})(1.602\times10^{-19}\,\text{J/eV})K}}=\frac{1.226\times10^{-9}\,\text{m}\cdot\text{eV}^{1/2}}{\sqrt{K}}$$
$$=\frac{1.226\ \text{nm}\cdot\text{eV}^{1/2}}{\sqrt{K}}.$$

ANALYZE With $\lambda = 590$ nm, the above equation can be inverted to give

$$K=\left(\frac{1.226\,\text{nm}\cdot\text{eV}^{1/2}}{\lambda}\right)^2=\left(\frac{1.226\,\text{nm}\cdot\text{eV}^{1/2}}{590\,\text{nm}}\right)^2=4.32\times10^{-6}\,\text{eV}.$$

LEARN The analytical expression shows that the kinetic energy is proportional to $1/\lambda^2$. This is so because $K\sim p^2$, while $p\sim1/\lambda$.

52. Using Eq. 37-8, we find the Lorentz factor to be

$$\gamma=\frac{1}{\sqrt{1-(v/c)^2}}=\frac{1}{\sqrt{1-(0.9900)^2}}=7.0888\,.$$

With $p=\gamma mv$ (Eq. 37-41), the de Broglie wavelength of the protons is

$$\lambda = \frac{h}{p} = \frac{h}{\gamma mv} = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{(7.0888)(1.67\times10^{-27}\ \text{kg})(0.99\times3.00\times10^{8}\ \text{m/s})} = 1.89\times10^{-16}\ \text{m}.$$

The vertical distance between the second interference minimum and the center point is

$$y_2 = \left(1+\frac{1}{2}\right)\frac{\lambda L}{d} = \frac{3}{2}\frac{\lambda L}{d}$$

where L is the perpendicular distance between the slits and the screen. Therefore, the angle between the center of the pattern and the second minimum is given by

$$\tan\theta = \frac{y_2}{L} = \frac{3\lambda}{2d}.$$

Since $\lambda \ll d$, $\tan\theta \approx \theta$, and we obtain

$$\theta \approx \frac{3\lambda}{2d} = \frac{3(1.89\times10^{-16}\ \text{m})}{2(4.00\times10^{-9}\ \text{m})} = 7.07\times10^{-8}\,\text{rad} = (4.0\times10^{-6})^\circ.$$

53. (a) The momentum of the photon is given by $p = E/c$, where E is its energy. Its wavelength is

$$\lambda = \frac{h}{p} = \frac{hc}{E} = \frac{1240\,\text{eV}\cdot\text{nm}}{1.00\,\text{eV}} = 1240\ \text{nm}.$$

(b) The momentum of the electron is given by $p = \sqrt{2mK}$, where K is its kinetic energy and m is its mass. Its wavelength is

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}}.$$

If K is given in electron volts, then

$$\lambda = \frac{6.626\times10^{-34}\,\text{J}\cdot\text{s}}{\sqrt{2(9.109\times10^{-31}\,\text{kg})(1.602\times10^{-19}\,\text{J/eV})K}} = \frac{1.226\times10^{-9}\,\text{m}\cdot\text{eV}^{1/2}}{\sqrt{K}} = \frac{1.226\text{nm}\cdot\text{eV}^{1/2}}{\sqrt{K}}.$$

For $K = 1.00\ \text{eV}$, we have

$$\lambda = \frac{1.226\text{nm}\cdot\text{eV}^{1/2}}{\sqrt{1.00\,\text{eV}}} = 1.23\ \text{nm}.$$

(c) For the photon,

$$\lambda = \frac{hc}{E} = \frac{1240\text{eV}\cdot\text{nm}}{1.00\times10^{9}\,\text{eV}} = 1.24\times10^{-6}\,\text{nm} = 1.24\ \text{fm}.$$

(d) Relativity theory must be used to calculate the wavelength for the electron. According to Eq. 38-51, the momentum p and kinetic energy K are related by

$$(pc)^2 = K^2 + 2Kmc^2.$$

Thus,

$$pc=\sqrt{K^2+2Kmc^2}=\sqrt{\left(1.00\times10^9\text{ eV}\right)^2+2\left(1.00\times10^9\text{eV}\right)\left(0.511\times10^6\text{ eV}\right)}$$
$$=1.00\times10^9\text{ eV}.$$

The wavelength is

$$\lambda=\frac{h}{p}=\frac{hc}{pc}=\frac{1240\text{eV}\cdot\text{nm}}{1.00\times10^9\text{ eV}}=1.24\times10^{-6}\text{ nm}=1.24\text{ fm}.$$

54. (a) The momentum of the electron is

$$p=\frac{h}{\lambda}=\frac{6.63\times10^{-34}\text{ J}\cdot\text{s}}{0.20\times10^{-9}\text{ m}}=3.3\times10^{-24}\text{ kg}\cdot\text{m/s}.$$

(b) The momentum of the photon is the same as that of the electron: $p=3.3\times10^{-24}\text{kg}\cdot\text{m/s}$.

(c) The kinetic energy of the electron is

$$K_e=\frac{p^2}{2m_e}=\frac{\left(3.3\times10^{-24}\text{kg}\cdot\text{m/s}\right)^2}{2\left(9.11\times10^{-31}\text{kg}\right)}=6.0\times10^{-18}\text{ J}=38\text{ eV}.$$

(d) The kinetic energy of the photon is

$$K_{\text{ph}}=pc=\left(3.3\times10^{-24}\text{ kg}\cdot\text{m/s}\right)\left(2.998\times10^8\text{ m/s}\right)=9.9\times10^{-16}\text{ J}=6.2\text{ keV}.$$

55. (a) Setting $\lambda=h/p=h/\sqrt{(E/c)^2-m_e^2c^2}$, we solve for $K = E - m_ec^2$:

$$K=\sqrt{\left(\frac{hc}{\lambda}\right)^2+m_e^2c^4}-m_ec^2=\sqrt{\left(\frac{1240\text{eV}\cdot\text{nm}}{10\times10^{-3}\text{nm}}\right)^2+\left(0.511\text{MeV}\right)^2}-0.511\text{MeV}$$
$$=0.015\text{ MeV}=15\text{ keV}.$$

(b) Using the value $hc=1240\text{eV}\cdot\text{nm}$

$$E=\frac{hc}{\lambda}=\frac{1240\text{ eV}\cdot\text{nm}}{10\times10^{-3}\text{ nm}}=1.2\times10^5\text{ eV}=120\text{ keV}.$$

(c) The electron microscope is more suitable, as the required energy of the electrons is much less than that of the photons.

56. (a) Since $K = 7.5\,\text{MeV} \ll m_\alpha c^2 = 4(932\,\text{MeV})$, we may use the nonrelativistic formula $p = \sqrt{2m_\alpha K}$. Using Eq. 38-43 (and noting that 1240 eV·nm = 1240 MeV·fm), we obtain

$$\lambda = \frac{h}{p} = \frac{hc}{\sqrt{2m_\alpha c^2 K}} = \frac{1240\,\text{MeV}\cdot\text{fm}}{\sqrt{2(4\text{u})(931.5\,\text{MeV/u})(7.5\,\text{MeV})}} = 5.2\,\text{fm}.$$

(b) Since $\lambda = 5.2\,\text{fm} \ll 30\,\text{fm}$, to a fairly good approximation, the wave nature of the α particle does not need to be taken into consideration.

57. The wavelength associated with the unknown particle is

$$\lambda_p = \frac{h}{p_p} = \frac{h}{m_p v_p},$$

where p_p is its momentum, m_p is its mass, and v_p is its speed. The classical relationship $p_p = m_p v_p$ was used. Similarly, the wavelength associated with the electron is $\lambda_e = h/(m_e v_e)$, where m_e is its mass and v_e is its speed. The ratio of the wavelengths is

$$\lambda_p/\lambda_e = (m_e v_e)/(m_p v_p),$$

so

$$m_p = \frac{v_e \lambda_e}{v_p \lambda_p} m_e = \frac{9.109\times10^{-31}\,\text{kg}}{3\left(1.813\times10^{-4}\right)} = 1.675\times10^{-27}\,\text{kg}.$$

According to Appendix B, this is the mass of a neutron.

58. (a) We use the value $hc = 1240\,\text{nm}\cdot\text{eV}$:

$$E_{\text{photon}} = \frac{hc}{\lambda} = \frac{1240\,\text{nm}\cdot\text{eV}}{1.00\,\text{nm}} = 1.24\,\text{keV}.$$

(b) For the electron, we have

$$K = \frac{p^2}{2m_e} = \frac{(h/\lambda)^2}{2m_e} = \frac{(hc/\lambda)^2}{2m_e c^2} = \frac{1}{2(0.511\,\text{MeV})}\left(\frac{1240\,\text{eV}\cdot\text{nm}}{1.00\,\text{nm}}\right)^2 = 1.50\ \text{eV}.$$

(c) In this case, we find

$$E_{\text{photon}} = \frac{1240\,\text{nm}\cdot\text{eV}}{1.00\times10^{-6}\,\text{nm}} = 1.24\times10^{9}\,\text{eV} = 1.24\ \text{GeV}.$$

(d) For the electron (recognizing that 1240 eV·nm = 1240 MeV·fm)

$$\begin{aligned} K &= \sqrt{p^2c^2 + \left(m_ec^2\right)^2} - m_ec^2 = \sqrt{(hc/\lambda)^2 + \left(m_ec^2\right)^2} - m_ec^2 \\ &= \sqrt{\left(\frac{1240\,\text{MeV}\cdot\text{fm}}{1.00\,\text{fm}}\right)^2 + (0.511\,\text{MeV})^2} - 0.511\,\text{MeV} \\ &= 1.24\times10^{3}\ \text{MeV} = 1.24\,\text{GeV}. \end{aligned}$$

We note that at short λ (large K) the kinetic energy of the electron, calculated with the relativistic formula, is about the same as that of the photon. This is expected since now $K \approx E \approx pc$ for the electron, which is the same as $E = pc$ for the photon.

59. (a) We solve v from $\lambda = h/p = h/(m_pv)$:

$$v = \frac{h}{m_p\lambda} = \frac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{\left(1.6705\times10^{-27}\ \text{kg}\right)\left(0.100\times10^{-12}\ \text{m}\right)} = 3.96\times10^{6}\ \text{m/s}.$$

(b) We set $eV = K = \frac{1}{2}m_pv^2$ and solve for the voltage:

$$V = \frac{m_pv^2}{2e} = \frac{\left(1.6705\times10^{-27}\,\text{kg}\right)\left(3.96\times10^{6}\,\text{m/s}\right)^2}{2\left(1.60\times10^{-19}\,\text{C}\right)} = 8.18\times10^{4}\,\text{V} = 81.8\ \text{kV}.$$

60. The wave function is now given by

$$\Psi(x,t) = \psi_0 e^{-i(kx+\omega t)}.$$

This function describes a plane matter wave traveling in the negative x direction. An example of the actual particles that fit this description is a free electron with linear momentum $\vec{p} = -(hk/2\pi)\hat{\text{i}}$ and kinetic energy

$$K = \frac{p^2}{2m_e} = \frac{h^2k^2}{8\pi^2 m_e}\ .$$

61. **THINK** In this problem we solve a special case of the Schrödinger's equation where the potential energy is $U(x) = U_0 = \text{constant}$.

EXPRESS For $U = U_0$, Schrödinger's equation becomes

$$\frac{d^2\psi}{dx^2}+\frac{8\pi^2 m}{h^2}[E-U_0]\psi=0.$$

We substitute $\psi=\psi_0 e^{ikx}$.

ANALYZE The second derivative is $\frac{d^2\psi}{dx^2}=-k^2\psi_0 e^{ikx}=-k^2\psi.$ The result is

$$-k^2\psi+\frac{8\pi^2 m}{h^2}[E-U_0]\psi=0.$$

Solving for k, we obtain

$$k=\sqrt{\frac{8\pi^2 m}{h^2}[E-U_0]}=\frac{2\pi}{h}\sqrt{2m[E-U_0]}.$$

LEARN Another way to realize this is to note that with a constant potential energy $U(x)=U_0$, we can simply redefine the total energy as $E'=E-U_0$, and the Schrödinger's equation looks just like the free-particle case:

$$\frac{d^2\psi}{dx^2}+\frac{8\pi^2 mE'}{h^2}\psi=0.$$

The solution is $\psi=\psi_0\exp(ik'x),$ where

$$k'^2=\frac{8\pi^2 mE'}{h^2}\ \Rightarrow\ k=\frac{2\pi}{h}\sqrt{2mE'}=\frac{2\pi}{h}\sqrt{2m(E-U_0)}\,.$$

62. We plug Eq. 38-17 into Eq. 38-16, and note that

$$\frac{d\psi}{dx}=\frac{d}{dx}\left(Ae^{ikx}+Be^{-ikx}\right)=ikAe^{ikx}-ikBe^{-ikx}.$$

Also,

$$\frac{d^2\psi}{dx^2}=\frac{d}{dx}\left(ikAe^{ikx}-ikBe^{-ikx}\right)=-k^2\ Ae^{ikx}-k^2\ Be^{ikx}.$$

Thus,

$$\frac{d^2\psi}{dx^2}+k^2\psi=-k^2\ Ae^{ikx}-k^2 Be^{ikx}+k^2\ \left(Ae^{ikx}+Be^{-ikx}\right)=0.$$

63. (a) Using Euler's formula $e^{i\phi}=\cos\phi+i\sin\phi$, we rewrite $\psi(x)$ as

$$\psi(x) = \psi_0 e^{ikx} = \psi_0 (\cos kx + i \sin kx) = (\psi_0 \cos kx) + i(\psi_0 \sin kx) = a + ib,$$

where $a = \psi_0 \cos kx$ and $b = \psi_0 \sin kx$ are both real quantities.

(b) The time-dependent wave function is

$$\begin{aligned}\psi(x,t) &= \psi(x)e^{-i\omega t} = \psi_0 e^{ikx} e^{-i\omega t} = \psi_0 e^{i(kx-\omega t)} \\ &= [\psi_0 \cos(kx - \omega t)] + i[\psi_0 \sin(kx - \omega t)].\end{aligned}$$

64. **THINK** The angular wave number k is related to the wavelength λ by $k = 2\pi/\lambda$.

EXPRESS The wavelength is related to the particle momentum p by $\lambda = h/p$, so $k = 2\pi p/h$. Now, the kinetic energy K and the momentum are related by $K = p^2/2m$, where m is the mass of the particle.

ANALYZE Thus, we have $p = \sqrt{2mK}$ and

$$k = \frac{2\pi}{\lambda} = \frac{2\pi p}{h} = \frac{2\pi\sqrt{2mK}}{h}.$$

LEARN The expression obtained above applies to the case of a free particle only. In the presence of interaction, the potential energy is nonzero, and the functional form of k will change. For example, as shown in Problem 38-57, when $U(x) = U_0$, the angular wave number becomes

$$k = \frac{2\pi}{h}\sqrt{2m(E - U_0)}\ .$$

65. (a) The product nn^* can be rewritten as

$$\begin{aligned}nn^* &= (a+ib)(a+ib)^* = (a+ib)(a^* + i^* b^*) = (a+ib)(a-ib) \\ &= a^2 + iba - iab + (ib)(-ib) = a^2 + b^2,\end{aligned}$$

which is always real since both a and b are real.

(b) Straightforward manipulation gives

$$\begin{aligned}|nm| &= |(a+ib)(c+id)| = |ac + iad + ibc + (-i)^2 bd| = |(ac - bd) + i(ad + bc)| \\ &= \sqrt{(ac-bd)^2 + (ad+bc)^2} = \sqrt{a^2c^2 + b^2d^2 + a^2d^2 + b^2c^2}.\end{aligned}$$

However, since

$$|n||m| = |a+ib||c+id| = \sqrt{a^2+b^2}\sqrt{c^2+d^2}$$
$$= \sqrt{a^2c^2+b^2d^2+a^2d^2+b^2c^2},$$

we conclude that $|nm| = |n|\,|m|$.

66. (a) The wave function is now given by

$$\Psi(x,t) = \psi_0\ e^{i(kx-\omega t)} + e^{-i(kx+\omega t)}\ = \psi_0 e^{-i\omega t}(e^{ikx}+e^{-ikx}).$$

Thus,

$$|\Psi(x,t)|^2 = \left|\psi_0 e^{-i\omega t}(e^{ikx}+e^{-ikx})\right|^2 = \left|\psi_0 e^{-i\omega t}\right|^2\left|e^{ikx}+e^{-ikx}\right|^2 = \psi_0^2\left|e^{ikx}+e^{-ikx}\right|^2$$
$$= \psi_0^2\,|(\cos kx + i\sin kx) + (\cos kx - i\sin kx)|^2 = 4\psi_0^2(\cos kx)^2$$
$$= 2\psi_0^2(1+\cos 2kx).$$

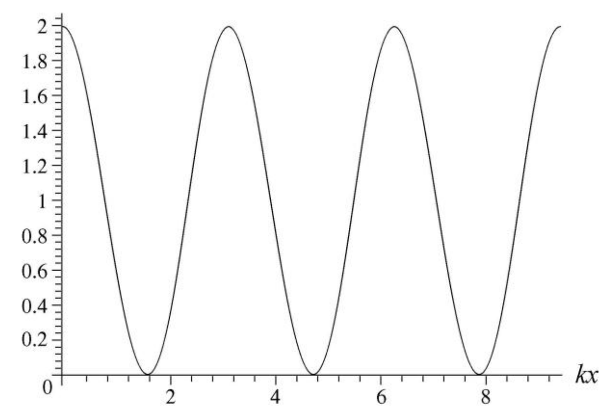
(b) Consider two plane matter waves, each with the same amplitude $\psi_0/\sqrt{2}$ and traveling in opposite directions along the x axis. The combined wave Ψ is a standing wave:

$$\Psi(x,t) = \psi_0 e^{i(kx-\omega t)} + \psi_0 e^{-i(kx+\omega t)} = \psi_0(e^{ikx}+e^{-ikx})e^{-i\omega t} = (2\psi_0\cos kx)e^{-i\omega t}.$$

Thus, the squared amplitude of the matter wave is

$$|\Psi(x,t)|^2 = (2\psi_0\cos kx)^2\left|e^{-i\omega t}\right|^2 = 2\psi_0^2(1+\cos 2kx),$$

which is shown to the right.



(c) We set $\left|\Psi(x,t)\right|^2 = 2\psi_0^2(1+\cos 2kx) = 0$ to obtain $\cos(2kx) = -1$. This gives

$$2kx = 2\left(\frac{2\pi}{\lambda}\right) = (2n+1)\pi, \quad (n = 0, 1, 2, 3, \ldots)$$

We solve for x:

$$x = \frac{1}{4}(2n+1)\lambda\ .$$

(d) The most probable positions for finding the particle are where $\left|\Psi(x,t)\right| \propto (1+\cos 2kx)$ reaches its maximum. Thus $\cos 2kx = 1$, or

$$2kx = 2\left(\frac{2\pi}{\lambda}\right) = 2n\pi, \quad (n = 0, 1, 2, 3, \ldots)$$

We solve for x and find $x = \frac{1}{2}n\lambda$.

67. If the momentum is measured at the same time as the position, then

$$\Delta p \approx \frac{\hbar}{\Delta x} = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{2\pi(50\,\text{pm})} = 2.1\times10^{-24}\ \text{kg}\cdot\text{m/s} .$$

68. (a) Using the value $hc = 1240\text{nm}\cdot\text{eV}$, we have

$$E = \frac{hc}{\lambda} = \frac{1240\,\text{nm}\cdot\text{eV}}{10.0\times10^{-3}\ \text{nm}} = 124\,\text{keV} .$$

(b) The kinetic energy gained by the electron is equal to the energy decrease of the photon:

$$\begin{aligned}\Delta E &= \Delta\left(\frac{hc}{\lambda}\right) = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda+\Delta\lambda}\right) = \left(\frac{hc}{\lambda}\right)\left(\frac{\Delta\lambda}{\lambda+\Delta\lambda}\right) = \frac{E}{1+\lambda/\Delta\lambda} \\ &= \frac{E}{1+\frac{\lambda}{\lambda_C(1-\cos\phi)}} = \frac{124\text{keV}}{1+\frac{10.0\text{pm}}{(2.43\text{pm})(1-\cos180°)}} \\ &= 40.5\text{keV}.\end{aligned}$$

(c) It is impossible to "view" an atomic electron with such a high-energy photon, because with the energy imparted to the electron the photon would have knocked the electron out of its orbit.

69. We use the uncertainty relationship $\Delta x\Delta p \geq \hbar$. Letting $\Delta x = \lambda$, the de Broglie wavelength, we solve for the minimum uncertainty in p:

$$\Delta p = \frac{\hbar}{\Delta x} = \frac{h}{2\pi\lambda} = \frac{p}{2\pi}$$

where the de Broglie relationship $p = h/\lambda$ is used. We use $1/2\pi = 0.080$ to obtain $\Delta p = 0.080p$. We would expect the measured value of the momentum to lie between $0.92p$ and $1.08p$. Measured values of zero, $0.5p$, and $2p$ would all be surprising.

70. (a) The potential energy of the electron is $U_b = qV = (-e)(-200\text{ V}) = 200\,\text{eV}$, so its kinetic energy is

$$K = E - U_b = 500\ \text{eV} - 200\ \text{eV} = 300\ \text{eV}.$$

(b) Using non-relativistic regime approximation, $K = \frac{1}{2}mv^2 = p^2/2m$, we find the momentum of the electron to be

$$p = \sqrt{2mK} = \sqrt{2(9.11\times10^{-31}\text{ kg})(300\text{ eV})(1.6\times10^{-19}\text{ J/eV})} = 9.35\times10^{-24}\text{ kg}\cdot\text{m/s}$$

(c) The speed of the electron is

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(300\text{ eV})(1.6\times10^{-19}\text{ J/eV})}{9.11\times10^{-31}\text{ kg}}} = 1.03\times10^{7}\text{ m/s}.$$

(d) The corresponding de Broglie wavelength is

$$\lambda = \frac{h}{p} = \frac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{9.35\times10^{-24}\text{ kg}\cdot\text{m/s}} = 7.08\times10^{-11}\text{m}.$$

(e) The angular wave number is

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{7.08\times10^{-11}\text{m}} = 8.87\times10^{10}\text{m}^{-1}.$$

71. (a) The angular wave number in region 1 is

$$k = \frac{2\pi}{h}\sqrt{2mE} = \frac{2\pi}{6.626\times10^{-34}\text{ J}\cdot\text{s}}\sqrt{2(9.11\times10^{-31}\text{ kg})(800\text{ eV})(1.6\times10^{-19}\text{ J/eV})}$$
$$= 1.45\times10^{11}\text{ m}^{-1}$$

(b) The angular wave number in region 2 is

$$k_b = \frac{2\pi}{h}\sqrt{2m(E-U_b)} = \frac{2\pi}{6.626\times10^{-34}\text{ J}\cdot\text{s}}\sqrt{2(9.11\times10^{-31}\text{ kg})(800\text{ eV}-200\text{ eV})(1.6\times10^{-19}\text{ J/eV})}$$
$$= \frac{k}{2} = 7.24\times10^{10}\text{ m}^{-1}$$

(c) The wave functions in the two regions can be written as

$$\psi_1(x) = Ae^{ikx} + Be^{-ikx}, \quad \psi_2(x) = Ce^{ik_b x}$$

Matching the boundary conditions leads to

$$A + B = C$$
$$Ak - Bk = Ck_b$$

Since $k_b = k/2$, the above equations can be solved to give $(B/A) = 1/3$ and $(C/A) = 4/3.$ The reflection coefficient is

$$R = \frac{|B|^2}{|A|^2} = \frac{1}{9} = 0.111.$$

(d) With $N_0 = 5.00\times10^5$ electrons in the incident beam, the number reflected is

$$N_R = RN_0 = \left(\frac{1}{9}\right)(5.00\times10^5) = 5.56\times10^4.$$

72. (a) The angular wave number in region 1 is given by

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{(h/p)} = \frac{2\pi p}{h} = \frac{2\pi mv}{h} = \frac{2\pi(9.11\times10^{-31}\text{ kg})(1.60\times10^{7}\text{ m/s})}{6.626\times10^{-34}\text{ J}\cdot\text{s}} = 1.38\times10^{11}\text{ m}^{-1}$$

(b) The energy of the electron in region 1 is

$$E = K = \frac{1}{2}mv^2 = \frac{1}{2}(9.11\times10^{-31}\text{ kg})(1.60\times10^{7}\text{ m/s})^2 = 1.17\times10^{-16}\text{ J} = 728.8\text{ eV}.$$

In region 2 where $V = -500$ V, the kinetic energy of the electron is

$$K_b = E - U_b = 728.8\text{ eV} - 500\text{ eV} = 228.8\text{ eV}.$$

and the corresponding angular wave number is

$$k_b = \frac{2\pi}{h}\sqrt{2m(E-U_b)} = \frac{2\pi}{h}\sqrt{2mK_b} = \frac{2\pi}{6.626\times10^{-34}\text{ J}\cdot\text{s}}\sqrt{2(9.11\times10^{-31}\text{ kg})(228.8\text{ eV})(1.6\times10^{-19}\text{ J/eV})}$$
$$= 7.74\times10^{10}\text{ m}^{-1}$$

(c) The wave functions in the two regions can be written as

$$\psi_1(x) = Ae^{ikx} + Be^{-ikx}, \quad \psi_2(x) = Ce^{ik_b x}$$

Matching the boundary conditions leads to

$$A + B = C$$
$$Ak - Bk = Ck_b$$

Solving for B and C in terms of A gives

$$\frac{B}{A}=\frac{1-k_b/k}{1+k_b/k}, \qquad \frac{C}{A}=\frac{2}{1+k_b/k}.$$

With $k_b/k=(7.74\times10^{10}\ \text{m}^{-1})/(1.38\times10^{11}\ \text{m}^{-1})=0.56,$ we find the reflection coefficient to be

$$R=\frac{|B|^2}{|A|^2}=\left(\frac{1-k_b/k}{1+k_b/k}\right)^2=\left(\frac{1-0.56}{1+0.56}\right)^2=0.0794$$

(d) With $N_0=3.00\times10^9$ electrons in the incident beam, the number reflected is

$$N_R=RN_0=\left(0.0794\right)(3.00\times10^9)=2.38\times10^8.$$

73. The energy of the electron in region 1 is

$$E=K=\frac{1}{2}mv^2=\frac{1}{2}(9.11\times10^{-31}\ \text{kg})(900\text{m/s})^2=3.69\times10^{-25}\ \text{J}=2.306\ \mu\text{eV}.$$

The angular wave number in region 1 is

$$k=\frac{2\pi}{\lambda}=\frac{2\pi}{(h/p)}=\frac{2\pi p}{h}=\frac{2\pi mv}{h}=\frac{2\pi(9.11\times10^{-31}\ \text{kg})(900\text{m/s})}{6.626\times10^{-34}\,\text{J}\cdot\text{s}}=7.77\times10^6\,\text{m}^{-1}$$

In region 2 where $V = -1.25\ \mu\text{V}$, the kinetic energy of the electron is

$$K_b=E-U_b=2.306\ \mu\text{eV}-1.25\ \mu\text{eV}=1.056\ \mu\text{eV}.$$

and the corresponding angular wave number is

$$k_b=\frac{2\pi}{h}\sqrt{2m(E-U_b)}=\frac{2\pi}{h}\sqrt{2mK_b}=\frac{2\pi}{6.626\times10^{-34}\,\text{J}\cdot\text{s}}\sqrt{2(9.11\times10^{-31}\ \text{kg})(1.056\ \mu\text{eV})(1.6\times10^{-25}\ \text{J}/\mu\text{eV})}$$
$$=5.258\times10^6\ \text{m}^{-1}$$

The ratio of the two wave numbers is $k_b/k=(5.258\times10^6\ \text{m}^{-1})/(7.77\times10^6\ \text{m}^{-1})=0.6767.$ The reflection coefficient is

$$R=\frac{|B|^2}{|A|^2}=\left(\frac{1-k_b/k}{1+k_b/k}\right)^2=\left(\frac{1-0.6767}{1+0.6767}\right)^2=0.0372,$$

which leads to the following transmission coefficient:

$$T=1-R=1-0.0372=0.9628.$$

Thus, we find the current on the other side of the step boundary to be

$$I_t = TI_0 = (0.9628)(5.00\ \text{mA}) = 4.81\ \text{mA}.$$

74. With

$$T \approx e^{-2bL} = \exp\left(-2L\sqrt{\frac{8\pi^2 m(U_b - E)}{h^2}}\right),$$

we have

$$E = U_b - \frac{1}{2m}\left(\frac{h\ln T}{4\pi L}\right)^2 = 6.0\,\text{eV} - \frac{1}{2(0.511\,\text{MeV})}\left[\frac{(1240\,\text{eV}\cdot\text{nm})(\ln 0.001)}{4\pi(0.70\,\text{nm})}\right]^2$$
$$= 5.1\,\text{eV}.$$

75. (a) The transmission coefficient T for a particle of mass m and energy E that is incident on a barrier of height U_b and width L is given by

$$T = e^{-2bL},$$

where

$$b = \sqrt{\frac{8\pi^2 m(U_b - E)}{h^2}}.$$

For the proton, we have

$$b = \sqrt{\frac{8\pi^2(1.6726\times10^{-27}\,\text{kg})(10\,\text{MeV} - 3.0\,\text{MeV})(1.6022\times10^{-13}\ \text{J/MeV})}{(6.6261\times10^{-34}\,\text{J}\cdot\text{s})^2}}$$
$$= 5.8082\times10^{14}\,\text{m}^{-1}.$$

This gives $bL = (5.8082\times10^{14}\ \text{m}^{-1})(10\times10^{-15}\ \text{m}) = 5.8082,$ and

$$T = e^{-2(5.8082)} = 9.02\times10^{-6}.$$

The value of b was computed to a greater number of significant digits than usual because an exponential is quite sensitive to the value of the exponent.

(b) Mechanical energy is conserved. Before the proton reaches the barrier, it has a kinetic energy of 3.0 MeV and a potential energy of zero. After passing through the barrier, the proton again has a potential energy of zero, thus a kinetic energy of 3.0 MeV.

(c) Energy is also conserved for the reflection process. After reflection, the proton has a potential energy of zero, and thus a kinetic energy of 3.0 MeV.

(d) The mass of a deuteron is 2.0141 u = 3.3454×10^{-27} kg, so

$$b=\sqrt{\frac{8\pi^2\left(3.3454\times10^{-27}\,\text{kg}\right)\left(10\,\text{MeV}-3.0\,\text{MeV}\right)\left(1.6022\times10^{-13}\ \text{J/MeV}\right)}{\left(6.6261\times10^{-34}\,\text{J}\cdot\text{s}\right)^2}}$$
$$=8.2143\times10^{14}\,\text{m}^{-1}.$$

This gives $bL=\left(8.2143\times10^{14}\,\text{m}^{-1}\right)\left(10\times10^{-15}\,\text{m}\right)=8.2143,$ and $T=e^{-2(8.2143)}=7.33\times10^{-8}$.

(e) As in the case of a proton, mechanical energy is conserved. Before the deuteron reaches the barrier, it has a kinetic energy of 3.0 MeV and a potential energy of zero. After passing through the barrier, the deuteron again has a potential energy of zero, thus a kinetic energy of 3.0 MeV.

(f) Energy is also conserved for the reflection process. After reflection, the deuteron has a potential energy of zero, and thus a kinetic energy of 3.0 MeV.

76. (a) The rate at which incident protons arrive at the barrier is

$$n=1.0\,\text{kA}/1.60\times10^{-19}\,\text{C}=6.25\times10^{21}/\text{s}\,.$$

Letting nTt = 1, we find the waiting time t:

$$t=\left(nT\right)^{-1}=\frac{1}{n}\exp\left(2L\sqrt{\frac{8\pi^2 m_p\left(U_b-E\right)}{h^2}}\right)$$
$$=\left(\frac{1}{6.25\times10^{21}/\text{s}}\right)\exp\left(\frac{2\pi\left(0.70\,\text{nm}\right)}{1240\,\text{eV}\cdot\text{nm}}\sqrt{8\left(938\,\text{MeV}\right)\left(6.0\,\text{eV}-5.0\,\text{eV}\right)}\right)$$
$$=3.37\times10^{111}\,\text{s}\approx10^{104}\,\text{y},$$

which is much longer than the age of the universe.

(b) Replacing the mass of the proton with that of the electron, we obtain the corresponding waiting time for an electron:

$$t=\left(nT\right)^{-1}=\frac{1}{n}\exp\left[2L\sqrt{\frac{8\pi^2 m_e\left(U_b-E\right)}{h^2}}\right]$$
$$=\left(\frac{1}{6.25\times10^{21}/\text{s}}\right)\exp\left[\frac{2\pi\left(0.70\,\text{nm}\right)}{1240\,\text{eV}\cdot\text{nm}}\sqrt{8\left(0.511\,\text{MeV}\right)\left(6.0\,\text{eV}-5.0\,\text{eV}\right)}\right]$$
$$=2.1\times10^{-19}\,\text{s}.$$

The enormous difference between the two waiting times is the result of the difference between the masses of the two kinds of particles.

77. **THINK** Even though $E < U_b$, barrier tunneling can still take place quantum mechanically with finite probability.

EXPRESS If m is the mass of the particle and E is its energy, then the transmission coefficient for a barrier of height U_b and width L is given by $T = e^{-2bL}$, where

$$b = \sqrt{\frac{8\pi^2 m(U_b - E)}{h^2}}.$$

If the change ΔU_b in U_b is small (as it is), the change in the transmission coefficient is given by

$$\Delta T = \frac{dT}{dU_b}\Delta U_b = -2LT\frac{db}{dU_b}\Delta U_b.$$

Now,

$$\frac{db}{dU_b} = \frac{1}{2\sqrt{U_b - E}}\sqrt{\frac{8\pi^2 m}{h^2}} = \frac{1}{2(U_b - E)}\sqrt{\frac{8\pi^2 m(U_b - E)}{h^2}} = \frac{b}{2(U_b - E)}.$$

Thus,

$$\Delta T = -LTb\frac{\Delta U_b}{U_b - E}.$$

ANALYZE (a) With

$$b = \sqrt{\frac{8\pi^2 (9.11\times10^{-31}\text{ kg})(6.8\text{ eV} - 5.1\text{ eV})(1.6022\times10^{-19}\text{ J/eV})}{(6.6261\times10^{-34}\text{ J}\cdot\text{s})^2}} = 6.67\times10^{9}\text{ m}^{-1},$$

we have $bL = (6.67\times10^{9}\text{ m}^{-1})(750\times10^{-12}\text{ m}^{-1}) = 5.0,$ and

$$\frac{\Delta T}{T} = -bL\frac{\Delta U_b}{U_b - E} = -(5.0)\frac{(0.010)(6.8\,\text{eV})}{6.8\,\text{eV} - 5.1\,\text{eV}} = -0.20.$$

There is a 20% decrease in the transmission coefficient.

(b) The change in the transmission coefficient is given by

$$\Delta T = \frac{dT}{dL}\Delta L = -2be^{-2bL}\Delta L = -2bT\Delta L$$

and

$$\frac{\Delta T}{T} = -2b\Delta L = -2\left(6.67\times10^{9}\ \text{m}^{-1}\right)\left(0.010\right)\left(750\times10^{-12}\ \text{m}\right) = -0.10\ .$$

There is a 10% decrease in the transmission coefficient.

(c) The change in the transmission coefficient is given by

$$\Delta T = \frac{dT}{dE}\Delta E = -2Le^{-2bL}\frac{db}{dE}\Delta E = -2LT\frac{db}{dE}\Delta E.$$

Now, $db/dE = -db/dU_b = -b/2\left(U_b - E\right)$, so

$$\frac{\Delta T}{T} = bL\frac{\Delta E}{U_b - E} = \left(5.0\right)\frac{\left(0.010\right)\left(5.1\text{eV}\right)}{6.8\,\text{eV} - 5.1\text{eV}} = 0.15\ .$$

There is a 15% increase in the transmission coefficient.

LEARN Increasing the barrier height or the barrier thickness reduces the probability of transmission, while increasing the kinetic energy of the electron increases the probability.

78. The energy of the electron in region 1 is

$$E = K = \frac{1}{2}mv^2 = \frac{1}{2}(9.11\times10^{-31}\ \text{kg})(1200\ \text{m/s})^2 = 6.56\times10^{-25}\ \text{J} = 4.0995\ \mu\text{eV}.$$

The angular wave number in region 1 is

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{(h/p)} = \frac{2\pi p}{h} = \frac{2\pi mv}{h} = \frac{2\pi(9.11\times10^{-31}\ \text{kg})(1200\ \text{m/s})}{6.626\times10^{-34}\,\text{J}\cdot\text{s}} = 1.036\times10^{7}\,\text{m}^{-1}$$

The transmission coefficient for a barrier of height U_b and width L is given by

$$T = e^{-2bL},$$

where

$$b = \sqrt{\frac{8\pi^2 m\left(U_b - E\right)}{h^2}} = \sqrt{\frac{8\pi^2\left(9.11\times10^{-31}\ \text{kg}\right)\left(4.719\ \mu\text{eV} - 4.0995\ \mu\text{eV}\right)\left(1.6022\times10^{-25}\ \text{J}/\mu\text{eV}\right)}{\left(6.6261\times10^{-34}\,\text{J}\cdot\text{s}\right)^2}}$$

$$= 4.0298\times10^{6}\,\text{m}^{-1}.$$

Thus,

$$T = \exp(-2bL) = \exp\left[-2(4.0298\times10^{6}\,\text{m}^{-1})(200\times10^{-9}\,\text{m}^{-1})\right] = e^{-1.612} = 0.1995,$$

and the current transmitted is

$$I_t = TI_0 = (0.1995)(9.00 \text{ mA}) = 1.795 \text{ mA} .$$

79. (a) Since $p_x = p_y = 0$, $\Delta p_x = \Delta p_y = 0$. Thus from Eq. 38-20 both Δx and Δy are infinite. It is therefore impossible to assign a y or z coordinate to the position of an electron.

(b) Since it is independent of y and z the wave function $\Psi(x)$ should describe a plane wave that extends infinitely in both the y and z directions. Also from Fig. 38-12 we see that $|\Psi(x)|^2$ extends infinitely along the x axis. Thus the matter wave described by $\Psi(x)$ extends throughout the entire three-dimensional space.

80. Using the value $hc = 1240 \text{eV}\cdot\text{nm}$, we obtain

$$E = \frac{hc}{\lambda} = \frac{1240 \text{ eV}\cdot\text{nm}}{21\times10^{7} \text{ nm}} = 5.9\times10^{-6} \text{ eV} = 5.9\mu\text{eV}.$$

81. We substitute the classical relationship between momentum p and velocity v, $v = p/m$ into the classical definition of kinetic energy, $K = \frac{1}{2}mv^2$ to obtain $K = p^2/2m$. Here m is the mass of an electron. Thus $p = \sqrt{2mK}$. The relationship between the momentum and the de Broglie wavelength λ is $\lambda = h/p$, where h is the Planck constant. Thus,

$$\lambda = \frac{h}{\sqrt{2mK}} .$$

If K is given in electron volts, then

$$\lambda = \frac{6.626\times10^{-34} \text{ J}\cdot\text{s}}{\sqrt{2(9.109\times10^{-31} \text{ kg})(1.602\times10^{-19} \text{ J/eV})K}} = \frac{1.226\times10^{-9} \text{ m}\cdot\text{eV}^{1/2}}{\sqrt{K}}$$
$$= \frac{1.226 \text{nm}\cdot\text{eV}^{1/2}}{\sqrt{K}}.$$

82. We rewrite Eq. 38-9 as

$$\frac{h}{m\lambda} - \frac{h}{m\lambda'}\cos\phi = \frac{v}{\sqrt{1-(v/c)^2}}\cos\theta ,$$

and Eq. 38-10 as

$$\frac{h}{m\lambda'}\sin\phi = \frac{v}{\sqrt{1-(v/c)^2}}\sin\theta .$$

We square both equations and add up the two sides:

$$\left(\frac{h}{m}\right)^2\left[\left(\frac{1}{\lambda}-\frac{1}{\lambda'}\cos\phi\right)^2+\left(\frac{1}{\lambda'}\sin\phi\right)^2\right]=\frac{v^2}{1-(v/c)^2}\ ,$$

where we use $\sin^2\theta+\cos^2\theta=1$ to eliminate θ. Now the right-hand side can be written as

$$\frac{v^2}{1-(v/c)^2}=-c^2\left[1-\frac{1}{1-(v/c)^2}\right],$$

so

$$\frac{1}{1-(v/c)^2}=\left(\frac{h}{mc}\right)^2\left[\left(\frac{1}{\lambda}-\frac{1}{\lambda'}\cos\phi\right)^2+\left(\frac{1}{\lambda'}\sin\phi\right)^2\right]+1\ .$$

Now we rewrite Eq. 38-8 as

$$\frac{h}{mc}\left(\frac{1}{\lambda}-\frac{1}{\lambda'}\right)+1=\frac{1}{\sqrt{1-(v/c)^2}}\ .$$

If we square this, then it can be directly compared with the previous equation we obtained for $[1-(v/c)^2]^{-1}$. This yields

$$\left[\frac{h}{mc}\left(\frac{1}{\lambda}-\frac{1}{\lambda'}\right)+1\right]^2=\left(\frac{h}{mc}\right)^2\left[\left(\frac{1}{\lambda}-\frac{1}{\lambda'}\cos\phi\right)^2+\left(\frac{1}{\lambda'}\sin\phi\right)^2\right]+1\ .$$

We have so far eliminated θ and v. Working out the squares on both sides and noting that $\sin^2\phi+\cos^2\phi=1$, we get

$$\lambda'-\lambda=\Delta\lambda=\frac{h}{mc}(1-\cos\phi)\ .$$

83. (a) The average kinetic energy is

$$K=\frac{3}{2}kT=\frac{3}{2}\left(1.38\times10^{-23}\ \text{J/K}\right)\left(300\,\text{K}\right)=6.21\times10^{-21}\ \text{J}=3.88\times10^{-2}\ \text{eV}.$$

(b) The de Broglie wavelength is

$$\lambda=\frac{h}{\sqrt{2m_nK}}=\frac{6.63\times10^{-34}\,\text{J}\cdot\text{s}}{\sqrt{2\left(1.675\times10^{-27}\,\text{kg}\right)\left(6.21\times10^{-21}\,\text{J}\right)}}=1.46\times10^{-10}\,\text{m}.$$

84. (a) The average de Broglie wavelength is

$$\lambda_{avg} = \frac{h}{p_{avg}} = \frac{h}{\sqrt{2mK_{avg}}} = \frac{h}{\sqrt{2m(3kT/2)}} = \frac{hc}{\sqrt{2(mc^2)kT}}$$
$$= \frac{1240\,\text{eV}\cdot\text{nm}}{\sqrt{3(4)(938\,\text{MeV})(8.62\times10^{-5}\,\text{eV/K})(300\,\text{K})}}$$
$$= 7.3\times10^{-11}\,\text{m} = 73\,\text{pm}.$$

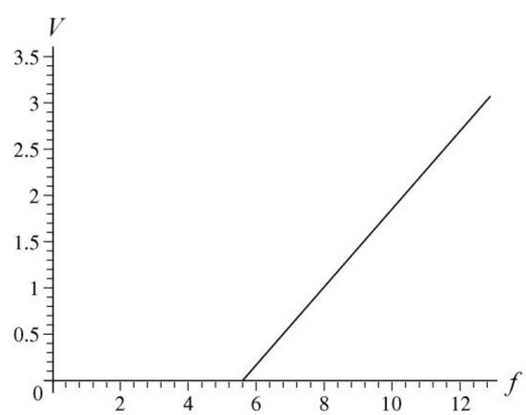
(b) The average separation is

$$d_{avg} = \frac{1}{\sqrt[3]{n}} = \frac{1}{\sqrt[3]{p/kT}} = \sqrt[3]{\frac{(1.38\times10^{-23}\,\text{J/K})(300\text{K})}{1.01\times10^5\,\text{Pa}}} = 3.4\,\text{nm}.$$

(c) Yes, since $\lambda_{avg} \ll d_{avg}$.

85. (a) We calculate frequencies from the wavelengths (expressed in SI units) using Eq. 38-1. Our plot of the points and the line that gives the least squares fit to the data is shown below. The vertical axis is in volts and the horizontal axis, when multiplied by 10^{14}, gives the frequencies in Hertz.

From our least squares fit procedure, we determine the slope to be 4.14×10^{-15} V·s, which, upon multiplying by e, gives 4.14×10^{-15} eV·s. The result is in very good agreement with the value given in Eq. 38-3.



(b) Our least squares fit procedure can also determine the y-intercept for that line. The y-intercept is the negative of the photoelectric work function. In this way, we find Φ = 2.31 eV.

86. We note that

$$|e^{ikx}|^2 = (e^{ikx})^*(e^{ikx}) = e^{-ikx}e^{ikx} = 1.$$

Referring to Eq. 38-14, we see therefore that $|\psi|^2 = |\Psi|^2$.

87. From Sample Problem — "Compton scattering of light by electrons," we have

$$\frac{\Delta E}{E}=\frac{\Delta\lambda}{\lambda+\Delta\lambda}=\frac{(h/mc)(1-\cos\phi)}{\lambda'}=\frac{hf'}{mc^2}(1-\cos\phi)$$

where we use the fact that $\lambda + \Delta\lambda = \lambda' = c/f'$.

88. The de Broglie wavelength for the bullet is

$$\lambda=\frac{h}{p}=\frac{h}{mv}=\frac{6.63\times10^{-34}\ \text{J.s}}{(40\times10^{-3}\ \text{kg})(1000\,\text{m/s})}=1.7\times10^{-35}\ \text{m}\ .$$

89. (a) Since

$$E_{ph} = h/\lambda = 1240\ \text{eV}\cdot\text{nm}/680\ \text{nm} = 1.82\ \text{eV} < \Phi = 2.28\ \text{eV},$$

there is no photoelectric emission.

(b) The cutoff wavelength is the longest wavelength of photons that will cause photoelectric emission. In sodium, this is given by $E_{ph} = hc/\lambda_{max} = \Phi$, or

$$\lambda_{max} = hc/\Phi = (1240\ \text{eV}\cdot\text{nm})/2.28\ \text{eV} = 544\ \text{nm}.$$

(c) This corresponds to the color green.

90. **THINK** We apply Heisenberg's uncertainty principle to calculate the uncertainty in position.

EXPRESS The uncertainty principle states that $\Delta x\Delta p \geq \hbar$, where Δx and Δp represent the intrinsic uncertainties in measuring the position and momentum, respectively. The uncertainty in the momentum is

$$\Delta p = m\,\Delta v = (0.50\ \text{kg})(1.0\ \text{m/s}) = 0.50\ \text{kg}\cdot\text{m/s},$$

where Δv is the uncertainty in the velocity.

ANALYZE Solving the uncertainty relationship $\Delta x\Delta p \geq \hbar$ for the minimum uncertainty in the coordinate x, we obtain

$$\Delta x=\frac{\hbar}{\Delta p}=\frac{0.60\,\text{J}\cdot\text{s}}{2\pi\left(0.50\,\text{kg}\cdot\text{m/s}\right)}=0.19\,\text{m}.$$

LEARN Heisenberg's uncertainty principle implies that it is impossible to simultaneously measure a particle's position and momentum with infinite accuracy.

Chapter 39

1. According to Eq. 39-4, $E_n \propto L^{-2}$. As a consequence, the new energy level E'_n satisfies

$$\frac{E'_n}{E_n} = \left(\frac{L'}{L}\right)^{-2} = \left(\frac{L}{L'}\right)^{2} = \frac{1}{2},$$

which gives $L' = \sqrt{2}L.$ Thus, the ratio is $L'/L = \sqrt{2} = 1.41.$

2. (a) The ground-state energy is

$$E_1 = \left(\frac{h^2}{8m_e L^2}\right)n^2 = \left(\frac{\left(6.63\times10^{-34}\,\text{J}\cdot\text{s}\right)^2}{8(9.11\times10^{-31}\,\text{kg})\left(200\times10^{-12}\,\text{m}\right)^2}\right)(1)^2 = 1.51\times10^{-18}\,\text{J}$$
$$= 9.42\,\text{eV}.$$

(b) With $m_p = 1.67 \times 10^{-27}$ kg, we obtain

$$E_1 = \left(\frac{h^2}{8m_p L^2}\right)n^2 = \left(\frac{\left(6.63\times10^{-34}\,\text{J}\cdot\text{s}\right)^2}{8(1.67\times10^{-27}\,\text{kg})\left(200\times10^{-12}\,\text{m}\right)^2}\right)(1)^2 = 8.225\times10^{-22}\,\text{J}$$
$$= 5.13\times10^{-3}\,\text{eV}.$$

3. Since $E_n \propto L^{-2}$ in Eq. 39-4, we see that if L is doubled, then E_1 becomes (2.6 eV)$(2)^{-2}$ = 0.65 eV.

4. We first note that since $h = 6.626 \times 10^{-34}$ J·s and $c = 2.998 \times 10^{8}$ m/s,

$$hc = \frac{\left(6.626\times10^{-34}\,\text{J}\cdot\text{s}\right)\left(2.998\times10^{8}\,\text{m/s}\right)}{\left(1.602\times10^{-19}\,\text{J/eV}\right)\left(10^{-9}\,\text{m/nm}\right)} = 1240\,\text{eV}\cdot\text{nm}.$$

Using the mc^2 value for an electron from Table 37-3 (511×10^3 eV), Eq. 39-4 can be rewritten as

$$E_n = \frac{n^2h^2}{8mL^2} = \frac{n^2(hc)^2}{8(mc^2)L^2}.$$

The energy to be absorbed is therefore

$$\Delta E = E_4 - E_1 = \frac{\left(4^2 - 1^2\right)h^2}{8m_e L^2} = \frac{15\left(hc\right)^2}{8\left(m_e c^2\right)L^2} = \frac{15\left(1240\,\text{eV}\cdot\text{nm}\right)^2}{8\left(511\times10^3\,\text{eV}\right)\left(0.250\,\text{nm}\right)^2} = 90.3\,\text{eV}.$$

5. We can use the mc^2 value for an electron from Table 37-3 (511×10^3 eV) and hc = 1240 eV · nm by writing Eq. 39-4 as

$$E_n = \frac{n^2 h^2}{8mL^2} = \frac{n^2\left(hc\right)^2}{8\left(mc^2\right)L^2}.$$

For $n = 3$, we set this expression equal to 4.7 eV and solve for L:

$$L = \frac{n\left(hc\right)}{\sqrt{8\left(mc^2\right)E_n}} = \frac{3\left(1240\,\text{eV}\cdot\text{nm}\right)}{\sqrt{8\left(511\times10^3\,\text{eV}\right)\left(4.7\,\text{eV}\right)}} = 0.85\,\text{nm}.$$

6. With $m = m_p = 1.67 \times 10^{-27}$ kg, we obtain

$$E_1 = \left(\frac{h^2}{8mL^2}\right)n^2 = \left(\frac{\left(6.63\times10^{-34}\,\text{J.s}\right)^2}{8(1.67\times10^{-27}\,\text{kg})\left(100\times10^{12}\,\text{m}\right)^2}\right)(1)^2 = 3.29\times10^{-21}\,\text{J} = 0.0206\,\text{eV}.$$

Alternatively, we can use the mc^2 value for a proton from Table 37-3 (938×10^6 eV) and hc = 1240 eV · nm by writing Eq. 39-4 as

$$E_n = \frac{n^2 h^2}{8mL^2} = \frac{n^2\left(hc\right)^2}{8\left(m_p c^2\right)L^2}.$$

This alternative approach is perhaps easier to plug into, but it is recommended that both approaches be tried to find which is most convenient.

7. To estimate the energy, we use Eq. 39-4, with $n = 1$, L equal to the atomic diameter, and m equal to the mass of an electron:

$$E = n^2\frac{h^2}{8mL^2} = \frac{(1)^2\left(6.63\times10^{-34}\,\text{J}\cdot\text{s}\right)^2}{8\left(9.11\times10^{-31}\,\text{kg}\right)\left(1.4\times10^{-14}\,\text{m}\right)^2} = 3.07\times10^{-10}\,\text{J} = 1920\,\text{MeV} \approx 1.9\ \text{GeV}.$$

8. The frequency of the light that will excite the electron from the state with quantum number n_i to the state with quantum number n_f is

$$f = \frac{\Delta E}{h} = \frac{h}{8mL^2}\left(n_f^2 - n_i^2\right)$$

and the wavelength of the light is

$$\lambda = \frac{c}{f} = \frac{8mL^2c}{h\left(n_f^2 - n_i^2\right)}.$$

The width of the well is

$$L = \sqrt{\frac{\lambda hc(n_f^2 - n_i^2)}{8mc^2}}.$$

The longest wavelength shown in Figure 39-27 is $\lambda = 80.78$ nm, which corresponds to a jump from $n_i = 2$ to $n_f = 3$. Thus, the width of the well is

$$L = \sqrt{\frac{\lambda hc(n_f^2 - n_i^2)}{8mc^2}} = \sqrt{\frac{(80.78\text{ nm})(1240\,\text{eV}\cdot\text{nm})(3^2 - 2^2)}{8(511\times10^3\,\text{eV})}} = 0.350\,\text{nm} = 350\text{ pm}.$$

9. We can use the mc^2 value for an electron from Table 37-3 (511×10^3 eV) and hc = 1240 eV · nm by rewriting Eq. 39-4 as

$$E_n = \frac{n^2h^2}{8mL^2} = \frac{n^2(hc)^2}{8\left(mc^2\right)L^2}.$$

(a) The first excited state is characterized by $n = 2$, and the third by $n' = 4$. Thus,

$$\Delta E = \frac{(hc)^2}{8\left(mc^2\right)L^2}\left(n'^2 - n^2\right) = \frac{(1240\,\text{eV}\cdot\text{nm})^2}{8\left(511\times10^3\,\text{eV}\right)(0.250\,\text{nm})^2}\left(4^2 - 2^2\right) = (6.02\,\text{eV})(16-4)$$
$$= 72.2\,\text{eV}.$$

Now that the electron is in the $n' = 4$ level, it can "drop" to a lower level (n'') in a variety of ways. Each of these drops is presumed to cause a photon to be emitted of wavelength

$$\lambda = \frac{hc}{E_{n'} - E_{n''}} = \frac{8\left(mc^2\right)L^2}{hc\left(n'^2 - n''^2\right)}.$$

For example, for the transition $n' = 4$ to $n'' = 3$, the photon emitted would have wavelength

$$\lambda = \frac{8\left(511\times10^3\,\text{eV}\right)(0.250\,\text{nm})^2}{(1240\,\text{eV}\cdot\text{nm})\left(4^2 - 3^2\right)} = 29.4\,\text{nm},$$

and once it is then in level $n'' = 3$ it might fall to level $n''' = 2$ emitting another photon. Calculating in this way all the possible photons emitted during the de-excitation of this system, we obtain the following results:

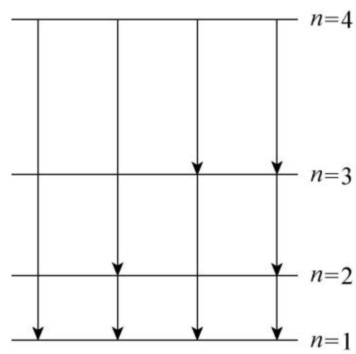
(b) The shortest wavelength that can be emitted is $\lambda_{4\to1} = 13.7\,\text{nm}$.

(c) The second shortest wavelength that can be emitted is $\lambda_{4\to2} = 17.2\,\text{nm}$.

(d) The longest wavelength that can be emitted is $\lambda_{2\to1} = 68.7\,\text{nm}$.

(e) The second longest wavelength that can be emitted is $\lambda_{3\to2} = 41.2\,\text{nm}$.

(f) The possible transitions are shown next. The energy levels are not drawn to scale.



(g) A wavelength of 29.4 nm corresponds to $4\to3$ transition. Thus, it could make either the $3\to1$ transition or the pair of transitions: $3\to2$ and $2\to1$. The longest wavelength that can be emitted is $\lambda_{2\to1} = 68.7\,\text{nm}$.

(h) The shortest wavelength that can next be emitted is $\lambda_{3\to1} = 25.8\,\text{nm}$.

10. Let the quantum numbers of the pair in question be n and $n + 1$, respectively. Then

$$E_{n+1} - E_n = E_1\,(n+1)^2 - E_1 n^2 = (2n+1)E_1.$$

Letting

$$E_{n+1} - E_n = (2n+1)E_1 = 3(E_4 - E_3) = 3\left(4^2 E_1 - 3^2 E_1\right) = 21E_1,$$

we get $2n + 1 = 21$, or $n = 10$. Thus,

(a) the higher quantum number is $n + 1 = 10 + 1 = 11$, and

(b) the lower quantum number is $n = 10$.

(c) Now letting

$$E_{n+1} - E_n = (2n+1)E_1 = 2(E_4 - E_3) = 2\left(4^2 E_1 - 3^2 E_1\right) = 14E_1,$$

we get $2n + 1 = 14$, which does not have an integer-valued solution. So it is impossible to find the pair of energy levels that fits the requirement.

11. Let the quantum numbers of the pair in question be n and $n + 1$, respectively. We note that

$$E_{n+1} - E_n = \frac{(n+1)^2 h^2}{8mL^2} - \frac{n^2 h^2}{8mL^2} = \frac{(2n+1)h^2}{8mL^2}$$

Therefore, $E_{n+1} - E_n = (2n + 1)E_1$. Now

$$E_{n+1} - E_n = E_5 = 5^2 E_1 = 25E_1 = (2n+1)E_1,$$

which leads to $2n + 1 = 25$, or $n = 12$. Thus,

(a) The higher quantum number is $n + 1 = 12 + 1 = 13$.

(b) The lower quantum number is $n = 12$.

(c) Now let

$$E_{n+1} - E_n = E_6 = 6^2 E_1 = 36E_1 = (2n+1)E_1,$$

which gives $2n + 1 = 36$, or $n = 17.5$. This is not an integer, so it is impossible to find the pair that fits the requirement.

12. The energy levels are given by $E_n = n^2h^2/8mL^2$, where h is the Planck constant, m is the mass of an electron, and L is the width of the well. The frequency of the light that will excite the electron from the state with quantum number n_i to the state with quantum number n_f is

$$f = \frac{\Delta E}{h} = \frac{h}{8mL^2}\left(n_f^2 - n_i^2\right)$$

and the wavelength of the light is

$$\lambda = \frac{c}{f} = \frac{8mL^2 c}{h\left(n_f^2 - n_i^2\right)}.$$

We evaluate this expression for $n_i = 1$ and $n_f = 2, 3, 4$, and 5, in turn. We use $h = 6.626 \times 10^{-34}$ J · s, $m = 9.109 \times 10^{-31}$kg, and $L = 250 \times 10^{-12}$ m, and obtain the following results:

(a) 6.87×10^{-8} m for $n_f = 2$, (the longest wavelength).

(b) 2.58×10^{-8} m for $n_f = 3$, (the second longest wavelength).

(c) 1.37×10^{-8} m for $n_f = 4$, (the third longest wavelength).

13. The position of maximum probability density corresponds to the center of the well: $x = L/2 = (200 \text{ pm})/2 = 100 \text{ pm}.$

(a) The probability of detection at x is given by Eq. 39-11:

$$p(x) = \psi_n^2(x)dx = \left[\sqrt{\frac{2}{L}}\sin\left(\frac{n\pi}{L}x\right)\right]^2 dx = \frac{2}{L}\sin^2\left(\frac{n\pi}{L}x\right)dx$$

For $n = 3$, $L = 200 \text{ pm}$, and $dx = 2.00 \text{ pm}$ (width of the probe), the probability of detection at $x = L/2 = 100 \text{ pm}$ is

$$p(x = L/2) = \frac{2}{L}\sin^2\left(\frac{3\pi}{L}\cdot\frac{L}{2}\right)dx = \frac{2}{L}\sin^2\left(\frac{3\pi}{2}\right)dx = \frac{2}{L}dx = \frac{2}{200 \text{ pm}}\left(2.00 \text{ pm}\right) = 0.020.$$

(b) With $N = 1000$ independent insertions, the number of times we expect the electron to be detected is $n = Np = (1000)(0.020) = 20$.

14. From Eq. 39-11, the condition of zero probability density is given by

$$\sin\left(\frac{n\pi}{L}x\right) = 0 \;\Rightarrow\; \frac{n\pi}{L}x = m\pi$$

where m is an integer. The fact that $x = 0.300L$ and $x = 0.400L$ have zero probability density implies

$$\sin\left(0.300n\pi\right) = \sin\left(0.400n\pi\right) = 0$$

which can be satisfied for $n = 10m$, where $m = 1, 2, \ldots$ However, since the probability density is nonzero between $x = 0.300L$ and $x = 0.400L$, we conclude that the electron is in the $n = 10$ state. The change of energy after making a transition to $n' = 9$ is then equal to

$$|\Delta E| = \frac{h^2}{8mL^2}\left(n^2 - n'^2\right) = \frac{\left(6.63\times10^{-34} \text{ J}\cdot\text{s}\right)^2}{8\left(9.11\times10^{-31} \text{ kg}\right)\left(2.00\times10^{-10} \text{ m}\right)^2}\left(10^2 - 9^2\right) = 2.86\times10^{-17} \text{ J}.$$

15. **THINK** The probability that the electron is found in any interval is given by $P = \int |\psi|^2 dx,$ where the integral is over the interval.

EXPRESS If the interval width Δx is small, the probability can be approximated by $P = |\psi|^2 \Delta x$, where the wave function is evaluated for the center of the interval, say. For an electron trapped in an infinite well of width L, the ground state probability density is

$$|\psi|^2 = \frac{2}{L}\sin^2\left(\frac{\pi x}{L}\right),$$

so

$$P = \left(\frac{2\Delta x}{L}\right)\sin^2\left(\frac{\pi x}{L}\right).$$

ANALYZE (a) We take L = 100 pm, x = 25 pm, and Δx = 5.0 pm. Then,

$$P = \left[\frac{2(5.0\,\text{pm})}{100\,\text{pm}}\right]\sin^2\left[\frac{\pi(25\,\text{pm})}{100\,\text{pm}}\right] = 0.050.$$

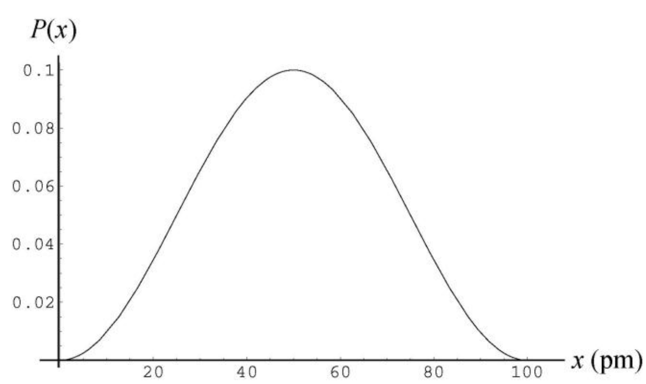
(b) We take L = 100 pm, x = 50 pm, and Δx = 5.0 pm. Then,

$$P = \left[\frac{2(5.0\,\text{pm})}{100\,\text{pm}}\right]\sin^2\left[\frac{\pi(50\,\text{pm})}{100\,\text{pm}}\right] = 0.10.$$

(c) We take L = 100 pm, x = 90 pm, and Δx = 5.0 pm. Then,

$$P = \left[\frac{2(5.0\,\text{pm})}{100\,\text{pm}}\right]\sin^2\left[\frac{\pi(90\,\text{pm})}{100\,\text{pm}}\right] = 0.0095.$$

LEARN The probability as a function of x is plotted next. As expected, the probability of detecting the electron is highest near the center of the well at $x = L/2$ = 50 pm.



16. We follow Sample Problem — "Detection potential in a 1D infinite potential well" in the presentation of this solution. The integration result quoted below is discussed in a little more detail in that Sample Problem. We note that the arguments of the sine functions used below are in radians.

(a) The probability of detecting the particle in the region $0 \le x \le L/4$ is

$$\left(\frac{2}{L}\right)\left(\frac{L}{\pi}\right)\int_0^{\pi/4}\sin^2 y\,dy = \frac{2}{\pi}\left(\frac{y}{2}-\frac{\sin 2y}{4}\right)\Bigg|_0^{\pi/4} = 0.091.$$

(b) As expected from symmetry,

$$\left(\frac{2}{L}\right)\left(\frac{L}{\pi}\right)\int_{\pi/4}^{\pi}\sin^2 y\,dy = \frac{2}{\pi}\left(\frac{y}{2}-\frac{\sin 2y}{4}\right)\Bigg|_{\pi/4}^{\pi} = 0.091.$$

(c) For the region $L/4 \le x \le 3L/4$, we obtain

$$\left(\frac{2}{L}\right)\left(\frac{L}{\pi}\right)\int_{\pi/4}^{3\pi/4}\sin^2 y\,dy = \frac{2}{\pi}\left(\frac{y}{2}-\frac{\sin 2y}{4}\right)\Bigg|_{\pi/4}^{3\pi/4} = 0.82$$

which we could also have gotten by subtracting the results of part (a) and (b) from 1; that is, 1 – 2(0.091) = 0.82.

17. According to Fig. 39-9, the electron’s initial energy is 106 eV. After the additional energy is absorbed, the total energy of the electron is 106 eV + 400 eV = 506 eV. Since it is in the region $x > L$, its potential energy is 450 eV, so its kinetic energy must be 506 eV – 450 eV = 56 eV.

18. From Fig. 39-9, we see that the sum of the kinetic and potential energies in that particular finite well is 233 eV. The potential energy is zero in the region $0 < x < L$. If the kinetic energy of the electron is detected while it is in that region (which is the only region where this is likely to happen), we should find K = 233 eV.

19. Using $E = hc/\lambda = (1240\,\text{eV}\cdot\text{nm})/\lambda$, the energies associated with λ_a, λ_b and λ_c are

$$E_a = \frac{hc}{\lambda_a} = \frac{1240\,\text{eV}\cdot\text{nm}}{14.588\text{ nm}} = 85.00\text{ eV}$$

$$E_b = \frac{hc}{\lambda_b} = \frac{1240\,\text{eV}\cdot\text{nm}}{4.8437\text{ nm}} = 256.0\text{ eV}$$

$$E_c = \frac{hc}{\lambda_c} = \frac{1240\,\text{eV}\cdot\text{nm}}{2.9108\text{ nm}} = 426.0\text{ eV}.$$

The ground-state energy is

$$E_1 = E_4 - E_c = 450.0\text{ eV} - 426.0\text{ eV} = 24.0\text{ eV}.$$

Since $E_a = E_2 - E_1$, the energy of the first excited state is

$$E_2 = E_1 + E_a = 24.0\text{ eV} + 85.0\text{ eV} = 109\text{ eV}.$$

20. The smallest energy a photon can have corresponds to a transition from the non-quantized region to E_3. Since the energy difference between E_3 and E_4 is

$$\Delta E = E_4 - E_3 = 9.0\ \text{eV} - 4.0\ \text{eV} = 5.0\ \text{eV}\,,$$

the energy of the photon is $E_{\text{photon}} = K + \Delta E = 2.00\ \text{eV} + 5.00\ \text{eV} = 7.00\ \text{eV}$.

21. Schrödinger's equation for the region $x > L$ is

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}\ E - U_0\ \ \psi = 0.$$

If $\psi = De^{2kx}$, then $d^2\psi/dx^2 = 4k^2De^{2kx} = 4k^2\psi$ and

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}\ E - U_0\ \ \psi = 4k^2\psi + \frac{8\pi^2 m}{h^2}\ E - U_0\ \ \psi.$$

This is zero provided

$$k = \frac{\pi}{h}\sqrt{2m(U_0 - E)}.$$

The proposed function satisfies Schrödinger's equation provided k has this value. Since U_0 is greater than E in the region $x > L$, the quantity under the radical is positive. This means k is real. If k is positive, however, the proposed function is physically unrealistic. It increases exponentially with x and becomes large without bound. The integral of the probability density over the entire x-axis must be unity. This is impossible if ψ is the proposed function.

22. We can use the mc^2 value for an electron from Table 37-3 (511×10^3 eV) and hc = 1240 eV · nm by writing Eq. 39-20 as

$$E_{nx,ny} = \frac{2h^2}{8m}\left(\frac{n_x^2}{L_x^2} + \frac{n_y^2}{L_y^2}\right) = \frac{(hc)^2}{8(mc^2)}\left(\frac{n_x^2}{L_x^2} + \frac{n_y^2}{L_y^2}\right).$$

For $n_x = n_y = 1$, we obtain

$$E_{1,1} = \frac{(1240\,\text{eV}\cdot\text{nm})^2}{8(511\times10^3\text{eV})}\left(\frac{1}{(0.800\,\text{nm})^2} + \frac{1}{(1.600\,\text{nm})^2}\right) = 0.734\ \text{eV}.$$

23. We can use the mc^2 value for an electron from Table 37-3 (511×10^3 eV) and hc = 1240 eV · nm by writing Eq. 39-21 as

$$E_{nx,ny,nz} = \frac{2h^2}{8m}\left(\frac{n_x^2}{L_x^2}+\frac{n_y^2}{L_y^2}+\frac{n_z^2}{L_z^2}\right) = \frac{(hc)^2}{8(mc^2)}\left(\frac{n_x^2}{L_x^2}+\frac{n_y^2}{L_y^2}+\frac{n_z^2}{L_z^2}\right).$$

For $n_x = n_y = n_z = 1$, we obtain

$$E_{1,1} = \frac{(1240\,\text{eV}\cdot\text{nm})^2}{8(511\times 10^3\,\text{eV})}\left(\frac{1}{(0.800\,\text{nm})^2}+\frac{1}{(1.600\,\text{nm})^2}+\frac{1}{(0.390\,\text{nm})^2}\right) = 3.21\text{ eV}.$$

24. The statement that there are three probability density maxima along $x = L_x/2$ implies that $n_y = 3$ (see for example, Figure 39-6). Since the maxima are separated by 2.00 nm, the width of L_y is $L_y = n_y(2.00\text{ nm}) = 6.00\text{ nm}$. Similarly, from the information given along $y = L_y/2$, we find $n_x = 5$ and $L_x = n_x(3.00\text{ nm}) = 15.0\text{ nm}$. Thus, using Eq. 39-20, the energy of the electron is

$$\begin{aligned}E_{n_x,n_y} &= \frac{h^2}{8m}\left(\frac{n_x^2}{L_x^2}+\frac{n_y^2}{L_y^2}\right) = \frac{(6.63\times 10^{-34}\text{ J}\cdot\text{s})^2}{8(9.11\times 10^{-31}\text{ kg})}\left[\frac{1}{(3.00\times 10^{-9}\text{ m})^2}+\frac{1}{(2.00\times 10^{-9}\text{ m})^2}\right]\\ &= 2.2\times 10^{-20}\text{ J}.\end{aligned}$$

25. The discussion on the probability of detection for the one-dimensional case can be readily extended to two dimensions. In analogy to Eq. 39-10, the normalized wave function in two dimensions can be written as

$$\begin{aligned}\psi_{n_x,n_y}(x,y) &= \psi_{n_x}(x)\psi_{n_y}(y) = \sqrt{\frac{2}{L_x}}\sin\left(\frac{n_x\pi}{L_x}x\right)\cdot\sqrt{\frac{2}{L_y}}\sin\left(\frac{n_y\pi}{L_y}y\right)\\ &= \sqrt{\frac{4}{L_xL_y}}\sin\left(\frac{n_x\pi}{L_x}x\right)\sin\left(\frac{n_y\pi}{L_y}y\right).\end{aligned}$$

The probability of detection by a probe of dimension $\Delta x\Delta y$ placed at (x, y) is

$$p(x,y) = |\psi_{n_x,n_y}(x,y)|^2\,\Delta x\Delta y = \frac{4(\Delta x\Delta y)}{L_xL_y}\sin^2\left(\frac{n_x\pi}{L_x}x\right)\sin^2\left(\frac{n_y\pi}{L_y}y\right).$$

With $L_x = L_y = L = 150\text{ pm}$ and $\Delta x = \Delta y = 5.00\text{ pm}$, the probability of detecting an electron in $(n_x, n_y) = (1,3)$ state by placing a probe at $(0.200L, 0.800L)$ is

$$p = \frac{4(\Delta x \Delta y)}{L_x L_y} \sin^2\left(\frac{n_x \pi}{L_x} x\right) \sin^2\left(\frac{n_y \pi}{L_y} y\right) = \frac{4(5.00 \text{ pm})^2}{(150 \text{ pm})^2} \sin^2\left(\frac{\pi}{L} \cdot 0.200L\right) \sin^2\left(\frac{3\pi}{L} 0.800L\right)$$

$$= 4\left(\frac{5.00 \text{ pm}}{150 \text{ pm}}\right)^2 \sin^2(0.200\pi) \sin^2(2.40\pi) = 1.4 \times 10^{-3}.$$

26. We are looking for the values of the ratio

$$\frac{E_{nx,ny}}{h^2/8mL^2} = L^2\left(\frac{n_x^2}{L_x^2} + \frac{n_y^2}{L_y^2}\right) = \left(n_x^2 + \frac{1}{4} n_y^2\right)$$

and the corresponding differences.

(a) For $n_x = n_y = 1$, the ratio becomes $1 + \frac{1}{4} = 1.25.$

(b) For $n_x = 1$ and $n_y = 2$, the ratio becomes $1 + \frac{1}{4}(4) = 2.00.$ One can check (by computing other (n_x, n_y) values) that this is the next to lowest energy in the system.

(c) The lowest set of states that are degenerate are (n_x, n_y) = (1, 4) and (2, 2). Both of these states have that ratio equal to $1 + \frac{1}{4}(16) = 5.00.$

(d) For $n_x = 1$ and $n_y = 3$, the ratio becomes $1 + \frac{1}{4}(9) = 3.25.$ One can check (by computing other (n_x, n_y) values) that this is the lowest energy greater than that computed in part (b). The next higher energy comes from (n_x, n_y) = (2, 1) for which the ratio is $4 + \frac{1}{4}(1) = 4.25.$ The difference between these two values is 4.25 – 3.25 = 1.00.

27. **THINK** The energy levels of an electron trapped in a regular corral with widths L_x and L_y are given by Eq. 39-20:

$$E_{n_x,n_y} = \frac{h^2}{8m}\left[\frac{n_x^2}{L_x^2} + \frac{n_y^2}{L_y^2}\right].$$

EXPRESS With $L_x = L$ and $L_y = 2L$, we have

$$E_{n_x,n_y} = \frac{h^2}{8m}\left[\frac{n_x^2}{L_x^2} + \frac{n_y^2}{L_y^2}\right] = \frac{h^2}{8mL^2}\left[n_x^2 + \frac{n_y^2}{4}\right].$$

Thus, in units of $h^2/8mL^2$, the energy levels are given by $n_x^2 + n_y^2/4$. The lowest five levels are $E_{1,1}$ = 1.25, $E_{1,2}$ = 2.00, $E_{1,3}$ = 3.25, $E_{2,1}$ = 4.25, and $E_{2,2} = E_{1,4}$ = 5.00. It is clear that there are no other possible values for the energy less than 5.

The frequency of the light emitted or absorbed when the electron goes from an initial state i to a final state f is $f = (E_f - E_i)/h$, and in units of $h/8mL^2$ is simply the difference in the values of $n_x^2 + n_y^2/4$ for the two states. The possible frequencies are as follows:

$$
\begin{aligned}
&0.75(1,2 \to 1,1), 2.00(1,3 \to 1,1), 3.00(2,1 \to 1,1),\\
&3.75(2,2 \to 1,1), 1.25(1,3 \to 1,2), 2.25(2,1 \to 1,2), 3.00(2,2 \to 1,2), 1.00(2,1 \to 1,3),\\
&1.75(2,2 \to 1,3), 0.75(2,2 \to 2,1),
\end{aligned}
$$

all in units of $h/8mL^2$.

ANALYZE (a) From the above, we see that there are 8 different frequencies.

(b) The lowest frequency is, in units of $h/8mL^2$, 0.75 (2, 2$\to$2,1).

(c) The second lowest frequency is, in units of $h/8mL^2$, 1.00 (2, 1$\to$1,3).

(d) The third lowest frequency is, in units of $h/8mL^2$, 1.25 (1, 3$\to$1,2).

(e) The highest frequency is, in units of $h/8mL^2$, 3.75 (2, 2$\to$1,1).

(f) The second highest frequency is, in units of $h/8mL^2$, 3.00 (2, 2$\to$1,2) or (2, 1$\to$1,1).

(g) The third highest frequency is, in units of $h/8mL^2$, 2.25 (2, 1$\to$1,2).

LEARN In general, when the electron makes a transition from (n_x, n_y) to a higher level (n'_x, n'_y), the frequency of photon it emits or absorbs is given by

$$
\begin{aligned}
f &= \frac{\Delta E}{h} = \frac{E_{n'_x,n'_y} - E_{n_x,n_y}}{h} = \frac{h}{8mL^2}\left(n_x'^2 + \frac{n_y'^2}{4}\right) - \frac{h}{8mL^2}\left(n_x^2 + \frac{n_y^2}{4}\right)\\
&= \frac{h}{8mL^2}\left[\left(n_x'^2 - n_x^2\right) + \frac{1}{4}\left(n_y'^2 - n_y^2\right)\right].
\end{aligned}
$$

28. We are looking for the values of the ratio

$$
\frac{E_{n_x,n_y,n_z}}{h^2/8mL^2} = L^2\left(\frac{n_x^2}{L_x^2} + \frac{n_y^2}{L_y^2} + \frac{n_z^2}{L_z^2}\right) = \left(n_x^2 + n_y^2 + n_z^2\right)
$$

and the corresponding differences.

(a) For $n_x = n_y = n_z = 1$, the ratio becomes 1 + 1 + 1 = 3.00.

(b) For $n_x = n_y = 2$ and $n_z = 1$, the ratio becomes 4 + 4 + 1 = 9.00. One can check (by computing other (n_x, n_y, n_z) values) that this is the third lowest energy in the system. One can also check that this same ratio is obtained for (n_x, n_y, n_z) = (2, 1, 2) and (1, 2, 2).

(c) For $n_x = n_y = 1$ and $n_z = 3$, the ratio becomes 1 + 1 + 9 = 11.00. One can check (by computing other (n_x, n_y, n_z) values) that this is three "steps" up from the lowest energy in the system. One can also check that this same ratio is obtained for (n_x, n_y, n_z) = (1, 3, 1) and (3, 1, 1). If we take the difference between this and the result of part (b), we obtain 11.0 – 9.00 = 2.00.

(d) For $n_x = n_y = 1$ and $n_z = 2$, the ratio becomes 1 + 1 + 4 = 6.00. One can check (by computing other (n_x, n_y, n_z) values) that this is the next to the lowest energy in the system. One can also check that this same ratio is obtained for (n_x, n_y, n_z) = (2, 1, 1) and (1, 2, 1). Thus, three states (three arrangements of (n_x, n_y, n_z) values) have this energy.

(e) For $n_x = 1$, $n_y = 2$ and $n_z = 3$, the ratio becomes 1 + 4 + 9 = 14.0. One can check (by computing other (n_x, n_y, n_z) values) that this is five "steps" up from the lowest energy in the system. One can also check that this same ratio is obtained for (n_x, n_y, n_z) = (1, 3, 2), (2, 3, 1), (2, 1, 3), (3, 1, 2) and (3, 2, 1). Thus, six states (six arrangements of (n_x, n_y, n_z) values) have this energy.

29. The ratios computed in Problem 39-28 can be related to the frequencies emitted using $f = \Delta E/h$, where each level E is equal to one of those ratios multiplied by $h^2/8mL^2$. This effectively involves no more than a cancellation of one of the factors of h. Thus, for a transition from the second excited state (see part (b) of Problem 39-28) to the ground state (treated in part (a) of that problem), we find

$$f = (9.00 - 3.00)\left(\frac{h}{8mL^2}\right) = (6.00)\left(\frac{h}{8mL^2}\right).$$

In the following, we omit the $h/8mL^2$ factors. For a transition between the fourth excited state and the ground state, we have f = 12.00 – 3.00 = 9.00. For a transition between the third excited state and the ground state, we have f = 11.00 – 3.00 = 8.00. For a transition between the third excited state and the first excited state, we have f = 11.00 – 6.00 = 5.00. For a transition between the fourth excited state and the third excited state, we have f = 12.00 – 11.00 = 1.00. For a transition between the third excited state and the second excited state, we have f = 11.00 – 9.00 = 2.00. For a transition between the second excited state and the first excited state, we have f = 9.00 – 6.00 = 3.00, which also results from some other transitions.

(a) From the above, we see that there are 7 frequencies.

(b) The lowest frequency is, in units of $h/8mL^2$, 1.00.

(c) The second lowest frequency is, in units of $h/8mL^2$, 2.00.

(d) The third lowest frequency is, in units of $h/8mL^2$, 3.00.

(e) The highest frequency is, in units of $h/8mL^2$, 9.00.

(f) The second highest frequency is, in units of $h/8mL^2$, 8.00.

(g) The third highest frequency is, in units of $h/8mL^2$, 6.00.

30. In analogy to Eq. 39-10, the normalized wave function in two dimensions can be written as

$$\psi_{n_x,n_y}(x,y) = \psi_{n_x}(x)\psi_{n_y}(y) = \sqrt{\frac{2}{L_x}}\sin\left(\frac{n_x\pi}{L_x}x\right)\cdot\sqrt{\frac{2}{L_y}}\sin\left(\frac{n_y\pi}{L_y}y\right)$$
$$= \sqrt{\frac{4}{L_xL_y}}\sin\left(\frac{n_x\pi}{L_x}x\right)\sin\left(\frac{n_y\pi}{L_y}y\right).$$

The probability of detection by a probe of dimension $\Delta x\Delta y$ placed at (x, y) is

$$p(x,y) = \left|\psi_{n_x,n_y}(x,y)\right|^2 \Delta x\Delta y = \frac{4(\Delta x\Delta y)}{L_xL_y}\sin^2\left(\frac{n_x\pi}{L_x}x\right)\sin^2\left(\frac{n_y\pi}{L_y}y\right).$$

A detection probability of 0.0450 of a ground-state electron ($n_x = n_y = 1$) by a probe of area $\Delta x\Delta y = 400\text{ pm}^2$ placed at $(x,y) = (L/8, L/8)$ implies

$$0.0450 = \frac{4(400\text{ pm}^2)}{L^2}\sin^2\left(\frac{\pi}{L}\cdot\frac{L}{8}\right)\sin^2\left(\frac{\pi}{L}\cdot\frac{L}{8}\right) = 4\left(\frac{20\text{ pm}}{L}\right)^2\sin^4\left(\frac{\pi}{8}\right).$$

Solving for *L*, we get $L = 27.6\text{ pm}$.

31. **THINK** The Lyman series is associated with transitions to or from the $n = 1$ level of the hydrogen atom, while the Balmer series is for transitions to or from the $n = 2$ level.

EXPRESS The energy *E* of the photon emitted when a hydrogen atom jumps from a state with principal quantum number n' to a state with principal quantum number $n < n'$ is given by

$$E = A\left(\frac{1}{n^2} - \frac{1}{n'^2}\right)$$

where $A = 13.6$ eV. The frequency *f* of the electromagnetic wave is given by $f = E/h$ and the wavelength is given by $\lambda = c/f$. Thus,

$$\frac{1}{\lambda}=\frac{f}{c}=\frac{E}{hc}=\frac{A}{hc}\left(\frac{1}{n^2}-\frac{1}{n'^2}\right).$$

ANALYZE The shortest wavelength occurs at the series limit, for which $n' = \infty$. For the Balmer series, $n=2$ and the shortest wavelength is $\lambda_B = 4hc/A$. For the Lyman series, $n=1$ and the shortest wavelength is $\lambda_L = hc/A$. The ratio is $\lambda_B/\lambda_L = 4.0$.

LEARN The energy of the photon emitted associated with the transition of an electron from $n'=\infty \to n=2$ (to become bound) is

$$E_{\infty\to 2}=\frac{13.6\text{ eV}}{2^2}=3.4\text{ eV}.$$

Similarly, the energy associated with the transition of an electron from $n'=\infty \to n=1$ (to become bound) is

$$E_{1\to\infty}=\frac{13.6\text{ eV}}{1^2}=13.6\text{ eV}.$$

32. The difference between the energy absorbed and the energy emitted is

$$E_{\text{photon absorbed}}-E_{\text{photon emitted}}=\frac{hc}{\lambda_{\text{absorbed}}}-\frac{hc}{\lambda_{\text{emitted}}}.$$

Thus, using $hc = 1240$ eV · nm, the net energy absorbed is

$$hc\Delta\left(\frac{1}{\lambda}\right)=(1240\,\text{eV}\cdot\text{nm})\left(\frac{1}{375\,\text{nm}}-\frac{1}{580\,\text{nm}}\right)=1.17\,\text{eV}.$$

33. (a) Since energy is conserved, the energy E of the photon is given by $E = E_i - E_f$, where E_i is the initial energy of the hydrogen atom and E_f is the final energy. The electron energy is given by $(-13.6\text{ eV})/n^2$, where n is the principal quantum number. Thus,

$$E=E_3-E_1=\frac{-13.6\,\text{eV}}{(3)^2}-\frac{-13.6\,\text{eV}}{(1)^2}=12.1\,\text{eV}.$$

(b) The photon momentum is given by

$$p=\frac{E}{c}=\frac{(12.1\,\text{eV})(1.60\times10^{-19}\text{ J/eV})}{3.00\times10^{8}\text{ m/s}}=6.45\times10^{-27}\text{ kg}\cdot\text{m/s}.$$

(c) Using $hc = 1240$ eV · nm, the wavelength is $\lambda=\dfrac{hc}{E}=\dfrac{1240\,\text{eV}\cdot\text{nm}}{12.1\,\text{eV}}=102\,\text{nm}.$

34. (a) We use Eq. 39-44. At r = 0, $P(r) \propto r^2 = 0$.

(b) At $r = a$, $P(r)=\frac{4}{a^3}a^2e^{-2a/a}=\frac{4e^{-2}}{a}=\frac{4e^{-2}}{5.29\times10^{-2}\text{ nm}}=10.2\,\text{nm}^{-1}$.

(c) At $r = 2a$, $P(r)=\frac{4}{a^3}(2a)^2e^{-4a/a}=\frac{16e^{-4}}{a}=\frac{16e^{-4}}{5.29\times10^{-2}\text{ nm}}=5.54\,\text{nm}^{-1}$.

35. (a) We use Eq. 39-39. At $r = a$,

$$\psi^2(r)=\left(\frac{1}{\sqrt{\pi}a^{3/2}}e^{-a/a}\right)^2=\frac{1}{\pi a^3}e^{-2}=\frac{1}{\pi\left(5.29\times10^{-2}\text{ nm}\right)^3}e^{-2}=291\,\text{nm}^{-3}.$$

(b) We use Eq. 39-44. At $r = a$,

$$P(r)=\frac{4}{a^3}a^2e^{-2a/a}=\frac{4e^{-2}}{a}=\frac{4e^{-2}}{5.29\times10^{-2}\text{ nm}}=10.2\,\text{nm}^{-1}.$$

36. (a) The energy level corresponding to the probability density distribution shown in Fig. 39-21 is the n = 2 level. Its energy is given by

$$E_2=-\frac{13.6\,\text{eV}}{2^2}=-3.4\,\text{eV}.$$

(b) As the electron is removed from the hydrogen atom the final energy of the proton-electron system is zero. Therefore, one needs to supply at least 3.4 eV of energy to the system in order to bring its energy up from E_2 = – 3.4 eV to zero. (If more energy is supplied, then the electron will retain some kinetic energy after it is removed from the atom.)

37. **THINK** The energy of the hydrogen atom is quantized.

EXPRESS If kinetic energy is not conserved, some of the neutron's initial kinetic energy could be used to excite the hydrogen atom. The least energy that the hydrogen atom can accept is the difference between the first excited state (n = 2) and the ground state (n = 1). Since the energy of a state with principal quantum number n is –(13.6 eV)/n^2, the smallest excitation energy is

$$\Delta E=E_2-E_1=\frac{-13.6\,\text{eV}}{(2)^2}-\frac{-13.6\,\text{eV}}{(1)^2}=10.2\,\text{eV}\ .$$

ANALYZE The neutron, with a kinetic energy of 6.0 eV, does not have sufficient kinetic energy to excite the hydrogen atom, so the hydrogen atom is left in its ground state and all the initial kinetic energy of the neutron ends up as the final kinetic energies of the neutron and atom. The collision must be elastic.

LEARN The minimum kinetic energy the neutron must have in order to excite the hydrogen atom is 10.2 eV.

38. From Eq. 39-6, $\Delta E = hf = \left(4.14\times10^{-15}\text{ eV}\cdot\text{s}\right)\left(6.2\times10^{14}\text{ Hz}\right) = 2.6\,\text{eV}$.

39. **THINK** The radial probability function for the ground state of hydrogen is

$$P(r) = (4r^2/a^3)e^{-2r/a},$$

where a is the Bohr radius.

EXPRESS We want to evaluate the integral $\int_0^\infty P(r)\,dr$. Equation 15 in the integral table of Appendix E is an integral of this form:

$$\int_0^\infty x^n e^{-ax}\,dx = \frac{n!}{a^{n+1}}\,.$$

ANALYZE We set n = 2 and replace a in the given formula with 2/a and x with r. Then

$$\int_0^\infty P(r)dr = \frac{4}{a^3}\int_0^\infty r^2 e^{-2r/a}dr = \frac{4}{a^3}\frac{2}{(2/a)^3} = 1.$$

LEARN The integral over the radial probability function $P(r)$ must be equal to 1. This means that in a hydrogen atom, the electron must be somewhere in the space surrounding the nucleus.

40. (a) The calculation is shown in Sample Problem — "Light emission from a hydrogen atom." The difference in the values obtained in parts (a) and (b) of that Sample Problem is 122 nm – 91.4 nm ≈ 31 nm.

(b) We use Eq. 39-1. For the Lyman series,

$$\Delta f = \frac{2.998\times10^8\text{ m/s}}{91.4\times10^{-9}\text{ m}} - \frac{2.998\times10^8\text{ m/s}}{122\times10^{-9}\text{ m}} = 8.2\times10^{14}\text{ Hz}\,.$$

(c) Figure 39-18 shows that the width of the Balmer series is 656.3 nm – 364.6 nm ≈ 292 nm $\approx 0.29\ \mu\text{m}$.

(d) The series limit can be obtained from the $\infty \to 2$ transition:

$$\Delta f = \frac{2.998\times10^{8}\ \text{m/s}}{364.6\times10^{-9}\,\text{m}} - \frac{2.998\times10^{8}\ \text{m/s}}{656.3\times10^{-9}\,\text{m}} = 3.65\times10^{14}\,\text{Hz} \approx 3.7\times10^{14}\,\text{Hz}.$$

41. Since Δr is small, we may calculate the probability using $p = P(r)\,\Delta r$, where $P(r)$ is the radial probability density. The radial probability density for the ground state of hydrogen is given by Eq. 39-44:

$$P(r) = \left(\frac{4r^2}{a^3}\right)e^{-2r/a}$$

where a is the Bohr radius.

(a) Here, $r = 0.500a$ and $\Delta r = 0.010a$. Then,

$$P = \left(\frac{4r^2\Delta r}{a^3}\right)e^{-2r/a} = 4(0.500)^2(0.010)e^{-1} = 3.68\times10^{-3} \approx 3.7\times10^{-3}.$$

(b) We set $r = 1.00a$ and $\Delta r = 0.010a$. Then,

$$P = \left(\frac{4r^2\Delta r}{a^3}\right)e^{-2r/a} = 4(1.00)^2(0.010)e^{-2} = 5.41\times10^{-3} \approx 5.4\times10^{-3}.$$

42. Conservation of linear momentum of the atom-photon system requires that

$$p_{\text{recoil}} = p_{\text{photon}} \Rightarrow m_p v_{\text{recoil}} = \frac{hf}{c}$$

where we use Eq. 39-7 for the photon and use the classical momentum formula for the atom (since we expect its speed to be much less than c). Thus, from Eq. 39-6 and Table 37-3,

$$v_{\text{recoil}} = \frac{\Delta E}{m_p c} = \frac{E_4 - E_1}{\left(m_p c^2\right)/c} = \frac{\left(-13.6\text{eV}\right)\left(4^{-2} - 1^{-2}\right)}{\left(938\times10^{6}\,\text{eV}\right)/\left(2.998\times10^{8}\ \text{m/s}\right)} = 4.1\,\text{m/s}\,.$$

43. (a) and (b) Letting $a = 5.292 \times 10^{-11}$ m be the Bohr radius, the potential energy becomes

$$U = -\frac{e^2}{4\pi\varepsilon_0 a} = \frac{\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.602\times10^{-19}\ \text{C}\right)^2}{5.292\times10^{-11}\ \text{m}} = -4.36\times10^{-18}\ \text{J} = -27.2\,\text{eV}\,.$$

The kinetic energy is $K = E - U$ = (– 13.6 eV) – (– 27.2 eV) = 13.6 eV.

44. (a) Since E_2 = – 0.85 eV and E_1 = – 13.6 eV + 10.2 eV = – 3.4 eV, the photon energy is

$$E_{\text{photon}} = E_2 - E_1 = -0.85\ \text{eV} - (-3.4\ \text{eV}) = 2.6\ \text{eV}.$$

(b) From

$$E_2 - E_1 = (-13.6\,\text{eV})\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right) = 2.6\ \text{eV}$$

we obtain

$$\frac{1}{n_2^2} - \frac{1}{n_1^2} = \frac{2.6\ \text{eV}}{13.6\,\text{eV}} \approx -\frac{3}{16} = \frac{1}{4^2} - \frac{1}{2^2}.$$

Thus, $n_2 = 4$ and $n_1 = 2$. So the transition is from the $n = 4$ state to the $n = 2$ state. One can easily verify this by inspecting the energy level diagram of Fig. 39-18. Thus, the higher quantum number is $n_2 = 4$.

(c) The lower quantum number is $n_1 = 2$.

45. **THINK** The probability density is given by $|\psi_{n\ell m_\ell}(r,\theta)|^2$, where $\psi_{n\ell m_\ell}(r,\theta)$ is the wave function.

EXPRESS To calculate $|\psi_{n\ell m_\ell}|^2 = \psi^*_{n\ell m_\ell}\,\psi_{n\ell m_\ell}$, we multiply the wave function by its complex conjugate. If the function is real, then $\psi^*_{n\ell m_\ell} = \psi_{n\ell m_\ell}$. Note that $e^{+i\phi}$ and $e^{-i\phi}$ are complex conjugates of each other, and $e^{i\phi}\,e^{-i\phi} = e^0 = 1$.

ANALYZE (a) ψ_{210} is real. Squaring it gives the probability density:

$$|\psi_{210}|^2 = \frac{r^2}{32\pi a^5} e^{-r/a} \cos^2\theta.$$

(b) Similarly,

$$|\psi_{21+1}|^2 = \frac{r^2}{64\pi a^5} e^{-r/a} \sin^2\theta$$

and

$$|\psi_{21-1}|^2 = \frac{r^2}{64\pi a^5} e^{-r/a} \sin^2\theta.$$

The last two functions lead to the same probability density.

(c) For $m_\ell = 0$, the probability density $|\psi_{210}|^2$ decreases with radial distance from the nucleus. With the $\cos^2\theta$ factor, $|\psi_{210}|^2$ is greatest along the z axis where $\theta = 0$. This is consistent with the dot plot of Fig. 39-23(a).

Similarly, for $m_\ell = \pm 1$, the probability density $|\psi_{21\pm1}|^2$ decreases with radial distance from the nucleus. With the $\sin^2\theta$ factor, $|\psi_{21\pm1}|^2$ is greatest in the *xy*-plane where θ = 90°. This is consistent with the dot plot of Fig. 39-23(b).

(d) The total probability density for the three states is the sum:

$$|\psi_{210}|^2 + |\psi_{21+1}|^2 + |\psi_{21-1}|^2 = \frac{r^2}{32\pi a^5} e^{-r/a}\left[\cos^2\theta + \frac{1}{2}\sin^2\theta + \frac{1}{2}\sin^2\theta\right] = \frac{r^2}{32\pi a^5} e^{-r/a}.$$

The trigonometric identity $\cos^2\theta + \sin^2\theta = 1$ is used. We note that the total probability density does not depend on θ or ϕ; it is spherically symmetric.

LEARN The wave functions discussed above are for the hydrogen states with n = 2 and $\ell = 1$. Since the angular momentum is nonzero, the probability densities are not spherically symmetric, but depend on both r and θ.

46. From Sample Problem — " Probability of detection of the electron in a hydrogen atom," we know that the probability of finding the electron in the ground state of the hydrogen atom inside a sphere of radius r is given by

$$p(r) = 1 - e^{-2x}\left(1 + 2x + 2x^2\right)$$

where $x = r/a$. Thus the probability of finding the electron between the two shells indicated in this problem is given by

$$p(a < r < 2a) = p(2a) - p(a) = \left[1 - e^{-2x}\left(1 + 2x + 2x^2\right)\right]_{x=2} - \left[1 - e^{-2x}\left(1 + 2x + 2x^2\right)\right]_{x=1}$$
$$= 0.439.$$

47. As illustrated in Fig. 39-24, the quantum number n in question satisfies $r = n^2 a$. Letting r = 1.0 mm, we solve for n:

$$n = \sqrt{\frac{r}{a}} = \sqrt{\frac{1.0 \times 10^{-3}\ \text{m}}{5.29 \times 10^{-11}\ \text{m}}} \approx 4.3 \times 10^3.$$

48. Using Eq. 39-6 and hc = 1240 eV · nm, we find

$$\Delta E = E_{\text{photon}} = \frac{hc}{\lambda} = \frac{1240\,\text{eV}\cdot\text{nm}}{121.6\,\text{nm}} = 10.2\,\text{eV}\ .$$

Therefore, n_{low} = 1, but what precisely is n_{high}?

$$E_{\text{high}} = E_{\text{low}} + \Delta E \quad \Rightarrow \quad -\frac{13.6\text{eV}}{n^2} = -\frac{13.6\text{eV}}{1^2} + 10.2\text{eV}$$

which yields n = 2 (this is confirmed by the calculation found from Sample Problem — "Light emission from a hydrogen atom). Thus, the transition is from the n = 2 to the n = 1 state.

(a) The higher quantum number is n = 2.

(b) The lower quantum number is n = 1.

(c) Referring to Fig. 39-18, we see that this must be one of the Lyman series transitions.

49. (a) We take the electrostatic potential energy to be zero when the electron and proton are far removed from each other. Then, the final energy of the atom is zero and the work done in pulling it apart is $W = -E_i$, where E_i is the energy of the initial state. The energy of the initial state is given by $E_i = (-13.6\text{ eV})/n^2$, where n is the principal quantum number of the state. For the ground state, n = 1 and W = 13.6 eV.

(b) For the state with n = 2, $W = (13.6\text{ eV})/(2)^2 = 3.40$ eV.

50. Using Eq. 39-6 and hc = 1240 eV · nm, we find

$$\Delta E = E_{\text{photon}} = \frac{hc}{\lambda} = \frac{1240\text{ eV}\cdot\text{nm}}{106.6\text{ nm}} = 12.09\text{ eV}.$$

Therefore, n_{low} = 1, but what precisely is n_{high}?

$$E_{\text{high}} = E_{\text{low}} + \Delta E \quad \Rightarrow \quad -\frac{13.6\text{ eV}}{n^2} = -\frac{13.6\text{ eV}}{1^2} + 12.09\text{ eV}$$

which yields n = 3. Thus, the transition is from the n = 3 to the n = 1 state.

(a) The higher quantum number is n = 3.

(b) The lower quantum number is n = 1.

(c) Referring to Fig. 39-18, we see that this must be one of the Lyman series transitions.

51. According to Sample Problem — " Probability of detection of the electron in a hydrogen atom," the probability the electron in the ground state of a hydrogen atom can be found inside a sphere of radius r is given by

$$p(r) = 1 - e^{-2x}\left(1 + 2x + 2x^2\right)$$

where $x = r/a$ and a is the Bohr radius. We want $r = a$, so x = 1 and

$$p(a) = 1 - e^{-2}(1+2+2) = 1 - 5e^{-2} = 0.323.$$

The probability that the electron can be found outside this sphere is 1 – 0.323 = 0.677. It can be found outside about 68% of the time.

52. (a) $\Delta E = -(13.6\text{ eV})(4^{-2} - 1^{-2}) = 12.8$ eV.

(b) There are 6 possible energies associated with the transitions 4 $\to$ 3, 4 $\to$ 2, 4 $\to$ 1, 3 $\to$ 2, 3 $\to$ 1 and 2 $\to$1.

(c) The greatest energy is $E_{4\to 1} = 12.8$ eV.

(d) The second greatest energy is $E_{3\to 1} = -(13.6\text{eV})\left(3^{-2} - 1^{-2}\right) = 12.1$ eV .

(e) The third greatest energy is $E_{2\to 1} = -(13.6\text{eV})\left(2^{-2} - 1^{-2}\right) = 10.2$ eV .

(f) The smallest energy is $E_{4\to 3} = -(13.6\text{eV})\left(4^{-2} - 3^{-2}\right) = 0.661$ eV .

(g) The second smallest energy is $E_{3\to 2} = -(13.6\text{eV})\left(3^{-2} - 2^{-2}\right) = 1.89$ eV .

(h) The third smallest energy is $E_{4\to 2} = -(13.6\text{eV})\left(4^{-2} - 2^{-2}\right) = 2.55$ eV.

53. **THINK** The ground state of the hydrogen atom corresponds to n = 1, $\ell = 0$, and $m_\ell = 0$.

EXPRESS The proposed wave function is

$$\psi = \frac{1}{\sqrt{\pi} a^{3/2}} e^{-r/a}$$

where a is the Bohr radius. Substituting this into the right side of Schrödinger's equation, our goal is to show that the result is zero.

ANALYZE The derivative is

$$\frac{d\psi}{dr} = -\frac{1}{\sqrt{\pi} a^{5/2}} e^{-r/a}$$

so

$$r^2 \frac{d\psi}{dr} = -\frac{r^2}{\sqrt{\pi} a^{5/2}} e^{-r/a}$$

and

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2 \frac{d\psi}{dr}\right) = \frac{1}{\sqrt{\pi} a^{5/2}}\left[-\frac{2}{r} + \frac{1}{a}\right] e^{-r/a} = \frac{1}{a}\left[-\frac{2}{r} + \frac{1}{a}\right]\psi.$$

The energy of the ground state is given by $E = -me^4/8\varepsilon_0^2 h^2$ and the Bohr radius is given by $a = h^2\varepsilon_0/\pi me^2$, so $E = -e^2/8\pi\varepsilon_0 a$. The potential energy is given by

$$U = -e^2/4\pi\varepsilon_0 r\,,$$

so

$$\frac{8\pi^2 m}{h^2}\;E-U\;\psi = \frac{8\pi^2 m}{h^2}\left[-\frac{e^2}{8\pi\varepsilon_0 a}+\frac{e^2}{4\pi\varepsilon_0 r}\right]\psi = \frac{8\pi^2 m}{h^2}\frac{e^2}{8\pi\varepsilon_0}\left[-\frac{1}{a}+\frac{2}{r}\right]\psi$$

$$= \frac{\pi me^2}{h^2\varepsilon_0}\left[-\frac{1}{a}+\frac{2}{r}\right]\psi = \frac{1}{a}\left[-\frac{1}{a}+\frac{2}{r}\right]\psi.$$

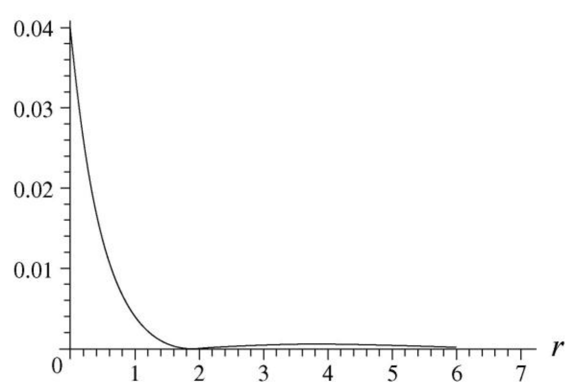
The two terms in Schrödinger's equation cancel, and the proposed function ψ satisfies that equation.

LEARN The radial probability density of the ground state of hydrogen atom is given by Eq. 39-44:

$$P(r) = |\psi|^2\,(4\pi r^2) = \frac{1}{\pi a^3}e^{-2r/a}(4\pi r^2) = \frac{4}{a^3}r^2 e^{-2r/a}\,.$$

A plot of $P(r)$ is shown in Fig. 39-20.

54. (a) The plot shown below for $|\psi_{200}(r)|^2$ is to be compared with the dot plot of Fig. 39-21. We note that the horizontal axis of our graph is labeled "r," but it is actually r/a (that is, it is in units of the parameter a). Now, in the plot below there is a high central peak between r = 0 and $r \sim 2a$, corresponding to the densely dotted region around the center of the dot plot of Fig. 39-21. Outside this peak is a region of near-zero values centered at $r = 2a$, where $\psi_{200} = 0$. This is represented in the dot plot by the empty ring surrounding the central peak. Further outside is a broader, flatter, low peak that reaches its maximum value at $r = 4a$. This corresponds to the outer ring with near-uniform dot density, which is lower than that of the central peak.



(b) The extrema of $\psi^2(r)$ for $0 < r < \infty$ may be found by squaring the given function, differentiating with respect to r, and setting the result equal to zero:

$$-\frac{1}{32}\frac{(r-2a)(r-4a)}{a^6\pi}e^{-r/a} = 0$$

which has roots at $r = 2a$ and $r = 4a$. We can verify directly from the plot above that $r = 4a$ is indeed a local maximum of $\psi^2_{200}(r)$. As discussed in part (a), the other root ($r = 2a$) is a local minimum.

(c) Using Eq. 39-43 and Eq. 39-41, the radial probability is

$$P_{200}(r) = 4\pi r^2\psi^2_{200}(r) = \frac{r^2}{8a^3}\left(2-\frac{r}{a}\right)^2 e^{-r/a}.$$

(d) Let $x = r/a$. Then

$$\int_0^\infty P_{200}(r)dr = \int_0^\infty \frac{r^2}{8a^3}\left(2-\frac{r}{a}\right)^2 e^{-r/a}dr = \frac{1}{8}\int_0^\infty x^2(2-x)^2e^{-x}dx = \int_0^\infty (x^4-4x^3+4x^2)e^{-x}dx$$
$$= \frac{1}{8}[4!-4(3!)+4(2!)] = 1$$

where we have used the integral formula $\int_0^\infty x^n e^{-x}dx = n!$.

55. The radial probability function for the ground state of hydrogen is

$$P(r) = (4r^2/a^3)e^{-2r/a},$$

where a is the Bohr radius. (See Eq. 39-44.) The integral table of Appendix E may be used to evaluate the integral $r_{\text{avg}} = \int_0^\infty rP(r)\,dr$. Setting $n = 3$ and replacing a in the given formula with $2/a$ (and x with r), we obtain

$$r_{\text{avg}} = \int_0^\infty rP(r)\,dr = \frac{4}{a^3}\int_0^\infty r^3e^{-2r/a}dr = \frac{4}{a^3}\frac{6}{(2/a)^4} = 1.5a\,.$$

56. (a) The allowed energy values are given by $E_n = n^2h^2/8mL^2$. The difference in energy between the state n and the state $n + 1$ is

$$\Delta E_{\text{adj}} = E_{n+1} - E_n = \left[(n+1)^2 - n^2\right]\frac{h^2}{8mL^2} = \frac{(2n+1)h^2}{8mL^2}$$

and

$$\frac{\Delta E_{\text{adj}}}{E} = \left[\frac{(2n+1)h^2}{8mL^2}\right]\left(\frac{8mL^2}{n^2h^2}\right) = \frac{2n+1}{n^2}.$$

As n becomes large, $2n+1 \to 2n$ and $(2n+1)/n^2 \to 2n/n^2 = 2/n$.

(b) No. As $n \to \infty$, ΔE_{adj} and E do not approach 0, but $\Delta E_{\text{adj}}/E$ does.

(c) No. See part (b).

(d) Yes. See part (b).

(e) $\Delta E_{\text{adj}}/E$ is a better measure than either ΔE_{adj} or E alone of the extent to which the quantum result is approximated by the classical result.

57. From Eq. 39-4,

$$E_{n+2} - E_n = \left(\frac{h^2}{8mL^2}\right)(n+2)^2 - \left(\frac{h^2}{8mL^2}\right)n^2 = \left(\frac{h^2}{2mL^2}\right)(n+1).$$

58. (a) and (b) In the region $0 < x < L$, $U_0 = 0$, so Schrödinger's equation for the region is

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}E\psi = 0$$

where $E > 0$. If $\psi^2(x) = B\sin^2 kx$, then $\psi(x) = B'\sin kx$, where B' is another constant satisfying $B'^2 = B$. Thus,

$$\frac{d^2\psi}{dx^2} = -k^2 B' \sin kx = -k^2\psi(x)$$

and

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}E\psi = -k^2\psi + \frac{8\pi^2 m}{h^2}E\psi.$$

This is zero provided that $k^2 = \dfrac{8\pi^2 mE}{h^2}$. The quantity on the right-hand side is positive, so k is real and the proposed function satisfies Schrödinger's equation. In this case, there exists no physical restriction as to the sign of k. It can assume either positive or negative values. Thus, $k = \pm\dfrac{2\pi}{h}\sqrt{2mE}$.

59. **THINK** For a finite well, the electron matter wave can penetrate the walls of the well. Thus, the wave function outside the well is not zero, but decreases exponentially with distance.

EXPRESS Schrödinger's equation for the region $x > L$ is

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}\ E - U_0\ \psi = 0,$$

where $E - U_0 < 0$. If $\psi^2(x) = Ce^{-2kx}$, then $\psi(x) = \sqrt{C}\, e^{-kx}$.

ANALYZE (a) and (b) Thus,

$$\frac{d^2\psi}{dx^2} = 4k^2\sqrt{C}e^{-kx} = 4k^2\psi$$

and

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}\ E - U_0\ \psi = k^2\psi + \frac{8\pi^2 m}{h^2}\ E - U_0\ \psi.$$

This is zero provided that $k^2 = \frac{8\pi^2 m}{h^2}\ U_0 - E$. Choosing the positive root, we have

$$k = \frac{2\pi}{h}\sqrt{2m\left(U_0 - E\right)}.$$

LEARN Note that the quantity $U_0 - E$ is positive, so k is real and the proposed function satisfies Schrödinger's equation. If k is negative, however, the proposed function would be physically unrealistic. It would increase exponentially with x. Since the integral of the probability density over the entire x axis must be finite, ψ diverging as $x \to \infty$ would be unacceptable.

60. We can use the mc^2 value for an electron from Table 37-3 (511×10^3 eV) and hc = 1240 eV · nm by writing Eq. 39-4 as

$$E_n = \frac{n^2 h^2}{8mL^2} = \frac{n^2 (hc)^2}{8(mc^2)L^2}.$$

(a) With $L = 3.0 \times 10^9$ nm, the energy difference is

$$E_2 - E_1 = \frac{1240^2}{8(511\times10^3)(3.0\times10^9)^2}(2^2 - 1^2) = 1.3\times10^{-19}\text{ eV}.$$

(b) Since $(n + 1)^2 - n^2 = 2n + 1$, we have

$$\Delta E = E_{n+1} - E_n = \frac{h^2}{8mL^2}(2n+1) = \frac{(hc)^2}{8(mc^2)L^2}(2n+1).$$

Setting this equal to 1.0 eV, we solve for n:

$$n=\frac{4\left(mc^2\right)L^2\Delta E}{\left(hc\right)^2}-\frac{1}{2}=\frac{4\left(511\times10^3\,\text{eV}\right)\left(3.0\times10^9\,\text{nm}\right)^2\left(1.0\text{eV}\right)}{\left(1240\text{eV}\cdot\text{nm}\right)^2}-\frac{1}{2}\approx1.2\times10^{19}.$$

(c) At this value of n, the energy is

$$E_n=\frac{1240^2}{8\left(511\times10^3\right)\left(3.0\times10^9\right)^2}\left(6\times10^{18}\right)^2\approx6\times10^{18}\text{ eV}.$$

Thus,

$$\frac{E_n}{mc^2}=\frac{6\times10^{18}\,\text{eV}}{511\times10^3\,\text{eV}}=1.2\times10^{13}.$$

(d) Since $E_n/mc^2\gg1$, the energy is indeed in the relativistic range.

61. (a) We recall that a derivative with respect to a dimensional quantity carries the (reciprocal) units of that quantity. Thus, the first term in Eq. 39-18 has dimensions of ψ multiplied by dimensions of x^{-2}. The second term contains no derivatives, does contain ψ, and involves several other factors that turn out to have dimensions of x^{-2}:

$$\frac{8\pi^2m}{h^2}\left[E-U\left(x\right)\right]\;\Rightarrow\;\frac{\text{kg}}{\left(\text{J}\cdot\text{s}\right)^2}\left[\text{J}\right]$$

assuming SI units. Recalling from Eq. 7-9 that J = kg·m^2/s^2, then we see the above is indeed in units of m^{-2} (which means dimensions of x^{-2}).

(b) In one-dimensional quantum physics, the wave function has units of m$^{-½}$, as shown in Eq. 39-17. Thus, since each term in Eq. 39-18 has units of ψ multiplied by units of x^{-2}, then those units are m$^{-1/2}$· m^{-2} = m$^{-2.5}$.

62. (a) The "home-base" energy level for the Balmer series is n = 2. Thus the transition with the least energetic photon is the one from the n = 3 level to the n = 2 level. The energy difference for this transition is

$$\Delta E=E_3-E_2=-\left(13.6\,\text{eV}\right)\left(\frac{1}{3^2}-\frac{1}{2^2}\right)=1.889\,\text{eV}\ .$$

Using hc = 1240 eV · nm, the corresponding wavelength is

$$\lambda=\frac{hc}{\Delta E}=\frac{1240\,\text{eV}\cdot\text{nm}}{1.889\,\text{eV}}=658\,\text{nm}\ .$$

(b) For the series limit, the energy difference is

$$\Delta E = E_\infty - E_2 = -(13.6\,\text{eV})\left(\frac{1}{\infty^2} - \frac{1}{2^2}\right) = 3.40\,\text{eV} .$$

The corresponding wavelength is then $\lambda = \dfrac{hc}{\Delta E} = \dfrac{1240\,\text{eV}\cdot\text{nm}}{3.40\,\text{eV}} = 366\,\text{nm}$.

63. (a) The allowed values of ℓ for a given n are 0, 1, 2, ..., n – 1. Thus there are n different values of ℓ .

(b) The allowed values of m_ℓ for a given ℓ are $-\ell$, $-\ell$ + 1, ..., ℓ . Thus there are 2 ℓ + 1 different values of m_ℓ .

(c) According to part (a) above, for a given n there are n different values of ℓ . Also, each of these ℓ 's can have 2 ℓ + 1 different values of m_ℓ [see part (b) above]. Thus, the total number of m_ℓ 's is

$$\sum_{\ell=0}^{n-1}(2\ell+1) = n^2.$$

64. For n = 1

$$E_1 = -\frac{m_e e^4}{8\varepsilon_0^2 h^2} = -\frac{\left(9.11\times10^{-31}\,\text{kg}\right)\left(1.6\times10^{-19}\,\text{C}\right)^4}{8\left(8.85\times10^{-12}\,\text{F/m}\right)^2\left(6.63\times10^{-34}\,\text{J}\cdot\text{s}\right)^2\left(1.60\times10^{-19}\,\text{J/eV}\right)} = -13.6\,\text{eV} .$$

65. (a) The angular momentum of the diatomic gas is

$$L = I\omega = 2\times m(d/2)^2\omega = \frac{1}{2}md^2\omega .$$

If its angular momentum is quantized, i.e., restricted to $L = n\hbar,\ n = 1, 2, \ldots$ then

$$\frac{1}{2}md^2\omega = n\hbar = \frac{nh}{2\pi} \quad \Rightarrow \quad \omega = \frac{nh}{\pi md^2}$$

(b) The quantized rotational energies are

$$E_n = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{md^2}{2}\right)\left(\frac{nh}{\pi md^2}\right)^2 = \frac{n^2h^2}{4\pi^2 md^2}$$

66. The expression for the probability of detecting an electron in the ground state of hydrogen atom inside a sphere of radius r is given in Sample Problem 39.07:

$$p(x) = 1 - e^{-2x}(1 + 2x + 2x^2)$$

where $x = r/a_0$, with $a_0 = 5.292 \times 10^{-11}$m. Given that $r = 1.1 \times 10^{-15}$m,

$$x = (1.1 \times 10^{-15}\,\text{m})/(5.292 \times 10^{-11}\,\text{m}) = 2.079 \times 10^{-5}.$$

For small *x*, *p*(*x*) can be simplified as

$$p(x) = 1 - e^{-2x}\left(1 + 2x + 2x^2\right) \approx 1 - \left(1 - 2x + 2x^2 - \frac{4}{3}x^3 + \cdots\right)\left(1 + 2x + 2x^2\right) = \frac{4}{3}x^3$$
$$= \frac{4}{3}\left(2.079 \times 10^{-5}\right)^3 = 1.2 \times 10^{-14}.$$

67. (a) For a particle of mass *m* trapped inside a container of length *L*, he allowed energy values are given by $E_n = n^2h^2/8mL^2$. With an argon atom and $L = 0.20$ m, the energy difference between the lowest two levels is

$$\Delta E = E_2 - E_1 = \frac{h^2}{8mL^2}\left(2^2 - 1^2\right) = \frac{3h^2}{8mL^2} = \frac{3(6.626 \times 10^{-34}\,\text{J} \cdot \text{s})^2}{8(0.0399\,\text{kg}/6.02 \times 10^{23})(0.20\,\text{m})^2}$$
$$= 6.21 \times 10^{-41}\,\text{J} = 3.88 \times 10^{-22}\,\text{eV}.$$

(b) The thermal energy at $T = 300$ K is its average kinetic energy:

$$\bar{K} = \frac{3}{2}kT = (1.38 \times 10^{-23}\,\text{J/K})(300\,\text{K}) = 6.21 \times 10^{-21}\,\text{J} = 3.88 \times 10^{-2}\,\text{eV}.$$

Thus, the ratio is

$$\frac{\bar{K}}{\Delta E} = \frac{3.88 \times 10^{-2}\,\text{eV}}{3.9 \times 10^{-22}\,\text{eV}} = 10^{20}.$$

(c) The temperature at which $\bar{K} = \frac{3}{2}kT = \Delta E$ is

$$T = \frac{2(\Delta E)}{3k} = \frac{2(6.21 \times 10^{-41}\,\text{J})}{3(1.38 \times 10^{-23}\,\text{J/K})} = 3.0 \times 10^{-18}\,\text{K}.$$

68. The muon orbits the He^+ nucleus at a speed given by ($k = 1/4\pi\varepsilon_0$)

$$\frac{mv^2}{r} = \frac{Zke^2}{r^2} \quad \Rightarrow \quad v = \sqrt{\frac{Zke^2}{mr}}$$

With quantization condition $L = mvr = n\hbar$, the allowed values of the radius is

$$r_n = \frac{n^2\hbar^2}{Zke^2 m}$$

Its total energy is

$$E = K + U = \frac{1}{2}mv^2 - \frac{Zke^2}{r} = -\frac{Zke^2}{2r}$$

The energy of the muon ground state is given by

$$E_n = -\frac{Zke^2}{2r_n} = -\frac{m(Ze^2)^2}{8\varepsilon_0^2 h^2}\frac{1}{n^2}$$

Evaluating the constants gives

$$\begin{aligned} E_n &= -\frac{m(Ze^2)^2}{8\varepsilon_0^2 h^2}\frac{1}{n^2} = -\frac{(207\times 9.11\times 10^{-31}\ \text{kg})(2)^2(1.6\times 10^{-19}\ \text{C})^4}{8(8.85\times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)^2(6.626\times 10^{-34}\,\text{J}\cdot\text{s})^2}\frac{1}{n^2} \\ &= -\frac{1.8\times 10^{-15}\,\text{J}}{n^2} = -\frac{11.3\,\text{keV}}{n^2}. \end{aligned}$$

69. The Ritz combination principle can be readily understood by noting that the transition from $n = n_i$ to $n = n_f < n_i$ can be done in two steps, with an intermediate state n' :

$$\Delta E = E_{n_f} - E_{n_i} = (-13.6\,\text{eV})\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) = (-13.6\,\text{eV})\left(\frac{1}{n_f^2} - \frac{1}{n'^2}\right) + (-13.6\,\text{eV})\left(\frac{1}{n'^2} - \frac{1}{n_i^2}\right)$$

The transition $n_i = 3 \rightarrow n_f = 1$ associated with the second Lyman-series line can be thought of as $n_i = 3 \rightarrow n' = 2$ (first Balmer) followed by $n' = 2 \rightarrow n_f = 1$ (first Lyman). Another example would be $n_i = 4 \rightarrow n_f = 2$ (second Balmer), which can be thought of as $n_i = 4 \rightarrow n' = 3$ (first Paschen) followed by $n' = 3 \rightarrow n_f = 2$ (first Balmer).

70. (a) We use e_0 to denote the electric charge. The constant A can be calculated by integrating the charge density distribution:

$$-e_0 = \int \rho(r)dV = \int_0^\infty (Ae^{-2r/a_0})4\pi r^2 dr = 4\pi A a_0^3 \int_0^\infty x^2 e^{-2x} dx = \pi A a_0^3$$

which gives $A = -e_0 / \pi a_0^3$.

(b) We apply Gauss's to calculate the electric field at a distance r from the center of the atom. The charge enclosed by a Gaussian sphere of radius $r = a_0$, including the proton charge $+e_0$ at the center, is

$$q_{\text{enc}} = e_0 + \int \rho(r)dV = e_0 + \int_0^{a_0} (Ae^{-2r/a_0})4\pi r^2 dr = e_0 + 4\pi Aa_0^3 \int_0^1 x^2 e^{-2x} dx$$
$$= e_0 + \pi Aa_0^3 \left(1 - \frac{5}{e^2}\right) = e_0 + (-e_0)\left(1 - \frac{5}{e^2}\right) = (5e^{-2})e_0$$

Using Gauss's law, $\int \vec{E} \cdot d\vec{a} = q_{\text{enc}} / \varepsilon_0$, we obtain

$$E(4\pi a_0^2) = \frac{(5e^{-2})e_0}{\varepsilon_0} \quad \Rightarrow \quad E = \frac{(5e^{-2})e_0}{4\pi\varepsilon_0 a_0^2}$$

(c) The net charge enclosed is positive, so the direction is radially outward.

71. (a) The charge enclosed by a sphere of radius r due to the uniform positive charge distribution is proportional to the volume: $q_{\text{enc}} = e(r/a_0)^3$. Using Gauss's law, $\int \vec{E} \cdot d\vec{a} = q_{\text{enc}} / \varepsilon_0$, the electric field at a radial distance r from the center of the atom is

$$E(4\pi r^2) = \frac{e}{\varepsilon_0}\left(\frac{r}{a_0}\right)^3 \quad \Rightarrow \quad E = \frac{e}{4\pi\varepsilon_0 a_0^3} r$$

and the force on the electron is $F = -eE = \dfrac{-e^2}{4\pi\varepsilon_0 a_0^3} r$. The negative sign means that the force points toward the center.

(b) Since $F = ma = md^2r/dt^2$,

$$m\frac{d^2r}{dt^2} = \frac{-e^2}{4\pi\varepsilon_0 a_0^3} r \qquad \Rightarrow \quad \frac{d^2r}{dt^2} + \omega^2 r = 0$$

and the angular frequency is

$$\omega = \sqrt{\frac{e^2}{4\pi\varepsilon_0 m a_0^3}} = \frac{e}{\sqrt{4\pi\varepsilon_0 m a_0^3}}.$$

72. (a) The electric potential is

$$V = \frac{kq}{r} = \frac{ke}{a_0} = \frac{8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2}{5.29\times10^{-11}\text{ m}} = 27.22\text{ V}$$

(b) The electric potential energy of the atom is

$$U = qV = -eV = -e(27.22\text{ V}) = -27.22\text{ eV}$$

(c) The electron moves in a circular orbit with

$$\frac{mv^2}{r}=\frac{ke^2}{r^2} \quad \Rightarrow \quad v=\sqrt{\frac{ke^2}{mr}}$$

Its kinetic energy at $r=a_0$ is

$$K=\frac{1}{2}mv^2=\frac{ke^2}{2a_0}=\frac{1}{2}(27.22\text{ eV})=13.6\text{ eV} .$$

(d) The total energy of the system is

$$E=K+U=\frac{1}{2}mv^2-\frac{ke^2}{a_0}=-\frac{ke^2}{2a_0}=-13.6\text{ eV} .$$

Therefore, the energy required to ionize the atom is +13.6 eV.

73. The energy is, after evaluating the constants,

$$\begin{aligned} E_{n_1,n_2,n_3} &= \frac{h^2}{8mL^2}\left(n_1^2+n_2^2+n_3^2\right)=\frac{(6.626\times10^{-34}\text{ J}\cdot\text{s})^2}{8(9.11\times10^{-31}\text{ kg})(0.25\times10^{-6}\text{ m})^2}\left(n_1^2+n_2^2+n_3^2\right) \\ &=(6.024\ \mu\text{eV})\left(n_1^2+n_2^2+n_3^2\right) \end{aligned}$$

The lowest five states correspond to $(n_1, n_2, n_3)=(1, 1, 1), (1, 2, 1), (1, 2, 2), (1, 3, 1)$ and $(2, 2, 2)$, and the energies are

$$\begin{aligned} E_{111} &= \frac{h^2}{8mL^2}\left(1^2+1^2+1^2\right)=3(6.024\ \mu\text{eV})=18.1\ \mu\text{eV} \\ E_{121} &= \frac{h^2}{8mL^2}\left(1^2+2^2+1^2\right)=6(6.024\ \mu\text{eV})=36.2\ \mu\text{eV} \\ E_{122} &= \frac{h^2}{8mL^2}\left(1^2+2^2+2^2\right)=9(6.024\ \mu\text{eV})=54.3\ \mu\text{eV} \\ E_{131} &= \frac{h^2}{8mL^2}\left(1^2+3^2+1^2\right)=11(6.024\ \mu\text{eV})=66.3\ \mu\text{eV} \\ E_{222} &= \frac{h^2}{8mL^2}\left(2^2+2^2+2^2\right)=12(6.024\ \mu\text{eV})=72.4\ \mu\text{eV} \end{aligned}$$

Chapter 40

1. The magnitude L of the orbital angular momentum $\vec{L}$ is given by Eq. 40-2: $L=\sqrt{\ell(\ell+1)}\hbar$. On the other hand, the components L_z are $L_z = m_\ell \hbar$, where $m_\ell = -\ell, \ldots + \ell$. Thus, the semi-classical angle is $\cos\theta = L_z / L$. The angle is the smallest when $m=\ell$, or

$$\cos\theta = \frac{\ell\hbar}{\sqrt{\ell(\ell+1)}\hbar} \quad \Rightarrow \quad \theta = \cos^{-1}\left(\frac{\ell}{\sqrt{\ell(\ell+1)}}\right).$$

With $\ell=5$, we have $\theta = \cos^{-1}(5/\sqrt{30}) = 24.1°$.

2. For a given quantum number n there are n possible values of ℓ, ranging from 0 to $n-1$. For each ℓ the number of possible electron states is $N_\ell = 2(2\ell + 1)$. Thus the total number of possible electron states for a given n is

$$N_n = \sum_{\ell=0}^{n-1} N_\ell = 2\sum_{\ell=0}^{n-1} (2\ell+1) = 2n^2.$$

Thus, in this problem, the total number of electron states is $N_n = 2n^2 = 2(5)^2 = 50$.

3. (a) We use Eq. 40-2:

$$L = \sqrt{\ell(\ell+1)}\hbar = \sqrt{3(3+1)}\left(1.055\times10^{-34}\,\mathrm{J\cdot s}\right) = 3.65\times10^{-34}\,\mathrm{J\cdot s}.$$

(b) We use Eq. 40-7: $L_z = m_\ell \hbar$. For the maximum value of L_z set $m_\ell = \ell$. Thus

$$\left[L_z\right]_{\max} = \ell\hbar = 3\left(1.055\times10^{-34}\,\mathrm{J\cdot s}\right) = 3.16\times10^{-34}\,\mathrm{J\cdot s}.$$

4. For a given quantum number n there are n possible values of ℓ, ranging from 0 to $n - 1$. For each ℓ the number of possible electron states is $N_\ell = 2(2\ell + 1)$. Thus, the total number of possible electron states for a given n is

$$N_n = \sum_{l=0}^{n-1} N_\ell = 2\sum_{l=0}^{n-1} (2\ell+1) = 2n^2.$$

(a) In this case $n = 4$, which implies $N_n = 2(4^2) = 32$.

(b) Now $n = 1$, so $N_n = 2(1^2) = 2$.

(c) Here $n = 3$, and we obtain $N_n = 2(3^2) = 18$.

(d) Finally, $n = 2 \rightarrow N_n = 2(2^2) = 8$.

5. (a) For a given value of the principal quantum number n, the orbital quantum number ℓ ranges from 0 to $n - 1$. For $n = 3$, there are three possible values: 0, 1, and 2.

(b) For a given value of ℓ, the magnetic quantum number m_ℓ ranges from $-\ell$ to $+\ell$. For $\ell = 1$, there are three possible values: – 1, 0, and +1.

6. For a given quantum number ℓ there are $(2\ell + 1)$ different values of m_ℓ. For each given m_ℓ the electron can also have two different spin orientations. Thus, the total number of electron states for a given ℓ is given by $N_\ell = 2(2\ell + 1)$.

(a) Now $\ell = 3$, so $N_\ell = 2(2 \times 3 + 1) = 14$.

(b) In this case, $\ell = 1$, which means $N_\ell = 2(2 \times 1 + 1) = 6$.

(c) Here $\ell = 1$, so $N_\ell = 2(2 \times 1 + 1) = 6$.

(d) Now $\ell = 0$, so $N_\ell = 2(2 \times 0 + 1) = 2$.

7. (a) Using Table 40-1, we find $\ell = [m_\ell]_{\max} = 4$.

(b) The smallest possible value of n is $n = \ell_{\max} + 1 \geq \ell + 1 = 5$.

(c) As usual, $m_s = \pm\frac{1}{2}$, so two possible values.

8. (a) For $\ell = 3$, the greatest value of m_ℓ is $m_\ell = 3$.

(b) Two states ($m_s = \pm\frac{1}{2}$) are available for $m_\ell = 3$.

(c) Since there are 7 possible values for m_ℓ : +3, +2, +1, 0, – 1, – 2, – 3, and two possible values for m_s, the total number of state available in the subshell $\ell = 3$ is 14.

9. **THINK** Knowing the value of ℓ, the orbital quantum number, allows us to determine the magnitudes of the angular momentum and the magnetic dipole moment.

EXPRESS The magnitude of the orbital angular momentum is

$$L=\sqrt{\ell(\ell+1)}\hbar.$$

Similarly, with $\vec{\mu}_{\text{orb}}=-\dfrac{e}{2m}\vec{L}$, the magnitude of $\vec{\mu}_{\text{orb}}$ is

$$\mu_{\text{orb}}=\frac{e\hbar}{2m}\sqrt{\ell(\ell+1)}=\mu_{\text{B}},$$

where $\mu_{\text{B}}=e\hbar/2m$ is the Bohr magneton.

ANALYZE (a) For $\ell=3$, we have

$$L=\sqrt{\ell(\ell+1)}\hbar=\sqrt{3(3+1)}\hbar=\sqrt{12}\hbar.$$

So the multiple is $\sqrt{12}\approx 3.46$.

(b) The magnitude of the orbital dipole moment is

$$\mu_{\text{orb}}=\sqrt{\ell(\ell+1)}\mu_B=\sqrt{12}\mu_B.$$

So the multiple is $\sqrt{12}\approx 3.46$.

(c) The largest possible value of m_ℓ is $m_\ell=\ell=3$.

(d) We use $L_z=m_\ell\hbar$ to calculate the *z* component of the orbital angular momentum. The multiple is $m_\ell=3$.

(e) We use $\mu_z=-m_\ell\mu_B$ to calculate the *z* component of the orbital magnetic dipole moment. The multiple is $-m_\ell=-3$.

(f) We use $\cos\theta=m_\ell\big/\sqrt{\ell(\ell+1)}$ to calculate the angle between the orbital angular momentum vector and the *z* axis. For $\ell=3$ and $m_\ell=3$, we have $\cos\theta=3/\sqrt{12}=\sqrt{3}/2$, or $\theta=30.0°$.

(g) For $\ell=3$ and $m_\ell=2$, we have $\cos\theta=2/\sqrt{12}=1/\sqrt{3}$, or $\theta=54.7°$.

(h) For $\ell=3$ and $m_\ell=-3$, $\cos\theta=-3/\sqrt{12}=-\sqrt{3}/2$, or $\theta=150°$.

LEARN Neither $\vec{L}$ nor $\vec{\mu}_{\text{orb}}$ can be measured in any way. We can, however, measure their *z* components.

10. (a) For n = 3 there are 3 possible values of ℓ: 0, 1, and 2.

(b) We interpret this as asking for the number of distinct values for m_ℓ (this ignores the multiplicity of any particular value). For each ℓ there are $2\ell + 1$ possible values of m_ℓ. Thus the number of possible m_ℓ's for $\ell = 2$ is $(2\ell + 1) = 5$. Examining the $\ell = 1$ and $\ell = 0$ cases cannot lead to any new (distinct) values for m_ℓ, so the answer is 5.

(c) Regardless of the values of n, ℓ and m_ℓ, for an electron there are always two possible values of $m_s: \pm\frac{1}{2}$.

(d) The population in the n = 3 shell is equal to the number of electron states in the shell, or $2n^2 = 2(3^2) = 18$.

(e) Each subshell has its own value of ℓ. Since there are three different values of ℓ for n = 3, there are three subshells in the n = 3 shell.

11. **THINK** We can only measure one component of $\vec{L}$, say L_z, but not all three components.

EXPRESS Since $L^2 = L_x^2 + L_y^2 + L_z^2, \sqrt{L_x^2 + L_y^2} = \sqrt{L^2 - L_z^2}$. Replacing L^2 with $\ell(\ell+1)\hbar^2$ and L_z with $m_\ell \hbar$, we obtain

$$\sqrt{L_x^2 + L_y^2} = \hbar\sqrt{\ell(\ell+1) - m_\ell^2} .$$

ANALYZE For a given value of ℓ, the greatest that m_ℓ can be is ℓ, so the smallest that $\sqrt{L_x^2 + L_y^2}$ can be is $\hbar\sqrt{\ell(\ell+1) - \ell^2} = \hbar\sqrt{\ell}$. The smallest possible magnitude of m_ℓ is zero, so the largest $\sqrt{L_x^2 + L_y^2}$ can be is $\hbar\sqrt{\ell(\ell+1)}$. Thus,

$$\hbar\sqrt{\ell} \le \sqrt{L_x^2 + L_y^2} \le \hbar\sqrt{\ell(\ell+1)} .$$

LEARN Once we have chosen to measure $\vec{L}$ along the z axis, the x- and y-components cannot be measured with infinite certainty.

12. The angular momentum of the rotating sphere, $\vec{L}_{\text{sphere}}$, is equal in magnitude but in opposite direction to $\vec{L}_{\text{atom}}$, the angular momentum due to the aligned atoms. The number of atoms in the sphere is $N = \dfrac{N_A m}{M}$, where $N_A = 6.02\times 10^{23}/\text{mol}$ is Avogadro's number and $M = 0.0558\,\text{kg/mol}$ is the molar mass of iron. The angular momentum due to the aligned atoms is

$$L_{\text{atom}} = 0.12N(m_s\hbar) = 0.12\frac{N_A m}{M}\frac{\hbar}{2}.$$

On the other hand, the angular momentum of the rotating sphere is (see Table 10-2 for I)

$$L_{\text{sphere}} = I\omega = \left(\frac{2}{5}mR^2\right)\omega.$$

Equating the two expressions, the mass m cancels out and the angular velocity is

$$\begin{aligned}\omega &= 0.12\frac{5N_A\hbar}{4MR^2} = 0.12\frac{5(6.02\times10^{23}/\text{mol})(6.63\times10^{-34}\ \text{J}\cdot\text{s}/2\pi)}{4(0.0558\ \text{kg/mol})(2.00\times10^{-3}\ \text{m})^2}.\\ &= 4.27\times10^{-5}\ \text{rad/s}\end{aligned}$$

13. **THINK** A gradient magnetic field gives rise to a magnetic force on the silver atom.

EXPRESS The force on the silver atom is given by

$$F_z = -\frac{dU}{dz} = -\frac{d}{dz}(-\mu_z B) = \mu_z\frac{dB}{dz}$$

where μ_z is the z component of the magnetic dipole moment of the silver atom, and B is the magnetic field. The acceleration is

$$a = \frac{F_z}{M} = \frac{\mu_z(dB/dz)}{M},$$

where M is the mass of a silver atom.

ANALYZE Using the data given in Sample Problem —"Beam separation in a Stern-Gerlach experiment," we obtain

$$a = \frac{(9.27\times10^{-24}\ \text{J/T})(1.4\times10^3\ \text{T/m})}{1.8\times10^{-25}\ \text{kg}} = 7.2\times10^4\ \text{m/s}^2.$$

LEARN The deflection of the silver atom is due to the interaction between the magnetic dipole moment of the atom and the magnetic field. However, if the field is uniform, then $dB/dz = 0$, and the silver atom will pass the poles undeflected.

14. (a) From Eq. 40-19,

$$F = \mu_B\left|\frac{dB}{dz}\right| = (9.27\times10^{-24}\ \text{J/T})(1.6\times10^2\ \text{T/m}) = 1.5\times10^{-21}\ \text{N}.$$

(b) The vertical displacement is

$$\Delta x = \frac{1}{2}at^2 = \frac{1}{2}\left(\frac{F}{m}\right)\left(\frac{l}{v}\right)^2 = \frac{1}{2}\left(\frac{1.5\times10^{-21}\,\text{N}}{1.67\times10^{-27}\,\text{kg}}\right)\left(\frac{0.80\text{m}}{1.2\times10^{5}\ \text{m/s}}\right)^2 = 2.0\times10^{-5}\,\text{m}.$$

15. The magnitude of the spin angular momentum is

$$S = \sqrt{s(s+1)}\hbar = \left(\sqrt{3}/2\right)\hbar\,,$$

where $s = \frac{1}{2}$ is used. The *z* component is either $S_z = \hbar/2$ or $-\hbar/2$.

(a) If $S_z = +\hbar/2$ the angle θ between the spin angular momentum vector and the positive *z* axis is

$$\theta = \cos^{-1}\left(\frac{S_z}{S}\right) = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right) = 54.7°\ .$$

(b) If $S_z = -\hbar/2$, the angle is θ = 180° – 54.7° = 125.3° $\approx$125°.

16. (a) From Fig. 40-10 and Eq. 40-18,

$$\Delta E = 2\mu_B B = \frac{2\left(9.27\times10^{-24}\ \text{J/T}\right)\left(0.50\,\text{T}\right)}{1.60\times10^{-19}\ \text{J/eV}} = 58\,\mu\text{eV}\ .$$

(b) From $\Delta E = hf$ we get

$$f = \frac{\Delta E}{h} = \frac{9.27\times10^{-24}\ \text{J}}{6.63\times10^{-34}\ \text{J}\cdot\text{s}} = 1.4\times10^{10}\ \text{Hz} = 14\,\text{GHz}\ .$$

(c) The wavelength is

$$\lambda = \frac{c}{f} = \frac{2.998\times10^{8}\ \text{m/s}}{1.4\times10^{10}\,\text{Hz}} = 2.1\ \text{cm}.$$

(d) The wave is in the short radio wave region.

17. The total magnetic field, $B = B_{\text{local}} + B_{\text{ext}}$, satisfies $\Delta E = hf = 2\mu B$ (see Eq. 40-22). Thus,

$$B_{\text{local}} = \frac{hf}{2\mu} - B_{\text{ext}} = \frac{\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(34\times10^{6}\ \text{Hz}\right)}{2\left(1.41\times10^{-26}\ \text{J/T}\right)} - 0.78\ \text{T} = 19\ \text{mT}\ .$$

18. We let $\Delta E = 2\mu_B B_{\text{eff}}$ (based on Fig. 40-10 and Eq. 40-18) and solve for B_{eff}:

$$B_{\text{eff}} = \frac{\Delta E}{2\mu_B} = \frac{hc}{2\lambda\mu_B} = \frac{1240\,\text{nm}\cdot\text{eV}}{2\left(21\times10^{-7}\text{ nm}\right)\left(5.788\times10^{-5}\text{ eV/T}\right)} = 51\text{ mT} .$$

19. The energy of a magnetic dipole in an external magnetic field $\vec{B}$ is $U = -\vec{\mu}\cdot\vec{B} = -\mu_z B$, where $\vec{\mu}$ is the magnetic dipole moment and μ_z is its component along the field. The energy required to change the moment direction from parallel to antiparallel is $\Delta E = \Delta U = 2\mu_z B$. Since the z component of the spin magnetic moment of an electron is the Bohr magneton μ_B,

$$\Delta E = 2\mu_B B = 2\left(9.274\times10^{-24}\text{ J/T}\right)\left(0.200\,\text{T}\right) = 3.71\times10^{-24}\text{ J} .$$

The photon wavelength is

$$\lambda = \frac{c}{f} = \frac{hc}{\Delta E} = \frac{\left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)\left(2.998\times10^{8}\text{ m/s}\right)}{3.71\times10^{-24}\text{ J}} = 5.35\times10^{-2}\text{ m} .$$

20. Using Eq. 39-20 we find that the lowest four levels of the rectangular corral (with this specific "aspect ratio") are nondegenerate, with energies $E_{1,1}$ = 1.25, $E_{1,2}$ = 2.00, $E_{1,3}$ = 3.25, and $E_{2,1}$ = 4.25 (all of these understood to be in "units" of $h^2/8mL^2$). Therefore, obeying the Pauli principle, we have

$$E_{\text{ground}} = 2E_{1,1} + 2E_{1,2} + 2E_{1,3} + E_{2,1} = 2(1.25) + 2(2.00) + 2(3.25) + 4.25$$

which means (putting the "unit" factor back in) that the lowest possible energy of the system is $E_{\text{ground}} = 17.25(h^2/8mL^2)$. Thus, the multiple of $h^2/8mL^2$ is 17.25.

21. Because of the Pauli principle (and the requirement that we construct a state of lowest possible total energy), two electrons fill the n = 1, 2, 3 levels and one electron occupies the n = 4 level. Thus, using Eq. 39-4,

$$\begin{aligned} E_{\text{ground}} &= 2E_1 + 2E_2 + 2E_3 + E_4 \\ &= 2\left(\frac{h^2}{8mL^2}\right)(1)^2 + 2\left(\frac{h^2}{8mL^2}\right)(2)^2 + 2\left(\frac{h^2}{8mL^2}\right)(3)^2 + \left(\frac{h^2}{8mL^2}\right)(4)^2 \\ &= (2+8+18+16)\left(\frac{h^2}{8mL^2}\right) = 44\left(\frac{h^2}{8mL^2}\right). \end{aligned}$$

Thus, the multiple of $h^2/8mL^2$ is 44.

22. Due to spin degeneracy ($m_s = \pm 1/2$), each state can accommodate two electrons. Thus, in the energy-level diagram shown, two electrons can be placed in the ground state with energy $E_1 = 4(h^2/8mL^2)$, six can occupy the "triple state" with $E_2 = 6(h^2/8mL^2)$,

and so forth. With 11 electrons, the lowest energy configuration consists of two electrons with $E_1 = 4(h^2/8mL^2)$, six electrons with $E_2 = 6(h^2/8mL^2)$, and three electrons with $E_3 = 7(h^2/8mL^2)$. Thus, we find the ground-state energy of the 11-electron system to be

$$\begin{aligned} E_{\text{ground}} &= 2E_1 + 6E_2 + 3E_3 = 2\left(\frac{4h^2}{8mL^2}\right) + 6\left(\frac{6h^2}{8mL^2}\right) + 3\left(\frac{7h^2}{8mL^2}\right) \\ &= \left[(2)(4) + (6)(6) + (3)(7)\right]\left(\frac{h^2}{8mL^2}\right) = 65\left(\frac{h^2}{8mL^2}\right). \end{aligned}$$

The first excited state of the 11-electron system consists of two electrons with $E_1 = 4(h^2/8mL^2)$, five electrons with $E_2 = 6(h^2/8mL^2)$, and four electrons with $E_3 = 7(h^2/8mL^2)$. Thus, its energy is

$$\begin{aligned} E_{\text{1st excited}} &= 2E_1 + 5E_2 + 4E_3 = 2\left(\frac{4h^2}{8mL^2}\right) + 5\left(\frac{6h^2}{8mL^2}\right) + 4\left(\frac{7h^2}{8mL^2}\right) \\ &= \left[(2)(4) + (5)(6) + (4)(7)\right]\left(\frac{h^2}{8mL^2}\right) = 66\left(\frac{h^2}{8mL^2}\right). \end{aligned}$$

Thus, the multiple of $h^2/8mL^2$ is 66.

23. **THINK** With eight electrons, the ground-state energy of the system is the sum of the energies of the individual electrons in the system's ground-state configuration.

EXPRESS In terms of the quantum numbers n_x, n_y, and n_z, the single-particle energy levels are given by

$$E_{n_x,n_y,n_z} = \frac{h^2}{8mL^2}\left(n_x^2 + n_y^2 + n_z^2\right).$$

The lowest single-particle level corresponds to $n_x = 1$, $n_y = 1$, and $n_z = 1$ and is $E_{1,1,1} = 3(h^2/8mL^2)$. There are two electrons with this energy, one with spin up and one with spin down. The next lowest single-particle level is three-fold degenerate in the three integer quantum numbers. The energy is

$$E_{1,1,2} = E_{1,2,1} = E_{2,1,1} = 6(h^2/8mL^2).$$

Each of these states can be occupied by a spin up and a spin down electron, so six electrons in all can occupy the states. This completes the assignment of the eight electrons to single-particle states.

ANALYZE The ground state energy of the system is

$$E_{\text{gr}} = (2)(3)(h^2/8mL^2) + (6)(6)(h^2/8mL^2) = 42(h^2/8mL^2).$$

Thus, the multiple of $h^2/8mL^2$ is 42.

LEARN We summarize the ground-state configuration and the energies (in multiples of $h^2/8mL^2$) in the chart below:

n_x	n_y	n_z	m_s	energy
1	1	1	–1/2, + 1/2	3 + 3
1	1	2	–1/2, + 1/2	6 + 6
1	2	1	–1/2, + 1/2	6 + 6
2	1	1	–1/2, + 1/2	6 + 6
			total	42

24. (a) Using Eq. 39-20 we find that the lowest five levels of the rectangular corral (with this specific "aspect ratio") have energies

$$E_{1,1} = 1.25,\ E_{1,2} = 2.00,\ E_{1,3} = 3.25,\ E_{2,1} = 4.25,\ E_{2,2} = 5.00$$

(all of these understood to be in "units" of $h^2/8mL^2$). It should be noted that the energy level we denote $E_{2,2}$ actually corresponds to two energy levels ($E_{2,2}$ and $E_{1,4}$; they are degenerate), but that will not affect our calculations in this problem. The configuration that provides the lowest system energy higher than that of the ground state has the first three levels filled, the fourth one empty, and the fifth one half-filled:

$$E_{\text{first excited}} = 2E_{1,1} + 2E_{1,2} + 2E_{1,3} + E_{2,2} = 2(1.25) + 2(2.00) + 2(3.25) + 5.00$$

which means (putting the "unit" factor back in) the energy of the first excited state is $E_{\text{first excited}} = 18.00(h^2/8mL^2)$. Thus, the multiple of $h^2/8mL^2$ is 18.00.

(b) The configuration that provides the next higher system energy has the first two levels filled, the third one half-filled, and the fourth one filled:

$$E_{\text{second excited}} = 2E_{1,1} + 2E_{1,2} + E_{1,3} + 2E_{2,1} = 2(1.25) + 2(2.00) + 3.25 + 2(4.25)$$

which means (putting the "unit" factor back in) the energy of the second excited state is

$$E_{\text{second excited}} = 18.25(h^2/8mL^2).$$

Thus, the multiple of $h^2/8mL^2$ is 18.25.

(c) Now, the configuration that provides the *next* higher system energy has the first two levels filled, with the next three levels half-filled:

$$E_{\text{third excited}} = 2E_{1,1} + 2E_{1,2} + E_{1,3} + E_{2,1} + E_{2,2} = 2(1.25) + 2(2.00) + 3.25 + 4.25 + 5.00$$

which means (putting the "unit" factor back in) the energy of the third excited state is $E_{\text{third excited}} = 19.00(h^2/8mL^2)$. Thus, the multiple of $h^2/8mL^2$ is 19.00.

(d) The energy states of this problem and Problem 40-22 are suggested below:

___________________ third excited $19.00(h^2/8mL^2)$

___________________ second excited $18.25(h^2/8mL^2)$

___________________ first excited $18.00(h^2/8mL^2)$

___________________ ground state $17.25(h^2/8mL^2)$

25. (a) Promoting one of the electrons (described in Problem 40-21) to a not-fully occupied higher level, we find that the configuration with the least total energy greater than that of the ground state has the n = 1 and 2 levels still filled, but now has only one electron in the n = 3 level; the remaining two electrons are in the n = 4 level. Thus,

$$\begin{aligned} E_{\text{first excited}} &= 2E_1 + 2E_2 + E_3 + 2E_4 \\ &= 2\left(\frac{h^2}{8mL^2}\right)(1)^2 + 2\left(\frac{h^2}{8mL^2}\right)(2)^2 + \left(\frac{h^2}{8mL^2}\right)(3)^2 + 2\left(\frac{h^2}{8mL^2}\right)(4)^2 \\ &= (2+8+9+32)\left(\frac{h^2}{8mL^2}\right) = 51\left(\frac{h^2}{8mL^2}\right). \end{aligned}$$

Thus, the multiple of $h^2/8mL^2$ is 51.

(b) Now, the configuration which provides the next higher total energy, above that found in part (a), has the bottom three levels filled (just as in the ground state configuration) and has the seventh electron occupying the n = 5 level:

$$\begin{aligned} E_{\text{second excited}} &= 2E_1 + 2E_2 + 2E_3 + E_5 \\ &= 2\left(\frac{h^2}{8mL^2}\right)(1)^2 + 2\left(\frac{h^2}{8mL^2}\right)(2)^2 + 2\left(\frac{h^2}{8mL^2}\right)(3)^2 + \left(\frac{h^2}{8mL^2}\right)(5)^2 \\ &= (2+8+18+25)\left(\frac{h^2}{8mL^2}\right) = 53\left(\frac{h^2}{8mL^2}\right). \end{aligned}$$

Thus, the multiple of $h^2/8mL^2$ is 53.

(c) The third excited state has the n = 1, 3, 4 levels filled, and the n = 2 level half-filled:

$$\begin{aligned}E_{\text{third excited}} &= 2E_1 + E_2 + 2E_3 + 2E_4 \\ &= 2\left(\frac{h^2}{8mL^2}\right)(1)^2 + \left(\frac{h^2}{8mL^2}\right)(2)^2 + 2\left(\frac{h^2}{8mL^2}\right)(3)^2 + 2\left(\frac{h^2}{8mL^2}\right)(4)^2 \\ &= (2+4+18+32)\left(\frac{h^2}{8mL^2}\right) = 56\left(\frac{h^2}{8mL^2}\right).\end{aligned}$$

Thus, the multiple of $h^2/8mL^2$ is 56.

(d) The energy states of this problem and Problem 40-21 are suggested below:

____________________ third excited $56(h^2/8mL^2)$

____________________ second excited $53(h^2/8mL^2)$

____________________ first excited $51(h^2/8mL^2)$

____________________ ground state $44(h^2/8mL^2)$

26. The energy levels are given by

$$E_{n_x,n_y,n_z} = \frac{h^2}{8m}\left(\frac{n_x^2}{L_x^2} + \frac{n_y^2}{L_y^2} + \frac{n_z^2}{L_z^2}\right) = \frac{h^2}{8mL^2}\left(n_x^2 + n_y^2 + n_z^2\right).$$

The Pauli principle requires that no more than two electrons be in the lowest energy level (at $E_{1,1,1} = 3(h^2/8mL^2)$ with $n_x = n_y = n_z = 1$), but — due to their degeneracies — as many as six electrons can be in the next three levels,

$$E' = E_{1,1,2} = E_{1,2,1} = E_{2,1,1} = 6(h^2/8mL^2)$$

$$E'' = E_{1,2,2} = E_{2,2,1} = E_{2,1,2} = 9(h^2/8mL^2)$$

$$E''' = E_{1,1,3} = E_{1,3,1} = E_{3,1,1} = 11(h^2/8mL^2).$$

Using Eq. 39-21, the level above those can only hold two electrons:

$$E_{2,2,2} = (2^2 + 2^2 + 2^2)(h^2/8mL^2) = 12(h^2/8mL^2).$$

And the next higher level can hold as much as twelve electrons and has energy

$$E'''' = 14(h^2/8mL^2).$$

(a) The configuration that provides the lowest system energy higher than that of the ground state has the first level filled, the second one with one vacancy, and the third one with one occupant:

$$E_{\text{first excited}} = 2E_{1,1,1} + 5E' + E'' = 2(3) + 5(6) + 9$$

which means (putting the "unit" factor back in) the energy of the first excited state is

$$E_{\text{first excited}} = 45(h^2/8mL^2).$$

Thus, the multiple of $h^2/8mL^2$ is 45.

(b) The configuration that provides the next higher system energy has the first level filled, the second one with one vacancy, the third one empty, and the fourth one with one occupant:

$$E_{\text{second excited}} = 2E_{1,1,1} + 5E' + E'' = 2(3) + 5(6) + 11$$

which means (putting the "unit" factor back in) the energy of the second excited state is $E_{\text{second excited}} = 47(h^2/8mL^2)$. Thus, the multiple of $h^2/8mL^2$ is 47.

(c) Now, there are a couple of configurations that provide the *next* higher system energy. One has the first level filled, the second one with one vacancy, the third and fourth ones empty, and the fifth one with one occupant:

$$E_{\text{third excited}} = 2E_{1,1,1} + 5E' + E''' = 2(3) + 5(6) + 12$$

which means (putting the "unit" factor back in) the energy of the third excited state is $E_{\text{third excited}} = 48(h^2/8mL^2)$. Thus, the multiple of $h^2/8mL^2$ is 48. The other configuration with this same total energy has the first level filled, the second one with two vacancies, and the third one with one occupant.

(d) The energy states of this problem and Problem 40-25 are suggested below:

________________ third excited $48(h^2/8mL^2)$

________________ second excited $47(h^2/8mL^2)$

________________ first excited $45(h^2/8mL^2)$

________________ ground state $42(h^2/8mL^2)$

27. **THINK** The four quantum numbers (n, ℓ, m_ℓ, m_s) identify the quantum states of individual electrons in a multi-electron atom.

EXPRESS A lithium atom has three electrons. The first two electrons have quantum numbers (1, 0, 0, $\pm 1/2$). All states with principal quantum number n = 1 are filled. The next lowest states have n = 2.

The orbital quantum number can have the values $\ell = 0$ or 1 and of these, the $\ell = 0$ states have the lowest energy. The magnetic quantum number must be $m_\ell = 0$ since this is the only possibility if $\ell = 0$. The spin quantum number can have either of the values $m_s = -\frac{1}{2}$ or $+\frac{1}{2}$. Since there is no external magnetic field, the energies of these two states are the same.

ANALYZE (a) Therefore, in the ground state, the quantum numbers of the third electron are either $n = 2, \ell = 0, m_\ell = 0, m_s = -\frac{1}{2}$ or $n = 2, \ell = 0, m_\ell = 0, m_s = +\frac{1}{2}$. That is, (n, ℓ, m_ℓ, m_s) = (2,0,0, +1/2) and (2,0,0, –1/2).

(b) The next lowest state in energy is an n = 2, $\ell = 1$ state. All n = 3 states are higher in energy. The magnetic quantum number can be $m_\ell = -1, 0, \text{or} +1$; the spin quantum number can be $m_s = -\frac{1}{2}$ or $+\frac{1}{2}$. Thus, (n, ℓ, m_ℓ, m_s) = (2,1,1, +1/2), (2,1,1, –1/2), $(2,1,0,+1/2)$, $(2,1,0,-1/2)$, $(2,1,-1,+1/2)$ and $(2,1,-1,-1/2)$.

LEARN No two electrons can have the same set of quantum numbers, as required by the Pauli exclusion principle.

28. For a given value of the principal quantum number n, there are n possible values of the orbital quantum number ℓ, ranging from 0 to n – 1. For any value of ℓ, there are $2\ell + 1$ possible values of the magnetic quantum number m_ℓ, ranging from $-\ell$ to $+\ell$. Finally, for each set of values of ℓ and m_ℓ, there are two states, one corresponding to the spin quantum number $m_s = -\frac{1}{2}$ and the other corresponding to $m_s = +\frac{1}{2}$. Hence, the total number of states with principal quantum number n is

$$N = 2\sum_{\ell=0}^{n-1}(2\ell + 1).$$

Now

$$\sum_{\ell=0}^{n-1} 2\ell = 2\sum_{\ell=0}^{n-1} \ell = 2\frac{n}{2}(n-1) = n(n-1),$$

since there are n terms in the sum and the average term is (n – 1)/2. Furthermore,

$$\sum_{\ell=0}^{n-1} 1 = n \ .$$

Thus, $N = 2\left[n(n-1)+n\right] = 2n^2$.

29. The total number of possible electron states for a given quantum number *n* is

$$N_n = \sum_{\ell=0}^{n-1} N_\ell = 2\sum_{\ell=0}^{n-1}(2\ell+1) = 2n^2.$$

Thus, if we ignore any electron-electron interaction, then with 110 electrons, we would have two electrons in the $n=1$ shell, eight in the $n=2$ shell, 18 in the $n=3$ shell, 32 in the $n=4$ shell, and the remaining 50 ($=110-2-8-18-32$) in the $n=5$ shell. The 50 electrons would be placed in the subshells in the order $s, p, d, f, g, h, \ldots$ and the resulting configuration is $5s^2 5p^6 5d^{10} 5f^{14} 5g^{18}$. Therefore, the spectroscopic notation for the quantum number ℓ of the last electron would be *g*.

Note, however, when the electron-electron interaction is considered, the ground-state electronic configuration of darmstadtium actually is $[\mathrm{Rn}]5f^{14}6d^9 7s^1$, where

$$[\mathrm{Rn}]: 1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2 4p^6 4d^{10} 4f^{14} 5s^2 5p^6 5d^{10} 6s^2 6p^6$$

represents the inner-shell electrons.

30. When a helium atom is in its ground state, both of its electrons are in the 1*s* state. Thus, for each of the electrons, n = 1, ℓ = 0, and m_ℓ = 0. One of the electrons is spin up $\left(m_s = +\frac{1}{2}\right)$ while the other is spin down $\left(m_s = -\frac{1}{2}\right)$. Thus,

(a) the quantum numbers (n, ℓ, m_ℓ, m_s) for the spin-up electron are (1,0,0,+1/2), and

(b) the quantum numbers (n, ℓ, m_ℓ, m_s) for the spin-down electron are (1,0,0,–1/2).

31. The first three shells (*n* = 1 through 3), which can accommodate a total of 2 + 8 + 18 = 28 electrons, are completely filled. For selenium (*Z* = 34) there are still 34 – 28 = 6 electrons left. Two of them go to the 4*s* subshell, leaving the remaining four in the highest occupied subshell, the 4*p* subshell.

(a) The highest occupied subshell is 4*p*.

(b) There are four electrons in the 4*p* subshell.

For bromine (*Z* = 35) the highest occupied subshell is also the 4*p* subshell, which contains five electrons.

(c) The highest occupied subshell is 4*p*.

(d) There are five electrons in the 4*p* subshell.

For krypton (Z = 36) the highest occupied subshell is also the 4*p* subshell, which now accommodates six electrons.

(e) The highest occupied subshell is 4*p*.

(f) There are six electrons in the 4*p* subshell.

32. (a) The number of different m_ℓ's is $2\ell+1=3$, ($m_\ell=1,0,-1$) and the number of different m_s's is 2, which we denote as +1/2 and −1/2. The allowed states are $(m_{\ell 1}, m_{s1}, m_{\ell 2}, m_{s2})$ = (1, +1/2, 1, −1/2), (1, +1/2, 0, +1/2), (1, +1/2, 0, −1/2), (1, +1/2, −1, +1/2), (1, +1/2, −1, −1/2), (1, −1/2, 0, +1/2), (1, −1/2, 0, −1/2), (1, −1/2, −1, +1/2), (1, −1/2, −1, −1/2), (0, +1/2, 0, −1/2), (0, +1/2, −1, +1/2), (0, +1/2, −1, −1/2), (0, −1/2, −1, + 1/2), (0, −1/2, −1, −1/2), (−1, +1/2, −1, −1/2). So, there are 15 states.

(b) There are six states disallowed by the exclusion principle, in which both electrons share the quantum numbers: $(m_{\ell 1}, m_{s1}, m_{\ell 2}, m_{s2})$=(1, +1/2, 1, +1/2), (1, −1/2, 1, −1/2), (0, +1/2, 0, +1/2), (0, −1/2, 0, −1/2), (−1, +1/2, −1, +1/2), (−1, −1/2, −1, −1/2). So, if the Pauli exclusion principle is not applied, then there would be 15 + 6 = 21 allowed states.

33. The kinetic energy gained by the electron is eV, where V is the accelerating potential difference. A photon with the minimum wavelength (which, because of $E = hc/\lambda$, corresponds to maximum photon energy) is produced when all of the electron's kinetic energy goes to a single photon in an event of the kind depicted in Fig. 40-15. Thus, with $hc = 1240\,\text{eV}\cdot\text{nm}$,

$$eV = \frac{hc}{\lambda_{\min}} = \frac{1240\,\text{eV}\cdot\text{nm}}{0.10\,\text{nm}} = 1.24\times 10^4\ \text{eV} .$$

Therefore, the accelerating potential difference is $V = 1.24 \times 10^4$ V = 12.4 kV.

34. With hc = 1240 eV·nm = 1240 keV·pm, for the K_α line from iron, the energy difference is

$$\Delta E = \frac{hc}{\lambda} = \frac{1240\,\text{keV}\cdot\text{pm}}{193\text{pm}} = 6.42\ \text{keV}.$$

We remark that for the hydrogen atom the corresponding energy difference is

$$\Delta E_{12} = -(13.6\,\text{eV})\left(\frac{1}{2^2}-\frac{1}{1^1}\right) = 10\,\text{eV} .$$

That this difference is much greater in iron is due to the fact that its atomic nucleus contains 26 protons, exerting a much greater force on the K- and L-shell electrons than that provided by the single proton in hydrogen.

35. **THINK** X-rays are produced when a solid target (silver in this case) is bombarded with electrons whose kinetic energies are in the keV range.

EXPRESS The wavelength is $\lambda_{\min} = hc/K_0$, where K_0 is the initial kinetic energy of the incident electron.

ANALYZE (a) With hc = 1240 $\text{eV}\cdot\text{nm}$, we obtain

$$\lambda_{\min} = \frac{hc}{K_0} = \frac{1240\,\text{eV}\cdot\text{nm}}{35\times10^3\,\text{eV}} = 3.54\times10^{-2}\,\text{nm} = 35.4\,\text{pm}\ .$$

(b) A K_α photon results when an electron in a target atom jumps from the L-shell to the K-shell. The energy of this photon is

$$E = 25.51\text{ keV} - 3.56\text{ keV} = 21.95\text{ keV}$$

and its wavelength is

$$\lambda_{K\alpha} = hc/E = (1240\ \text{eV}\cdot\text{nm})/(21.95\times10^3\text{ eV}) = 5.65\times10^{-2}\text{ nm} = 56.5\text{ pm}.$$

(c) A K_β photon results when an electron in a target atom jumps from the M-shell to the K-shell. The energy of this photon is 25.51 keV – 0.53 keV = 24.98 keV and its wavelength is

$$\lambda_{K\beta} = (1240\,\text{eV}\cdot\text{nm})/(24.98\times10^3\text{ eV}) = 4.96\times10^{-2}\text{ nm} = 49.6\text{ pm}.$$

LEARN Note that the cut-off wavelength $\lambda_{\min}$ is characteristic of the incident electrons, not of the target material.

36. (a) We use $eV = hc/\lambda_{\min}$ (see Eq. 40-23 and Eq. 38-4). With hc = 1240 eV·nm = 1240 keV·pm, the mean value of $\lambda_{\min}$ is

$$\lambda_{\min} = \frac{hc}{eV} = \frac{1240\,\text{keV}\cdot\text{pm}}{50.0\,\text{keV}} = 24.8\,\text{pm}\ .$$

(b) The values of λ for the K_α and K_β lines do not depend on the external potential and are therefore unchanged.

37. Suppose an electron with total energy E and momentum p spontaneously changes into a photon. If energy is conserved, the energy of the photon is E and its momentum has magnitude E/c. Now the energy and momentum of the electron are related by

$$E^2 = (pc)^2 + (mc^2)^2 \quad\Rightarrow\quad pc = \sqrt{E^2 - (mc^2)^2}\ .$$

Since the electron has nonzero mass, E/c and p cannot have the same value. Hence, momentum cannot be conserved. A third particle must participate in the interaction, primarily to conserve momentum. It does, however, carry off some energy.

38. From the data given in the problem, we calculate frequencies (using Eq. 38-1), take their square roots, look up the atomic numbers (see Appendix F), and do a least-squares fit to find the slope: the result is 5.02×10^7 with the odd-sounding unit of a square root of a hertz. We remark that the least squares procedure also returns a value for the y-intercept of this statistically determined "best-fit" line; that result is negative and would appear on a graph like Fig. 40-17 to be at about – 0.06 on the vertical axis. Also, we can estimate the slope of the Moseley line shown in Fig. 40-17:

$$\frac{(1.95-0.50)10^9\,\text{Hz}^{1/2}}{40-11} \approx 5.0\times 10^7\,\text{Hz}^{1/2}\ .$$

39. **THINK** The frequency of an x-ray emission is proportional to $(Z - 1)^2$, where Z is the atomic number of the target atom.

EXPRESS The ratio of the wavelength λ_{Nb} for the K_α line of niobium to the wavelength λ_{Ga} for the K_α line of gallium is given by

$$\lambda_{\text{Nb}}/\lambda_{\text{Ga}} = (Z_{\text{Ga}}-1)^2 \big/ (Z_{\text{Nb}}-1)^2 ,$$

where Z_{Nb} is the atomic number of niobium (41) and Z_{Ga} is the atomic number of gallium (31). Thus, $\lambda_{\text{Nb}}/\lambda_{\text{Ga}} = (30)^2 \big/ (40)^2 = 9/16 \approx 0.563$.

LEARN The frequency of the K_α line is given by Eq. 40-26:

$$f = (2.46\times 10^{15}\ \text{Hz})(Z-1)^2 .$$

40. (a) According to Eq. 40-26, $f \propto (Z-1)^2$, so the ratio of energies is (using Eq. 38-2)

$$\frac{f}{f'} = \left(\frac{Z-1}{Z'-1}\right)^2 .$$

(b) We refer to Appendix F. Applying the formula from part (a) to $Z = 92$ and $Z' = 13$, we obtain

$$\frac{E}{E'} = \frac{f}{f'} = \left(\frac{Z-1}{Z'-1}\right)^2 = \left(\frac{92-1}{13-1}\right)^2 = 57.5 .$$

(c) Applying this to $Z = 92$ and $Z' = 3$, we obtain

$$\frac{E}{E'} = \left(\frac{92-1}{3-1}\right)^2 = 2.07\times 10^3 .$$

41. We use Eq. 36-31, Eq. 39-6, and hc = 1240 eV·nm = 1240 keV·pm. Letting $2d\sin\theta = m\lambda = mhc/\Delta E$, where θ = 74.1°, we solve for d:

$$d = \frac{mhc}{2\Delta E\sin\theta} = \frac{(1)(1240\ \text{keV}\cdot\text{nm})}{2(8.979\ \text{keV} - 0.951\ \text{keV})(\sin 74.1°)} = 80.3\ \text{pm} .$$

42. Using hc = 1240 eV·nm = 1240 keV·pm, the energy difference $E_L - E_M$ for the x-ray atomic energy levels of molybdenum is

$$\Delta E = E_L - E_M = \frac{hc}{\lambda_L} - \frac{hc}{\lambda_M} = \frac{1240\ \text{keV}\cdot\text{pm}}{63.0\ \text{pm}} - \frac{1240\ \text{keV}\cdot\text{pm}}{71.0\ \text{pm}} = 2.2\ \text{keV} .$$

43. (a) An electron must be removed from the K-shell, so that an electron from a higher energy shell can drop. This requires an energy of 69.5 keV. The accelerating potential must be at least 69.5 kV.

(b) After it is accelerated, the kinetic energy of the bombarding electron is 69.5 keV. The energy of a photon associated with the minimum wavelength is 69.5 keV, so its wavelength is

$$\lambda_{\min} = \frac{1240\ \text{eV}\cdot\text{nm}}{69.5\times 10^3\ \text{eV}} = 1.78\times 10^{-2}\ \text{nm} = 17.8\ \text{pm} .$$

(c) The energy of a photon associated with the K_α line is 69.5 keV – 11.3 keV = 58.2 keV and its wavelength is

$$\lambda_{K\alpha} = (1240\ \text{eV}\cdot\text{nm})/(58.2\times 10^3\ \text{eV}) = 2.13\times 10^{-2}\ \text{nm} = 21.3\ \text{pm}.$$

(d) The energy of a photon associated with the K_β line is

$$E = 69.5\ \text{keV} - 2.30\ \text{keV} = 67.2\ \text{keV}$$

and its wavelength is, using hc = 1240 eV·nm,

$$\lambda_{K\beta} = hc/E = (1240\ \text{eV}\cdot\text{nm})/(67.2\times 10^3\ \text{eV}) = 1.85\times 10^{-2}\ \text{nm} = 18.5\ \text{pm}.$$

44. (a) and (b) Let the wavelength of the two photons be λ_1 and $\lambda_2 = \lambda_1 + \Delta\lambda$. Then,

$$eV = \frac{hc}{\lambda_1} + \frac{hc}{\lambda_1 + \Delta\lambda} \quad \Rightarrow \quad \lambda_1 = \frac{-(\Delta\lambda/\lambda_0 - 2) \pm \sqrt{(\Delta\lambda/\lambda_0)^2 + 4}}{2/\Delta\lambda} .$$

Here, $\Delta\lambda$ = 130 pm and

$$\lambda_0 = hc/eV = 1240 \text{ keV}\cdot\text{pm}/20 \text{ keV} = 62 \text{ pm},$$

where we have used hc = 1240 eV·nm = 1240 keV·pm. We choose the plus sign in the expression for λ_1 (since $\lambda_1 > 0$) and obtain

$$\lambda_1 = \frac{-\left(130\,\text{pm}/62\,\text{pm}-2\right)+\sqrt{\left(130\,\text{pm}/62\,\text{pm}\right)^2+4}}{2/62\ \text{pm}} = 87\ \text{pm}\,.$$

The energy of the electron after its first deceleration is

$$K = K_i - \frac{hc}{\lambda_1} = 20\ \text{keV} - \frac{1240\ \text{keV}\cdot\text{pm}}{87\ \text{pm}} = 5.7\ \text{keV}\ .$$

(c) The energy of the first photon is $E_1 = \dfrac{hc}{\lambda_1} = \dfrac{1240\ \text{keV}\cdot\text{pm}}{87\ \text{pm}} = 14\ \text{keV}\ .$

(d) The wavelength associated with the second photon is

$$\lambda_2 = \lambda_1 + \Delta\lambda = 87\,\text{pm} + 130\,\text{pm} = 2.2\times10^2\ \text{pm}\ .$$

(e) The energy of the second photon is $E_2 = \dfrac{hc}{\lambda_2} = \dfrac{1240\ \text{keV}\cdot\text{pm}}{2.2\times10^2\,\text{pm}} = 5.7\ \text{keV}.$

45. The initial kinetic energy of the electron is K_0 = 50.0 keV. After the first collision, the kinetic energy is K_1 = 25 keV; after the second, it is K_2 = 12.5 keV; and after the third, it is zero.

(a) The energy of the photon produced in the first collision is 50.0 keV – 25.0 keV = 25.0 keV. The wavelength associated with this photon is

$$\lambda = \frac{hc}{E} = \frac{1240\,\text{eV}\cdot\text{nm}}{25.0\times10^3\ \text{eV}} = 4.96\times10^{-2}\ \text{nm} = 49.6\ \text{pm}$$

where we have used hc = 1240 eV·nm.

(b) The energies of the photons produced in the second and third collisions are each 12.5 keV and their wavelengths are

$$\lambda = \frac{1240\,\text{eV}\cdot\text{nm}}{12.5\times10^3\ \text{eV}} = 9.92\times10^{-2}\ \text{nm} = 99.2\,\text{pm}\ .$$

46. The transition is from n = 2 to n = 1, so Eq. 40-26 combined with Eq. 40-24 yields

$$f = \left(\frac{m_e e^4}{8\varepsilon_0^2 h^3}\right)\left(\frac{1}{1^2} - \frac{1}{2^2}\right)(Z-1)^2$$

so that the constant in Eq. 40-27 is

$$C = \sqrt{\frac{3m_e e^4}{32\varepsilon_0^2 h^3}} = 4.9673\times10^7\ \text{Hz}^{1/2}$$

using the values in the next-to-last column in the table in Appendix B (but note that the power of ten is given in the middle column).

We are asked to compare the results of Eq. 40-27 (squared, then multiplied by the accurate values of h/e found in Appendix B to convert to x-ray energies) with those in the table of K_α energies (in eV) given at the end of the problem. We look up the corresponding atomic numbers in Appendix F.

(a) For Li, with Z = 3, we have

$$E_{\text{theory}} = \frac{h}{e}C^2(Z-1)^2 = \frac{6.6260688\times10^{-34}\ \text{J}\cdot\text{s}}{1.6021765\times10^{-19}\ \text{J/eV}}\left(4.9673\times10^7\ \text{Hz}^{1/2}\right)^2(3-1)^2 = 40.817\,\text{eV}.$$

The percentage deviation is

$$\text{percentage deviation} = 100\left(\frac{E_{\text{theory}} - E_{\text{exp}}}{E_{\text{exp}}}\right) = 100\left(\frac{40.817 - 54.3}{54.3}\right) = -24.8\% \approx -25\%.$$

In subsequent calculations, we use the steps outlined above.

(b) For Be, with Z = 4, the percentage deviation is –15%.

(c) For B, with Z = 5, the percentage deviation is –11%.

(d) For C, with Z = 6, the percentage deviation is –7.9%.

(e) For N, with Z = 7, the percentage deviation is –6.4%.

(f) For O, with Z = 8, the percentage deviation is –4.7%.

(g) For F, with Z = 9, the percentage deviation is –3.5%.

(h) For Ne, with Z = 10, the percentage deviation is –2.6%.

(i) For Na, with Z = 11, the percentage deviation is –2.0%.

(j) For Mg, with Z = 12, the percentage deviation is –1.5%.

Note that the trend is clear from the list given above: the agreement between theory and experiment becomes better as Z increases. One might argue that the most questionable step in Section 40-10 is the replacement $e^4 \to (Z-1)^2 e^4$ and ask why this could not equally well be $e^4 \to (Z-.9)^2 e^4$ or $e^4 \to (Z-.8)^2 e^4$. For large Z, these subtleties would not matter so much as they do for small Z, since $Z - \xi \approx Z$ for $Z >> \xi$.

47. Let the power of the laser beam be P and the energy of each photon emitted be E. Then, the rate of photon emission is

$$R = \frac{P}{E} = \frac{P}{hc/\lambda} = \frac{P\lambda}{hc} = \frac{(5.0\times10^{-3}\,\text{W})\,(0.80\times10^{-6}\,\text{m})}{(6.63\times10^{-34}\,\text{J}\cdot\text{s})\,(2.998\times10^{8}\ \text{m/s})} = 2.0\times10^{16}\ \text{s}^{-1}.$$

48. The Moon is a distance $R = 3.82 \times 10^8$ m from Earth (see Appendix C). We note that the "cone" of light has apex angle equal to 2θ. If we make the small angle approximation (equivalent to using Eq. 36-14), then the diameter D of the spot on the Moon is

$$D = 2R\theta = 2R\left(\frac{1.22\lambda}{d}\right) = \frac{2(3.82\times10^{8}\,\text{m})\,(1.22)\,(600\times10^{-9}\,\text{m})}{0.12\,\text{m}} = 4.7\times10^{3}\,\text{m} = 4.7\,\text{km}.$$

49. Let the range of frequency of the microwave be Δf. Then the number of channels that could be accommodated is

$$N = \frac{\Delta f}{10\ \text{MHz}} = \frac{(2.998\times10^{8}\ \text{m/s})\,\left[(450\ \text{nm})^{-1} - (650\ \text{nm})^{-1}\right]}{10\ \text{MHz}} = 2.1\times10^{7}.$$

The higher frequencies of visible light would allow many more channels to be carried compared with using the microwave.

50. From Eq. 40-29, $N_2/N_1 = e^{-(E_2-E_1)/kT}$. We solve for T:

$$T = \frac{E_2 - E_1}{k\ln(N_1/N_2)} = \frac{3.2\,\text{eV}}{(1.38\times10^{-23}\ \text{J/K})\,\ln\left(2.5\times10^{15}/6.1\times10^{13}\right)} = 1.0\times10^{4}\ \text{K}.$$

51. **THINK** The number of atoms in a state with energy E is proportional to $e^{-E/kT}$, where T is the temperature on the Kelvin scale and k is the Boltzmann constant.

EXPRESS Thus, the ratio of the number of atoms in the thirteenth excited state to the number in the eleventh excited state is

$$\frac{n_{13}}{n_{11}} = \frac{e^{-E_{13}/kT}}{e^{-E_{11}/kT}} = e^{-(E_{13}-E_{11})/kT} = e^{-\Delta E/kT},$$

where $\Delta E = E_{13} - E_{11}$ is the difference in the energies:

$$\Delta E = E_{13} - E_{11} = 2(1.2 \text{ eV}) = 2.4 \text{ eV}.$$

ANALYZE For the given temperature, $kT = (8.62 \times 10^{-2}$ eV/K)(2000 K) = 0.1724 eV. Hence,

$$\frac{n_{13}}{n_{11}} = e^{-2.4/0.1724} = 9.0 \times 10^{-7}.$$

LEARN The 13th excited state has higher energy than the 11th excited state. Therefore, we expect fewer atoms to be in the 13th excited state.

52. The energy of the laser pulse is

$$E_p = P\Delta t = (2.80 \times 10^6 \text{ J/s})(0.500 \times 10^{-6} \text{ s}) = 1.400 \text{ J}.$$

Since the energy carried by each photon is

$$E = \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})(2.998 \times 10^8 \text{ m/s})}{424 \times 10^{-9} \text{ m}} = 4.69 \times 10^{-19} \text{ J},$$

the number of photons emitted in each pulse is

$$N = \frac{E_p}{E} = \frac{1.400 \text{J}}{4.69 \times 10^{-19} \text{ J}} = 3.0 \times 10^{18} \text{ photons.}$$

With each atom undergoing stimulated emission only once, the number of atoms contributed to the pulse is also 3.0×10^{18}.

53. Let the power of the laser beam be P and the energy of each photon emitted be E. Then, the rate of photon emission is

$$R = \frac{P}{E} = \frac{P}{hc/\lambda} = \frac{P\lambda}{hc} = \frac{\left(2.3 \times 10^{-3} \text{ W}\right)\left(632.8 \times 10^{-9} \text{ m}\right)}{\left(6.63 \times 10^{-34} \text{ J} \cdot \text{s}\right)\left(2.998 \times 10^8 \text{ m/s}\right)} = 7.3 \times 10^{15} \text{ s}^{-1}.$$

54. According to Sample Problem — "Population inversion in a laser," the population ratio at room temperature is $N_x/N_0 = 1.3 \times 10^{-38}$. Let the number of moles of the lasing material needed be n; then $N_0 = nN_A$, where N_A is the Avogadro constant. Also $N_x = 10$. We solve for n:

$$n = \frac{N_x}{\left(1.3\times10^{-38}\right)N_A} = \frac{10}{\left(1.3\times10^{-38}\right)\left(6.02\times10^{23}\right)} = 1.3\times10^{15}\ \text{mol.}$$

55. (a) If t is the time interval over which the pulse is emitted, the length of the pulse is

$$L = ct = (3.00 \times 10^{8}\ \text{m/s})(1.20 \times 10^{-11}\ \text{s}) = 3.60 \times 10^{-3}\ \text{m.}$$

(b) If E_p is the energy of the pulse, E is the energy of a single photon in the pulse, and N is the number of photons in the pulse, then $E_p = NE$. The energy of the pulse is

$$E_p = (0.150\ \text{J})/(1.602 \times 10^{-19}\ \text{J/eV}) = 9.36 \times 10^{17}\ \text{eV}$$

and the energy of a single photon is $E = (1240\ \text{eV}\cdot\text{nm})/(694.4\ \text{nm}) = 1.786\ \text{eV}$. Hence,

$$N = \frac{E_p}{E} = \frac{9.36\times10^{17}\ \text{eV}}{1.786\ \text{eV}} = 5.24\times10^{17}\ \text{photons.}$$

56. Consider two levels, labeled 1 and 2, with $E_2 > E_1$. Since $T = -|T| < 0$,

$$\frac{N_2}{N_1} = e^{-(E_2-E_1)/kT} = e^{-|E_2-E_1|/(-k|T|)} = e^{|E_2-E_1|/k|T|} > 1.$$

Thus, $N_2 > N_1$; this is population inversion. We solve for T:

$$T = -|T| = -\frac{E_2 - E_1}{k\ln\left(N_2/N_1\right)} = -\frac{2.26\ \text{eV}}{\left(8.62\times10^{-5}\ \text{eV/K}\right)\ln\left(1+0.100\right)} = -2.75\times10^{5}\ \text{K.}$$

57. (a) We denote the upper level as level 1 and the lower one as level 2. From $N_1/N_2 = e^{-(E_2-E_1)/kT}$ we get (using $hc = 1240$ eV·nm)

$$N_1 = N_2e^{-(E_1-E_2)/kT} = N_2e^{-hc/\lambda kT} = \left(4.0\times10^{20}\right)\exp\left[-\frac{1240\,\text{eV}\cdot\text{nm}}{(580\,\text{nm})(8.62\times10^{-5}\,\text{eV/K})(300\,\text{K})}\right]$$
$$= 5.0\times10^{-16} << 1,$$

so practically no electron occupies the upper level.

(b) With $N_1 = 3.0 \times 10^{20}$ atoms emitting photons and $N_2 = 1.0 \times 10^{20}$ atoms absorbing photons, then the net energy output is

$$E=(N_1-N_2)E_{\text{photon}}=(N_1-N_2)\frac{hc}{\lambda}=(2.0\times10^{20})\frac{(6.63\times10^{-34}\,\text{J}\cdot\text{s})(2.998\times10^{8}\ \text{m/s})}{580\times10^{-9}\,\text{m}}$$
$$=68\,\text{J}.$$

58. For the nth harmonic of the standing wave of wavelength λ in the cavity of width L we have $n\lambda = 2L$, so $n\Delta\lambda + \lambda\Delta n = 0$. Let $\Delta n = \pm1$ and use $\lambda = 2L/n$ to obtain

$$|\Delta\lambda|=\frac{\lambda|\Delta n|}{n}=\frac{\lambda}{n}=\lambda\left(\frac{\lambda}{2L}\right)=\frac{(533\ \text{nm})^2}{2(8.0\times10^{7}\ \text{nm})}=1.8\times10^{-12}\ \text{m}=1.8\ \text{pm}.$$

59. For stimulated emission to take place, we need a long-lived state above a short-lived state in both atoms. In addition, for the light emitted by A to cause stimulated emission of B, an energy match for the transitions is required. The above conditions are fulfilled for the transition from the 6.9 eV state (lifetime 3 ms) to 3.9 eV state (lifetime 3 μs) in A, and the transition from 10.8 eV (lifetime 3 ms) to 7.8 eV (lifetime 3 μs) in B. Thus, the energy per photon of the stimulated emission of B is $10.8\,\text{eV}-7.8\,\text{eV}=3.0\,\text{eV}$.

60. (a) The radius of the central disk is

$$R=\frac{1.22\,f\,\lambda}{d}=\frac{(1.22)(3.50\ \text{cm})(515\ \text{nm})}{3.00\ \text{mm}}=7.33\ \mu\text{m}.$$

(b) The average power flux density in the incident beam is

$$\frac{P}{\pi d^2/4}=\frac{4(5.00\,\text{W})}{\pi(3.00\,\text{mm})^2}=7.07\times10^{5}\,\text{W/m}^2.$$

(c) The average power flux density in the central disk is

$$\frac{(0.84)P}{\pi R^2}=\frac{(0.84)(5.00\,\text{W})}{\pi(7.33\,\mu\text{m})^2}=2.49\times10^{10}\,\text{W/m}^2.$$

61. (a) If both mirrors are perfectly reflecting, there is a node at each end of the crystal. With one end partially silvered, there is a node very close to that end. We assume nodes at both ends, so there are an integer number of half-wavelengths in the length of the crystal. The wavelength in the crystal is $\lambda_c = \lambda/n$, where λ is the wavelength in a vacuum and n is the index of refraction of ruby. Thus $N(\lambda/2n) = L$, where N is the number of standing wave nodes, so

$$N=\frac{2nL}{\lambda}=\frac{2(1.75)(0.0600\ \text{m})}{694\times10^{-9}\ \text{m}}=3.03\times10^{5}.$$

(b) Since $\lambda = c/f$, where f is the frequency, $N = 2nLf/c$ and $\Delta N = (2nL/c)\Delta f$. Hence,

$$\Delta f = \frac{c\Delta N}{2nL} = \frac{(2.998\times10^8 \text{ m/s})(1)}{2(1.75)(0.0600 \text{ m})} = 1.43\times10^9 \text{ Hz}.$$

(c) The speed of light in the crystal is c/n and the round-trip distance is $2L$, so the round-trip travel time is $2nL/c$. This is the same as the reciprocal of the change in frequency.

(d) The frequency is

$$f = c/\lambda = (2.998\times10^8 \text{ m/s})/(694\times10^{-9} \text{ m}) = 4.32\times10^{14} \text{ Hz}$$

and the fractional change in the frequency is

$$\Delta f/f = (1.43\times10^9 \text{ Hz})/(4.32\times10^{14} \text{ Hz}) = 3.31\times10^{-6}.$$

62. The energy carried by each photon is

$$E = \frac{hc}{\lambda} = \frac{(6.63\times10^{-34} \text{ J}\cdot\text{s})(2.998\times10^8 \text{ m/s})}{694\times10^{-9} \text{ m}} = 2.87\times10^{-19} \text{ J}.$$

Now, the photons emitted by the Cr ions in the excited state can be absorbed by the ions in the ground state. Thus, the average power emitted during the pulse is

$$P = \frac{(N_1 - N_0)E}{\Delta t} = \frac{(0.600-0.400)(4.00\times10^{19})(2.87\times10^{-19} \text{ J})}{2.00\times10^{-6} \text{ s}} = 1.1\times10^6 \text{ J/s}$$

or 1.1×10^6 W.

63. Due to spin degeneracy ($m_s = \pm1/2$), each state can accommodate two electrons. Thus, in the energy-level diagram shown, two electrons can be placed in the ground state with energy $E_1 = 3(h^2/8mL^2)$, six can occupy the "triple state" with $E_2 = 6(h^2/8mL^2)$, and so forth. With 22 electrons in the system, the lowest energy configuration consists of two electrons with $E_1 = 3(h^2/8mL^2)$, six electrons with $E_2 = 6(h^2/8mL^2)$, six electrons with $E_3 = 9(h^2/8mL^2)$, six electrons with $E_4 = 11(h^2/8mL^2)$, and two electrons with $E_5 = 12(h^2/8mL^2)$. Thus, we find the ground-state energy of the 22-electron system to be

$$\begin{aligned}E_{\text{ground}} &= 2E_1 + 6E_2 + 6E_3 + 6E_4 + 2E_5 \\ &= 2\left(\frac{3h^2}{8mL^2}\right) + 6\left(\frac{6h^2}{8mL^2}\right) + 6\left(\frac{9h^2}{8mL^2}\right) + 6\left(\frac{11h^2}{8mL^2}\right) + 2\left(\frac{12h^2}{8mL^2}\right) \\ &= \left[(2)(3) + (6)(6) + (6)(9) + (6)(11) + (2)(12)\right]\left(\frac{h^2}{8mL^2}\right) \\ &= 186\left(\frac{h^2}{8mL^2}\right).\end{aligned}$$

Thus, the multiple of $h^2/8mL^2$ is 186.

64. (a) In the lasing action the molecules are excited from energy level E_0 to energy level E_2. Thus the wavelength λ of the sunlight that causes this excitation satisfies

$$\Delta E = E_2 - E_0 = \frac{hc}{\lambda},$$

which gives (using hc = 1240 eV·nm)

$$\lambda = \frac{hc}{E_2 - E_0} = \frac{1240 \text{ eV}\cdot\text{nm}}{0.289 \text{ eV} - 0} = 4.29\times 10^3 \text{ nm} = 4.29\ \mu\text{m}.$$

(b) Lasing occurs as electrons jump down from the higher energy level E_2 to the lower level E_1. Thus the lasing wavelength λ' satisfies

$$\Delta E' = E_2 - E_1 = \frac{hc}{\lambda'},$$

which gives

$$\lambda' = \frac{hc}{E_2 - E_1} = \frac{1240 \text{ eV}\cdot\text{nm}}{0.289 \text{ eV} - 0.165 \text{ eV}} = 1.00\times 10^4 \text{ nm} = 10.0\ \mu\text{m}.$$

(c) Both λ and λ' belong to the infrared region of the electromagnetic spectrum.

65. (a) Using hc = 1240 eV·nm,

$$\Delta E = hc\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right) = (1240\,\text{eV}\cdot\text{nm})\left(\frac{1}{588.995\,\text{nm}} - \frac{1}{589.592\,\text{nm}}\right) = 2.13\,\text{meV}\ .$$

(b) From $\Delta E = 2\mu_B B$ (see Fig. 40-10 and Eq. 40-18), we get

$$B = \frac{\Delta E}{2\mu_B} = \frac{2.13\times 10^{-3}\,\text{eV}}{2\left(5.788\times 10^{-5}\,\text{eV/T}\right)} = 18\,\text{T}\ .$$

66. (a) The energy difference between the two states 1 and 2 was equal to the energy of the photon emitted. Since the photon frequency was f = 1666 MHz, its energy was given by

$$hf = (4.14 \times 10^{-15}\ \text{eV}\cdot\text{s})(1666\ \text{MHz}) = 6.90 \times 10^{-6}\ \text{eV}.$$

Thus,

$$E_2 - E_1 = hf = 6.90\times10^{-6}\ \text{eV} = 6.90\ \mu\text{eV}.$$

(b) The emission was in the *radio* region of the electromagnetic spectrum.

67. Letting $eV = hc/\lambda_{\min}$ (see Eq. 40-23 and Eq. 38-4), we get

$$\lambda_{\min} = \frac{hc}{eV} = \frac{1240\,\text{nm}\cdot\text{eV}}{eV} = \frac{1240\,\text{pm}\cdot\text{keV}}{eV} = \frac{1240\,\text{pm}}{V}$$

where V is measured in kV.

68. (a) The distance from the Earth to the Moon is $d_{em} = 3.82 \times 10^8$ m (see Appendix C). Thus, the time required is given by

$$t = \frac{2d_{em}}{c} = \frac{2\left(3.82\times10^8\ \text{m}\right)}{2.998\times10^8\ \text{m/s}} = 2.55\ \text{s}.$$

(b) We denote the uncertainty in time measurement as δt and let $2\delta d_{es}$ = 15 cm. Then, since $d_{em} \propto t$, $\delta t/t = \delta d_{em}/d_{em}$. We solve for δt:

$$\delta t = \frac{t\delta d_{em}}{d_{em}} = \frac{(2.55\ \text{s})(0.15\ \text{m})}{2\left(3.82\times10^8\ \text{m}\right)} = 5.0\times10^{-10}\ \text{s}.$$

(c) The angular divergence of the beam is

$$\theta = 2\tan^{-1}\left(\frac{1.5\times10^3}{d_{em}}\right) = 2\tan^{-1}\left(\frac{1.5\times10^3}{3.82\times10^8}\right) = (4.5\times10^{-4})^\circ\,.$$

69. **THINK** The intensity at the target is given by $I = P/A$, where P is the power output of the source and A is the area of the beam at the target. We want to compute I and compare the result with 10^8 W/m^2.

EXPRESS The laser beam spreads because diffraction occurs at the aperture of the laser. Consider the part of the beam that is within the central diffraction maximum. The angular position of the edge is given by $\sin\theta = 1.22\lambda/d$, where λ is the wavelength and d is the diameter of the aperture. At the target, a distance D away, the radius of the beam is

$r = D\tan\theta$. Since θ is small, we may approximate both $\sin\theta$ and $\tan\theta$ by θ, in radians. Then,

$$r = D\theta = 1.22D\lambda/d.$$

ANALYZE (a) Thus, we find the intensity to be

$$I = \frac{P}{\pi r^2} = \frac{Pd^2}{\pi(1.22D\lambda)^2} = \frac{\left(5.0\times10^6\,\mathrm{W}\right)\left(4.0\,\mathrm{m}\right)^2}{\pi\left[1.22\left(3000\times10^3\,\mathrm{m}\right)\left(3.0\times10^{-6}\,\mathrm{m}\right)\right]^2} = 2.1\times10^5\ \mathrm{W/m^2},$$

not great enough to destroy the missile.

(b) We solve for the wavelength in terms of the intensity and substitute $I = 1.0\times10^8$ W/m^2:

$$\lambda = \frac{d}{1.22D}\sqrt{\frac{P}{\pi I}} = \frac{4.0\,\mathrm{m}}{1.22(3000\times10^3\,\mathrm{m})}\sqrt{\frac{5.0\times10^6\,\mathrm{W}}{\pi(1.0\times10^8\,\mathrm{W/m^2})}} = 1.40\times10^{-7}\ \mathrm{m} = 140\,\mathrm{nm}.$$

LEARN The wavelength corresponds to the x-rays on the electromagnetic spectrum.

70. (a) From Fig. 40-14 we estimate the wavelengths corresponding to the K_β line to be λ_β = 63.0 pm. Using hc = 1240 eV·nm = 1240 keV·pm, we have

$$E_\beta = (1240\ \mathrm{keV\cdot pm})/(63.0\ \mathrm{pm}) = 19.7\ \mathrm{keV} \approx 20\ \mathrm{keV}.$$

(b) For K_α, with λ_α = 70.0 pm, $E_\alpha = \dfrac{hc}{\lambda_\alpha} = \dfrac{1240\,\mathrm{keV\cdot pm}}{70.0\,\mathrm{pm}} = 17.7\,\mathrm{keV} \approx 18\ \mathrm{keV}$.

(c) Both Zr and Nb can be used, since E_α < 18.00 eV < E_β and E_α < 18.99 eV < E_β. According to the hint given in the problem statement, Zr is the best choice.

(d) Nb is the second best choice.

71. The principal quantum number n must be greater than 3. The magnetic quantum number m_ℓ can have any of the values – 3, – 2, – 1, 0, +1, +2, or +3. The spin quantum number can have either of the values $-\frac{1}{2}$ or $+\frac{1}{2}$.

72. For a given shell with quantum number n the total number of available electron states is $2n^2$. Thus, for the first four shells (n = 1 through 4) the numbers of available states are 2, 8, 18, and 32 (see Appendix G). Since 2 + 8 + 18 + 32 = 60 < 63, according to the "logical" sequence the first four shells would be completely filled in an europium atom, leaving 63 – 60 = 3 electrons to partially occupy the n = 5 shell. Two of these three electrons would fill up the 5s subshell, leaving only one remaining electron in the only partially filled subshell (the 5p subshell). In chemical reactions this electron would have the tendency to be transferred to another element, leaving the remaining 62 electrons in

chemically stable, completely filled subshells. This situation is very similar to the case of sodium, which also has only one electron in a partially filled shell (the 3*s* shell).

73. **THINK** One femtosecond (fs) is equal to 10^{-15} s.

EXPRESS The length of the pulse's wave train is given by $L = c\Delta t$, where Δt is the duration of the laser. Thus, the number of wavelengths contained in the pulse is

$$N = \frac{L}{\lambda} = \frac{c\Delta t}{\lambda}.$$

ANALYZE (a) With λ = 500 nm and $\Delta t = 10\times10^{-15}$ s, we have

$$N = \frac{L}{\lambda} = \frac{(3.0\times10^{8}\text{ m/s})(10\times10^{-15}\text{ s})}{500\times10^{-9}\text{ m}} = 6.0.$$

(b) We solve for X from 10 fm/1 m = 1 s/X:

$$X = \frac{(1\text{ s})(1\text{ m})}{10\times10^{-15}\text{ m}} = \frac{1\text{ s}}{(10\times10^{-15})(3.15\times10^{7}\text{ s/y})} = 3.2\times10^{6}\text{ y}.$$

LEARN Femtosecond lasers have important applications in areas such as micro-machining and optical data storage.

74. One way to think of the units of h is that, because of the equation $E = hf$ and the fact that f is in cycles/second, then the "explicit" units for h should be J·s/cycle. Then, since 2π rad/cycle is a conversion factor for $\text{cycles}\rightarrow\text{radians}$, $\hbar = h/2\pi$ can be thought of as the Planck constant expressed in terms of radians instead of cycles. Using the precise values stated in Appendix B,

$$\hbar = \frac{h}{2\pi} = \frac{6.62606876\times10^{-34}\,\text{J}\cdot\text{s}}{2\pi} = 1.05457\times10^{-34}\,\text{J}\cdot\text{s} = \frac{1.05457\times10^{-34}\,\text{J}\cdot\text{s}}{1.6021765\times10^{-19}\text{ J/eV}}$$
$$= 6.582\times10^{-16}\,\text{eV}\cdot\text{s}.$$

75. Without the spin degree of freedom the number of available electron states for each shell would be reduced by half. So the values of Z for the noble gas elements would become half of what they are now: Z = 1, 5, 9, 18, 27, and 43. Of this set of numbers, the only one that coincides with one of the familiar noble gas atomic numbers (Z = 2, 10, 18, 36, 54, and 86) is 18. Thus, argon would be the only one that would remain "noble."

76. (a) The value of ℓ satisfies $\sqrt{\ell(\ell+1)}\hbar \approx \sqrt{\ell^2}\hbar = \ell\hbar = L$, so $\ell \simeq L/\hbar \simeq 3\times10^{74}$.

(b) The number is $2\ell + 1 \approx 2(3 \times 10^{74}) = 6 \times 10^{74}$.

(c) Since

$$\cos\theta_{\min} = \frac{m_{\ell\,\max}\hbar}{\sqrt{\ell(\ell+1)}\hbar} = \frac{1}{\sqrt{\ell(\ell+1)}} \approx 1 - \frac{1}{2\ell} = 1 - \frac{1}{2(3\times 10^{74})}$$

or $\cos\theta_{\min} \simeq 1 - \theta_{\min}^2/2 \approx 1 - 10^{-74}/6$, we have

$$\theta_{\min} \simeq \sqrt{10^{-74}/3} = 6\times 10^{-38}\text{ rad}.$$

The correspondence principle requires that all the quantum effects vanish as $\hbar \to 0$. In this case $\hbar/L$ is extremely small so the quantization effects are barely existent, with $\theta_{\min} \simeq 10^{-38}\text{ rad} \simeq 0$.

77. We use $eV = hc/\lambda_{\min}$ (see Eq. 40-23 and Eq. 38-4):

$$h = \frac{eV\lambda_{\min}}{c} = \frac{(1.60\times 10^{-19}\text{ C})(40.0\times 10^3\text{ eV})(31.1\times 10^{-12}\text{ m})}{2.998\times 10^8\text{ m/s}} = 6.63\times 10^{-34}\text{ J}\cdot\text{s}.$$

78. Using hc = 1240 eV·nm, we find the energy difference to be

$$\Delta E = hc\left(\frac{1}{\lambda_A} - \frac{1}{\lambda_B}\right) = (1240\text{ eV}\cdot\text{nm})\left(\frac{1}{500\text{ nm}} - \frac{1}{510\text{ nm}}\right) = 0.049\text{ eV}.$$

79. (a) Using $E = -\partial V/\partial r$, we find the electric field to be

$$E = -\frac{\partial V}{\partial r} = -\frac{\partial}{\partial r}\left[\frac{Ze}{4\pi\varepsilon_0}\left(\frac{1}{r} - \frac{3}{2R} + \frac{r^2}{2R^3}\right)\right] = \frac{Ze}{4\pi\varepsilon_0}\left(\frac{1}{r^2} - \frac{r}{R^3}\right)$$

(b) The electric field at $r = R$ vanishes: $E(r=R) = \dfrac{Ze}{4\pi\varepsilon_0}\left(\dfrac{1}{R^2} - \dfrac{R}{R^3}\right) = 0$. Since $V = 0$ outside the sphere, we conclude that the electric field is zero in the region $r \geq R$.

(c) At $r = R$, the electric potential is

$$V(r=R) = \frac{Ze}{4\pi\varepsilon_0}\left(\frac{1}{R} - \frac{3}{2R} + \frac{R^2}{2R^3}\right) = 0$$

The electric potential outside the sphere is also zero.

Chapter 41

1. According to Eq. 41-9, the Fermi energy is given by

$$E_F = \left(\frac{3}{16\sqrt{2}\pi}\right)^{2/3} \frac{h^2}{m} n^{2/3}$$

where n is the number of conduction electrons per unit volume, m is the mass of an electron, and h is the Planck constant. This can be written $E_F = An^{2/3}$, where

$$A = \left(\frac{3}{16\sqrt{2}\pi}\right)^{2/3} \frac{h^2}{m} = \left(\frac{3}{16\sqrt{2}\pi}\right)^{2/3} \frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})^2}{9.109\times10^{-31}\ \text{kg}} = 5.842\times10^{-38}\ \text{J}^2\cdot\text{s}^2/\text{kg}\ .$$

Since $1\ \text{J} = 1\ \text{kg}\cdot\text{m}^2/\text{s}^2$, the units of A can be taken to be $\text{m}^2\cdot\text{J}$. Dividing by 1.602×10^{-19} J/eV, we obtain $A = 3.65\times10^{-19}\ \text{m}^2\cdot\text{eV}$.

2. Equation 41-5 gives

$$N(E) = \frac{8\sqrt{2}\pi m^{3/2}}{h^3} E^{1/2}$$

for the density of states associated with the conduction electrons of a metal. This can be written

$$N(E) = CE^{1/2}$$

where

$$C = \frac{8\sqrt{2}\pi m^{3/2}}{h^3} = \frac{8\sqrt{2}\pi(9.109\times10^{-31}\ \text{kg})^{3/2}}{(6.626\times10^{-34}\ \text{J}\cdot\text{s})^3} = 1.062\times10^{56}\ \text{kg}^{3/2}/\text{J}^3\cdot\text{s}^3$$
$$= 6.81\times10^{27}\ \text{m}^{-3}\cdot(\text{eV})^{-2/3}.$$

Thus,

$$N(E) = CE^{1/2} = \left[6.81\times10^{27}\ \text{m}^{-3}\cdot(\text{eV})^{-2/3}\right](8.0\,\text{eV})^{1/2} = 1.9\times10^{28}\ \text{m}^{-3}\cdot\text{eV}^{-1}\ .$$

This is consistent with that shown in Fig. 41-6.

3. The number of atoms per unit volume is given by $n = d/M$, where d is the mass density of copper and M is the mass of a single copper atom. Since each atom contributes one conduction electron, n is also the number of conduction electrons per unit volume. Since the molar mass of copper is $A = 63.54\,\text{g/mol}$,

$$M = A/N_A = (63.54\,\text{g/mol})/(6.022\times10^{23}\ \text{mol}^{-1}) = 1.055\times10^{-22}\ \text{g}\,.$$

Thus,

$$n = \frac{8.96\,\text{g/cm}^3}{1.055\times10^{-22}\,\text{g}} = 8.49\times10^{22}\,\text{cm}^{-3} = 8.49\times10^{28}\,\text{m}^{-3}\,.$$

4. Let E_1 = 63 meV + E_F and E_2 = – 63 meV + E_F. Then according to Eq. 41-6,

$$P_1 = \frac{1}{e^{(E_1-E_F)/kT}+1} = \frac{1}{e^x+1}$$

where $x = (E_1 - E_F)/kT$. We solve for e^x:

$$e^x = \frac{1}{P_1} - 1 = \frac{1}{0.090} - 1 = \frac{91}{9}\,.$$

Thus,

$$P_2 = \frac{1}{e^{(E_2-E_F)/kT}+1} = \frac{1}{e^{-(E_1-E_F)/kT}+1} = \frac{1}{e^{-x}+1} = \frac{1}{(91/9)^{-1}+1} = 0.91,$$

where we use $E_2 - E_F$ = – 63 meV = $E_F - E_1$ = – ($E_1 - E_F$).

5. (a) Equation 41-5 gives

$$N(E) = \frac{8\sqrt{2}\pi m^{3/2}}{h^3}E^{1/2}$$

for the density of states associated with the conduction electrons of a metal. This can be written

$$N(E) = CE^{1/2}$$

where

$$C = \frac{8\sqrt{2}\pi m^{3/2}}{h^3} = \frac{8\sqrt{2}\pi(9.109\times10^{-31}\,\text{kg})^{3/2}}{(6.626\times10^{-34}\,\text{J}\cdot\text{s})^3} = 1.062\times10^{56}\,\text{kg}^{3/2}/\text{J}^3\cdot\text{s}^3.$$

(b) Now, $1\,\text{J} = 1\text{kg}\cdot\text{m}^2/\text{s}^2$ (think of the equation for kinetic energy $K = \frac{1}{2}mv^2$), so 1 kg = 1 J·s^2·m^{-2}. Thus, the units of C can be written as

$$(\text{J}\cdot\text{s}^2)^{3/2}\cdot(\text{m}^{-2})^{3/2}\cdot\text{J}^{-3}\cdot\text{s}^{-3} = \text{J}^{-3/2}\cdot\text{m}^{-3}.$$

This means

$$C = (1.062\times10^{56}\,\text{J}^{-3/2}\cdot\text{m}^{-3})(1.602\times10^{-19}\,\text{J/eV})^{3/2} = 6.81\times10^{27}\,\text{m}^{-3}\cdot\text{eV}^{-3/2}\,.$$

(c) If E = 5.00 eV, then

$$N(E) = (6.81\times10^{27}\,\text{m}^{-3}\cdot\text{eV}^{-3/2})(5.00\,\text{eV})^{1/2} = 1.52\times10^{28}\,\text{eV}^{-1}\cdot\text{m}^{-3}\,.$$

6. We note that $n = 8.43 \times 10^{28}\ \text{m}^{-3} = 84.3\ \text{nm}^{-3}$. From Eq. 41-9,

$$E_F = \frac{0.121(hc)^2}{m_e c^2} n^{2/3} = \frac{0.121(1240\,\text{eV}\cdot\text{nm})^2}{511\times 10^3\ \text{eV}}(84.3\,\text{nm}^{-3})^{2/3} = 7.0\ \text{eV}$$

where we have used $hc = 1240\,\text{eV}\cdot\text{nm}$.

7. **THINK** This problem deals with occupancy probability $P(E)$, the probability that an energy level will be occupied by an electron.

EXPRESS A plot of $P(E)$ as a function of E is shown in Fig. 41-7. From the figure, we see that at $T = 0$ K, $P(E)$ is unity for $E \le E_F$, where E_F is the Fermi energy, and zero for $E > E_F$. On the other hand, the probability that a state with energy E is occupied at temperature T is given by

$$P(E) = \frac{1}{e^{(E-E_F)/kT}+1}$$

where k is the Boltzmann constant and E_F is the Fermi energy.

ANALYZE (a) At absolute temperature $T = 0$, the probability is zero that any state with energy above the Fermi energy is occupied.

(b) Now, $E - E_F = 0.0620$ eV, and

$$(E - E_F)/kT = (0.0620\,\text{eV})/(8.62\times 10^{-5}\ \text{eV/K})(320\ \text{K}) = 2.248 .$$

We find $P(E)$ to be

$$P(E) = \frac{1}{e^{2.248}+1} = 0.0955 .$$

See Appendix B for the value of k.

LEARN When $E = E_F$, the occupancy probability is $P(E_F) = 0.5$. Thus, one may think of the Fermi energy as the energy of a quantum state that has a probability 0.5 of being occupied by an electron.

8. We note that there is one conduction electron per atom and that the molar mass of gold is $197\,\text{g/mol}$. Therefore, combining Eqs. 41-2, 41-3, and 41-4 leads to

$$n = \frac{(19.3\,\text{g/cm}^3)(10^6\ \text{cm}^3/\text{m}^3)}{(197\,\text{g/mol})/(6.02\times 10^{23}\ \text{mol}^{-1})} = 5.90\times 10^{28}\ \text{m}^{-3} .$$

9. **THINK** According to Appendix F the molar mass of silver is M = 107.870 g/mol and the density is ρ = 10.49 g/cm^3. Silver is monovalent.

EXPRESS The mass of a silver atom is, dividing the molar mass by Avogadro's number:

$$M_0 = \frac{M}{N_A} = \frac{107.870\times10^{-3}\text{ kg/mol}}{6.022\times10^{23}\text{ mol}^{-1}} = 1.791\times10^{-25}\text{ kg} .$$

Since silver is monovalent, there is one valence electron per atom (see Eq. 41-2).

ANALYZE (a) The number density is

$$n = \frac{\rho}{M_0} = \frac{10.49\times10^{-3}\text{ kg/m}^3}{1.791\times10^{-25}\text{ kg}} = 5.86\times10^{28}\text{ m}^{-3} .$$

This is the same as the number density of conduction electrons.

(b) The Fermi energy is

$$E_F = \frac{0.121h^2}{m}n^{2/3} = \frac{(0.121)(6.626\times10^{-34}\text{ J}\cdot\text{s})^2}{9.109\times10^{-31}\text{kg}} = (5.86\times10^{28}\text{ m}^{-3})^{2/3}$$
$$= 8.80\times10^{-19}\text{ J} = 5.49\text{eV}.$$

(c) Since $E_F = \frac{1}{2}mv_F^2$,

$$v_F = \sqrt{\frac{2E_F}{m}} = \sqrt{\frac{2(8.80\times10^{-19}\text{ J})}{9.109\times10^{-31}\text{ kg}}} = 1.39\times10^6\text{ m/s} .$$

(d) The de Broglie wavelength is

$$\lambda = \frac{h}{mv_F} = \frac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{(9.109\times10^{-31}\text{ kg})(1.39\times10^6\text{m/s})} = 5.22\times10^{-10}\text{ m}.$$

LEARN Once the number density of conduction electrons is known, the Fermi energy for a particular metal can be calculated using Eq. 41-9.

10. The probability P_h that a state is occupied by a hole is the same as the probability the state is *unoccupied* by an electron. Since the total probability that a state is either occupied or unoccupied is 1, we have $P_h + P = 1$. Thus,

$$P_h = 1 - \frac{1}{e^{(E-E_F)/kT}+1} = \frac{e^{(E-E_F)/kT}}{1+e^{(E-E_F)/kT}} = \frac{1}{e^{-(E-E_F)/kT}+1} .$$

11. We use

$$N_O(E) = N(E)P(E) = CE^{1/2}\left[e^{(E-E_F)/kT}+1\right]^{-1},$$

where

$$C = \frac{8\sqrt{2}\pi m^{3/2}}{h^3} = \frac{8\sqrt{2}\pi(9.109\times10^{-31}\,\text{kg})^{3/2}}{(6.626\times10^{-34}\,\text{J}\cdot\text{s})^3} = 1.062\times10^{56}\,\text{kg}^{3/2}/\text{J}^3\cdot\text{s}^3$$
$$= 6.81\times10^{27}\,\text{m}^{-3}\cdot(\text{eV})^{-3/2}.$$

(a) At E = 4.00 eV,

$$N_O = \frac{\left(6.81\times10^{27}\,\text{m}^{-3}\cdot(\text{eV})^{-3/2}\right)(4.00\,\text{eV})^{1/2}}{\exp\left((4.00\,\text{eV}-7.00\,\text{eV})/[(8.62\times10^{-5}\,\text{eV/K})(1000\,\text{K})]\right)+1} = 1.36\times10^{28}\,\text{m}^{-3}\cdot\text{eV}^{-1}.$$

(b) At E = 6.75 eV,

$$N_O = \frac{\left(6.81\times10^{27}\,\text{m}^{-3}\cdot(\text{eV})^{-3/2}\right)(6.75\,\text{eV})^{1/2}}{\exp\left((6.75\,\text{eV}-7.00\,\text{eV})/[(8.62\times10^{-5}\,\text{eV/K})(1000\,\text{K})]\right)+1} = 1.68\times10^{28}\,\text{m}^{-3}\cdot\text{eV}^{-1}.$$

(c) Similarly, at E = 7.00 eV, the value of $N_o(E)$ is $9.01\times10^{27}\,\text{m}^{-3}\cdot\text{eV}^{-1}$.

(d) At E = 7.25 eV, the value of $N_o(E)$ is $9.56\times10^{26}\,\text{m}^{-3}\cdot\text{eV}^{-1}$.

(e) At E = 9.00 eV, the value of $N_o(E)$ is $1.71\times10^{18}\,\text{m}^{-3}\cdot\text{eV}^{-1}$.

12. The molar mass of carbon is m = 12.01115 g/mol and the mass of the Earth is M_e = 5.98×10^{24} kg. Thus, the number of carbon atoms in a diamond as massive as the Earth is $N = (M_e/m)N_A$, where N_A is the Avogadro constant. From the result of Sample Problem – "Probability of electron excitation in an insulator," the probability in question is given by

$$P = N_e^{-E_g/kT} = \left(\frac{M_e}{m}\right)N_A e^{-E_g/kT} = \left(\frac{5.98\times10^{24}\,\text{kg}}{12.01115\,\text{g/mol}}\right)(6.02\times10^{23}/\text{mol})(3\times10^{-93})$$
$$= 9\times10^{-43} \approx 10^{-42}.$$

13. (a) Equation 41-6 leads to

$$E = E_F + kT\ln(P^{-1}-1) = 7.00\,\text{eV} + (8.62\times10^{-5}\,\text{eV/K})(1000\,\text{K})\ln\left(\frac{1}{0.900}-1\right) = 6.81\,\text{eV}.$$

(b) $N(E) = CE^{1/2} = \left(6.81\times10^{27}\,\text{m}^{-3}\cdot\text{eV}^{-3/2}\right)(6.81\,\text{eV})^{1/2} = 1.77\times10^{28}\,\text{m}^{-3}\cdot\text{eV}^{-1}.$

(c) $N_O(E) = P(E)N(E) = (0.900)(1.77\times10^{28}\,\text{m}^{-3}\cdot\text{eV}^{-1}) = 1.59\times10^{28}\,\text{m}^{-3}\cdot\text{eV}^{-1}$.

14. (a) The volume per cubic meter of sodium occupied by the sodium ions is

$$V_{\text{Na}} = \frac{(971\text{kg})(6.022\times10^{23}/\text{mol})(4\pi/3)(98.0\times10^{-12}\,\text{m})^3}{(23.0\,\text{g}/\text{mol})} = 0.100\,\text{m}^3,$$

so the fraction available for conduction electrons is $1-(V_{\text{Na}}/1.00\,\text{m}^3) = 1-0.100 = 0.900$, or 90.0%.

(b) For copper, we have

$$V_{\text{Cu}} = \frac{(8960\,\text{kg})(6.022\times10^{23}/\text{mol})(4\pi/3)(135\times10^{-12}\,\text{m})^3}{(63.5\,\text{g}/\text{mol})} = 0.1876\,\text{m}^{-3}\ .$$

Thus, the fraction is $1-(V_{\text{Cu}}/1.00\,\text{m}^3) = 1-0.876 = 0.124$, or 12.4%.

(c) Sodium, because the electrons occupy a greater portion of the space available.

15. **THINK** The Fermi-Dirac occupation probability is given by $P_{\text{FD}} = 1/\left(e^{\Delta E/kT}+1\right)$, and the Boltzmann occupation probability is given by $P_{\text{B}} = e^{-\Delta E/kT}$.

EXPRESS Let *f* be the fractional difference. Then

$$f = \frac{P_{\text{B}} - P_{\text{FD}}}{P_{\text{B}}} = \frac{e^{-\Delta E/kT} - \frac{1}{e^{\Delta E/kT}+1}}{e^{-\Delta E/kT}}\ .$$

Using a common denominator and a little algebra yields $f = \dfrac{e^{-\Delta E/kT}}{e^{-\Delta E/kT}+1}$. The solution for $e^{-\Delta E/kT}$ is

$$e^{-\Delta E/kT} = \frac{f}{1-f}\ .$$

We take the natural logarithm of both sides and solve for *T*. The result is

$$T = \frac{\Delta E}{k\ln\left(\dfrac{f}{1-f}\right)}\ .$$

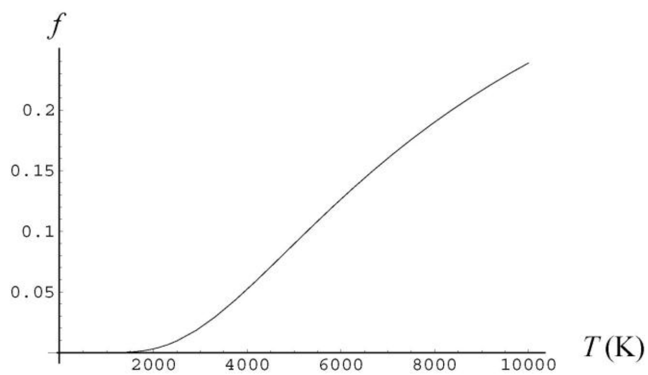
ANALYZE (a) Letting *f* equal 0.01, we evaluate the expression for *T*:

$$T=\frac{(1.00\,\text{eV})(1.60\times10^{-19}\,\text{J/eV})}{(1.38\times10^{-23}\,\text{J/K})\ln\left(\dfrac{0.010}{1-0.010}\right)}=2.50\times10^{3}\,\text{K}.$$

(b) We set f equal to 0.10 and evaluate the expression for T:

$$T=\frac{(1.00\,\text{eV})(1.60\times10^{-19}\,\text{J/eV})}{(1.38\times10^{-23}\,\text{J/K})\ln\left(\dfrac{0.10}{1-0.10}\right)}=5.30\times10^{3}\,\text{K}.$$

LEARN The fractional difference as a function of T is plotted below:



With a given ΔE, the difference increases with T.

16. (a) The ideal gas law in the form of Eq. 20-9 leads to $p = NkT/V = n_0kT$. Thus, we solve for the molecules per cubic meter:

$$n_0=\frac{p}{kT}=\frac{(1.0\,\text{atm})(1.0\times10^{5}\,\text{Pa/atm})}{(1.38\times10^{-23}\,\text{J/K})(273\,\text{K})}=2.7\times10^{25}\,\text{m}^{-3}\,.$$

(b) Combining Eqs. 41-2, 41-3, and 41-4 leads to the conduction electrons per cubic meter in copper:

$$n=\frac{8.96\times10^{3}\,\text{kg/m}^{3}}{(63.54)(1.67\times10^{-27}\,\text{kg})}=8.43\times10^{28}\,\text{m}^{-3}\,.$$

(c) The ratio is $n/n_0 = (8.43\times10^{28}\,\text{m}^{-3})/(2.7\times10^{25}\,\text{m}^{-3}) = 3.1\times10^{3}$.

(d) We use $d_{\text{avg}} = n^{-1/3}$. For case (a), $d_{\text{avg},\,0} = (2.7\times10^{25}\,\text{m}^{-3})^{-1/3} = 3.3$ nm.

(e) For case (b), $d_{\text{avg}} = (8.43\times10^{28}\,\text{m}^{-3})^{-1/3} = 0.23$ nm.

17. Let N be the number of atoms per unit volume and n be the number of free electrons per unit volume. Then, the number of free electrons per atom is n/N. We use the result of Problem 41-1 to find n: $E_F = An^{2/3}$, where $A = 3.65 \times 10^{-19}\ \text{m}^2 \cdot \text{eV}$. Thus,

$$n = \left(\frac{E_F}{A}\right)^{3/2} = \left(\frac{11.6\,\text{eV}}{3.65\times10^{-19}\,\text{m}^2\cdot\text{eV}}\right)^{3/2} = 1.79\times10^{29}\,\text{m}^{-3}\ .$$

If M is the mass of a single aluminum atom and d is the mass density of aluminum, then $N = d/M$. Now,

$$M = (27.0\ \text{g/mol})/(6.022 \times 10^{23}\ \text{mol}^{-1}) = 4.48 \times 10^{-23}\ \text{g},$$

so

$$N = (2.70\ \text{g/cm}^3)/(4.48 \times 10^{-23}\ \text{g}) = 6.03 \times 10^{22}\ \text{cm}^{-3} = 6.03 \times 10^{28}\ \text{m}^{-3}.$$

Thus, the number of free electrons per atom is

$$\frac{n}{N} = \frac{1.79\times10^{29}\,\text{m}^{-3}}{6.03\times10^{28}\,\text{m}^{-3}} = 2.97 \approx 3.$$

18. The mass of the sample is

$$m = \rho V = (9.0\ \text{g/cm}^3)(40.0\ \text{cm}^3) = 360\ \text{g}\,,$$

which is equivalent to

$$n = \frac{m}{M} = \frac{360\ \text{g}}{60\ \text{g/mol}} = 6.0\ \text{mol}\,.$$

Since the atoms are bivalent (each contributing two electrons), there are 12.0 moles of conduction electrons, or

$$N = nN_\text{A} = (12.0\ \text{mol})(6.02\times10^{23}\,/\,\text{mol}) = 7.2\times10^{24}\,.$$

19. (a) We evaluate $P(E) = 1/(e^{(E-E_F)/kT}+1)$ for the given value of E, using

$$kT = \frac{(1.381\times10^{-23}\ \text{J / K})(273\,\text{K})}{1.602\times10^{-19}\ \text{J / eV}} = 0.02353\,\text{eV}\ .$$

For $E = 4.4$ eV, $(E - E_F)/kT = (4.4\ \text{eV} - 5.5\ \text{eV})/(0.02353\ \text{eV}) = -46.25$ and

$$P(E) = \frac{1}{e^{-46.25}+1} = 1.0.$$

(b) Similarly, for $E = 5.4$ eV, $P(E) = 0.986 \approx 0.99$.

(c) For E = 5.5 eV, $P(E)$ = 0.50.

(d) For E = 5.6 eV, $P(E)$ = 0.014.

(e) For E = 6.4 eV, $P(E) = 2.447 \times 10^{-17} \approx 2.4 \times 10^{-17}$.

(f) Solving $P = 1/(e^{\Delta E/kT} + 1)$ for $e^{\Delta E/kT}$, we get

$$e^{\Delta E/kT} = \frac{1}{P} - 1 .$$

Now, we take the natural logarithm of both sides and solve for T. The result is

$$T = \frac{\Delta E}{k \ln\left(\frac{1}{P} - 1\right)} = \frac{(5.6\,\text{eV} - 5.5\,\text{eV})(1.602\times10^{-19}\,\text{J/eV})}{(1.381\times10^{-23}\,\text{J/K})\ln\left(\frac{1}{0.16} - 1\right)} = 699\,\text{K} \approx 7.0\times10^{2}\,\text{K}.$$

20. The probability that a state with energy E is occupied at temperature T is given by

$$P(E) = \frac{1}{e^{(E-E_F)/kT} + 1}$$

where k is the Boltzmann constant and E_F is the Fermi energy. Now,

$$E - E_F = 6.10\text{ eV} - 5.00\text{ eV} = 1.10\text{ eV}$$

and

$$\frac{E - E_F}{kT} = \frac{1.10\,\text{eV}}{(8.62\times10^{-5}\,\text{eV/K})(1500\,\text{K})} = 8.51\,,$$

so

$$P(E) = \frac{1}{e^{8.51} + 1} = 2.01\times10^{-4}\,.$$

From Fig. 41-6, we find the density of states at 6.0 eV to be about $N(E) = 1.7\times10^{28}\,/\,\text{m}^3\cdot\text{eV}.$ Thus, using Eq. 41-7, the density of occupied states is

$$N_\text{O}(E) = N(E)P(E) = (1.7\times10^{28}\,/\,\text{m}^3\cdot\text{eV})(2.01\times10^{-4}) = 3.42\times10^{24}\,/\,\text{m}^3\cdot\text{eV}.$$

Within energy range of $\Delta E = 0.0300\,\text{eV}$ and a volume $V = 5.00\times10^{-8}\ \text{m}^3,$ the number of occupied states is

$$\begin{aligned}\begin{pmatrix}\text{number}\\ \text{states}\end{pmatrix} &= N_\text{O}(E)V\Delta E = (3.42\times10^{24}\,/\,\text{m}^3\cdot\text{eV})(5.00\times10^{-8}\ \text{m}^3)(0.0300\,\text{eV})\\ &= 5.1\times10^{15}\,.\end{aligned}$$

21. (a) At T = 300 K, $f = \dfrac{3kT}{2E_F} = \dfrac{3(8.62\times10^{-5}\,\text{eV/K})(300\,\text{K})}{2(7.0\,\text{eV})} = 5.5\times10^{-3}$.

(b) At T = 1000 K, $f = \dfrac{3kT}{2E_F} = \dfrac{3(8.62\times10^{-5}\,\text{eV/K})(1000\,\text{K})}{2(7.0\,\text{eV})} = 1.8\times10^{-2}$.

(c) Many calculators and most math software packages (here we use MAPLE) have built-in numerical integration routines. Setting up ratios of integrals of Eq. 41-7 and canceling common factors, we obtain

$$frac = \frac{\int_{E_F}^{\infty}\sqrt{E}\,/\,(e^{(E-E_F)/kT}+1)dE}{\int_0^{\infty}\sqrt{E}\,/\,(e^{(E-E_F)/kT}+1)dE}$$

where $k = 8.62\times10^{-5}$ eV/K. We use the Fermi energy value for copper (E_F = 7.0 eV) and evaluate this for T = 300 K and T = 1000 K; we find *frac* = 0.00385 and *frac* = 0.0129, respectively.

22. The fraction f of electrons with energies greater than the Fermi energy is (approximately) given in Problem 41-21:

$$f = \frac{3kT/2}{E_F}$$

where T is the temperature on the Kelvin scale, k is the Boltzmann constant, and E_F is the Fermi energy. We solve for T:

$$T = \frac{2fE_F}{3k} = \frac{2(0.013)(4.70\,\text{eV})}{3(8.62\times10^{-5}\,\text{eV/K})} = 472\text{ K}.$$

23. The average energy of the conduction electrons is given by

$$E_{\text{avg}} = \frac{1}{n}\int_0^{\infty} EN(E)P(E)dE$$

where n is the number of free electrons per unit volume, $N(E)$ is the density of states, and $P(E)$ is the occupation probability. The density of states is proportional to $E^{1/2}$, so we may write $N(E) = CE^{1/2}$, where C is a constant of proportionality. The occupation probability is one for energies below the Fermi energy and zero for energies above. Thus,

$$E_{\text{avg}} = \frac{C}{n}\int_0^{E_F} E^{3/2}dE = \frac{2C}{5n}E_F^{5/2} .$$

Now

$$n=\int_0^\infty N(E)P(E)dE=C\int_0^{E_F}E^{1/2}dE=\frac{2C}{3}E_F^{3/2}.$$

We substitute this expression into the formula for the average energy and obtain

$$E_{\text{avg}}=\left(\frac{2C}{5}\right)E_F^{5/2}\left(\frac{3}{2CE_F^{3/2}}\right)=\frac{3}{5}E_F\ .$$

24. From Eq. 41-9, we find the number of conduction electrons per unit volume to be

$$n=\frac{16\sqrt{2}\pi}{3}\left(\frac{m_eE_F}{h^2}\right)^{3/2}=\frac{16\sqrt{2}\pi}{3}\left(\frac{(m_ec^2)E_F}{(hc)^2}\right)^{3/2}=\frac{16\sqrt{2}\pi}{3}\left(\frac{(0.511\times10^6\,\text{eV})(5.0\,\text{eV})}{(1240\,\text{eV}\cdot\ \text{nm})^2}\right)^{3/2}$$
$$=50.9/\text{nm}^3=5.09\times10^{28}/\text{m}^3$$
$$\approx 8.4\times10^4\ \text{mol/m}^3.$$

Since the atom is bivalent, the number density of the atom is

$$n_{\text{atom}}=n/2=4.2\times10^4\ \text{mol/m}^3.$$

Thus, the mass density of the atom is

$$\rho=n_{\text{atom}}M=(4.2\times10^4\ \text{mol/m}^3)(20.0\,\text{g/mol})=8.4\times10^5\ \text{g/m}^3=0.84\,\text{g/cm}^3.$$

25. (a) Using Eq. 41-4, the energy released would be

$$E=NE_{\text{avg}}=\frac{(3.1\text{g})}{(63.54\,\text{g}/\text{mol})/(6.02\times10^{23}/\text{mol})}\left(\frac{3}{5}\right)(7.0\text{eV})(1.6\times10^{-19}\,\text{J/eV})$$
$$=1.97\times10^4\,\text{J}.$$

(b) Keeping in mind that a watt is a joule per second, we have

$$t=\frac{E}{P}=\frac{1.97\times10^4\,\text{J}}{100\text{J/s}}=197\,\text{s}.$$

26. Let the energy of the state in question be an amount ΔE above the Fermi energy E_F. Then, Eq. 41-6 gives the occupancy probability of the state as

$$P=\frac{1}{e^{(E_F+\Delta E-E_F)/kT}+1}=\frac{1}{e^{\Delta E/kT}+1}\ .$$

We solve for ΔE to obtain

$$\Delta E = kT\ln\left(\frac{1}{P}-1\right) = (1.38\times10^{23}\text{ J/K})(300\text{ K})\ln\left(\frac{1}{0.10}-1\right) = 9.1\times10^{-21}\text{ J},$$

which is equivalent to 5.7×10^{-2} eV = 57 meV.

27. (a) Combining Eqs. 41-2, 41-3, and 41-4 leads to the conduction electrons per cubic meter in zinc:

$$n = \frac{2(7.133\text{ g/cm}^3)}{(65.37\text{ g/mol})/(6.02\times10^{23}\text{ mol})} = 1.31\times10^{23}\text{ cm}^{-3} = 1.31\times10^{29}\text{ m}^{-3}.$$

(b) From Eq. 41-9,

$$E_F = \frac{0.121h^2}{m_e}n^{2/3} = \frac{0.121(6.63\times10^{-34}\text{ J}\cdot\text{s})^2(1.31\times10^{29}\text{ m}^{-3})^{2/3}}{(9.11\times10^{-31}\text{ kg})(1.60\times10^{-19}\text{ J/eV})} = 9.43\text{eV}.$$

(c) Equating the Fermi energy to $\frac{1}{2}m_e v_F^2$ we find (using the m_ec^2 value in Table 37-3)

$$v_F = \sqrt{\frac{2E_Fc^2}{m_ec^2}} = \sqrt{\frac{2(9.43\text{eV})(2.998\times10^8\text{ m/s})^2}{511\times10^3\text{ eV}}} = 1.82\times10^6\text{ m/s}.$$

(d) The de Broglie wavelength is

$$\lambda = \frac{h}{m_e v_F} = \frac{6.63\times10^{-34}\text{ J}\cdot\text{s}}{(9.11\times10^{-31}\text{ kg})(1.82\times10^6\text{ m/s})} = 0.40\text{ nm}.$$

28. Combining Eqs. 41-2, 41-3, and 41-4, the number density of conduction electrons in gold is

$$n = \frac{(19.3\text{ g/cm}^3)(6.02\times10^{23}\text{/mol})}{(197\text{ g/mol})} = 5.90\times10^{22}\text{ cm}^{-3} = 59.0\text{ nm}^{-3}.$$

Now, using $hc = 1240\text{ eV}\cdot\text{nm}$, Eq. 41-9 leads to

$$E_F = \frac{0.121(hc)^2}{(m_ec^2)}n^{2/3} = \frac{0.121(1240\text{ eV}\cdot\text{nm})^2}{511\times10^3\text{ eV}}(59.0\text{ nm}^{-3})^{2/3} = 5.52\text{ eV}.$$

29. Let the volume be $v = 1.00 \times 10^{-6}$ m^3. Then,

$$K_{\text{total}} = NE_{\text{avg}} = n\nu E_{\text{avg}} = (8.43\times10^{28}\,\text{m}^{-3})(1.00\times10^{-6}\,\text{m}^3)\left(\frac{3}{5}\right)(7.00\,\text{eV})(1.60\times10^{-19}\,\text{J/eV})$$
$$= 5.71\times10^{4}\,\text{J} = 57.1\text{ kJ}.$$

30. The probability that a state with energy E is occupied at temperature T is given by

$$P(E) = \frac{1}{e^{(E-E_F)/kT}+1}$$

where k is the Boltzmann constant and

$$E_F = \frac{0.121h^2}{m_e}n^{2/3} = \frac{0.121(6.626\times10^{-34}\,\text{J}\cdot\text{s})^2}{9.11\times10^{-31}\,\text{kg}}(1.70\times10^{28}\,\text{m}^{-3})^{2/3} = 3.855\times10^{-19}\,\text{J}$$

is the Fermi energy. Now,

$$E - E_F = 4.00\times10^{-19}\text{ J} - 3.855\times10^{-19}\text{ J} = 1.45\times10^{-20}\text{ J}$$

and

$$\frac{E-E_F}{kT} = \frac{1.45\times10^{-20}\text{ J}}{(1.38\times10^{-23}\,\text{J/K})(200\text{K})} = 5.2536,$$

so

$$P(E) = \frac{1}{e^{5.2536}+1} = 5.20\times10^{-3}.$$

Next, for the density of states associated with the conduction electrons of a metal, Eq. 41-5 gives

$$N(E) = \frac{8\sqrt{2}\pi m^{3/2}}{h^3}E^{1/2} = \frac{8\sqrt{2}\pi(9.109\times10^{-31}\,\text{kg})^{3/2}}{(6.626\times10^{-34}\,\text{J}\cdot\text{s})^3}\left(4.00\times10^{-19}\,\text{J}\right)^{1/2}$$
$$= \left(1.062\times10^{56}\,\text{kg}^{3/2}/\text{J}^3\cdot\text{s}^3\right)\left(4.00\times10^{-19}\,\text{J}\right)^{1/2}$$
$$= 6.717\times10^{46}\ /\,\text{m}^3\cdot\text{J}$$

where we have used 1 kg =1 J$\cdot$s$^2\cdot$m^{-2} for unit conversion. Thus, using Eq. 41-7, the density of occupied states is

$$N_{\text{O}}(E) = N(E)P(E) = (6.717\times10^{46}\,/\,\text{m}^3\cdot\text{J})(5.20\times10^{-3}) = 3.49\times10^{44}\,/\,\text{m}^3\cdot\text{J}.$$

Within energy range of $\Delta E = 3.20\times10^{-20}$ J and a volume $V = 6.00\times10^{-6}$ m^3, the number of occupied states is

$$\begin{pmatrix}\text{number}\\ \text{states}\end{pmatrix} = N_O(E)V\Delta E = (3.49\times10^{44}/\text{m}^3\cdot\ \text{J})(6.00\times10^{-6}\ \text{m}^3)(3.20\times10^{-20}\ \text{J})$$
$$= 6.7\times10^{19}.$$

31. **THINK** The valence band and the conduction band are separated by an energy gap.

EXPRESS Since the electron jumps from the conduction band to the valence band, the energy of the photon equals the energy gap between those two bands. The photon energy is given by $hf = hc/\lambda$, where f is the frequency of the electromagnetic wave and λ is its wavelength.

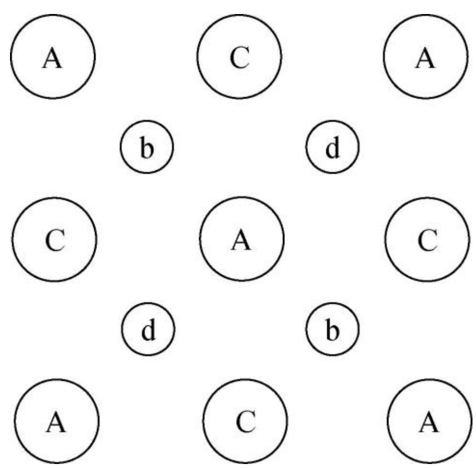
ANALYZE (a) Thus, $E_g = hc/\lambda$ and

$$\lambda = \frac{hc}{E_g} = \frac{(6.63\times10^{-34}\,\text{J}\cdot\text{s})(2.998\times10^{8}\,\text{m/s})}{(5.5\text{eV})(1.60\times10^{-19}\,\text{J/eV})} = 2.26\times10^{-7}\ \text{m} = 226\,\text{nm}.$$

(b) These photons are in the ultraviolet portion of the electromagnetic spectrum.

LEARN Note that photons from other transitions have a greater energy, so their waves have shorter wavelengths.

32. Each arsenic atom is connected (by covalent bonding) to four gallium atoms, and each gallium atom is similarly connected to four arsenic atoms. The "depth" of their very nontrivial lattice structure is, of course, not evident in a flattened-out representation such as shown for silicon in Fig. 41-10.



Still we try to convey some sense of this (in the [1, 0, 0] view shown — for those who might be familiar with Miller indices) by using letters to indicate the depth: A for the closest atoms (to the observer), b for the next layer deep, C for further into the page, d for the last layer seen, and E (not shown) for the atoms that are at the deepest layer (and are behind the A's) needed for our description of the structure. The capital letters are used for the gallium atoms, and the small letters for the arsenic.

Consider the arsenic atom (with the letter b) near the upper left; it has covalent bonds with the two A's and the two C's near it. Now consider the arsenic atom (with the letter d) near the upper right; it has covalent bonds with the two C's, which are near it, and with the two E's (which are behind the A's which are near :+).

(a) The 3p, 3d, and 4s subshells of both arsenic and gallium are filled. They both have partially filled 4p subshells. An isolated, neutral arsenic atom has three electrons in the 4p subshell, and an isolated, neutral gallium atom has one electron in the 4p subshell. To supply the total of eight shared electrons (for the four bonds connected to each ion in the lattice), not only the electrons from 4p must be shared but also the electrons from 4s. The core of the gallium ion has charge $q = +3e$ (due to the "loss" of its single 4p and two 4s electrons).

(b) The core of the arsenic ion has charge $q = +5e$ (due to the "loss" of the three 4p and two 4s electrons).

(c) As remarked in part (a), there are two electrons shared in each of the covalent bonds. This is the same situation that one finds for silicon (see Fig. 41-10).

33. (a) At the bottom of the conduction band E = 0.67 eV. Also E_F = 0.67 eV/2 = 0.335 eV. So the probability that the bottom of the conduction band is occupied is

$$P(E)=\frac{1}{\exp\left(\dfrac{E-E_{\rm F}}{kT}\right)+1}=\frac{1}{\exp\left(\dfrac{0.67\,\text{eV}-0.335\,\text{eV}}{(8.62\times10^{-5}\ \text{eV/K})(290\,\text{K})}\right)+1}=1.5\times10^{-6}\ .$$

(b) At the top of the valence band E = 0, so the probability that the state is *unoccupied* is given by

$$1-P(E)=1-\frac{1}{e^{(E-E_{\rm F})/kT}+1}=\frac{1}{e^{-(E-E_{\rm F})/kT}+1}=\frac{1}{e^{-(0-0.335\,\text{eV})/\left[(8.62\times10^{-5}\,\text{eV/K})(290\,\text{K})\right]}+1}$$

$$=1.5\times10^{-6}\ .$$

34. (a) The number of electrons in the valence band is

$$N_{\rm ev}=N_vP(E_v)=\frac{N_v}{e^{(E_v-E_{\rm F})/kT}+1}\ .$$

Since there are a total of N_v states in the valence band, the number of holes in the valence band is

$$N_{\rm hv}=N_v-N_{\rm ev}=N_v\left[1-\frac{1}{e^{(E_v-E_{\rm F})/kT}+1}\right]=\frac{N_v}{e^{-(E_v-E_{\rm F})/kT}+1}\ .$$

Now, the number of electrons in the conduction band is

$$N_{\text{ec}} = N_c P(E_c) = \frac{N_c}{e^{(E_c - E_F)/kT} + 1} ,$$

Hence, from $N_{\text{ev}} = N_{\text{hc}}$, we get

$$\frac{N_v}{e^{-(E_v - E_F)/kT} + 1} = \frac{N_c}{e^{(E_c - E_F)/kT} + 1} .$$

(b) In this case, $e^{(E_c - E_F)/kT} \gg 1$ and $e^{-(E_v - E_F)/kT} \gg 1$. Thus, from the result of part (a),

$$\frac{N_c}{e^{(E_c - E_F)/kT}} \approx \frac{N_v}{e^{-(E_v - E_F)/kT}} ,$$

or $e^{(E_v - E_c + 2E_F)/kT} \approx N_v / N_c$. We solve for E_F:

$$E_F \approx \frac{1}{2}(E_c + E_v) + \frac{1}{2} kT \ln\left(\frac{N_v}{N_c}\right).$$

35. **THINK** Doping silicon with phosphorus increases the number of electrons in the conduction band.

EXPRESS Sample Problem — "Doping silicon with phosphorus" gives the fraction of silicon atoms that must be replaced by phosphorus atoms. We find the number the silicon atoms in 1.0 g, then the number that must be replaced, and finally the mass of the replacement phosphorus atoms. The molar mass of silicon is $M_{\text{Si}} = 28.086$ g/mol, so the mass of one silicon atom is

$$m_{0,\text{Si}} = M_{\text{Si}} / N_A = (28.086 \text{ g/mol})/(6.022 \times 10^{23} \text{ mol}^{-1}) = 4.66 \times 10^{-23} \text{ g}$$

and the number of atoms in 1.0 g is

$$N_{\text{Si}} = m_{\text{Si}} / m_{0,\text{Si}} = (1.0 \text{ g})/(4.66 \times 10^{-23} \text{ g}) = 2.14 \times 10^{22}.$$

According to the Sample Problem, one of every 5×10^6 silicon atoms is replaced with a phosphorus atom. This means there will be

$$N_{\text{P}} = (2.14 \times 10^{22})/(5 \times 10^6) = 4.29 \times 10^{15}$$

phosphorus atoms in 1.0 g of silicon.

ANALYZE The molar mass of phosphorus is $M_{\text{P}} = 30.9758$ g/mol so the mass of a phosphorus atom is

$$m_{0,\mathrm{P}} = M_\mathrm{P} / N_A = (30.9758 \text{ g/mol})/(6.022 \times 10^{-23} \text{ mol}^{-1}) = 5.14 \times 10^{-23} \text{ g}.$$

The mass of phosphorus that must be added to 1.0 g of silicon is

$$m_\mathrm{P} = N_\mathrm{P} m_{0,\mathrm{P}} = (4.29 \times 10^{15})(5.14 \times 10^{-23} \text{ g}) = 2.2 \times 10^{-7} \text{ g}.$$

LEARN The phosphorus atom is a *donor* atom since it donates an electron to the conduction band. Semiconductors doped with donor atoms are called *n*-type semiconductors.

36. (a) The Fermi level is above the top of the silicon valence band.

(b) Measured from the top of the valence band, the energy of the donor state is

$$E = 1.11 \text{ eV} - 0.11 \text{ eV} = 1.0 \text{ eV}.$$

We solve E_F from Eq. 41-6:

$$\begin{aligned} E_F &= E - kT \ln\left[P^{-1} - 1\right] = 1.0\,\mathrm{eV} - \left(8.62\times10^{-5}\ \mathrm{eV/K}\right)\left(300\,\mathrm{K}\right)\ln\left[\left(5.00\times10^{-5}\right)^{-1} - 1\right] \\ &= 0.744\,\mathrm{eV}. \end{aligned}$$

(c) Now E = 1.11 eV, so

$$P(E) = \frac{1}{e^{(E-E_F)/kT} + 1} = \frac{1}{e^{(1.11\mathrm{eV}-0.744\mathrm{eV})/\left[\left(8.62\times10^{-5}\ \mathrm{eV/K}\right)(300\mathrm{K})\right]} + 1} = 7.13\times10^{-7}.$$

37. (a) The probability that a state with energy E is occupied is given by

$$P(E) = \frac{1}{e^{(E-E_F)/kT} + 1}$$

where E_F is the Fermi energy, T is the temperature on the Kelvin scale, and k is the Boltzmann constant. If energies are measured from the top of the valence band, then the energy associated with a state at the bottom of the conduction band is E = 1.11 eV. Furthermore,

$$kT = (8.62 \times 10^{-5} \text{ eV/K})(300 \text{ K}) = 0.02586 \text{ eV}.$$

For pure silicon, E_F = 0.555 eV and

$$(E - E_F)/kT = (0.555 \text{ eV})/(0.02586 \text{ eV}) = 21.46.$$

Thus,

$$P(E) = \frac{1}{e^{21.46} + 1} = 4.79\times10^{-10}.$$

(b) For the doped semiconductor,

$$(E - E_F)/kT = (0.11 \text{ eV})/(0.02586 \text{ eV}) = 4.254$$

and

$$P(E) = \frac{1}{e^{4.254}+1} = 1.40 \times 10^{-2}.$$

(c) The energy of the donor state, relative to the top of the valence band, is 1.11 eV – 0.15 eV = 0.96 eV. The Fermi energy is 1.11 eV – 0.11 eV = 1.00 eV. Hence,

$$(E - E_F)/kT = (0.96 \text{ eV} - 1.00 \text{ eV})/(0.02586 \text{ eV}) = -1.547$$

and

$$P(E) = \frac{1}{e^{-1.547}+1} = 0.824.$$

38. (a) The semiconductor is *n*-type, since each phosphorus atom has one more valence electron than a silicon atom.

(b) The added charge carrier density is

$$n_{\text{P}} = 10^{-7}\, n_{\text{Si}} = 10^{-7}\,(5 \times 10^{28} \text{ m}^{-3}) = 5 \times 10^{21} \text{ m}^{-3}.$$

(c) The ratio is

$$(5 \times 10^{21} \text{ m}^{-3})/[2(5 \times 10^{15} \text{ m}^{-3})] = 5 \times 10^{5}.$$

Here the factor of 2 in the denominator reflects the contribution to the charge carrier density from *both* the electrons in the conduction band *and* the holes in the valence band.

39. **THINK** The valence band and the conduction band are separated by an energy gap E_g. An electron must acquire E_g in order to make the transition to the conduction band.

EXPRESS Since the energy received by each electron is exactly E_g, the difference in energy between the bottom of the conduction band and the top of the valence band, the number of electrons that can be excited across the gap by a single photon of energy E is

$$N = E / E_g.$$

ANALYZE With E_g = 1.1 eV and E = 662 keV, we obtain

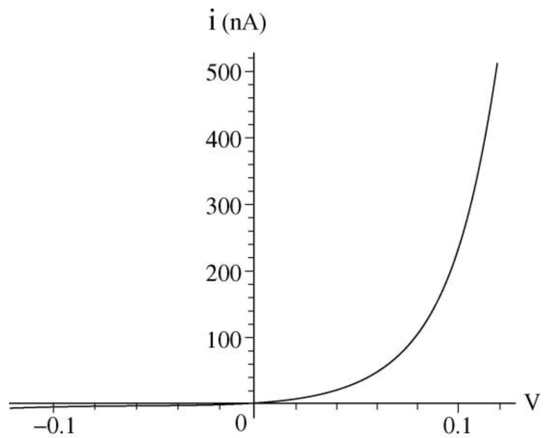
$$N = (662 \times 10^{3} \text{ eV})/(1.1 \text{ eV}) = 6.0 \times 10^{5}.$$

Since each electron that jumps the gap leaves a hole behind, this is also the number of electron-hole pairs that can be created.

LEARN The wavelength of the photon is

$$\lambda = \frac{hc}{E} = \frac{1240 \text{ nm}\cdot\text{eV}}{662\times10^{3} \text{ eV}} = 1.87\times10^{-3} \text{ nm} = 1.87 \text{ pm}.$$

40. (a) The vertical axis in the graph below is the current in nanoamperes:



(b) The ratio is

$$\frac{I\big|_{v=+0.50\,\text{V}}}{I\big|_{v=-0.50\,\text{V}}} = \frac{I_0\left[\exp\left(\dfrac{+0.50\,\text{eV}}{(8.62\times10^{-5} \text{ eV/K})(300\,\text{K})}\right)-1\right]}{I_0\left[\exp\left(\dfrac{-0.50\,\text{eV}}{(8.62\times10^{-5} \text{ eV/K})(300\,\text{K})}\right)-1\right]} = 2.5\times10^{8}.$$

41. The valence band is essentially filled and the conduction band is essentially empty. If an electron in the valence band is to absorb a photon, the energy it receives must be sufficient to excite it across the band gap. Photons with energies less than the gap width are not absorbed and the semiconductor is transparent to this radiation. Photons with energies greater than the gap width are absorbed and the semiconductor is opaque to this radiation. Thus, the width of the band gap is the same as the energy of a photon associated with a wavelength of 295 nm. Noting that $hc = 1240\,\text{eV}\cdot\text{nm}$, we obtain

$$E_{\text{gap}} = \frac{1240\,\text{eV}\cdot\text{nm}}{\lambda} = \frac{1240\,\text{eV}\cdot\text{nm}}{295\,\text{nm}} = 4.20\,\text{eV}.$$

42. Since (using $hc = 1240\,\text{eV}\cdot\text{nm}$)

$$E_{\text{photon}} = \frac{hc}{\lambda} = \frac{1240\,\text{eV}\cdot\text{nm}}{140\,\text{nm}} = 8.86\,\text{eV} > 7.6\,\text{eV},$$

the light will be absorbed by the KCI crystal. Thus, the crystal is opaque to this light.

43. We denote the maximum dimension (side length) of each transistor as $\ell_{\max}$, the size of the chip as *A*, and the number of transistors on the chip as *N*. Then $A = N\ell_{\max}^2$. Therefore,

$$\ell_{\max} = \sqrt{\frac{A}{N}} = \sqrt{\frac{(1.0\,\text{in.} \times 0.875\,\text{in.})\left(2.54\times10^{-2}\,\text{m/in.}\right)^2}{3.5\times10^6}} = 1.3\times10^{-5}\,\text{m} = 13\,\mu\text{m}.$$

44. (a) According to Chapter 25, the capacitance is $C = \kappa\varepsilon_0 A/d$. In our case $\kappa = 4.5$, $A = (0.50\ \mu\text{m})^2$, and $d = 0.20\ \mu\text{m}$, so

$$C = \frac{\kappa\varepsilon_0 A}{d} = \frac{(4.5)\left(8.85\times10^{-12}\,\text{F/m}\right)(0.50\,\mu\text{m})^2}{0.20\,\mu\text{m}} = 5.0\times10^{-17}\,\text{F}.$$

(b) Let the number of elementary charges in question be *N*. Then, the total amount of charges that appear in the gate is $q = Ne$. Thus, $q = Ne = CV$, which gives

$$N = \frac{CV}{e} = \frac{\left(5.0\times10^{-17}\,\text{F}\right)(1.0\,\text{V})}{1.6\times10^{-19}\,\text{C}} = 3.1\times10^2.$$

45. **THINK** We differentiate the occupancy probability $P(E)$ with respect to *E* to explore the properties of $P(E)$.

EXPRESS The probability that a state with energy *E* is occupied at temperature *T* is given by

$$P(E) = \frac{1}{e^{(E-E_F)/kT}+1}$$

where *k* is the Boltzmann constant and E_F is the Fermi energy.

ANALYZE (a) The derivative of $P(E)$ is

$$\frac{dP}{dE} = \frac{-1}{[e^{(E-E_F)/kT}+1]^2}\frac{d}{dE}e^{(E-E_F)/kT} = \frac{-1}{[e^{(E-E_F)/kT}+1]^2}\frac{1}{kT}e^{(E-E_F)/kT}\ .$$

For $E = E_F$, we readily obtain the desired result:

$$\left.\frac{dP}{dE}\right|_{E=E_F} = \frac{-1}{[e^{(E_E-E_F)/kT}+1]^2}\frac{1}{kT}e^{(E_F-E_F)/kT} = -\frac{1}{4kT}\ .$$

(b) The equation of a line may be written as $y = m(x - x_0)$ where $m = -1/4kT$ is the slope, and x_0 is the *x*-intercept (which is what we are asked to solve for). It is clear that $P(E_F) = 1/2$, so our equation of the line, evaluated at $x = E_F$, becomes

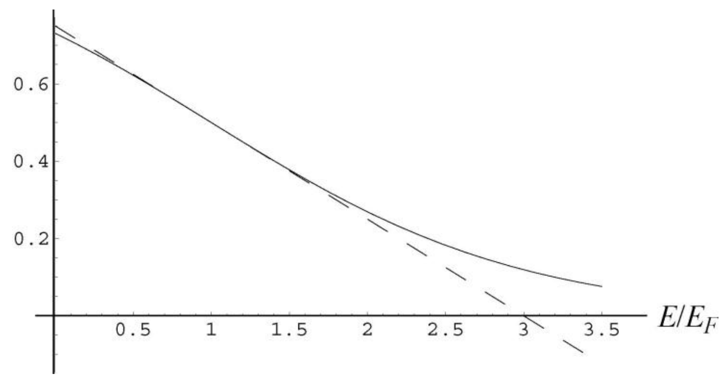
$$1/2 = (-1/4kT)(E_F - x_0),$$

which leads to $x_0 = E_F + 2kT$.

LEARN The straight line can be rewritten as

$$y = \frac{1}{2} - \frac{1}{4kT}\left(E - E_F\right).$$

A plot of $P(E)$ (solid line) and $y(E)$ (dashed line) in units of E_F / kT. The straight line passes the horizontal axis at $E/E_F = 3$.



46. (a) For copper, Eq. 41-10 leads to

$$\frac{d\rho}{dT} = [\rho\alpha]_{\text{Cu}} = (2\times10^{-8}\ \Omega\cdot\text{m})(4\times10^{-3}\,\text{K}^{-1}) = 8\times10^{-11}\ \Omega\cdot\text{m}/\text{K}\ .$$

(b) For silicon,

$$\frac{d\rho}{dT} = [\rho\alpha]_{\text{Si}} = (3\times10^{3}\ \Omega\cdot\text{m})(-70\times10^{-3}\,\text{K}^{-1}) = -2.1\times10^{2}\ \Omega\cdot\text{m}/\text{K}\ .$$

47. The description in the problem statement implies that an atom is at the center point C of the regular tetrahedron, since its four *neighbors* are at the four vertices. The side length for the tetrahedron is given as a = 388 pm. Since each face is an equilateral triangle, the "altitude" of each of those triangles (which is not to be confused with the altitude of the tetrahedron itself) is $h' = \frac{1}{2}a\sqrt{3}$ (this is generally referred to as the "slant height" in the solid geometry literature). At a certain location along the line segment representing the "slant height" of each face is the center C' of the face. Imagine this line segment starting at atom A and ending at the midpoint of one of the sides. Knowing that this line segment bisects the 60° angle of the equilateral face, it is easy to see that C' is a distance $AC' = a/\sqrt{3}$. If we draw a line from C' all the way to the farthest point on the

tetrahedron (this will land on an atom we label *B*), then this new line is the altitude *h* of the tetrahedron. Using the Pythagorean theorem,

$$h = \sqrt{a^2 - (AC')^2} = \sqrt{a^2 - \left(\frac{a}{\sqrt{3}}\right)^2} = a\sqrt{\frac{2}{3}}\ .$$

Now we include coordinates: imagine atom *B* is on the +*y* axis at $y_b = h = a\sqrt{2/3}$, and atom *A* is on the +*x* axis at $x_a = AC' = a/\sqrt{3}$. Then point *C'* is the origin. The tetrahedron center point *C* is on the *y* axis at some value y_c, which we find as follows: *C* must be equidistant from *A* and *B*, so

$$y_b - y_c = \sqrt{x_a^2 + y_c^2} \quad \Rightarrow \quad a\sqrt{\frac{2}{3}} - y_c = \sqrt{\left(\frac{a}{\sqrt{3}}\right)^2 + y_c^2}$$

which yields $y_c = a/2\sqrt{6}$.

(a) In unit vector notation, using the information found above, we express the vector starting at *C* and going to *A* as

$$\vec{r}_{ac} = x_a\hat{\mathrm{i}} + (-y_c)\hat{\mathrm{j}} = \frac{\mathrm{a}}{\sqrt{3}}\hat{\mathrm{i}} - \frac{a}{2\sqrt{6}}\hat{\mathrm{j}}\ .$$

Similarly, the vector starting at *C* and going to *B* is

$$\vec{r}_{bc} = (y_b - y_c)\hat{\mathrm{j}} = \tfrac{a}{2}\sqrt{3/2}\,\hat{\mathrm{j}}\,.$$

Therefore, using Eq. 3-20,

$$\theta = \cos^{-1}\left(\frac{\vec{r}_{ac}\cdot\vec{r}_{bc}}{|\vec{r}_{ac}||\vec{r}_{bc}|}\right) = \cos^{-1}\left(-\frac{1}{3}\right)$$

which yields $\theta = 109.5°$ for the angle between adjacent bonds.

(b) The length of vector $\vec{r}_{bc}$ (which is, of course, the same as the length of $\vec{r}_{ac}$) is

$$|\,\vec{r}_{bc}\,| = \frac{a}{2}\sqrt{\frac{3}{2}} = \frac{388\text{pm}}{2}\sqrt{\frac{3}{2}} = 237.6\ \text{pm} \approx 238\ \text{pm}.$$

We note that in the solid geometry literature, the distance $\frac{a}{2}\sqrt{\frac{3}{2}}$ is known as the circumradius of the regular tetrahedron.

48. According to Eq. 41-6,

$$P(E_F+\Delta E)=\frac{1}{e^{(E_F+\Delta E-E_F)/kT}+1}=\frac{1}{e^{\Delta E/kT}+1}=\frac{1}{e^x+1}$$

where $x=\Delta E/kT$. Also,

$$P(E_F+\Delta E)=\frac{1}{e^{(E_F-\Delta E-E_F)/kT}+1}=\frac{1}{e^{-\Delta E/kT}+1}=\frac{1}{e^{-x}+1}\ .$$

Thus,

$$P(E_F+\Delta E)+P(E_F-\Delta E)=\frac{1}{e^x+1}+\frac{1}{e^{-x}+1}=\frac{e^x+1+e^{-x}+1}{(e^{-x}+1)(e^x+1)}=1\ .$$

A special case of this general result can be found in Problem 41-4, where ΔE = 63 meV and

$$P(E_F + 63\text{ meV}) + P(E_F - 63\text{ meV}) = 0.090 + 0.91 = 1.0.$$

49. (a) Setting $E = E_F$ (see Eq. 41-9), Eq. 41-5 becomes

$$N(E_F)=\frac{8\pi m\sqrt{2m}}{h^3}\left(\frac{3}{16\pi\sqrt{2}}\right)^{1/3}\frac{h}{\sqrt{m}}n^{1/3}\ .$$

Noting that $16\sqrt{2}=2^4 2^{1/2}=2^{9/2}$ so that the cube root of this is $2^{3/2}=2\sqrt{2}$, we are able to simplify the above expression and obtain

$$N(E_F)=\frac{4m}{h^2}\sqrt[3]{3\pi^2 n}$$

which is equivalent to the result shown in the problem statement. Since the desired numerical answer uses eV units, we multiply numerator and denominator of our result by c^2 and make use of the mc^2 value for an electron in Table 37-3 as well as the value $hc=1240\,\text{eV}\cdot\text{nm}$:

$$N(E_F)=\left(\frac{4mc^2}{(hc)^2}\sqrt[3]{3\pi^2}\right)n^{1/3}=\left(\frac{4(511\times10^3\text{ eV})}{(1240\,\text{eV}\cdot\text{nm})^2}\sqrt[3]{3\pi^2}\right)n^{1/3}=(4.11\,\text{nm}^{-2}\cdot\text{eV}^{-1})\,n^{1/3}$$

which is equivalent to the value indicated in the problem statement.

(b) Since there are 10^{27} cubic nanometers in a cubic meter, then the result of Problem 41-3 may be written as

$$n=8.49\times10^{28}\,\text{m}^{-3}=84.9\,\text{nm}^{-3}\ .$$

The cube root of this is $n^{1/3}\approx 4.4$/nm. Hence, the expression in part (a) leads to

$$N(E_F)=(4.11\text{nm}^{-2}\cdot\text{eV}^{-1})(4.4\text{nm}^{-1})=18\text{nm}^{-3}\cdot\text{eV}^{-1}=1.8\times10^{28}\,\text{m}^{-3}\cdot\text{eV}^{-1}.$$

If we multiply this by 10^{27} m^3/nm^3, we see this compares very well with the curve in Fig. 41-6 evaluated at 7.0 eV.

50. If we use the approximate formula discussed in Problem 41-21, we obtain

$$frac = \frac{3(8.62\times10^{-5}\text{ eV / K})(961+273\text{K})}{2(5.5\text{eV})} \approx 0.03\ .$$

The numerical approach is briefly discussed in part (c) of Problem 41-21. Although the problem does not ask for it here, we remark that numerical integration leads to a fraction closer to 0.02.

51. We equate E_F with $\frac{1}{2}m_e v_F^2$ and write our expressions in such a way that we can make use of the electron mc^2 value found in Table 37-3:

$$v_F = \sqrt{\frac{2E_F}{m}} = c\sqrt{\frac{2E_F}{mc^2}} = (3.0\times10^5\text{ km/s})\sqrt{\frac{2(7.0\text{eV})}{5.11\times10^5\text{ eV}}} = 1.6\times10^3\ \text{ km/s}\ .$$

52. The numerical factor $\left(\frac{3}{16\sqrt{2\pi}}\right)^{2/3}$ is approximately equal to 0.121.

53. We use the ideal gas law in the form of Eq. 20-9:

$$p = nkT = (8.43\times10^{28}\text{ m}^{-3})(1.38\times10^{-23}\text{J/K})(300\text{ K}) = 3.49\times10^{8}\text{Pa} = 3.49\times10^{3}\ \text{ atm}\ .$$

Chapter 42

1. Kinetic energy (we use the classical formula since v is much less than c) is converted into potential energy (see Eq. 24-43). From Appendix F or G, we find $Z = 3$ for lithium and $Z = 90$ for thorium; the charges on those nuclei are therefore $3e$ and $90e$, respectively. We manipulate the terms so that one of the factors of e cancels the "e" in the kinetic energy unit MeV, and the other factor of e is set to be 1.6×10^{-19} C. We note that $k = 1/4\pi\varepsilon_0$ can be written as 8.99×10^9 V·m/C. Thus, from energy conservation, we have

$$K = U \Rightarrow \quad r = \frac{kq_1q_2}{K} = \frac{\left(8.99\times10^9 \ \frac{\text{V}\cdot\text{m}}{\text{C}}\right)\left(3\times1.6\times10^{-19}\ \text{C}\right)(90e)}{3.00\times10^6\ \text{eV}}$$

which yields $r = 1.3 \times 10^{-13}$ m (or about 130 fm).

2. Our calculation is similar to that shown in Sample Problem — "Rutherford scattering of an alpha particle by a gold nucleus." We set

$$K = 5.30\,\text{MeV} = U = \left(1/4\pi\varepsilon_0\right)\left(q_\alpha q_{\text{Cu}} / r_{\min}\right)$$

and solve for the closest separation, $r_{\min}$:

$$\begin{aligned} r_{\min} &= \frac{q_\alpha q_{\text{Cu}}}{4\pi\varepsilon_0 K} = \frac{kq_\alpha q_{\text{Cu}}}{4\pi\varepsilon_0 K} = \frac{(2e)(29)\left(1.60\times10^{-19}\ \text{C}\right)\left(8.99\times10^9\ \text{V}\cdot\text{m/C}\right)}{5.30\times10^6\ \text{eV}} \\ &= 1.58\times10^{-14}\ \text{m} = 15.8\ \text{fm}. \end{aligned}$$

We note that the factor of e in $q_\alpha = 2e$ was not set equal to 1.60×10^{-19} C, but was instead allowed to cancel the "e" in the non-SI energy unit, electron-volt.

3. Kinetic energy (we use the classical formula since v is much less than c) is converted into potential energy. From Appendix F or G, we find $Z = 3$ for lithium and $Z = 110$ for Ds; the charges on those nuclei are therefore $3e$ and $110e$, respectively. From energy conservation, we have

$$K = U = \frac{1}{4\pi\varepsilon_0}\frac{q_{\text{Li}}q_{\text{Ds}}}{r}$$

which yields

$$r = \frac{1}{4\pi\varepsilon_0}\frac{q_{\text{Li}}q_{\text{Ds}}}{K} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(3\times1.6\times10^{-19}\ \text{C})(110\times1.6\times10^{-19}\ \text{C})}{(10.2\ \text{MeV})(1.60\times10^{-13}\ \text{J/MeV})}$$
$$= 4.65\times10^{-14}\ \text{m} = 46.5\ \text{fm}.$$

4. In order for the α particle to penetrate the gold nucleus, the separation between the centers of mass of the two particles must be no greater than

$$r = r_{\text{Cu}} + r_\alpha = 6.23\ \text{fm} + 1.80\ \text{fm} = 8.03\ \text{fm}.$$

Thus, the minimum energy K_α is given by

$$K_\alpha = U = \frac{1}{4\pi\varepsilon_0}\frac{q_\alpha q_{\text{Au}}}{r} = \frac{kq_\alpha q_{\text{Au}}}{r}$$
$$= \frac{\left(8.99\times10^9\ \text{V}\cdot\text{m/C}\right)\left(2e\right)\left(79\right)\left(1.60\times10^{-19}\ \text{C}\right)}{8.03\times10^{-15}\ \text{m}} = 28.3\times10^6\ \text{eV}.$$

We note that the factor of e in $q_\alpha = 2e$ was not set equal to 1.60×10^{-19} C, but was instead carried through to become part of the final units.

5. The conservation laws of (classical kinetic) energy and (linear) momentum determine the outcome of the collision (see Chapter 9). The final speed of the α particle is

$$v_{\alpha f} = \frac{m_\alpha - m_{\text{Au}}}{m_\alpha + m_{\text{Au}}} v_{\alpha i},$$

and that of the recoiling gold nucleus is

$$v_{\text{Au},f} = \frac{2m_\alpha}{m_\alpha + m_{\text{Au}}} v_{\alpha i}.$$

(a) Therefore, the kinetic energy of the recoiling nucleus is

$$K_{\text{Au},f} = \frac{1}{2}m_{\text{Au}}v_{\text{Au},f}^2 = \frac{1}{2}m_{\text{Au}}\left(\frac{2m_\alpha}{m_\alpha + m_{\text{Au}}}\right)^2 v_{\alpha i}^2 = K_{\alpha i}\,\frac{4m_{\text{Au}}m_\alpha}{\left(m_\alpha + m_{\text{Au}}\right)^2}$$
$$= \left(5.00\ \text{MeV}\right)\frac{4\left(197\ \text{u}\right)\left(4.00\ \text{u}\right)}{\left(4.00\ \text{u} + 197\ \text{u}\right)^2}$$
$$= 0.390\ \text{MeV}.$$

(b) The final kinetic energy of the alpha particle is

$$K_{\alpha f} = \frac{1}{2} m_\alpha v_{\alpha f}^2 = \frac{1}{2} m_\alpha \left(\frac{m_\alpha - m_{\text{Au}}}{m_\alpha + m_{\text{Au}}} \right)^2 v_{\alpha i}^2 = K_{\alpha i} \left(\frac{m_\alpha - m_{\text{Au}}}{m_\alpha + m_{\text{Au}}} \right)^2$$
$$= (5.00 \text{ MeV}) \left(\frac{4.00 \text{ u} - 197 \text{ u}}{4.00 \text{ u} + 197 \text{ u}} \right)^2$$
$$= 4.61 \text{ MeV}.$$

We note that $K_{af} + K_{\text{Au},f} = K_{\alpha i}$ is indeed satisfied.

6. (a) The atomic number Z = 39 corresponds to the element yttrium (see Appendix F and/or Appendix G).

(b) The atomic number Z = 53 corresponds to iodine.

(c) A detailed listing of stable nuclides (such as the Web site http://nucleardata.nuclear.lu.se/nucleardata) shows that the stable isotope of yttrium has 50 neutrons (this can also be inferred from the Molar Mass values listed in Appendix F).

(d) Similarly, the stable isotope of iodine has 74 neutrons.

(e) The number of neutrons left over is 235 – 127 – 89 = 19.

7. For ^{55}Mn the mass density is

$$\rho_m = \frac{M}{V} = \frac{0.055 \text{kg/mol}}{(4\pi/3)\left[(1.2\times10^{-15}\text{ m})(55)^{1/3}\right]^3 (6.02\times10^{23}/\text{mol})} = 2.3\times10^{17} \text{ kg/m}^3 .$$

(b) For ^{209}Bi,

$$\rho_m = \frac{M}{V} = \frac{0.209 \text{ kg / mol}}{(4\pi/3)\left[(1.2\times10^{-15}\text{ m})(209)^{1/3}\right]^3 (6.02\times10^{23}/\text{mol})} = 2.3\times10^{17} \text{ kg / m}^3.$$

(c) Since $V \propto r^3 = \left(r_0 A^{1/3}\right)^3 \propto A,$ we expect $\rho_m \propto A/V \propto A/A \approx \text{const.}$ for all nuclides.

(d) For ^{55}Mn, the charge density is

$$\rho_q = \frac{Ze}{V} = \frac{(25)(1.6\times10^{-19}\text{ C})}{(4\pi/3)\left[(1.2\times10^{-15}\text{ m})(55)^{1/3}\right]^3} = 1.0\times10^{25}\text{ C/m}^3.$$

(e) For ^{209}Bi, the charge density is

$$\rho_q = \frac{Ze}{V} = \frac{(83)\left(1.6\times10^{-19}\ \text{C}\right)}{(4\pi/3)\left(1.2\times10^{-15}\ \text{m}\right)(209)^{1/3}\ ^3} = 8.8\times10^{24}\ \text{C}/\text{m}^3.$$

Note that $\rho_q \propto Z/V \propto Z/A$ should gradually decrease since $A > 2Z$ for large nuclides.

8. (a) The mass number A is the number of nucleons in an atomic nucleus. Since $m_p \approx m_n$ the mass of the nucleus is approximately Am_p. Also, the mass of the electrons is negligible since it is much less than that of the nucleus. So $M \approx Am_p$.

(b) For ^{1}H, the approximate formula gives

$$M \approx Am_p = (1)(1.007276\ \text{u}) = 1.007276\ \text{u}.$$

The actual mass is (see Table 42-1) 1.007825 u. The percentage deviation committed is then

$$\delta = (1.007825\ \text{u} - 1.007276\ \text{u})/1.007825\ \text{u} = 0.054\% \approx 0.05\%.$$

(c) Similarly, for ^{31}P, $\delta = 0.81\%$.

(d) For ^{120}Sn, $\delta = 0.81\%$.

(e) For ^{197}Au, $\delta = 0.74\%$.

(f) For ^{239}Pu, $\delta = 0.71\%$.

(g) No. In a typical nucleus the binding energy per nucleon is several MeV, which is a bit less than 1% of the nucleon mass times c^2. This is comparable with the percent error calculated in parts (b) – (f) , so we need to use a more accurate method to calculate the nuclear mass.

9. (a) 6 protons, since $Z = 6$ for carbon (see Appendix F).

(b) 8 neutrons, since $A - Z = 14 - 6 = 8$ (see Eq. 42-1).

10. (a) Table 42-1 gives the atomic mass of ^{1}H as $m = 1.007825$ u. Therefore, the *mass excess* for ^{1}H is

$$\Delta = (1.007825\ \text{u} - 1.000000\ \text{u}) = 0.007825\ \text{u}.$$

(b) In the unit MeV/c^2,

$$\Delta = (1.007825\ \text{u} - 1.000000\ \text{u})(931.5\ \text{MeV}/c^2\cdot\text{u}) = +7.290\ \text{MeV}/c^2.$$

(c) The mass of the neutron is $m_\text{n} = 1.008665$ u. Thus, for the neutron,

$$\Delta = (1.008665\text{ u} - 1.000000\text{ u}) = 0.008665\text{ u}.$$

(d) In the unit MeV/c^2,

$$\Delta = (1.008665\text{ u} - 1.000000\text{ u})(931.5\text{ MeV}/\,c^2\cdot\text{u}) = +8.071\text{ MeV}/c^2.$$

(e) Appealing again to Table 42-1, we obtain, for ^{120}Sn,

$$\Delta = (119.902199\text{ u} - 120.000000\text{ u}) = -\,0.09780\text{ u}.$$

(f) In the unit MeV/c^2,

$$\Delta = (119.902199\text{ u} - 120.000000\text{ u})\,(931.5\text{ MeV}/\,c^2\cdot\text{u}) = -\,91.10\text{ MeV}/c^2.$$

11. **THINK** To resolve the detail of a nucleus, the de Broglie wavelength of the probe must be smaller than the size of the nucleus.

EXPRESS The de Broglie wavelength is given by $\lambda = h/p$, where p is the magnitude of the momentum. Since the kinetic energy K of the electron is much greater than its rest energy, relativistic formulation must be used. The kinetic energy and the momentum are related by Eq. 37-54:

$$pc = \sqrt{K^2 + 2Kmc^2}.$$

ANALYZE (a) With K = 200 MeV and mc^2 = 0.511 MeV, we obtain

$$pc = \sqrt{K^2 + 2Kmc^2} = \sqrt{(200\,\text{MeV})^2 + 2(200\,\text{MeV})(0.511\,\text{MeV})} = 200.5\,\text{MeV}.$$

Thus,

$$\lambda = \frac{hc}{pc} = \frac{1240\text{ eV}\cdot\text{nm}}{200.5\times10^6\,\text{eV}} = 6.18\times10^{-6}\,\text{nm} \approx 6.2\text{ fm}.$$

(b) The diameter of a copper nucleus, for example, is about 8.6 fm, just a little larger than the de Broglie wavelength of a 200-MeV electron. To resolve detail, the wavelength should be smaller than the target, ideally a tenth of the diameter or less. 200-MeV electrons are perhaps at the lower limit in energy for useful probes.

LEARN The more energetic the incident particle, the finer the details of the target that can be probed.

12. (a) Since $U > 0$, the energy represents a tendency for the sphere to blow apart.

(b) For ^{239}Pu, Q = 94e and R = 6.64 fm. Including a conversion factor for $\text{J} \to \text{eV}$ we obtain

$$U=\frac{3Q^2}{20\pi\varepsilon_0 r}=\frac{3\,94\left(1.60\times10^{-19}\text{ C}\right)^2\left(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)}{5\left(6.64\times10^{-15}\text{ m}\right)}\left(\frac{1\,\text{eV}}{1.60\times10^{-19}\text{ J}}\right)$$
$$=1.15\times10^9\text{ eV}=1.15\,\text{GeV}.$$

(c) Since Z = 94, the electrostatic potential per proton is 1.15 GeV/94 = 12.2 MeV/proton.

(d) Since A = 239, the electrostatic potential per nucleon is 1.15 GeV/239 = 4.81 MeV/nucleon.

(e) The strong force that binds the nucleus is very strong.

13. We note that the mean density and mean radius for the Sun are given in Appendix C. Since $\rho = M/V$ where $V \propto r^3$, we get $r \propto \rho^{-1/3}$. Thus, the new radius would be

$$r=R_s\left(\frac{\rho_s}{\rho}\right)^{1/3}=\left(6.96\times10^8\text{ m}\right)\left(\frac{1410\text{ kg/m}^3}{2\times10^{17}\text{ kg/m}^3}\right)^{1/3}=1.3\times10^4\text{ m}.$$

14. The binding energy is given by

$$\Delta E_{\text{be}}=\left[Zm_H+\left(A-Z\right)m_n-M_{\text{Am}}\right]c^2,$$

where Z is the atomic number (number of protons), A is the mass number (number of nucleons), m_H is the mass of a hydrogen atom, m_n is the mass of a neutron, and M_{Am} is the mass of a ${}^{244}_{95}\text{Am}$ atom. In principle, nuclear masses should be used, but the mass of the Z electrons included in ZM_H is canceled by the mass of the Z electrons included in M_{Am}, so the result is the same. First, we calculate the mass difference in atomic mass units:

$$\Delta m = (95)(1.007825\text{ u}) + (244 - 95)(1.008665\text{ u}) - (244.064279\text{ u}) = 1.970181\text{ u}.$$

Since 1 u is equivalent to 931.494013 MeV,

$$\Delta E_{\text{be}} = (1.970181\text{ u})(931.494013\text{ MeV/u}) = 1835.212\text{ MeV}.$$

Since there are 244 nucleons, the binding energy per nucleon is

$$\Delta E_{\text{ben}} = E/A = (1835.212\text{ MeV})/244 = 7.52\text{ MeV}.$$

15. (a) Since the nuclear force has a short range, any nucleon interacts only with its nearest neighbors, not with more distant nucleons in the nucleus. Let N be the number of neighbors that interact with any nucleon. It is independent of the number A of nucleons in the nucleus. The number of interactions in a nucleus is approximately NA, so the energy

associated with the strong nuclear force is proportional to NA and, therefore, proportional to A itself.

(b) Each proton in a nucleus interacts electrically with every other proton. The number of pairs of protons is $Z(Z - 1)/2$, where Z is the number of protons. The Coulomb energy is, therefore, proportional to $Z(Z - 1)$.

(c) As A increases, Z increases at a slightly slower rate but Z^2 increases at a faster rate than A and the energy associated with Coulomb interactions increases faster than the energy associated with strong nuclear interactions.

16. The binding energy is given by

$$\Delta E_{\text{be}} = \left[Z m_H + \left(A - Z \right) m_n - M_{\text{Eu}} \right] c^2 ,$$

where Z is the atomic number (number of protons), A is the mass number (number of nucleons), m_H is the mass of a hydrogen atom, m_n is the mass of a neutron, and M_{Eu} is the mass of a ${}^{152}_{63}\text{Eu}$ atom. In principle, nuclear masses should be used, but the mass of the Z electrons included in ZM_H is canceled by the mass of the Z electrons included in M_{Eu}, so the result is the same. First, we calculate the mass difference in atomic mass units:

$$\Delta m = (63)(1.007825 \text{ u}) + (152 - 63)(1.008665 \text{ u}) - (151.921742 \text{ u}) = 1.342418 \text{ u}.$$

Since 1 u is equivalent to 931.494013 MeV,

$$\Delta E_{\text{be}} = (1.342418 \text{ u})(931.494013 \text{ MeV/u}) = 1250.454 \text{ MeV}.$$

Since there are 152 nucleons, the binding energy per nucleon is

$$\Delta E_{\text{ben}} = E/A = (1250.454 \text{ MeV})/152 = 8.23 \text{ MeV}.$$

17. It should be noted that when the problem statement says the "masses of the proton and the deuteron are …" they are actually referring to the corresponding atomic masses (given to very high precision). That is, the given masses include the "orbital" electrons. As in many computations in this chapter, this circumstance (of implicitly including electron masses in what should be a purely nuclear calculation) does not cause extra difficulty in the calculation. Setting the gamma ray energy equal to ΔE_{be}, we solve for the neutron mass (with each term understood to be in u units):

$$\begin{aligned} m_{\text{n}} &= M_{\text{d}} - m_{\text{H}} + \frac{E_\gamma}{c^2} = 2.013553212 - 1.007276467 + \frac{2.2233}{931.502} \\ &= 1.0062769 + 0.0023868 \end{aligned}$$

which yields m_n = 1.0086637 u $\approx$ 1.0087 u.

18. The binding energy is given by

$$\Delta E_{\text{be}} = \left[Zm_H + (A - Z)m_n - M_{\text{Rf}} \right] c^2,$$

where Z is the atomic number (number of protons), A is the mass number (number of nucleons), m_H is the mass of a hydrogen atom, m_n is the mass of a neutron, and M_{Rf} is the mass of a ${}^{259}_{104}\text{Rf}$ atom. In principle, nuclear masses should be used, but the mass of the Z electrons included in ZM_H is canceled by the mass of the Z electrons included in M_{Rf}, so the result is the same. First, we calculate the mass difference in atomic mass units:

$$\Delta m = (104)(1.007825\text{ u}) + (259 - 104)(1.008665\text{ u}) - (259.10563\text{ u}) = 2.051245\text{ u}.$$

Since 1 u is equivalent to 931.494013 MeV,

$$\Delta E_{\text{be}} = (2.051245\text{ u})(931.494013\text{ MeV/u}) = 1910.722\text{ MeV}.$$

Since there are 259 nucleons, the binding energy per nucleon is

$$\Delta E_{\text{ben}} = E/A = (1910.722\text{ MeV})/259 = 7.38\text{ MeV}.$$

19. Let f_{24} be the abundance of ^{24}Mg, let f_{25} be the abundance of ^{25}Mg, and let f_{26} be the abundance of ^{26}Mg. Then, the entry in the periodic table for Mg is

$$24.312 = 23.98504 f_{24} + 24.98584 f_{25} + 25.98259 f_{26}.$$

Since there are only three isotopes, $f_{24} + f_{25} + f_{26} = 1$. We solve for f_{25} and f_{26}. The second equation gives $f_{26} = 1 - f_{24} - f_{25}$. We substitute this expression and f_{24} = 0.7899 into the first equation to obtain

$$24.312 = (23.98504)(0.7899) + 24.98584 f_{25} + 25.98259 - (25.98259)(0.7899) - 25.98259 f_{25}.$$

The solution is f_{25} = 0.09303. Then,

$$f_{26} = 1 - 0.7899 - 0.09303 = 0.1171.\ 78.99\%$$

of naturally occurring magnesium is ^{24}Mg.

(a) Thus, 9.303% is ^{25}Mg.

(b) 11.71% is ^{26}Mg.

20. From Appendix F and/or G, we find Z = 107 for bohrium, so this isotope has $N = A - Z$ = 262 – 107 = 155 neutrons. Thus,

$$\Delta E_{\text{ben}} = \frac{\left(Zm_{\text{H}} + Nm_n - m_{\text{Bh}}\right)c^2}{A}$$
$$= \frac{\left((107)(1.007825\,\text{u}) + (155)(1.008665\,\text{u}) - 262.1231\,\text{u}\right)(931.5\,\text{MeV/u})}{262}$$

which yields 7.31 MeV per nucleon.

21. **THINK** Binding energy is the difference in mass energy between a nucleus and its individual nucleons.

EXPRESS If a nucleus contains Z protons and N neutrons, its binding energy is given by Eq. 42-7:

$$\Delta E_{\text{be}} = \sum(mc^2) - Mc^2 = \left(Zm_H + Nm_n - M\right)c^2,$$

where m_H is the mass of a hydrogen atom, m_n is the mass of a neutron, and M is the mass of the atom containing the nucleus of interest.

ANALYZE (a) If the masses are given in atomic mass units, then mass excesses are defined by $\Delta_H = (m_H - 1)c^2, \Delta_n = (m_n - 1)c^2$, and $\Delta = (M - A)c^2$. This means $m_H c^2 = \Delta_H + c^2$, $m_n c^2 = \Delta_n + c^2$, and $Mc^2 = \Delta + Ac^2$. Thus,

$$\Delta E_{\text{be}} = \left(Z\Delta_H + N\Delta_n - \Delta\right) + \left(Z + N - A\right)c^2 = Z\Delta_H + N\Delta_n - \Delta,$$

where $A = Z + N$ is used.

(b) For ${}^{197}_{79}\text{Au}$, Z = 79 and N = 197 – 79 = 118. Hence,

$$\Delta E_{\text{be}} = (79)(7.29\,\text{MeV}) + (118)(8.07\,\text{MeV}) - (-31.2\,\text{MeV}) = 1560\,\text{MeV}.$$

This means the binding energy per nucleon is $\Delta E_{\text{ben}} = (1560\,\text{MeV})/197 = 7.92\,\text{MeV}$.

LEARN Using mass excesses (Δ_H, Δ_n, and Δ) instead of actual masses provides another convenient way of calculating the binding energy of a nucleus.

22. (a) The first step is to add energy to produce ${}^4\text{He} \to p + {}^3\text{H}$, which — to make the electrons "balance" — may be rewritten as ${}^4\text{He} \to {}^1\text{H} + {}^3\text{H}$. The energy needed is

$$\Delta E_1 = \left(m_{^3\text{H}} + m_{^1\text{H}} - m_{^4\text{He}}\right)c^2 = \left(3.01605\,\text{u} + 1.00783\,\text{u} - 4.00260\,\text{u}\right)(931.5\,\text{MeV/u})$$
$$= 19.8\,\text{MeV}.$$

(b) The second step is to add energy to produce ${}^3\text{H} \to n + {}^2\text{H}$. The energy needed is

$$\begin{aligned}\Delta E_2 &= \left(m_{{}^2\text{H}} + m_n - m_{{}^3\text{H}}\right)c^2 = \left(2.01410\,\text{u} + 1.00867\,\text{u} - 3.01605\,\text{u}\right)\left(931.5\,\text{MeV/u}\right) \\ &= 6.26\,\text{MeV}.\end{aligned}$$

(c) The third step: ${}^2\text{H} \to p + n$, which — to make the electrons "balance" — may be rewritten as ${}^2\text{H} \to {}^1\text{H} + n$. The work required is

$$\begin{aligned}\Delta E_3 &= \left(m_{{}^1\text{H}} + m_n - m_{{}^2\text{H}}\right)c^2 = \left(1.00783\,\text{u} + 1.00867\,\text{u} - 2.01410\,\text{u}\right)\left(931.5\,\text{MeV/u}\right) \\ &= 2.23\,\text{MeV}.\end{aligned}$$

(d) The total binding energy is

$$\Delta E_{\text{be}} = \Delta E_1 + \Delta E_2 + \Delta E_3 = 19.8\,\text{MeV} + 6.26\,\text{MeV} + 2.23\,\text{MeV} = 28.3\,\text{MeV}.$$

(e) The binding energy per nucleon is

$$\Delta E_{\text{ben}} = \Delta E_{\text{be}} / A = 28.3\,\text{MeV} / 4 = 7.07\,\text{MeV}.$$

(f) No, the answers do not match.

23. **THINK** The binding energy is given by

$$\Delta E_{\text{be}} = \left[Z m_H + (A - Z) m_n - M_{\text{Pu}}\right] c^2,$$

where Z is the atomic number (number of protons), A is the mass number (number of nucleons), m_H is the mass of a hydrogen atom, m_n is the mass of a neutron, and M_{Pu} is the mass of a ${}^{239}_{94}\text{Pu}$ atom.

EXPRESS In principle, nuclear masses should be used, but the mass of the Z electrons included in Zm_H is canceled by the mass of the Z electrons included in M_{Pu}, so the result is the same. First, we calculate the mass difference in atomic mass units:

$$\Delta m = (94)(1.00783\text{ u}) + (239 - 94)(1.00867\text{ u}) - (239.05216\text{ u}) = 1.94101\text{ u}.$$

Since the mass energy of 1 u is equivalent to 931.5 MeV,

$$\Delta E_{\text{be}} = (1.94101\text{ u})(931.5\text{ MeV/u}) = 1808\text{ MeV}.$$

ANALYZE With 239 nucleons, the binding energy per nucleon is

$$\Delta E_{\text{ben}} = E/A = (1808\text{ MeV})/239 = 7.56\text{ MeV}.$$

The result is the same as that given in Table 42-1.

LEARN An alternative way to calculate binding energy is to use mass excesses, as discussed in Problem 21. The formula is

$$\Delta E_{\text{be}} = Z\Delta_H + N\Delta_n - \Delta_{239},$$

where $\Delta_H = (m_H - 1)c^2$, $\Delta_n = (m_n - 1)c^2$, and $\Delta_{239} = (M_{\text{Pu}} - 239\text{ u})c^2$.

24. We first "separate" all the nucleons in one copper nucleus (which amounts to simply calculating the nuclear binding energy) and then figure the number of nuclei in the penny (so that we can multiply the two numbers and obtain the result). To begin, we note that (using Eq. 42-1 with Appendix F and/or G) the copper-63 nucleus has 29 protons and 34 neutrons. Thus,

$$\begin{aligned}\Delta E_{\text{be}} &= \big(29(1.007825\,\text{u}) + 34(1.008665\,\text{u}) - 62.92960\,\text{u}\big)(931.5\,\text{MeV/u})\\ &= 551.4\,\text{MeV}.\end{aligned}$$

To figure the number of nuclei (or, equivalently, the number of atoms), we adapt Eq. 42-21:

$$N_{\text{Cu}} = \left(\frac{3.0\,\text{g}}{62.92960\,\text{g/mol}}\right)(6.02\times10^{23}\,\text{atoms/mol}) \approx 2.9\times10^{22}\,\text{atoms}.$$

Therefore, the total energy needed is

$$N_{\text{Cu}}\Delta E_{\text{be}} = (551.4\,\text{MeV})(2.9\times10^{22}) = 1.6\times10^{25}\,\text{MeV}.$$

25. The rate of decay is given by $R = \lambda N$, where λ is the disintegration constant and N is the number of undecayed nuclei. In terms of the half-life $T_{1/2}$, the disintegration constant is $\lambda = (\ln 2)/T_{1/2}$, so

$$\begin{aligned}N &= \frac{R}{\lambda} = \frac{RT_{1/2}}{\ln 2} = \frac{(6000\,\text{Ci})(3.7\times10^{10}\,\text{s}^{-1}/\text{Ci})(5.27\,\text{y})(3.16\times10^{7}\,\text{s/y})}{\ln 2}\\ &= 5.33\times10^{22}\,\text{nuclei}.\end{aligned}$$

26. By the definition of half-life, the same has reduced to $\frac{1}{2}$ its initial amount after 140 d. Thus, reducing it to $\frac{1}{4} = \left(\frac{1}{2}\right)^2$ of its initial number requires that two half-lives have passed: $t = 2T_{1/2} = 280$ d.

27. (a) Since 60 y = 2(30 y) = $2T_{1/2}$, the fraction left is $2^{-2} = 1/4 = 0.250$.

(b) Since 90 y = 3(30 y) = $3T_{1/2}$, the fraction that remains is $2^{-3} = 1/8 = 0.125$.

28. (a) We adapt Eq. 42-21:

$$N_{\text{Pu}} = \left(\frac{0.002\,\text{g}}{239\,\text{g/mol}}\right)\left(6.02\times10^{23}\,\text{nuclei/mol}\right) \approx 5.04\times10^{18}\,\text{nuclei}.$$

(b) Eq. 42-20 leads to

$$R = \frac{N\ln 2}{T_{1/2}} = \frac{5\times10^{18}\ln 2}{2.41\times10^{4}\text{ y}} = 1.4\times10^{14}\text{ / y}$$

which is equivalent to 4.60×10^6/s = 4.60×10^6 Bq (the unit becquerel is defined as 1 decay/s).

29. **THINK** Half-life is the time is takes for the number of radioactive nuclei to decrease to half of its initial value.

EXPRESS The half-life $T_{1/2}$ and the disintegration constant λ are related by

$$T_{1/2} = (\ln 2)/\lambda.$$

ANALYZE (a) With $\lambda = 0.0108\text{ h}^{-1}$, we obtain

$$T_{1/2} = (\ln 2)/(0.0108\text{ h}^{-1}) = 64.2\text{ h}.$$

(b) At time t, the number of undecayed nuclei remaining is given by

$$N = N_0\,e^{-\lambda t} = N_0\,e^{-(\ln 2)t/T_{1/2}}.$$

We substitute $t = 3T_{1/2}$ to obtain

$$\frac{N}{N_0} = e^{-3\ln 2} = 0.125.$$

In each half-life, the number of undecayed nuclei is reduced by half. At the end of one half-life, $N = N_0/2$, at the end of two half-lives, $N = N_0/4$, and at the end of three half-lives, $N = N_0/8 = 0.125N_0$.

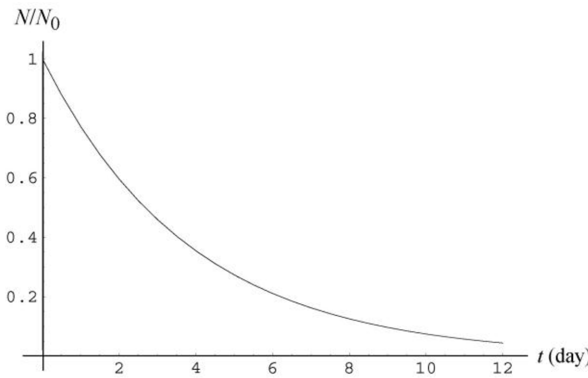
(c) We use

$$N = N_0\,e^{-\lambda t}.$$

Since 10.0 d is 240 h, $\lambda t = (0.0108\text{ h}^{-1})(240\text{ h}) = 2.592$ and

$$\frac{N}{N_0} = e^{-2.592} = 0.0749.$$

LEARN The fraction of the Hg sample remaining as a function of time (measured in days) is plotted below.



30. We note that t = 24 h is four times $T_{1/2}$ = 6.5 h. Thus, it has reduced by half, four-fold:

$$\left(\frac{1}{2}\right)^4\left(48\times10^{19}\right)=3.0\times10^{19}.$$

31. (a) The decay rate is given by $R = \lambda N$, where λ is the disintegration constant and N is the number of undecayed nuclei. Initially, $R=R_0=\lambda N_0$, where N_0 is the number of undecayed nuclei at that time. One must find values for both N_0 and λ. The disintegration constant is related to the half-life $T_{1/2}$ by

$$\lambda=(\ln 2)/T_{1/2}=(\ln 2)/(78\text{h})=8.89\times10^{-3}\,\text{h}^{-1}.$$

If M is the mass of the sample and m is the mass of a single atom of gallium, then $N_0 = M/m$. Now,

$$m = (67\text{ u})(1.661\times10^{-24}\text{ g/u}) = 1.113\times10^{-22}\text{ g}$$

and

$$N_0 = (3.4\text{ g})/(1.113\times10^{-22}\text{ g}) = 3.05\times10^{22}.$$

Thus,

$$R_0 = (8.89\times10^{-3}\text{ h}^{-1})\,(3.05\times10^{22}) = 2.71\times10^{20}\text{ h}^{-1} = 7.53\times10^{16}\text{ s}^{-1}.$$

(b) The decay rate at any time t is given by

$$R=R_0\,e^{-\lambda t}$$

where R_0 is the decay rate at t = 0. At t = 48 h, $\lambda t = (8.89\times10^{-3}\text{ h}^{-1})\,(48\text{ h}) = 0.427$ and

$$R = \left(7.53\times10^{16}\ \text{s}^{-1}\right)e^{-0.427} = 4.91\times10^{16}\ \text{s}^{-1}.$$

32. Using Eq. 42-15 with Eq. 42-18, we find the fraction remaining:

$$\frac{N}{N_0} = e^{-t\ln 2/T_{1/2}} = e^{-30\ln 2/29} = 0.49.$$

33. We note that 3.82 days is 330048 s, and that a becquerel is a disintegration per second (see Section 42-3). From Eq. 34-19, we have

$$\frac{N}{\mathcal{V}} = \frac{R}{\mathcal{V}}\frac{T_{1/2}}{\ln 2} = \left(1.55\times10^{5}\ \frac{\text{Bq}}{\text{m}^3}\right)\frac{330048\,\text{s}}{\ln 2} = 7.4\times10^{10}\ \frac{\text{atoms}}{\text{m}^3}$$

where we have divided by volume *v*. We estimate *v* (the volume breathed in 48 h = 2880 min) as follows:

$$\left(2\,\frac{\text{liters}}{\text{breath}}\right)\left(\frac{1\,\text{m}^3}{1000\,\text{L}}\right)\left(40\,\frac{\text{breaths}}{\text{min}}\right)\left(2880\,\text{min}\right)$$

which yields $v \approx 200\ \text{m}^3$. Thus, the order of magnitude of *N* is

$$\left(\frac{N}{\mathcal{V}}\right)\left(\mathcal{V}\right) \approx \left(7\times10^{10}\ \frac{\text{atoms}}{\text{m}^3}\right)\left(200\,\text{m}^3\right) \approx 1\times10^{13}\,\text{atoms}.$$

34. Combining Eqs. 42-20 and 42-21, we obtain

$$M_{\text{sam}} = N\frac{M_{\text{K}}}{M_{\text{A}}} = \left(\frac{RT_{1/2}}{\ln 2}\right)\left(\frac{40\,\text{g}/\text{mol}}{6.02\times10^{23}/\text{mol}}\right)$$

which gives 0.66 g for the mass of the sample once we plug in 1.7×10^5/s for the decay rate and 1.28×10^9 y = 4.04×10^{16} s for the half-life.

35. **THINK** We modify Eq. 42-11 to take into consideration the rate of production of the radionuclide.

EXPRESS If *N* is the number of undecayed nuclei present at time *t*, then

$$\frac{dN}{dt} = R - \lambda N$$

where *R* is the rate of production by the cyclotron and λ is the disintegration constant. The second term gives the rate of decay. Note the sign difference between *R* and λ*N*.

ANALYZE (a) Rearrange the equation slightly and integrate:

$$\int_{N_0}^{N} \frac{dN}{R-\lambda N} = \int_0^t dt$$

where N_0 is the number of undecayed nuclei present at time $t = 0$. This yields

$$-\frac{1}{\lambda}\ln\frac{R-\lambda N}{R-\lambda N_0} = t.$$

We solve for N:

$$N = \frac{R}{\lambda} + \left(N_0 - \frac{R}{\lambda}\right) e^{-\lambda t}.$$

After many half-lives, the exponential is small and the second term can be neglected. Then, $N = R/\lambda$.

(b) The result $N = R/\lambda$ holds regardless of the initial value N_0, because the dependence on N_0 shows up only in the second term, which is exponentially suppressed at large t.

LEARN At times that are long compared to the half-life, the rate of production equals the rate of decay and N is a constant. The nuclide is in secular equilibrium with its source.

36. We have one alpha particle (helium nucleus) produced for every plutonium nucleus that decays. To find the number that have decayed, we use Eq. 42-15, Eq. 42-18, and adapt Eq. 42-21:

$$N_0 - N = N_0\left(1 - e^{-t\ln 2/T_{1/2}}\right) = N_A \frac{12.0\,\mathrm{g/mol}}{239\,\mathrm{g/mol}}\left(1 - e^{-20000\ln 2/24100}\right)$$

where N_A is the Avogadro constant. This yields 1.32×10^{22} alpha particles produced. In terms of the amount of helium gas produced (assuming the α particles slow down and capture the appropriate number of electrons), this corresponds to

$$m_{\mathrm{He}} = \left(\frac{1.32\times10^{22}}{6.02\times10^{23}\,/\,\mathrm{mol}}\right)(4.0\,\mathrm{g/mol}) = 87.9\times10^{-3}\,\mathrm{g}.$$

37. Using Eq. 42-15 and Eq. 42-18 (and the fact that mass is proportional to the number of atoms), the amount decayed is

$$\begin{aligned}|\Delta m| &= m\big|_{t_f=16.0\,\mathrm{h}} - m\big|_{t_f=14.0\,\mathrm{h}} = m_0\left(1-e^{-t_i\ln 2/T_{1/2}}\right) - m_0\left(1-e^{-t_f\ln 2/T_{1/2}}\right)\\ &= m_0\left(e^{-t_f\ln 2/T_{1/2}} - e^{-t_i\ln 2/T_{1/2}}\right) = (5.50\,\mathrm{g})\left[e^{-(16.0\,h/12.7\,\mathrm{h})\ln 2} - e^{-(14.0\,\mathrm{h}/12.7\,\mathrm{h})\ln 2}\right]\\ &= 0.265\,\mathrm{g}.\end{aligned}$$

38. With $T_{1/2} = 3.0 \text{ h} = 1.08 \times 10^4 \text{ s}$, the decay constant is (using Eq. 42-18)

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{1.08 \times 10^4 \text{ s}} = 6.42 \times 10^{-5} \text{ / s}.$$

Thus, the number of isotope parents injected is

$$N = \frac{R}{\lambda} = \frac{(8.60 \times 10^{-6} \text{ Ci})(3.7 \times 10^{10} \text{ Bq/Ci})}{6.42 \times 10^{-5} \text{ / s}} = 4.96 \times 10^9.$$

39. (a) The sample is in secular equilibrium with the source, and the decay rate equals the production rate. Let R be the rate of production of ^{56}Mn and let λ be the disintegration constant. According to the result of Problem 42-35, $R = \lambda N$ after a long time has passed. Now, $\lambda N = 8.88 \times 10^{10}\ \text{s}^{-1}$, so $R = 8.88 \times 10^{10}\ \text{s}^{-1}$.

(b) We use $N = R/\lambda$. If $T_{1/2}$ is the half-life, then the disintegration constant is

$$\lambda = (\ln 2)/T_{1/2} = (\ln 2)/(2.58 \text{ h}) = 0.269 \text{ h}^{-1} = 7.46 \times 10^{-5} \text{ s}^{-1},$$

so $N = (8.88 \times 10^{10}\ \text{s}^{-1})/(7.46 \times 10^{-5}\ \text{s}^{-1}) = 1.19 \times 10^{15}$.

(c) The mass of a ^{56}Mn nucleus is

$$m = (56 \text{ u})\ (1.661 \times 10^{-24} \text{ g/u}) = 9.30 \times 10^{-23} \text{ g}$$

and the total mass of ^{56}Mn in the sample at the end of the bombardment is

$$Nm = (1.19 \times 10^{15})(9.30 \times 10^{-23} \text{ g}) = 1.11 \times 10^{-7} \text{ g}.$$

40. We label the two isotopes with subscripts 1 (for ^{32}P) and 2 (for ^{33}P). Initially, 10% of the decays come from ^{33}P, which implies that the initial rate $R_{02} = 9R_{01}$. Using Eq. 42-17, this means

$$R_{01} = \lambda_1 N_{01} = \frac{1}{9} R_{02} = \frac{1}{9} \lambda_2 N_{02}.$$

At time t, we have $R_1 = R_{01} e^{-\lambda_1 t}$ and $R_2 = R_{02} e^{-\lambda_2 t}$. We seek the value of t for which $R_1 = 9R_2$ (which means 90% of the decays arise from ^{33}P). We divide equations to obtain

$$\left(R_{01} / R_{02}\right) e^{-(\lambda_1 - \lambda_2)t} = 9,$$

and solve for t:

$$t = \frac{1}{\lambda_1 - \lambda_2} \ln\left(\frac{R_{01}}{9R_{02}}\right) = \frac{\ln\left(R_{01}/9R_{02}\right)}{\ln 2/T_{1/2_1} - \ln 2/T_{1/2_2}} = \frac{\ln\left[(1/9)^2\right]}{\ln 2\left[(14.3\text{d})^{-1} - (25.3\text{d})^{-1}\right]}$$
$$= 209\text{d}.$$

41. The number N of undecayed nuclei present at any time and the rate of decay R at that time are related by $R = \lambda N$, where λ is the disintegration constant. The disintegration constant is related to the half-life $T_{1/2}$ by $\lambda = (\ln 2)/T_{1/2}$, so $R = (N \ln 2)/T_{1/2}$ and

$$T_{1/2} = (N \ln 2)/R.$$

Since 15.0% by mass of the sample is ^{147}Sm, the number of ^{147}Sm nuclei present in the sample is

$$N = \frac{(0.150)(1.00\,\text{g})}{(147\,\text{u})\left(1.661\times10^{-24}\ \text{g/u}\right)} = 6.143\times10^{20}.$$

Thus,

$$T_{1/2} = \frac{\left(6.143\times10^{20}\right)\ln 2}{120\,\text{s}^{-1}} = 3.55\times10^{18}\ \text{s} = 1.12\times10^{11}\ \text{y}.$$

42. Adapting Eq. 42-21, we have

$$N_{\text{Kr}} = \frac{M_{\text{sam}}}{M_{\text{Kr}}} N_A = \left(\frac{20\times10^{-9}\ \text{g}}{92\,\text{g/mol}}\right)\left(6.02\times10^{23}\ \text{atoms/mol}\right) = 1.3\times10^{14}\ \text{atoms}.$$

Consequently, Eq. 42-20 leads to

$$R = \frac{N \ln 2}{T_{1/2}} = \frac{\left(1.3\times10^{14}\right)\ln 2}{1.84\,\text{s}} = 4.9\times10^{13}\ \text{Bq}.$$

43. Using Eq. 42-16 with Eq. 42-18, we find the initial activity:

$$R_0 = Re^{t\ln 2/T_{1/2}} = \left(7.4\times10^{8}\ \text{Bq}\right)e^{24\ln 2/83.61} = 9.0\times10^{8}\ \text{Bq}.$$

44. The number of atoms present initially at $t = 0$ is $N_0 = 2.00\times10^6$. From Fig. 42-19, we see that the number is halved at $t = 2.00\,\text{s}$. Thus, using Eq. 42-15, we find the decay constant to be

$$\lambda = \frac{1}{t}\ln\left(\frac{N_0}{N}\right) = \frac{1}{2.00\ \text{s}}\ln\left(\frac{N_0}{N_0/2}\right) = \frac{1}{2.00\ \text{s}}\ln 2 = 0.3466\ \text{s}^{-1}.$$

At $t = 27.0\ \text{s}$, the number of atoms remaining is

$$N = N_0 e^{-\lambda t} = (2.00\times10^6)e^{-(0.3466/\text{s})(27.0\,\text{s})} \approx 173.$$

Using Eq. 42-17, the decay rate is

$$R = \lambda N = (0.3466/\text{s})(173) \approx 60/\text{s} = 60\text{ Bq}.$$

45. (a) Equation 42-20 leads to

$$R = \frac{\ln 2}{T_{1/2}} N = \frac{\ln 2}{30.2\text{y}}\left(\frac{M_{\text{sam}}}{m_{\text{atom}}}\right) = \frac{\ln 2}{9.53\times10^8\text{ s}}\left(\frac{0.0010\text{kg}}{137\times1.661\times10^{-27}\text{ kg}}\right)$$
$$= 3.2\times10^{12}\text{ Bq}.$$

(b) Using the conversion factor $1\text{ Ci} = 3.7\times10^{10}$ Bq, $R = 3.2\times10^{12}\text{ Bq} = 86$ Ci.

46. (a) Molybdenum beta decays into technetium:

$$^{99}_{42}\text{Mo} \rightarrow ^{99}_{43}\text{Tc} + e^- + v$$

(b) Each decay corresponds to a photon produced when the technetium nucleus de-excites (note that the de-excitation half-life is much less than the beta decay half-life). Thus, the gamma rate is the same as the decay rate: 8.2×10^7/s.

(c) Equation 42-20 leads to

$$N = \frac{RT_{1/2}}{\ln 2} = \frac{(38/\text{s})(6.0\,\text{h})(3600\,\text{s/h})}{\ln 2} = 1.2\times10^6.$$

47. **THINK** The mass fraction of Ra in $RaCl_2$ is given by

$$\frac{M_{\text{Ra}}}{M_{\text{Ra}} + 2M_{\text{Cl}}}$$

where M_{Ra} is the molar mass of Ra and M_{Cl} is the molar mass of Cl.

EXPRESS We assume that the chlorine in the sample had the naturally occurring isotopic mixture, so the average molar mass is 35.453 g/mol, as given in Appendix F. Then, the mass of ^{226}Ra was

$$m = \frac{226}{226 + 2(35.453)}(0.10\,\text{g}) = 76.1\times10^{-3}\text{ g}.$$

ANALYZE (a) The mass of a ^{226}Ra nucleus is (226 u)(1.661×10^{-24} g/u) = 3.75×10^{-22} g, so the number of ^{226}Ra nuclei present was

$$N = (76.1 \times 10^{-3}\ \text{g})/(3.75 \times 10^{-22}\ \text{g}) = 2.03 \times 10^{20}.$$

(b) The decay rate is given by

$$R = N\lambda = (N \ln 2)/T_{1/2},$$

where λ is the disintegration constant, $T_{1/2}$ is the half-life, and N is the number of nuclei. The relationship $\lambda = (\ln 2)/T_{1/2}$ is used. Thus,

$$R = \frac{\left(2.03\times10^{20}\right)\ln 2}{(1600\,\text{y})\left(3.156\times10^{7}\ \text{s/y}\right)} = 2.79\times10^{9}\ \text{s}^{-1}.$$

LEARN Radium has 33 different known isotopes, four of which naturally occurring. ^{226}Ra, with a half-life of 1600 years, is the most stable isotope of radium.

48. (a) The nuclear reaction is written as $^{238}\text{U}\rightarrow{}^{234}\text{Th} + {}^{4}\text{He}$. The energy released is

$$\begin{aligned}\Delta E_1 &= \left(m_{\text{U}} - m_{\text{He}} - m_{\text{Th}}\right)c^2 \\ &= (238.05079\,\text{u} - 4.00260\,\text{u} - 234.04363\,\text{u})(931.5\,\text{MeV/u}) \\ &= 4.25\,\text{MeV}.\end{aligned}$$

(b) The reaction series consists of $^{238}\text{U} \rightarrow {}^{237}\text{U} + \text{n}$, followed by

$$\begin{aligned}^{237}\text{U} &\rightarrow {}^{236}\text{Pa} + \text{p} \\ ^{236}\text{Pa} &\rightarrow {}^{235}\text{Pa} + \text{n} \\ ^{235}\text{Pa} &\rightarrow {}^{234}\text{Th} + \text{p}\end{aligned}$$

The net energy released is then

$$\begin{aligned}\Delta E_2 &= \left(m_{^{238}\text{U}} - m_{^{237}\text{U}} - m_n\right)c^2 + \left(m_{^{237}\text{U}} - m_{^{236}\text{Pa}} - m_p\right)c^2 \\ &\quad + \left(m_{^{236}\text{Pa}} - m_{^{235}\text{Pa}} - m_n\right)c^2 + \left(m_{^{235}\text{Pa}} - m_{^{234}\text{Th}} - m_p\right)c^2 \\ &= \left(m_{^{238}\text{U}} - 2m_n - 2m_p - m_{^{234}\text{Th}}\right)c^2 \\ &= \left[238.05079\,\text{u} - 2(1.00867\,\text{u}) - 2(1.00783\,\text{u}) - 234.04363\,\text{u}\right](931.5\,\text{MeV/u}) \\ &= -24.1\,\text{MeV}.\end{aligned}$$

(c) This leads us to conclude that the binding energy of the α particle is

$$\left|\left(2m_n + 2m_p - m_{\text{He}}\right)c^2\right| = \left|-24.1\,\text{MeV} - 4.25\,\text{MeV}\right| = 28.3\,\text{MeV}.$$

49. **THINK** The time for half the original ^{238}U nuclei to decay is equal to 4.5×10^9 y, which is the half-life of ^{238}U.

EXPRESS The fraction of undecayed nuclei remaining after time t is given by

$$\frac{N}{N_0} = e^{-\lambda t} = e^{-(\ln 2)t/T_{1/2}}$$

where λ is the disintegration constant and $T_{1/2} = (\ln 2)/\lambda$ is the half-life.

(a) For ^{244}Pu at $t = 4.5 \times 10^9$ y,

$$\lambda t = \frac{(\ln 2)t}{T_{1/2}} = \frac{(\ln 2)(4.5\times10^9\text{ y})}{8.0\times10^7\text{ y}} = 39$$

and the fraction remaining is

$$\frac{N}{N_0} = e^{-39.0} \approx 1.2\times10^{-17}.$$

(b) For ^{248}Cm at $t = 4.5 \times 10^9$ y,

$$\frac{(\ln 2)t}{T_{1/2}} = \frac{(\ln 2)(4.5\times10^9\text{ y})}{3.4\times10^5\text{ y}} = 9170$$

and the fraction remaining is

$$\frac{N}{N_0} = e^{-9170} = 3.31\times10^{-3983}.$$

For any reasonably sized sample this is less than one nucleus and may be taken to be zero. A standard calculator probably cannot evaluate e^{-9170} directly. Our recommendation is to treat it as $(e^{-91.70})^{100}$.

LEARN Since $(T_{1/2})_{^{248}\text{Cm}} < (T_{1/2})_{^{244}\text{Pu}} < (T_{1/2})_{^{238}\text{U}}$, with $N/N_0 = e^{-(\ln 2)t/T_{1/2}}$, we have

$$(N/N_0)_{^{248}\text{Cm}} < (N/N_0)_{^{244}\text{Pu}} < (N/N_0)_{^{238}\text{U}}.$$

50. (a) The disintegration energy for uranium-235 "decaying" into thorium-232 is

$$\begin{aligned} Q_3 &= \left(m_{^{235}\text{U}} - m_{^{232}\text{Th}} - m_{^{3}\text{He}}\right)c^2 = \left(235.0439\,\text{u} - 232.0381\,\text{u} - 3.0160\,\text{u}\right)\left(931.5\,\text{MeV/u}\right) \\ &= -9.50\,\text{MeV}. \end{aligned}$$

(b) Similarly, the disintegration energy for uranium-235 decaying into thorium-231 is

$$Q_4 = \left(m_{^{235}\text{U}} - m_{^{231}\text{Th}} - m_{^{4}\text{He}}\right)c^2 = \left(235.0439\,\text{u} - 231.0363\,\text{u} - 4.0026\,\text{u}\right)\left(931.5\,\text{MeV/u}\right)$$
$$= 4.66\,\text{MeV}.$$

(c) Finally, the considered transmutation of uranium-235 into thorium-230 has a Q-value of

$$Q_5 = \left(m_{^{235}\text{U}} - m_{^{230}\text{Th}} - m_{^{5}\text{He}}\right)c^2 = \left(235.0439\,\text{u} - 230.0331\,\text{u} - 5.0122\,\text{u}\right)\left(931.5\,\text{MeV/u}\right)$$
$$= -1.30\,\text{MeV}.$$

Only the second decay process (the α decay) is spontaneous, as it releases energy.

51. Energy and momentum are conserved. We assume the residual thorium nucleus is in its ground state. Let K_α be the kinetic energy of the alpha particle and K_{Th} be the kinetic energy of the thorium nucleus. Then, $Q = K_\alpha + K_{\text{Th}}$. We assume the uranium nucleus is initially at rest. Then, conservation of momentum yields $0 = p_\alpha + p_{\text{Th}}$, where p_α is the momentum of the alpha particle and p_{Th} is the momentum of the thorium nucleus. Both particles travel slowly enough that the classical relationship between momentum and energy can be used. Thus $K_{\text{Th}} = p_{\text{Th}}^2/2m_{\text{Th}}$, where m_{Th} is the mass of the thorium nucleus. We substitute $p_{\text{Th}} = -p_\alpha$ and use $K_\alpha = p_\alpha^2/2m_\alpha$ to obtain $K_{\text{Th}} = (m_\alpha/m_{\text{Th}})K_\alpha$. Consequently,

$$Q = K_\alpha + \frac{m_\alpha}{m_{\text{Th}}}K_\alpha = \left(1 + \frac{m_\alpha}{m_{\text{Th}}}\right)K_\alpha = \left(1 + \frac{4.00\text{u}}{234\text{u}}\right)\left(4.196\text{MeV}\right) = 4.269\text{MeV}.$$

52. (a) For the first reaction

$$Q_1 = \left(m_{\text{Ra}} - m_{\text{Pb}} - m_{\text{C}}\right)c^2 = \left(223.01850\text{u} - 208.98107\text{u} - 14.00324\text{u}\right)\left(931.5\text{MeV/u}\right)$$
$$= 31.8\text{MeV}.$$

(b) For the second one

$$Q_2 = \left(m_{\text{Ra}} - m_{\text{Rn}} - m_{\text{He}}\right)c^2 = \left(223.01850\,\text{u} - 219.00948\,\text{u} - 4.00260\,\text{u}\right)\left(931.5\,\text{MeV/u}\right)$$
$$= 5.98\,\text{MeV}.$$

(c) From $U \propto q_1q_2/r$, we get

$$U_1 \approx U_2\left(\frac{q_{\text{Pb}}\,q_C}{q_{\text{Rn}}\,q_{\text{He}}}\right) = \left(30.0\,\text{MeV}\right)\frac{(82e)(6.0e)}{(86e)(2.0e)} = 86\,\text{MeV}.$$

53. **THINK** The energy released in the decay is the disintegration energy:

$$Q = M_i c^2 - M_f c^2 = (M_i - M_f)c^2 = -\Delta M\, c^2,$$

where $\Delta M = M_f - M_i$ is the change in mass due to the decay.

EXPRESS Let M_{Cs} be the mass of one atom of ${}^{137}_{55}\mathrm{Cs}$ and M_{Ba} be the mass of one atom of ${}^{137}_{56}\mathrm{Ba}$. The energy released is

$$Q = (M_{Cs} - M_{Ba})c^2 .$$

ANALYZE With M_{Cs} = 136.9071 u and M_{Ba} = 136.9058 u, we obtain

$$Q = [136.9071\,\mathrm{u} - 136.9058\,\mathrm{u}]c^2 = (0.0013\,\mathrm{u})c^2 = (0.0013\,\mathrm{u})(931.5\,\mathrm{MeV/u})$$
$$= 1.21\,\mathrm{MeV}.$$

LEARN In calculating Q above, we have used the atomic masses instead of nuclear masses. One can readily show that both lead to the same results. To obtain the nuclear masses, we subtract the mass of 55 electrons from M_{Cs} and the mass of 56 electrons from M_{Ba}. The energy released is

$$Q = [(M_{Cs} - 55m) - (M_{Ba} - 56m) - m]\, c^2,$$

where m is the mass of an electron (the last term in the bracket comes from the beta decay). Once cancellations have been made, $Q = (M_{Cs} - M_{Ba})c^2$, which is the same as before.

54. Assuming the neutrino has negligible mass, then

$$\Delta m c^2 = (\mathbf{m}_{\mathrm{Ti}} - \mathbf{m}_{\mathrm{V}} - m_e)c^2.$$

Now, since vanadium has 23 electrons (see Appendix F and/or G) and titanium has 22 electrons, we can add and subtract $22m_e$ to the above expression and obtain

$$\Delta m c^2 = (\mathbf{m}_{\mathrm{Ti}} + 22m_e - \mathbf{m}_{\mathrm{V}} - 23m_e)c^2 = (m_{\mathrm{Ti}} - m_{\mathrm{V}})c^2.$$

We note that our final expression for Δmc^2 involves the *atomic* masses, and that this assumes (due to the way they are usually tabulated) the atoms are in the ground states (which is certainly not the case here, as we discuss below). The question now is: do we set $Q = -\Delta mc^2$ as in Sample Problem —"Q value in a beta decay, suing masses?" The answer is "no." The atom is left in an excited (high energy) state due to the fact that an electron was captured from the lowest shell (where the absolute value of the energy, E_K, is quite large for large Z). To a very good approximation, the energy of the K-shell electron in Vanadium is equal to that in Titanium (where there is now a "vacancy" that must be filled by a readjustment of the whole electron cloud), and we write $Q = -\Delta m c^2 - E_K$ so that Eq. 42-26 still holds. Thus,

$$Q = \left(m_{\text{V}} - m_{\text{Ti}}\right)c^2 - E_{K.}$$

55. The decay scheme is $\text{n} \rightarrow \text{p} + \text{e}^- + \nu$. The electron kinetic energy is a maximum if no neutrino is emitted. Then,

$$K_{\text{max}} = (m_n - m_p - m_e)c^2,$$

where m_n is the mass of a neutron, m_p is the mass of a proton, and m_e is the mass of an electron. Since $m_p + m_e = m_{\text{H}}$, where m_{H} is the mass of a hydrogen atom, this can be written $K_{\text{max}} = (m_n - m_{\text{H}})c^2$. Hence,

$$K_{\text{max}} = (840 \times 10^{-6}\ \text{u})c^2 = (840 \times 10^{-6}\ \text{u})(931.5\ \text{MeV/u}) = 0.783\ \text{MeV}.$$

56. (a) We recall that $mc^2 = 0.511$ MeV from Table 37-3, and $hc = 1240$ MeV·fm. Using Eq. 37-54 and Eq. 38-13, we obtain

$$\begin{aligned}\lambda &= \frac{h}{p} = \frac{hc}{\sqrt{K^2 + 2Kmc^2}} \\ &= \frac{1240\,\text{MeV}\cdot\text{fm}}{\sqrt{(1.0\,\text{MeV})^2 + 2(1.0\,\text{MeV})(0.511\,\text{MeV})}} = 9.0 \times 10^2\ \text{fm}.\end{aligned}$$

(b) $r = r_0 A^{1/3} = (1.2\ \text{fm})(150)^{1/3} = 6.4$ fm.

(c) Since $\lambda \gg r$ the electron cannot be confined in the nuclide. We recall that at least $\lambda/2$ was needed in any particular direction, to support a standing wave in an "infinite well." A finite well is able to support *slightly* less than $\lambda/2$ (as one can infer from the ground state wave function in Fig. 39-6), but in the present case λ/r is far too big to be supported.

(d) A strong case can be made on the basis of the remarks in part (c), above.

57. (a) Since the positron has the same mass as an electron, and the neutrino has negligible mass, then

$$\Delta mc^2 = \left(\mathbf{m}_{\text{B}} + m_e - \mathbf{m}_{\text{C}}\right)c^2.$$

Now, since carbon has 6 electrons (see Appendix F and/or G) and boron has 5 electrons, we can add and subtract $6m_e$ to the above expression and obtain

$$\Delta mc^2 = \left(\mathbf{m}_{\text{B}} + 7m_e - \mathbf{m}_{\text{C}} - 6m_e\right)c^2 = \left(m_{\text{B}} + 2m_e - m_{\text{C}}\right)c^2.$$

We note that our final expression for Δmc^2 involves the *atomic* masses, as well an "extra" term corresponding to two electron masses. From Eq. 37-50 and Table 37-3, we obtain

$$Q = \left(m_{\text{C}} - m_{\text{B}} - 2m_e\right)c^2 = \left(m_{\text{C}} - m_{\text{B}}\right)c^2 - 2(0.511\,\text{MeV}).$$

(b) The disintegration energy for the positron decay of carbon-11 is

$$\begin{aligned}Q &= (11.011434\,\text{u} - 11.009305\,\text{u})(931.5\,\text{MeV/u}) - 1.022\,\text{MeV} \\ &= 0.961\,\text{MeV}.\end{aligned}$$

58. (a) The rate of heat production is

$$\begin{aligned}\frac{dE}{dt} &= \sum_{i=1}^{3} R_i Q_i = \sum_{i=1}^{3} \lambda_1 N_i Q_i = \sum_{i=1}^{3} \left(\frac{\ln 2}{T_{1/2_i}}\right) \frac{(1.00\,\text{kg}) f_i}{m_i} Q_i \\ &= \frac{(1.00\,\text{kg})(\ln 2)(1.60\times 10^{-13}\,\text{J / MeV})}{(3.15\times 10^{7}\,\text{s / y})(1.661\times 10^{-27}\,\text{kg / u})} \left[\frac{(4\times 10^{-6})(51.7\,\text{MeV})}{(238\,\text{u})(4.47\times 10^{9}\,\text{y})}\right. \\ &\qquad \left. + \frac{(13\times 10^{-6})(42.7\,\text{MeV})}{(232\,\text{u})(1.41\times 10^{10}\,\text{y})} + \frac{(4\times 10^{-6})(1.31\,\text{MeV})}{(40\,\text{u})(1.28\times 10^{9}\,\text{y})}\right] \\ &= 1.0\times 10^{-9}\,\text{W}.\end{aligned}$$

(b) The contribution to heating, due to radioactivity, is

$$P = (2.7 \times 10^{22}\,\text{kg})(1.0 \times 10^{-9}\,\text{W/kg}) = 2.7 \times 10^{13}\,\text{W},$$

which is very small compared to what is received from the Sun.

59. **THINK** The beta decay of ^{32}P is given by

$$^{32}\text{P} \rightarrow {}^{32}\text{S} + e^{-} + \bar{\nu}\ .$$

However, since the electron has the maximum possible kinetic energy, no (anti)neutrino is emitted.

EXPRESS Since momentum is conserved, the momentum of the electron and the momentum of the residual sulfur nucleus are equal in magnitude and opposite in direction. If p_e is the momentum of the electron and p_S is the momentum of the sulfur nucleus, then $p_S = -p_e$. The kinetic energy K_S of the sulfur nucleus is

$$K_S = p_S^2 / 2M_S = p_e^2 / 2M_S\,,$$

where M_S is the mass of the sulfur nucleus. Now, the electron's kinetic energy K_e is related to its momentum by the relativistic equation $(p_e c)^2 = K_e^2 + 2K_e mc^2$, where m is the mass of an electron.

ANALYZE With K_e = 1.71 MeV, the kinetic energy of the recoiling sulfur nucleus is

$$K_S = \frac{(p_e c)^2}{2M_S c^2} = \frac{K_e^2 + 2K_e mc^2}{2M_S c^2} = \frac{(1.71\,\text{MeV})^2 + 2(1.71\,\text{MeV})(0.511\,\text{MeV})}{2(32\,\text{u})(931.5\,\text{MeV/u})}$$
$$= 7.83\times10^{-5}\ \text{MeV} = 78.3\ \text{eV}$$

where mc^2 = 0.511 MeV is used for the electron (see Table 37-3).

LEARN The maximum kinetic energy of the electron is equal to the disintegration energy Q:

$$Q = K_{\text{max}}\,.$$

To show this, we use the following data: M_P = 31.97391 u and M_S = 31.97207 u. The result is

$$Q = [31.97391\,\text{u} - 31.97207\,\text{u}]c^2 = (0.00184\,\text{u})c^2 = (0.00184\,\text{u})(931.5\,\text{MeV/u})$$
$$= 1.71\,\text{MeV}.$$

60. We solve for t from $R = R_0 e^{-\lambda t}$:

$$t = \frac{1}{\lambda}\ln\frac{R_0}{R} = \left(\frac{5730\,\text{y}}{\ln 2}\right)\ln\left[\left(\frac{15.3}{63.0}\right)\left(\frac{5.00}{1.00}\right)\right] = 1.61\times10^3\ \text{y}.$$

61. (a) The mass of a ^{238}U atom is (238 u)(1.661×10^{-24} g/u) = 3.95×10^{-22} g, so the number of uranium atoms in the rock is

$$N_U = (4.20 \times 10^{-3}\ \text{g})/(3.95 \times 10^{-22}\ \text{g}) = 1.06 \times 10^{19}.$$

(b) The mass of a ^{206}Pb atom is (206 u)(1.661×10^{-24} g) = 3.42×10^{-22} g, so the number of lead atoms in the rock is

$$N_{Pb} = (2.135 \times 10^{-3}\ \text{g})/(3.42 \times 10^{-22}\ \text{g}) = 6.24 \times 10^{18}.$$

(c) If no lead was lost, there was originally one uranium atom for each lead atom formed by decay, in addition to the uranium atoms that did not yet decay. Thus, the original number of uranium atoms was

$$N_{U0} = N_U + N_{Pb} = 1.06 \times 10^{19} + 6.24 \times 10^{18} = 1.68 \times 10^{19}.$$

(d) We use

$$N_U = N_{U0} e^{-\lambda t}$$

where λ is the disintegration constant for the decay. It is related to the half-life $T_{1/2}$ by $\lambda = (\ln 2)/T_{1/2}$. Thus,

$$t = -\frac{1}{\lambda}\ln\left(\frac{N_U}{N_{U0}}\right) = -\frac{T_{1/2}}{\ln 2}\ln\left(\frac{N_U}{N_{U0}}\right) = -\frac{4.47\times10^9\text{ y}}{\ln 2}\ln\left(\frac{1.06\times10^{19}}{1.68\times10^{19}}\right) = 2.97\times10^9\text{ y}.$$

62. The original amount of ^{238}U the rock contains is given by

$$m_0 = me^{\lambda t} = (3.70\text{ mg})e^{(\ln 2)\left(260\times10^6\text{ y}\right)/\left(4.47\times10^9\text{ y}\right)} = 3.85\text{ mg}.$$

Thus, the amount of lead produced is

$$m' = (m_0 - m)\left(\frac{m_{206}}{m_{238}}\right) = (3.85\text{ mg} - 3.70\text{ mg})\left(\frac{206}{238}\right) = 0.132\text{ mg}.$$

63. We can find the age t of the rock from the masses of ^{238}U and ^{206}Pb. The initial mass of ^{238}U is

$$m_{U_0} = m_U + \frac{238}{206}m_{Pb}.$$

Therefore,

$$m_U = m_{U_0}e^{-\lambda_U t} = \left(m_U + m_{^{238}Pb}/206\right)e^{-(t\ln 2)/T_{1/2_U}}.$$

We solve for t:

$$t = \frac{T_{1/2_U}}{\ln 2}\ln\left(\frac{m_U + (238/206)m_{Pb}}{m_U}\right) = \frac{4.47\times10^9\text{ y}}{\ln 2}\ln\left[1+\left(\frac{238}{206}\right)\left(\frac{0.15\text{mg}}{0.86\text{mg}}\right)\right]$$
$$= 1.18\times10^9\text{ y}.$$

For the β decay of ^{40}K, the initial mass of ^{40}K is

$$m_{K_0} = m_K + (40/40)m_{Ar} = m_K + m_{Ar},$$

so

$$m_K = m_{K_0}e^{-\lambda_K t} = (m_K + m_{Ar})e^{-\lambda_K t}.$$

We solve for m_K:

$$m_K = \frac{m_{Ar}e^{-\lambda_K t}}{1-e^{-\lambda_K t}} = \frac{m_{Ar}}{e^{\lambda_K t}-1} = \frac{1.6\text{mg}}{e^{(\ln 2)\left(1.18\times10^9\text{ y}\right)/\left(1.25\times10^9\text{ y}\right)}-1} = 1.7\text{mg}.$$

64. We note that every calcium-40 atom and krypton-40 atom found now in the sample was once one of the original numbers of potassium atoms. Thus, using Eq. 42-14 and Eq. 42-18, we find

$$\ln\left(\frac{N_K}{N_K + N_{Ar} + N_{Ca}}\right) = -\lambda t \quad \Rightarrow \quad \ln\left(\frac{1}{1+1+8.54}\right) = -\frac{\ln 2}{T_{1/2}} t$$

which (with $T_{1/2} = 1.26 \times 10^9$ y) yields $t = 4.28 \times 10^9$ y.

65. **THINK** The activity of a radioactive sample expressed in curie (Ci) can be converted to SI units (Bq) as

$$1 \text{ curie} = 1 \text{ Ci} = 3.7 \times 10^{10} \text{ Bq} = 3.7 \times 10^{10} \text{ disintegrations/s.}$$

EXPRESS The decay rate R is related to the number of nuclei N by $R = \lambda N$, where λ is the disintegration constant. The disintegration constant is related to the half-life $T_{1/2}$ by

$$\lambda = \frac{\ln 2}{T_{1/2}} \quad \Rightarrow \quad N = \frac{R}{\lambda} = \frac{RT_{1/2}}{\ln 2} .$$

Since 1 Ci = 3.7×10^{10} disintegrations/s,

$$N = \frac{(250\,\text{Ci})(3.7\times10^{10}\text{ s}^{-1}/\text{Ci})(2.7\,\text{d})(8.64\times10^4\text{ s/d})}{\ln 2} = 3.11\times10^{18}.$$

ANALYZE The mass of a ^{198}Au atom is

$$M_0 = (198 \text{ u})(1.661 \times 10^{-24} \text{ g/u}) = 3.29 \times 10^{-22} \text{ g},$$

so the mass required is

$$M = N M_0 = (3.11 \times 10^{18})(3.29 \times 10^{-22} \text{ g}) = 1.02 \times 10^{-3} \text{ g} = 1.02 \text{ mg}.$$

LEARN The ^{198}Au atom undergoes beta decay and emit an electron:

$$^{198}\text{Au} \rightarrow {}^{198}\text{Hg} + e^- + \bar{\nu} .$$

66. The becquerel (Bq) and curie (Ci) are defined in Section 42-3.

(a) R = 8700/60 = 145 Bq.

(b) $R = \dfrac{145\,\text{Bq}}{3.7\times10^{10}\,\text{Bq/Ci}} = 3.92\times10^{-9}$ Ci.

67. The absorbed dose is

$$\text{absorbed dose} = \frac{2.00\times10^{-3}\ \text{J}}{4.00\ \text{kg}} = 5.00\times10^{-4}\ \text{J/kg} = 5.00\times10^{-4}\ \text{Gy}$$

where $1\ \text{J/kg} = 1\,\text{Gy}$. With $\text{RBE} = 5$, the dose equivalent is

$$\begin{aligned}\text{dose equivalent} &= \text{RBE}\cdot(5.00\times10^{-4}\ \text{Gy}) = 5(5.00\times10^{-4}\ \text{Gy}) = 2.50\times10^{-3}\ \text{Sv}\\ &= 2.50\ \text{mSv}.\end{aligned}$$

68. (a) Using Eq. 42-32, the energy absorbed is

$$\left(2.4\times10^{-4}\ \text{Gy}\right)\left(75\,\text{kg}\right) = 18\,\text{mJ}.$$

(b) The dose equivalent is

$$\left(2.4\times10^{-4}\,\text{Gy}\right)\left(12\right) = 2.9\times10^{-3}\,\text{Sv}.$$

(c) Using Eq. 42-33, we have $2.9\times10^{-3}\,\text{Sv} = 0.29\ \text{rem}$.

69. (a) Adapting Eq. 42-21, we find

$$N_0 = \frac{\left(2.5\times10^{-3}\ \text{g}\right)\left(6.02\times10^{23}\ /\ \text{mol}\right)}{239\,\text{g}\ /\ \text{mol}} = 6.3\times10^{18}.$$

(b) From Eq. 42-15 and Eq. 42-18,

$$|\,\Delta N\,| = N_0\left[1-e^{-t\ln 2/T_{1/2}}\right] = \left(6.3\times10^{18}\right)\left[1-e^{-(12\,\text{h})\ln 2/(24{,}100\,\text{y})(8760\,\text{h/y})}\right] = 2.5\times10^{11}.$$

(c) The energy absorbed by the body is

$$\left(0.95\right)E_\alpha\left|\Delta N\right| = \left(0.95\right)\left(5.2\,\text{MeV}\right)\left(2.5\times10^{11}\right)\left(1.6\times10^{-13}\ \text{J/MeV}\right) = 0.20\ \text{J}.$$

(d) On a per unit mass basis, the previous result becomes (according to Eq. 42-32)

$$\frac{0.20\,\text{mJ}}{85\,\text{kg}} = 2.3\times10^{-3}\ \text{J}\ /\ \text{kg} = 2.3\,\text{mGy}.$$

(e) Using Eq. 42-31, (2.3 mGy)(13) = 30 mSv.

70. From Eq. 19-24, we obtain

$$T = \frac{2}{3}\left(\frac{K_{\text{avg}}}{\text{k}}\right) = \frac{2}{3}\left(\frac{5.00\times10^{6}\,\text{eV}}{8.62\times10^{-5}\,\text{eV/K}}\right) = 3.87\times10^{10}\,\text{K}.$$

71. (a) Following Sample Problem — "Lifetime of a compound nucleus made by neutron capture," we compute

$$\Delta E \approx \frac{\hbar}{t_{\text{avg}}} = \frac{\left(4.14\times10^{-15}\,\text{eV}\cdot\text{fs}\right)/\,2\pi}{1.0\times10^{-22}\,\text{s}} = 6.6\times10^{6}\,\text{eV}.$$

(b) In order to fully distribute the energy in a fairly large nucleus, and create a "compound nucleus" equilibrium configuration, about 10^{-15} s is typically required. A reaction state that exists no more than about 10^{-22} s does not qualify as a compound nucleus.

72. (a) We compare both the proton numbers (atomic numbers, which can be found in Appendix F and/or G) and the neutron numbers (see Eq. 42-1) with the magic nucleon numbers (special values of either *Z* or *N*) listed in Section 42-8. We find that ^{18}O, ^{60}Ni, ^{92}Mo, ^{144}Sm, and ^{207}Pb each have a filled shell for either the protons or the neutrons (two of these, ^{18}O and ^{92}Mo, are explicitly discussed in that section).

(b) Consider ^{40}K, which has Z = 19 protons (which is one less than the magic number 20). It has N = 21 neutrons, so it has one neutron outside a closed shell for neutrons, and thus qualifies for this list. Others in this list include ^{91}Zr, ^{121}Sb, and ^{143}Nd.

(c) Consider ^{13}C, which has Z = 6 and N = 13 – 6 = 7 neutrons. Since 8 is a magic number, then ^{13}C has a vacancy in an otherwise filled shell for neutrons. Similar arguments lead to inclusion of ^{40}K, ^{49}Ti, ^{205}Tl, and ^{207}Pb in this list.

73. **THINK**A generalized formation reaction can be written $X + x \rightarrow Y$, where *X* is the target nucleus, *x* is the incident light particle, and *Y* is the excited compound nucleus (^{20}Ne).

EXPRESS We assume *X* is initially at rest. Then, conservation of energy yields

$$m_X c^2 + m_x c^2 + K_x = m_Y c^2 + K_Y + E_Y$$

where m_X, m_x, and m_Y are masses, K_x and K_Y are kinetic energies, and E_Y is the excitation energy of *Y*. Conservation of momentum yields $p_x = p_Y$. Now,

$$K_Y = \frac{p_Y^2}{2m_Y} = \frac{p_x^2}{2m_Y} = \left(\frac{m_x}{m_Y}\right)K_x$$

so

$$m_X c^2 + m_x c^2 + K_x = m_Y c^2 + \left(m_x / m_Y\right) K_x + E_Y$$

and

$$K_x = \frac{m_Y}{m_Y - m_x} \left(m_Y - m_X - m_x\right) c^2 + E_Y \ .$$

ANALYZE (a) Let x represent the alpha particle and X represent the ^{16}O nucleus. Then,

$$\begin{aligned}(m_Y - m_X - m_x)c^2 &= (19.99244 \text{ u} - 15.99491 \text{ u} - 4.00260 \text{ u})(931.5 \text{ MeV/u}) \\ &= -4.722 \text{ MeV}\end{aligned}$$

and

$$K_\alpha = \frac{19.99244\text{u}}{19.99244\text{u} - 4.00260\text{u}}\left(-4.722\text{MeV} + 25.0\text{MeV}\right) = 25.35\text{MeV} \approx 25.4 \text{ MeV}.$$

(b) Let x represent the proton and X represent the ^{19}F nucleus. Then,

$$\begin{aligned}(m_Y - m_X - m_x)c^2 &= (19.99244 \text{ u} - 18.99841 \text{ u} - 1.00783 \text{ u})(931.5 \text{ MeV/u}) \\ &= -12.85 \text{ MeV}\end{aligned}$$

and

$$K_\alpha = \frac{19.99244\,\text{u}}{19.99244\,\text{u} - 1.00783\,\text{u}}\left(-12.85\,\text{MeV} + 25.0\,\text{MeV}\right) = 12.80\,\text{MeV}.$$

(c) Let x represent the photon and X represent the ^{20}Ne nucleus. Since the mass of the photon is zero, we must rewrite the conservation of energy equation: if E_γ is the energy of the photon, then

$$E_\gamma + m_X c^2 = m_Y c^2 + K_Y + E_Y.$$

Since $m_X = m_Y$, this equation becomes $E_\gamma = K_Y + E_Y$. Since the momentum and energy of a photon are related by $p_\gamma = E_\gamma/c$, the conservation of momentum equation becomes $E_\gamma/c = p_Y$. The kinetic energy of the compound nucleus is

$$K_Y = \frac{p_Y^2}{2m_Y} = \frac{E_\gamma^2}{2m_Y c^2} \ .$$

We substitute this result into the conservation of energy equation to obtain

$$E_\gamma = \frac{E_\gamma^2}{2m_Y c^2} + E_Y.$$

This quadratic equation has the solutions

$$E_\gamma = m_Y c^2 \pm \sqrt{\left(m_Y c^2\right)^2 - 2m_Y c^2 E_Y}\,.$$

If the problem is solved using the relativistic relationship between the energy and momentum of the compound nucleus, only one solution would be obtained, the one corresponding to the negative sign above. Since

$$m_Y c^2 = (19.99244 \text{ u})(931.5 \text{ MeV/u}) = 1.862 \times 10^4 \text{ MeV},$$

we have

$$\begin{aligned} E_\gamma &= \left(1.862\times10^4 \text{ MeV}\right) - \sqrt{\left(1.862\times10^4 \text{ MeV}\right)^2 - 2\left(1.862\times10^4 \text{ MeV}\right)\left(25.0 \text{ MeV}\right)} \\ &= 25.0 \text{ MeV}. \end{aligned}$$

LEARN In part (c), the kinetic energy of the compound nucleus is

$$K_Y = \frac{E_\gamma^2}{2m_Y c^2} = \frac{(25.0 \text{ MeV})^2}{2(1.862\times10^4 \text{ MeV})} = 0.0168 \text{ MeV}$$

which is very small compared to E_Y = 25.0 MeV. Essentially all of the photon energy goes to excite the nucleus.

74. Using Eq. 42-15, the amount of uranium atoms and lead atoms present in the rock at time t is

$$\begin{aligned} N_{\text{U}} &= N_0 e^{-\lambda t} \\ N_{\text{Pb}} &= N_0 - N_{\text{U}} = N_0 - N_0 e^{-\lambda t} = N_0(1 - e^{-\lambda t}) \end{aligned}$$

and their ratio is

$$\frac{N_{\text{Pb}}}{N_{\text{U}}} = \frac{1 - e^{-\lambda t}}{e^{-\lambda t}} = e^{\lambda t} - 1.$$

The age of the rock is

$$t = \frac{1}{\lambda}\ln\left(1 + \frac{N_{\text{Pb}}}{N_{\text{U}}}\right) = \frac{T_{1/2}}{\ln 2}\ln\left(1 + \frac{N_{\text{Pb}}}{N_{\text{U}}}\right) = \frac{4.47\times10^9 \text{ y}}{\ln 2}\ln\left(1 + 0.30\right) = 1.69\times10^9 \text{ y}.$$

75. **THINK** We represent the unknown nuclide as ${}^{A}_{Z}X$, where A and Z are its mass number and atomic number, respectively.

EXPRESS The reaction equation can be written as

$${}^{A}_{Z}X + {}^{1}_{0}\text{n} \rightarrow {}^{0}_{-1}\text{e} + 2\,{}^{4}_{2}\text{He}.$$

Conservation of charge yields $Z + 0 = -1 + 4$ or $Z = 3$. Conservation of mass number yields $A + 1 = 0 + 8$ or $A = 7$.

ANALYZE According to the periodic table in Appendix G (also see Appendix F), lithium has atomic number 3, so the nuclide must be ${}^7_3\mathrm{Li}$.

LEARN Charge and mass number are conserved in the neutron-capture process. The intermediate nuclide is ${}^8\mathrm{Li}$, which is unstable and decays (via α and β^- modes) into two ${}^4\mathrm{He}$'s and an electron.

76. The dose equivalent is the product of the absorbed dose and the RBE factor, so the absorbed dose is

$$(\text{dose equivalent})/(\text{RBE}) = (250 \times 10^{-6}\ \text{Sv})/(0.85) = 2.94 \times 10^{-4}\ \text{Gy}.$$

But 1 Gy = 1 J/kg, so the absorbed dose is

$$\left(2.94\times10^{-4}\ \text{Gy}\right)\left(1\frac{\text{J}}{\text{kg}\cdot\text{Gy}}\right) = 2.94\times10^{-4}\ \text{J/kg}.$$

To obtain the total energy received, we multiply this by the mass receiving the energy:

$$E = (2.94 \times 10^{-4}\ \text{J/kg})(44\ \text{kg}) = 1.29 \times 10^{-2}\ \text{J} \approx 1.3 \times 10^{-2}\ \text{J}.$$

77. **THINK** The decay rate R is proportional to N, the number of radioactive nuclei.

EXPRESS According to Eq. 42-17, $R = \lambda N$, where λ is the decay constant. Since R is proportional to N, then $N/N_0 = R/R_0 = e^{-\lambda t}$. Since $\lambda = (\ln 2)/T_{1/2}$, the solution for t is

$$t = -\frac{1}{\lambda}\ln\left(\frac{R}{R_0}\right) = -\frac{T_{1/2}}{\ln 2}\ln\left(\frac{R}{R_0}\right).$$

ANALYZE With $T_{1/2}$ = 5730 y and R/R_0 = 0.020, we obtain

$$t = -\frac{T_{1/2}}{\ln 2}\ln\left(\frac{R}{R_0}\right) = -\frac{5730\ \text{y}}{\ln 2}\ln(0.020) = 3.2\times10^4\ \text{y}.$$

LEARN Radiocarbon dating based on the decay of ${}^{14}\mathrm{C}$ is one of the most widely used dating method in estimating the age of organic remains.

78. Let N_{AA0} be the number of element AA at $t = 0$. At a later time t, due to radioactive decay, we have

$$N_{\text{AA0}} = N_{\text{AA}} + N_{\text{BB}} + N_{\text{CC}}.$$

The decay constant is

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{8.00\text{ d}} = 0.0866/\text{d}.$$

Since $N_{BB}/N_{CC} = 2$, when $N_{CC}/N_{AA} = 1.50$, $N_{BB}/N_{AA} = 3.00$. Therefore, at time *t*,

$$N_{AA0} = N_{AA} + N_{BB} + N_{CC} = N_{AA} + 3.00N_{AA} + 1.50N_{AA} = 5.50N_{AA}.$$

Since $N_{AA} = N_{AA0}e^{-\lambda t}$, combining the two expressions leads to

$$\frac{N_{AA0}}{N_{AA}} = e^{\lambda t} = 5.50$$

which can be solved to give

$$t = \frac{\ln(5.50)}{\lambda} = \frac{\ln(5.50)}{0.0866/\text{d}} = 19.7\text{ d}.$$

79. **THINK** The count rate in the area in question is given by $R = \lambda N$, where λ is the decay constant and N is the number of radioactive nuclei.

EXPRESS Since the spreading is assumed uniform, the count rate R = 74,000/s is given by

$$R = \lambda N = \lambda(M/m)(a/A),$$

where M is the mass of ^{90}Sr produced, m is the mass of a single ^{90}Sr nucleus, A is the area over which fall out occurs, and a is the area in question. Since $\lambda = (\ln 2)/T_{1/2}$, the solution for a is

$$a = A\left(\frac{m}{M}\right)\left(\frac{R}{\lambda}\right) = \frac{AmRT_{1/2}}{M\ln 2}.$$

ANALYZE The molar mass of ^{90}Sr is 90g/mol. With M = 400 g and A = 2000 km^2, we find the area to be

$$a = \frac{AmRT_{1/2}}{M\ln 2} = \frac{\left(2000\times 10^6\text{ m}^2\right)\left(90\text{ g/mol}\right)\left(74,000/\text{s}\right)\left(29\text{ y}\right)\left(3.15\times 10^7\text{ s/y}\right)}{\left(400\text{ g}\right)\left(6.02\times 10^{23}/\text{mol}\right)\left(\ln 2\right)}$$
$$= 7.3\times 10^{-2}\text{ m}^{-2} = 730\text{ cm}^2.$$

LEARN The Chernobyl nuclear accident in 1986 contaminated a very large area with ^{90}Sr.

80. (a) Assuming a "target" area of one square meter, we establish a ratio:

$$\frac{\text{rate through you}}{\text{total rate upward}} = \frac{1\,\text{m}^2}{\left(2.6\times10^5\ \text{km}^2\right)\left(1000\,\text{m/km}\right)^2} = 3.8\times10^{-12}.$$

The SI unit becquerel is equivalent to a disintegration per second. With half the beta-decay electrons moving upward, we find

$$\text{rate through you} = \frac{1}{2}\left(1\times10^{16}/\text{s}\right)\left(3.8\times10^{-12}\right) = 1.9\times10^4/\text{s}$$

which implies (converting $\text{s}\rightarrow\text{h}$) that the rate of electrons you would intercept is $R_0 = 7 \times 10^7$/h. So in one hour, 7×10^7 electrons would be intercepted.

(b) Let D indicate the current year (2003, 2004, etc.). Combining Eq. 42-16 and Eq. 42-18, we find

$$R = R_0 e^{-t\ln 2/T_{1/2}} = \left(7\times10^7/\text{h}\right)e^{-(D-1996)\ln 2/(30.2\,\text{y})}.$$

81. The lines that lead toward the lower left are alpha decays, involving an atomic number change of $\Delta Z_\alpha = -2$ and a mass number change of $\Delta A_\alpha = -4$. The short horizontal lines toward the right are beta decays (involving electrons, not positrons) in which case A stays the same but the change in atomic number is $\Delta Z_\beta = +1$. Figure 42-20 shows three alpha decays and two beta decays; thus,

$$Z_f = Z_i + 3\Delta Z_\alpha + 2\Delta Z_\beta \text{ and } A_f = A_i + 3\Delta A_\alpha.$$

Referring to Appendix F or G, we find $Z_i = 93$ for neptunium, so

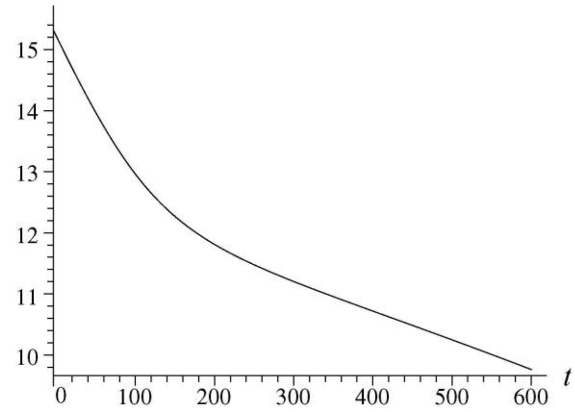
$$Z_f = 93 + 3(-2) + 2(1) = 89,$$

which indicates the element actinium. We are given $A_i = 237$, so $A_f = 237 + 3(-4) = 225$. Therefore, the final isotope is ^{225}Ac.

82. We note that 2.42 min = 145.2 s. We are asked to plot (with SI units understood)

$$\ln R = \ln\left(R_0 e^{-\lambda t} + R_0' e^{-\lambda' t}\right)$$

where $R_0 = 3.1 \times 10^5$, $R_0' = 4.1 \times 10^6$, $\lambda = \ln 2/145.2$, and $\lambda' = \ln 2/24.6$. Our plot is shown below.



We note that the magnitude of the slope for small t is λ' (the disintegration constant for ^{110}Ag), and for large t is λ (the disintegration constant for ^{108}Ag).

83. We note that hc = 1240 MeV·fm, and that the classical kinetic energy $\frac{1}{2}mv^2$ can be written directly in terms of the classical momentum $p = mv$ (see below). Letting

$$p \simeq \Delta p \simeq \Delta h / \Delta x \simeq h / r,$$

we get

$$E = \frac{p^2}{2m} \simeq \frac{(hc)^2}{2(mc^2)r^2} = \frac{(1240\,\text{MeV}\cdot\text{fm})^2}{2(938\,\text{MeV})\left[(1.2\,\text{fm})(100)^{1/3}\right]^2} \simeq 30\,\text{MeV}.$$

84. (a) The rate at which radium-226 is decaying is

$$R = \lambda N = \left(\frac{\ln 2}{T_{1/2}}\right)\left(\frac{M}{m}\right) = \frac{(\ln 2)(1.00\,\text{mg})(6.02\times 10^{23}\,/\,\text{mol})}{(1600\,\text{y})(3.15\times 10^{7}\,\text{s}/\text{y})(226\,\text{g}/\text{mol})} = 3.66\times 10^{7}\,\text{s}^{-1}.$$

The activity is 3.66×10^{7} Bq.

(b) The activity of ^{222}Rn is also 3.66×10^{7} Bq.

(c) From $R_{\text{Ra}} = R_{\text{Rn}}$ and $R = \lambda N = (\ln 2/T_{1/2})(M/m)$, we get

$$M_{\text{Rn}} = \left(\frac{T_{1/2_{\text{Rn}}}}{T_{1/2_{\text{Ra}}}}\right)\left(\frac{m_{\text{Rn}}}{m_{\text{Ra}}}\right)M_{\text{Ra}} = \frac{(3.82\text{d})(1.00\times 10^{-3}\,\text{g})(222\text{u})}{(1600\text{y})(365\text{d/y})(226\text{u})} = 6.42\times 10^{-9}\,\text{g}.$$

85. Although we haven't drawn the requested lines in the following table, we can indicate their slopes: lines of constant A would have – 45° slopes, and those of constant $N - Z$ would have 45°. As an example of the latter, the $N - Z = 20$ line (which is one of "eighteen-neutron excess") would pass through Cd-114 at the lower left corner up through Te-122 at the upper right corner. The first column corresponds to $N = 66$, and the

bottom row to Z = 48. The last column corresponds to N = 70, and the top row to Z = 52. Much of the information below (regarding values of $T_{1/2}$ particularly) was obtained from the Web sites http://nucleardata.nuclear.lu.se/nucleardata and http://www.nndc.bnl.gov/nndc/ensdf.

^{118}Te 6.0 days	^{119}Te 16.0 h	^{120}Te 0.1%	^{121}Te 19.4 days	^{122}Te 2.6%
^{117}Sb 2.8 h	^{118}Sb 3.6 min	^{119}Sb 38.2 s	^{120}Sb 15.9 min	^{121}Sb 57.2%
^{116}Sn 14.5%	^{117}Sn 7.7%	^{118}Sn 24.2%	^{119}Sn 8.6%	^{120}Sn 32.6%
^{115}In 95.7%	^{116}In 14.1 s	^{117}In 43.2 min	^{118}In 5.0 s	^{119}In 2.4 min
^{114}Cd 28.7%	^{115}Cd 53.5 h	^{116}Cd 7.5%	^{117}Cd 2.5 h	^{118}Cd 50.3 min

86. Using Eq. 42-3 ($r = r_0 A^{1/3}$), we estimate the nuclear radii of the alpha particle and Al to be

$$r_\alpha = (1.2\times10^{-15}\text{ m})(4)^{1/3} = 1.90\times10^{-15}\text{ m}$$
$$r_{\text{Al}} = (1.2\times10^{-15}\text{ m})(27)^{1/3} = 3.60\times10^{-15}\text{ m}.$$

The distance between the centers of the nuclei when their surfaces touch is

$$r = r_\alpha + r_{\text{Al}} = 1.90\times10^{-15}\text{ m} + 3.60\times10^{-15}\text{ m} = 5.50\times10^{-15}\text{ m}.$$

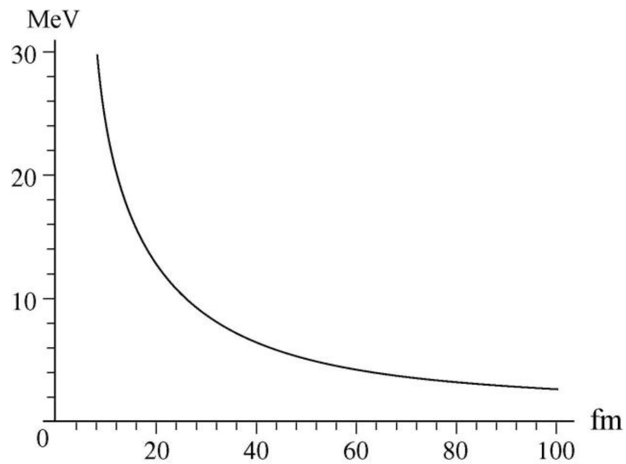
From energy conservation, the amount of energy required is

$$\begin{aligned} K &= \frac{1}{4\pi\varepsilon_0}\frac{q_\alpha q_{\text{Al}}}{r} = \frac{(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(2\times1.6\times10^{-19}\text{ C})(13\times1.6\times10^{-19}\text{ C})}{5.50\times10^{-15}\text{ m}} \\ &= 1.09\times10^{-12}\text{ J} = 6.79\times10^6\text{ eV} \end{aligned}$$

87. Equation 24-43 gives the electrostatic potential energy between two uniformly charged spherical charges (in this case q_1 = 2e and q_2 = 90e) with r being the distance between their centers. Assuming the "uniformly charged spheres" condition is met in this instance, we write the equation in such a way that we can make use of $k = 1/4\pi\varepsilon_0$ and the electronvolt unit:

$$U = k\frac{(2e)(90e)}{r} = \left(8.99\times10^{9}\ \frac{\text{V}\cdot\text{m}}{\text{C}}\right)\frac{\left(3.2\times10^{-19}\ \text{C}\right)(90e)}{r} = \frac{2.59\times10^{-7}}{r}\,\text{eV}$$

with r understood to be in meters. It is convenient to write this for r in femtometers, in which case $U = 259/r$ MeV. This is shown plotted below.



88. We take the speed to be constant, and apply the classical kinetic energy formula:

$$\begin{aligned} t &= \frac{d}{v} = \frac{d}{\sqrt{2K/m}} = 2r\sqrt{\frac{m_n}{2K}} = \frac{r}{c}\sqrt{\frac{2mc^2}{K}} \\ &\approx \frac{\left(1.2\times10^{-15}\,\text{m}\right)\left(100\right)^{1/3}}{3.0\times10^{8}\,\text{m/s}}\sqrt{\frac{2\left(938\text{MeV}\right)}{5\text{MeV}}} \\ &\approx 4\times10^{-22}\,\text{s}. \end{aligned}$$

89. We solve for A from Eq. 42-3:

$$A = \left(\frac{r}{r_0}\right)^3 = \left(\frac{3.6\,\text{fm}}{1.2\,\text{fm}}\right)^3 = 27.$$

90. The problem with Web-based services is that there are no guarantees of accuracy or that the Web page addresses will not change from the time this solution is written to the time someone reads this. Still, it is worth mentioning that a very accessible Web site for a wide variety of periodic table and isotope-related information is http://www.webelements.com. Two sites, http://nucleardata.nuclear.lu.se/nucleardata and http://www.nndc.bnl.gov/nndc/ensdf, are aimed more toward the nuclear professional. These are the sites where some of the information mentioned below was obtained.

(a) According to Appendix F, the atomic number 60 corresponds to the element neodymium (Nd). The first Web site mentioned above gives ^{142}Nd, ^{143}Nd, ^{144}Nd, ^{145}Nd, ^{146}Nd, ^{148}Nd, and ^{150}Nd in its list of naturally occurring isotopes. Two of these, ^{144}Nd and ^{150}Nd, are not perfectly stable, but their half-lives are much longer than the age of the universe (detailed information on their half-lives, modes of decay, etc. are available at the last two Web sites referred to, above).

(b) In this list, we are asked to put the nuclides that contain 60 neutrons and that are recognized to exist but not stable nuclei (this is why, for example, ^{108}Cd is not included here). Although the problem does not ask for it, we include the half-lives of the nuclides in our list, though it must be admitted that not all reference sources agree on those values (we picked ones we regarded as "most reliable"). Thus, we have ^{97}Rb (0.2 s), ^{98}Sr (0.7 s), ^{99}Y (2 s), ^{100}Zr (7 s), ^{101}Nb (7 s), ^{102}Mo (11 minutes), ^{103}Tc (54 s), ^{105}Rh (35 hours), ^{109}In (4 hours), ^{110}Sn (4 hours), ^{111}Sb (75 s), ^{112}Te (2 minutes), ^{113}I (7 s), ^{114}Xe (10 s), ^{115}Cs (1.4 s), and ^{116}Ba (1.4 s).

(c) We would include in this list: ^{60}Zn, ^{60}Cu, ^{60}Ni, ^{60}Co, ^{60}Fe, ^{60}Mn, ^{60}Cr, and ^{60}V.

91. (a) In terms of the original value of u, the newly defined u is greater by a factor of 1.007825. So the mass of ^{1}H would be 1.000000 u, the mass of ^{12}C would be

$$(12.000000/1.007825)\text{ u} = 11.90683\text{ u}.$$

(b) The mass of ^{238}U would be (238.050785/ 1.007825) u = 236.2025 u.

92. (a) The mass number A of a radionuclide changes by 4 in an α decay and is unchanged in a β decay. If the mass numbers of two radionuclides are given by $4n + k$ and $4n' + k$ (where k = 0, 1, 2, 3), then the heavier one can decay into the lighter one by a series of α (and β) decays, as their mass numbers differ by only an integer times 4. If $A = 4n + k$, then after α-decaying for m times, its mass number becomes

$$A = 4n + k - 4m = 4(n - m) + k,$$

still in the same chain.

(b) For ^{235}U, $235 = 58 \times 4 + 3 = 4n + 3$.

(c) For ^{236}U, $236 = 59 \times 4 = 4n$.

(d) For ^{238}U, $238 = 59 \times 4 + 2 = 4n + 2$.

(e) For ^{239}Pu, $239 = 59 \times 4 + 3 = 4n + 3$.

(f) For ^{240}Pu, $240 = 60 \times 4 = 4n$.

(g) For ^{245}Cm, $245 = 61 \times 4 + 1 = 4n + 1$.

(h) For ^{246}Cm, 246 = 61 × 4 + 2 = 4*n* + 2.

(i) For ^{249}Cf, 249 = 62 × 4 + 1 = 4*n* + 1.

(j) For ^{253}Fm, 253 = 63 × 4 + 1 = 4*n* + 1.

93. The disintegration energy is

$$\begin{aligned} Q &= (m_{\text{V}} - m_{\text{Ti}})c^2 - E_K \\ &= (48.94852\,\text{u} - 48.94787\,\text{u})(931.5\,\text{MeV/u}) - 0.00547\,\text{MeV} \\ &= 0.600\,\text{MeV}. \end{aligned}$$

94. We locate a nuclide from Table 42-1 by finding the coordinate (*N*, *Z*) of the corresponding point in Fig. 42-4. It is clear that all the nuclides listed in Table 42-1 are stable except the last two, ^{227}Ac and ^{239}Pu.

95. (a) We use $R = R_0 e^{-\lambda t}$ to find *t*:

$$t = \frac{1}{\lambda}\ln\frac{R_0}{R} = \frac{T_{1/2}}{\ln 2}\ln\frac{R_0}{R} = \frac{14.28\,\text{d}}{\ln 2}\ln\frac{3050}{170} = 59.5\,\text{d}.$$

(b) The required factor is

$$\frac{R_0}{R} = e^{\lambda t} = e^{t\ln 2/T_{1/2}} = e^{(3.48\text{d}/14.28\text{d})\ln 2} = 1.18.$$

96. (a) From the decay series, we know that N_{210}, the amount of ^{210}Pb nuclei, changes because of two decays: the decay from ^{226}Ra into ^{210}Pb at the rate $R_{226} = \lambda_{226}N_{226}$, and the decay from ^{210}Pb into ^{206}Pb at the rate $R_{210} = \lambda_{210}N_{210}$. The first of these decays causes N_{210} to increase while the second one causes it to decrease. Thus,

$$\frac{dN_{210}}{dt} = R_{226} - R_{210} = \lambda_{226}N_{226} - \lambda_{210}N_{210}.$$

(b) We set $dN_{210}/dt = R_{226} - R_{210} = 0$ to obtain $R_{226}/R_{210} = 1.00$.

(c) From $R_{226} = \lambda_{226}N_{226} = R_{210} = \lambda_{210}N_{210}$, we obtain

$$\frac{N_{226}}{N_{210}} = \frac{\lambda_{210}}{\lambda_{226}} = \frac{T_{1/2_{226}}}{T_{1/2_{210}}} = \frac{1.60\times 10^3\,\text{y}}{22.6\,\text{y}} = 70.8.$$

(d) Since only 1.00% of the ^{226}Ra remains, the ratio R_{226}/R_{210} is 0.00100 of that of the equilibrium state computed in part (b). Thus the ratio is (0.0100)(1) = 0.0100.

(e) This is similar to part (d) above. Since only 1.00% of the ^{226}Ra remains, the ratio N_{226}/N_{210} is 1.00% of that of the equilibrium state computed in part (c), or (0.0100)(70.8) = 0.708.

(f) Since the actual value of N_{226}/N_{210} is 0.09, which much closer to 0.0100 than to 1, the sample of the lead pigment cannot be 300 years old. So *Emmaus* is not a *Vermeer*.

97. (a) Replacing differentials with deltas in Eq. 42-12, we use the fact that ΔN = – 12 during Δt = 1.0 s to obtain

$$\frac{\Delta N}{N} = -\lambda \Delta t \quad \Rightarrow \quad \lambda = 4.8 \times 10^{-18}\,/\,\text{s}$$

where $N = 2.5 \times 10^{18}$, mentioned at the second paragraph of Section 42-3, is used.

(b) Equation 42-18 yields $T_{1/2} = \ln 2/\lambda = 1.4 \times 10^{17}$ s, or about 4.6 billion years.

Chapter 43

1. (a) Using Eq. 42-20 and adapting Eq. 42-21 to this sample, the number of fission-events per second is

$$R_{\text{fission}} = \frac{N \ln 2}{T_{1/2_{\text{fission}}}} = \frac{M_{\text{sam}} N_A \ln 2}{M_{\text{U}} T_{1/2_{\text{fission}}}}$$
$$= \frac{(1.0 \text{ g})(6.02 \times 10^{23} / \text{mol}) \ln 2}{(235 \text{ g/mol})(3.0 \times 10^{17} \text{ y})(365 \text{ d/y})} = 16 \text{ fissions/day}.$$

(b) Since $R \propto 1/T_{1/2}$ (see Eq. 42-20), the ratio of rates is

$$\frac{R_\alpha}{R_{\text{fission}}} = \frac{T_{1/2_{\text{fission}}}}{\text{T}_{1/2_\alpha}} = \frac{3.0 \times 10^{17} \text{ y}}{7.0 \times 10^{8} \text{ y}} = 4.3 \times 10^{8}.$$

2. When a neutron is captured by ^{237}Np it gains 5.0 MeV, more than enough to offset the 4.2 MeV required for ^{238}Np to fission. Consequently, ^{237}Np is fissionable by thermal neutrons.

3. The energy transferred is

$$\begin{aligned} Q &= (m_{\text{U238}} + m_n - m_{\text{U239}})c^2 \\ &= (238.050782 \text{ u} + 1.008664 \text{ u} - 239.054287 \text{ u})(931.5 \text{ MeV/u}) \\ &= 4.8 \text{ MeV}. \end{aligned}$$

4. Adapting Eq. 42-21, there are

$$N_{\text{Pu}} = \frac{M_{\text{sam}}}{M_{\text{Pu}}} NA = \left(\frac{1000 \text{ g}}{239 \text{ g/mol}}\right)(6.02 \times 10^{23} / \text{mol}) = 2.5 \times 10^{24}$$

plutonium nuclei in the sample. If they all fission (each releasing 180 MeV), then the total energy release is 4.54×10^{26} MeV.

5. The yield of one warhead is 2.0 megatons of TNT, or

$$\text{yield} = 2(2.6 \times 10^{28} \text{ MeV}) = 5.2 \times 10^{28} \text{ MeV}.$$

Since each fission event releases about 200 MeV of energy, the number of fissions is

$$N = \frac{5.2\times10^{28}\ \text{MeV}}{200\ \text{MeV}} = 2.6\times10^{26}.$$

However, this only pertains to the 8.0% of Pu that undergoes fission, so the total number of Pu is

$$N_0 = \frac{N}{0.080} = \frac{2.6\times10^{26}}{0.080} = 3.25\times10^{27} = 5.4\times10^{3}\ \text{mol}.$$

With $M = 0.239$ kg/mol, the mass of the warhead is

$$m = (5.4\times10^{3}\ \text{mol})(0.239\ \text{kg/mol}) = 1.3\times10^{3}\ \text{kg}.$$

6. We note that the sum of superscripts (mass numbers A) must balance, as well as the sum of Z values (where reference to Appendix F or G is helpful). A neutron has $Z = 0$ and $A = 1$. Uranium has $Z = 92$.

(a) Since xenon has $Z = 54$, then "Y" must have $Z = 92 - 54 = 38$, which indicates the element strontium. The mass number of "Y" is $235 + 1 - 140 - 1 = 95$, so "Y" is ^{95}Sr.

(b) Iodine has $Z = 53$, so "Y" has $Z = 92 - 53 = 39$, corresponding to the element yttrium (the symbol for which, coincidentally, is Y). Since $235 + 1 - 139 - 2 = 95$, then the unknown isotope is ^{95}Y.

(c) The atomic number of zirconium is $Z = 40$. Thus, $92 - 40 - 2 = 52$, which means that "X" has $Z = 52$ (tellurium). The mass number of "X" is $235 + 1 - 100 - 2 = 134$, so we obtain ^{134}Te.

(d) Examining the mass numbers, we find $b = 235 + 1 - 141 - 92 = 3$.

7. If R is the fission rate, then the power output is $P = RQ$, where Q is the energy released in each fission event. Hence,

$$R = P/Q = (1.0\ \text{W})/(200 \times 10^{6}\ \text{eV})(1.60 \times 10^{-19}\ \text{J/eV}) = 3.1 \times 10^{10}\ \text{fissions/s}.$$

8. (a) We consider the process $^{98}\text{Mo} \to {}^{49}\text{Sc} + {}^{49}\text{Sc}.$ The disintegration energy is

$$Q = (m_{\text{Mo}} - 2m_{\text{Sc}})c^2 = [97.90541\ \text{u} - 2(48.95002\ \text{u})](931.5\ \text{MeV/u}) = +5.00\ \text{MeV}.$$

(b) The fact that it is positive does not necessarily mean we should expect to find a great deal of molybdenum nuclei spontaneously fissioning; the energy barrier (see Fig. 43-3) is presumably higher and/or broader for molybdenum than for uranium.

9. (a) The mass of a single atom of ^{235}U is

$$m_0 = (235\ \text{u})(1.661 \times 10^{-27}\ \text{kg/u}) = 3.90 \times 10^{-25}\ \text{kg},$$

so the number of atoms in $m = 1.0$ kg is

$$N = m/m_0 = (1.0 \text{ kg})/(3.90 \times 10^{-25} \text{ kg}) = 2.56 \times 10^{24} \approx 2.6 \times 10^{24}.$$

An alternate approach (but essentially the same once the connection between the "u" unit and N_A is made) would be to adapt Eq. 42-21.

(b) The energy released by N fission events is given by $E = NQ$, where Q is the energy released in each event. For 1.0 kg of ^{235}U,

$$E = (2.56 \times 10^{24})(200 \times 10^6 \text{ eV})(1.60 \times 10^{-19} \text{ J/eV}) = 8.19 \times 10^{13} \text{ J} \approx 8.2 \times 10^{13} \text{ J}.$$

(c) If P is the power requirement of the lamp, then

$$t = E/P = (8.19 \times 10^{13} \text{ J})/(100 \text{ W}) = 8.19 \times 10^{11} \text{ s} = 2.6 \times 10^4 \text{ y}.$$

The conversion factor 3.156×10^7 s/y is used to obtain the last result.

10. The energy released is

$$\begin{aligned} Q &= (m_U + m_n - m_{Cs} - m_{Rb} - 2m_n)c^2 \\ &= (235.04392 \text{ u} - 1.00867 \text{ u} - 140.91963 \text{ u} - 92.92157 \text{ u})(931.5 \text{ MeV/u}) \\ &= 181 \text{ MeV}. \end{aligned}$$

11. If M_{Cr} is the mass of a ^{52}Cr nucleus and M_{Mg} is the mass of a ^{26}Mg nucleus, then the disintegration energy is

$$Q = (M_{Cr} - 2M_{Mg})c^2 = [51.94051 \text{ u} - 2(25.98259 \text{ u})](931.5 \text{ MeV/u}) = -23.0 \text{ MeV}.$$

12. (a) Consider the process $^{239}\text{U} + \text{n} \rightarrow {}^{140}\text{Ce} + {}^{99}\text{Ru} + \text{Ne}.$ We have

$$Z_f - Z_i = Z_{Ce} + Z_{Ru} - Z_U = 58 + 44 - 92 = 10.$$

Thus the number of beta-decay events is 10.

(b) Using Table 37-3, the energy released in this fission process is

$$\begin{aligned} Q &= (m_U + m_n - m_{Ce} - m_{Ru} - 10m_e)c^2 \\ &= (238.05079 \text{ u} + 1.00867 \text{ u} - 139.90543 \text{ u} - 98.90594 \text{ u})(931.5 \text{ MeV/u}) - 10(0.511 \text{ MeV}) \\ &= 226 \text{ MeV}. \end{aligned}$$

13. (a) The electrostatic potential energy is given by

$$U = \frac{1}{4\pi\varepsilon_0}\frac{Z_{\text{Xe}}Z_{\text{Sr}}e^2}{r_{\text{Xe}}+r_{\text{Sr}}}$$

where Z_{Xe} is the atomic number of xenon, Z_{Sr} is the atomic number of strontium, r_{Xe} is the radius of a xenon nucleus, and r_{Sr} is the radius of a strontium nucleus. Atomic numbers can be found either in Appendix F or Appendix G. The radii are given by $r = (1.2\text{ fm})A^{1/3}$, where A is the mass number, also found in Appendix F. Thus,

$$r_{\text{Xe}} = (1.2\text{ fm})(140)^{1/3} = 6.23\text{ fm} = 6.23 \times 10^{-15}\text{ m}$$

and

$$r_{\text{Sr}} = (1.2\text{ fm})(96)^{1/3} = 5.49\text{ fm} = 5.49 \times 10^{-15}\text{ m}.$$

Hence, the potential energy is

$$U = (8.99\times10^9\text{ V}\cdot\text{m/C})\frac{(54)(38)(1.60\times10^{-19}\text{ C})^2}{6.23\times10^{-15}\text{ m}+5.49\times10^{-15}\text{ m}} = 4.08\times10^{-11}\text{ J}$$
$$= 251\text{ MeV}.$$

(b) The energy released in a typical fission event is about 200 MeV, roughly the same as the electrostatic potential energy when the fragments are touching. The energy appears as kinetic energy of the fragments and neutrons produced by fission.

14. (a) The surface area a of a nucleus is given by

$$a \simeq 4\pi R^2 \simeq 4\pi\left(R_0 A^{1/3}\right)^2 \propto A^{2/3}.$$

Thus, the fractional change in surface area is

$$\frac{\Delta a}{a_i} = \frac{a_f - a_i}{a_i} = \frac{(140)^{2/3}+(96)^{2/3}}{(236)^{2/3}} - 1 = +0.25.$$

(b) Since $V \propto R^3 \propto (A^{1/3})^3 = A$, we have

$$\frac{\Delta V}{V} = \frac{V_f}{V_i} - 1 = \frac{140+96}{236} - 1 = 0.$$

(c) The fractional change in potential energy is

$$\frac{\Delta U}{U} = \frac{U_f}{U_i} - 1 = \frac{Q_{\text{Xe}}^2/R_{\text{Xe}} + Q_{\text{Sr}}^2/R_{\text{Sr}}}{Q_{\text{U}}^2/R_{\text{U}}} - 1 = \frac{(54)^2(140)^{-1/3}+(38)^2(96)^{-1/3}}{(92)^2(236)^{-1/3}} - 1$$
$$= -0.36.$$

15. **THINK** One megaton of TNT releases 2.6×10^{28} MeV of energy. The energy released in each fission event is about 200 MeV.

EXPRESS The energy yield of the bomb is

$$E = (66 \times 10^{-3} \text{ megaton})(2.6 \times 10^{28} \text{ MeV/ megaton}) = 1.72 \times 10^{27} \text{ MeV}.$$

At 200 MeV per fission event, the total number of fission events taking place is

$$(1.72 \times 10^{27} \text{ MeV})/(200 \text{ MeV}) = 8.58 \times 10^{24}.$$

Now, since only 4.0% of the ^{235}U nuclei originally present undergo fission, there must have been $(8.58 \times 10^{24})/(0.040) = 2.14 \times 10^{26}$ nuclei originally present.

ANALYZE (a) The mass of ^{235}U originally present was

$$(2.14 \times 10^{26})(235 \text{ u})(1.661 \times 10^{-27} \text{ kg/u}) = 83.7 \text{ kg} \approx 84 \text{ kg}.$$

(b) Two fragments are produced in each fission event, so the total number of fragments is

$$2(8.58 \times 10^{24}) = 1.72 \times 10^{25} \approx 1.7 \times 10^{25}.$$

(c) One neutron produced in a fission event is used to trigger the next fission event, so the average number of neutrons released to the environment in each event is 1.5. The total number released is

$$(8.58 \times 10^{24})(1.5) = 1.29 \times 10^{25} \approx 1.3 \times 10^{25}.$$

LEARN When one ^{235}U nucleus undergoes fission, the neutrons it produces (an average number of 2.5 neutrons per fission) can trigger other ^{235}U nuclei to fission, thereby setting up a chain reaction that allows an enormous amount of energy to be released.

16. (a) Using the result of Problem 43-4, the TNT equivalent is

$$\frac{(2.50 \text{ kg})(4.54 \times 10^{26} \text{ MeV/kg})}{2.6 \times 10^{28} \text{ MeV}/10^{6} \text{ ton}} = 4.4 \times 10^{4} \text{ ton} = 44 \text{ kton}.$$

(b) Assuming that this is a fairly inefficiently designed bomb, then much of the remaining 92.5 kg is probably “wasted” and was included perhaps to make sure the bomb did not “fizzle.” There is also an argument for having more than just the critical mass based on the short assembly time of the material during the implosion, but this so-called “super-critical mass,” as generally quoted, is much less than 92.5 kg, and does not necessarily have to be purely plutonium.

17. **THINK** We represent the unknown fragment as ${}^{A}_{Z}X$, where A and Z are its mass number and atomic number, respectively. Charge and mass number are conserved in the neutron-capture process.

EXPRESS The reaction can be written as

$${}^{235}_{92}\text{U} + {}^{1}_{0}\text{n} \to {}^{82}_{32}\text{Ge} + {}^{A}_{Z}X\ .$$

Conservation of charge yields 92 + 0 = 32 + Z, so Z = 60. Conservation of mass number yields 235 + 1 = 83 + A, so A = 153.

ANALYZE (a) Looking in Appendix F or G for nuclides with Z = 60, we find that the unknown fragment is ${}^{153}_{60}\text{Nd}$.

(b) We neglect the small kinetic energy and momentum carried by the neutron that triggers the fission event. Then,

$$Q = K_{\text{Ge}} + K_{\text{Nd}},$$

where K_{Ge} is the kinetic energy of the germanium nucleus and K_{Nd} is the kinetic energy of the neodymium nucleus. Conservation of momentum yields $\vec{p}_{\text{Ge}} + \vec{p}_{\text{Nd}} = 0$. Now, we can write the classical formula for kinetic energy in terms of the magnitude of the momentum vector:

$$K = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

which implies that

$$K_{\text{Nd}} = \frac{p_{\text{Nd}}^2}{2M_{\text{Nd}}} = \frac{p_{\text{Ge}}^2}{2M_{\text{Nd}}} = \frac{M_{\text{Ge}}}{M_{\text{Nd}}}\frac{p_{\text{Ge}}^2}{2M_{\text{Ge}}} = \frac{M_{\text{Ge}}}{M_{\text{Nd}}}K_{\text{Ge}}\ .$$

Thus, the energy equation becomes

$$Q = K_{\text{Ge}} + \frac{M_{\text{Ge}}}{M_{\text{Nd}}}K_{\text{Ge}} = \frac{M_{\text{Nd}} + M_{\text{Ge}}}{M_{\text{Nd}}}K_{\text{Ge}}$$

and

$$K_{\text{Ge}} = \frac{M_{\text{Nd}}}{M_{\text{Nd}} + M_{\text{Ge}}}Q = \frac{153\ \text{u}}{153\ \text{u} + 83\ \text{u}}(170\ \text{MeV}) = 110\ \text{MeV}.$$

(c) Similarly,

$$K_{\text{Nd}} = \frac{M_{\text{Ge}}}{M_{\text{Nd}} + M_{\text{Ge}}}Q = \frac{83\ \text{u}}{153\ \text{u} + 83\ \text{u}}(170\ \text{MeV}) = 60\ \text{MeV}.$$

(d) The initial speed of the germanium nucleus is

$$v_{\mathrm{Ge}}=\sqrt{\frac{2K_{\mathrm{Ge}}}{M_{\mathrm{Ge}}}}=\sqrt{\frac{2(110\times10^{6}\ \mathrm{eV})(1.60\times10^{-19}\ \mathrm{J/eV})}{(83\ \mathrm{u})(1.661\times10^{-27}\ \mathrm{kg/u})}}=1.60\times10^{7}\ \mathrm{m/s}.$$

(e) The initial speed of the neodymium nucleus is

$$v_{\mathrm{Nd}}=\sqrt{\frac{2K_{\mathrm{Nd}}}{M_{\mathrm{ND}}}}=\sqrt{\frac{2(60\times10^{6}\ \mathrm{eV})(1.60\times10^{-19}\ \mathrm{J/eV})}{(153\ \mathrm{u})(1.661\times10^{-27}\ \mathrm{kg/u})}}=8.69\times10^{6}\ \mathrm{m/s}.$$

LEARN By momentum conservation, the two fragments fly apart in opposite directions.

18. If P is the power output, then the energy E produced in the time interval Δt (= 3 y) is

$$\begin{aligned}E &= P\,\Delta t = (200\times10^{6}\ \mathrm{W})(3\ \mathrm{y})(3.156\times10^{7}\ \mathrm{s/y}) = 1.89\times10^{16}\ \mathrm{J}\\ &= (1.89\times10^{16}\ \mathrm{J})/(1.60\times10^{-19}\ \mathrm{J/eV}) = 1.18\times10^{35}\ \mathrm{eV}\\ &= 1.18\times10^{29}\ \mathrm{MeV}.\end{aligned}$$

At 200 MeV per event, this means $(1.18\times10^{29})/200 = 5.90\times10^{26}$ fission events occurred. This must be half the number of fissionable nuclei originally available. Thus, there were $2(5.90\times10^{26}) = 1.18\times10^{27}$ nuclei. The mass of a ^{235}U nucleus is

$$(235\ \mathrm{u})(1.661\times10^{-27}\ \mathrm{kg/u}) = 3.90\times10^{-25}\ \mathrm{kg},$$

so the total mass of ^{235}U originally present was $(1.18\times10^{27})(3.90\times10^{-25}\ \mathrm{kg}) = 462$ kg.

19. After each time interval t_{gen} the number of nuclides in the chain reaction gets multiplied by k. The number of such time intervals that has gone by at time t is t/t_{gen}. For example, if the multiplication factor is 5 and there were 12 nuclei involved in the reaction to start with, then after one interval 60 nuclei are involved. And after another interval 300 nuclei are involved. Thus, the number of nuclides engaged in the chain reaction at time t is $N(t)=N_0k^{t/t_{\mathrm{gen}}}$. Since $P\propto N$ we have

$$P(t)=P_0k^{t/t_{\mathrm{gen}}}.$$

20. We use the formula from Problem 43-19:

$$P(t)=P_0k^{t/t_{\mathrm{gen}}}=(400\,\mathrm{MW})(1.0003)^{(5.00\ \mathrm{min})(60\ \mathrm{s/min})/(0.00300\,\mathrm{s})}=8.03\times10^{3}\,\mathrm{MW}.$$

21. If R is the decay rate then the power output is $P = RQ$, where Q is the energy produced by each alpha decay. Now

$$R=\lambda N=N\ln 2/T_{1/2},$$

where λ is the disintegration constant and $T_{1/2}$ is the half-life. The relationship $\lambda = (\ln 2)/T_{1/2}$ is used. If M is the total mass of material and m is the mass of a single ^{238}Pu nucleus, then

$$N = \frac{M}{m} = \frac{1.00 \text{ kg}}{(238 \text{ u})(1.661\times 10^{-27} \text{ kg/u})} = 2.53\times 10^{24}.$$

Thus,

$$P = \frac{NQ\ln 2}{T_{1/2}} = \frac{(2.53\times 10^{24})(5.50\times 10^{6} \text{ eV})(1.60\times 10^{-19} \text{ J/eV})(\ln 2)}{(87.7 \text{ y})(3.156\times 10^{7} \text{ s/y})} = 557 \text{ W}.$$

22. We recall Eq. 43-6:

$$Q \approx 200 \text{ MeV} = 3.2 \times 10^{-11} \text{ J}.$$

It is important to bear in mind that watts multiplied by seconds give joules. From $E = Pt_{\text{gen}} = NQ$ we get the number of free neutrons:

$$N = \frac{Pt_{\text{gen}}}{Q} = \frac{(500\times 10^{6} \text{ W})(1.0\times 10^{-3} \text{ s})}{3.2\times 10^{-11} \text{ J}} = 1.6\times 10^{16}.$$

23. **THINK** The neutron generation time t_{gen} in a reactor is the average time needed for a fast neutron emitted in a fission event to be slowed to thermal energies by the moderator and then initiate another fission event.

EXPRESS Let P_0 be the initial power output, P be the final power output, k be the multiplication factor, t be the time for the power reduction, and t_{gen} be the neutron generation time. Then, according to the result of Problem 43-19,

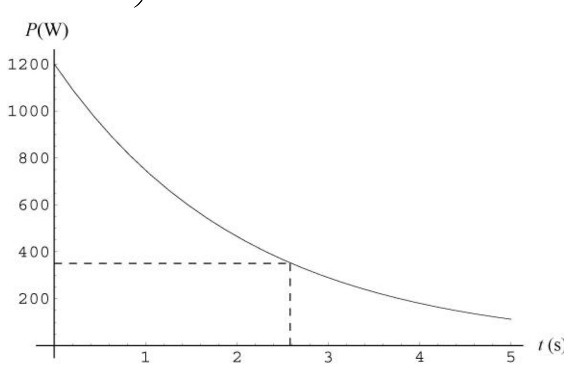
$$P = P_0 k^{t/t_{\text{gen}}}.$$

ANALYZE We divide by P_0, take the natural logarithm of both sides of the equation and solve for ln k:

$$\ln k = \frac{t_{\text{gen}}}{t}\ln\left(\frac{P}{P_0}\right) = \frac{1.3\times 10^{-3} \text{ s}}{2.6 \text{ s}}\ln\left(\frac{350 \text{ MW}}{1200 \text{ MW}}\right) = -0.0006161.$$

Hence, $k = e^{-0.0006161} = 0.99938$.

LEARN The power output as a function of time is shown to the right. Since the multiplication factor k is smaller than 1, the output decreases with time.



24. (a) We solve Q_{eff} from $P = RQ_{eff}$:

$$\begin{aligned} Q_{eff} &= \frac{P}{R} = \frac{P}{N\lambda} = \frac{mPT_{1/2}}{M \ln 2} \\ &= \frac{(90.0 \text{ u})(1.66\times10^{-27} \text{ kg/u})(0.93 \text{ W})(29 \text{ y})(3.15\times10^{7} \text{ s/y})}{(1.00\times10^{-3} \text{ kg})(\ln 2)(1.60\times10^{-13} \text{ J/MeV})} \\ &= 1.2 \text{ MeV}. \end{aligned}$$

(b) The amount of ^{90}Sr needed is

$$M = \frac{150 \text{ W}}{(0.050)(0.93 \text{ W/g})} = 3.2 \text{ kg}.$$

25. **THINK** Momentum is conserved in the collision process. In addition, energy is also conserved since the collision is elastic.

EXPRESS Let v_{ni} be the initial velocity of the neutron, v_{nf} be its final velocity, and v_f be the final velocity of the target nucleus. Then, since the target nucleus is initially at rest, conservation of momentum yields

$$m_n v_{ni} = m_n v_{nf} + m v_f$$

and conservation of energy yields

$$\frac{1}{2}m_n v_{ni}^2 = \frac{1}{2}m_n v_{nf}^2 + \frac{1}{2}m v_f^2.$$

We solve these two equations simultaneously for v_f. This can be done, for example, by using the conservation of momentum equation to obtain an expression for v_{nf} in terms of v_f and substituting the expression into the conservation of energy equation. We solve the resulting equation for v_f. We obtain $v_f = 2m_n v_{ni}/(m + m_n)$.

ANALYZE (a) The energy lost by the neutron is the same as the energy gained by the target nucleus, so

$$\Delta K = \frac{1}{2}m v_f^2 = \frac{1}{2}\frac{4m_n^2 m}{(m+m_n)^2} v_{ni}^2.$$

The initial kinetic energy of the neutron is $K = \frac{1}{2}m_n v_{ni}^2$, so

$$\frac{\Delta K}{K} = \frac{4m_n m}{(m+m_n)^2}.$$

(b) The mass of a neutron is 1.0 u and the mass of a hydrogen atom is also 1.0 u. (Atomic masses can be found in Appendix G.) Thus,

$$\frac{\Delta K}{K} = \frac{4(1.0\ \text{u})(1.0\ \text{u})}{(1.0\ \text{u} + 1.0\ \text{u})^2} = 1.0.$$

(c) Similarly, the mass of a deuterium atom is 2.0 u, so

$$(\Delta K)/K = 4(1.0\ \text{u})(2.0\ \text{u})/(2.0\ \text{u} + 1.0\ \text{u})^2 = 0.89.$$

(d) The mass of a carbon atom is 12 u, so

$$(\Delta K)/K = 4(1.0\ \text{u})(12\ \text{u})/(12\ \text{u} + 1.0\ \text{u})^2 = 0.28.$$

(e) The mass of a lead atom is 207 u, so

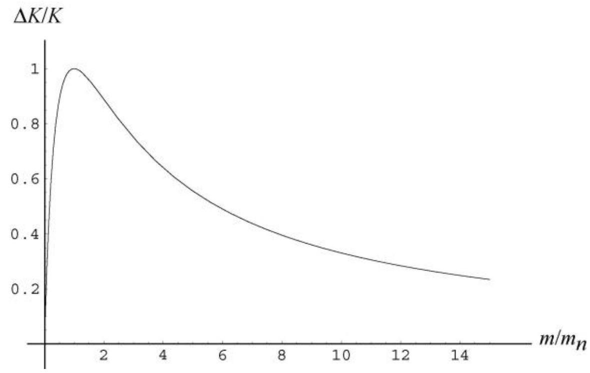
$$(\Delta K)/K = 4(1.0\ \text{u})(207\ \text{u})/(207\ \text{u} + 1.0\ \text{u})^2 = 0.019.$$

(f) During each collision, the energy of the neutron is reduced by the factor 1 – 0.89 = 0.11. If E_i is the initial energy, then the energy after n collisions is given by $E = (0.11)^n E_i$. We take the natural logarithm of both sides and solve for n. The result is

$$n = \frac{\ln(E/E_i)}{\ln 0.11} = \frac{\ln(0.025\ \text{eV}/1.00\ \text{eV})}{\ln 0.11} = 7.9 \approx 8.$$

The energy first falls below 0.025 eV on the eighth collision.

LEARN The fractional kinetic energy loss as a function of the mass of the stationary atom (in units of m/m_n) is plotted below.



From the plot, it is clear that the energy loss is greatest ($\Delta K/K = 1$) when the atom has the same as the neutron.

26. The ratio is given by

$$\frac{N_5(t)}{N_8(t)} = \frac{N_5(0)}{N_8(0)} e^{-(\lambda_5-\lambda_8)t},$$

or

$$t = \frac{1}{\lambda_8 - \lambda_5} \ln\left[\left(\frac{N_5(t)}{N_8(t)}\right)\left(\frac{N_8(0)}{N_5(0)}\right)\right] = \frac{1}{(1.55-9.85)10^{-10}\ \text{y}^{-1}} \ln[(0.0072)(0.15)^{-1}]$$
$$= 3.6\times10^9\ \text{y}.$$

27. (a) $P_{avg} = (15 \times 10^9\ \text{W}\cdot\text{y})/(200{,}000\ \text{y}) = 7.5 \times 10^4\ \text{W} = 75\ \text{kW}$.

(b) Using the result of Eq. 43-6, we obtain

$$M = \frac{m_U E_{total}}{Q} = \frac{(235\,\text{u})(1.66\times10^{-27}\,\text{kg/u})(15\times10^9\,\text{W}\cdot\text{y})(3.15\times10^7\,\text{s/y})}{(200\,\text{MeV})(1.6\times10^{-13}\,\text{J/MeV})} = 5.8\times10^3\,\text{kg}.$$

28. The nuclei of ^{238}U can capture neutrons and beta-decay. With a large amount of neutrons available due to the fission of ^{235}U, the probability for this process is substantially increased, resulting in a much higher decay rate for ^{238}U and causing the depletion of ^{238}U (and relative enrichment of ^{235}U).

29. **THINK** With a shorter half-life, ^{235}U has a greater decay rate than ^{238}U. Thus, if the ore contains only 0.72% of ^{235}U today, then the concentration must be higher in the far distant past.

EXPRESS Let t be the present time and t = 0 be the time when the ratio of ^{235}U to ^{238}U was 3.0%. Let N_{235} be the number of ^{235}U nuclei present in a sample now and $N_{235,0}$ be the number present at t = 0. Let N_{238} be the number of ^{238}U nuclei present in the sample now and $N_{238,0}$ be the number present at t = 0. The law of radioactive decay holds for each species, so

$$N_{235} = N_{235,0} e^{-\lambda_{235} t}$$

and

$$N_{238} = N_{238,0} e^{-\lambda_{238} t}.$$

Dividing the first equation by the second, we obtain

$$r = r_0 e^{-(\lambda_{235}-\lambda_{238})t}$$

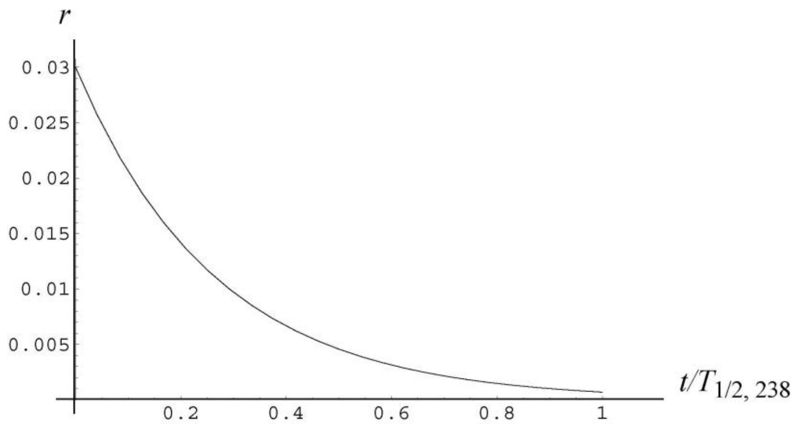
where $r = N_{235}/N_{238}$ (= 0.0072) and $r_0 = N_{235,0}/N_{238,0}$ (= 0.030). We solve for t:

$$t = -\frac{1}{\lambda_{235} - \lambda_{238}} \ln\left(\frac{r}{r_0}\right).$$

ANALYZE Now we use $\lambda_{235} = (\ln 2)/T_{1/2_{235}}$ and $\lambda_{238} = (\ln 2)/T_{1/2_{238}}$ to obtain

$$t = \frac{T_{1/2_{235}} T_{1/2_{238}}}{(T_{1/2_{238}} - T_{1/2_{235}})\ln 2} \ln\left(\frac{r}{r_0}\right) = -\frac{(7.0\times10^8\text{ y})(4.5\times10^9\text{ y})}{(4.5\times10^9\text{ y} - 7.0\times10^8\text{ y})\ln 2} \ln\left(\frac{0.0072}{0.030}\right)$$
$$= 1.7\times10^9\text{ y}.$$

LEARN How the ratio $r = N_{235}/N_{238}$ changes with time is plotted below. In the plot, we take the ratio to be 0.03 at t = 0. At $t = 1.7\times10^9$ y or $t/T_{1/2,238} = 0.378$, r is reduced to 0.072.



30. We are given the energy release per fusion (Q = 3.27 MeV = 5.24 × 10^{-13} J) and that a pair of deuterium atoms is consumed in each fusion event. To find how many pairs of deuterium atoms are in the sample, we adapt Eq. 42-21:

$$N_{d\text{ pairs}} = \frac{M_{\text{sam}}}{2M_d} N_\text{A} = \left(\frac{1000\text{ g}}{2(2.0\text{ g/mol})}\right)(6.02\times10^{23}\text{ /mol}) = 1.5\times10^{26}.$$

Multiplying this by Q gives the total energy released: 7.9 × 10^{13} J. Keeping in mind that a watt is a joule per second, we have

$$t = \frac{7.9\times10^{13}\text{ J}}{100\text{ W}} = 7.9\times10^{11}\text{ s} = 2.5\times10^4\text{ y}.$$

31. **THINK** Coulomb repulsion acts to prevent two charged particles from coming close enough to be within the range of their attractive nuclear force.

EXPRESS We take the height of the Coulomb barrier to be the value of the kinetic energy K each deuteron must initially have if they are to come to rest when their surfaces touch. If r is the radius of a deuteron, conservation of energy yields

$$2K = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{2r}.$$

ANALYZE With r = 2.1 fm, we have

$$K = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{4r} = (8.99\times10^9\ \text{V}\cdot\text{m/C})\frac{(1.60\times10^{-19}\ \text{C})^2}{4(2.1\times10^{-15}\ \text{m})} = 2.74\times10^{-14}\ \text{J} = 170\ \text{keV}.$$

LEARN The height of the Coulomb barrier depends on the charges and radii of the two interacting nuclei. Increasing the charge raises the barrier.

32. (a) Our calculation is identical to that in Sample Problem — "Fusion in a gas of protons and required temperature" except that we are now using *R* appropriate to two deuterons coming into "contact," as opposed to the *R* = 1.0 fm value used in the Sample Problem. If we use *R* = 2.1 fm for the deuterons, then our *K* is simply the *K* calculated in the Sample Problem, divided by 2.1:

$$K_{d+d} = \frac{K_{p+p}}{2.1} = \frac{360\ \text{keV}}{2.1} \approx 170\ \text{keV}.$$

Consequently, the voltage needed to accelerate each deuteron from rest to that value of *K* is 170 kV.

(b) Not all deuterons that are accelerated toward each other will come into "contact" and not all of those that do so will undergo nuclear fusion. Thus, a great many deuterons must be repeatedly encountering other deuterons in order to produce a macroscopic energy release. An accelerator needs a fairly good vacuum in its beam pipe, and a very large number flux is either impractical and/or very expensive. Regarding expense, there are other factors that have dissuaded researchers from using accelerators to build a controlled fusion "reactor," but those factors may become less important in the future — making the feasibility of accelerator "add-ons" to magnetic and inertial confinement schemes more cost-effective.

33. Our calculation is very similar to that in Sample Problem – "Fusion in a gas of protons and required temperature" except that we are now using *R* appropriate to two lithium-7 nuclei coming into "contact," as opposed to the *R* = 1.0 fm value used in the Sample Problem. If we use

$$R = r = r_0 A^{1/3} = (1.2\ \text{fm})\sqrt[3]{7} = 2.3\ \text{fm}$$

and $q = Ze = 3e$, then our *K* is given by (see the Sample Problem)

$$K = \frac{Z^2e^2}{16\pi\varepsilon_0 r} = \frac{3^2(1.6\times10^{-19}\ \text{C})^2}{16\pi(8.85\times10^{-12}\ \text{F/m})(2.3\times10^{15}\ \text{m})}$$

which yields 2.25×10^{-13} J = 1.41 MeV. We interpret this as the answer to the problem, though the term "Coulomb barrier height" as used here may be open to other interpretations.

34. From the expression for $n(K)$ given we may write $n(K) \propto K^{1/2}e^{-K/kT}$. Thus, with

$$k = 8.62 \times 10^{-5} \text{ eV/K} = 8.62 \times 10^{-8} \text{ keV/K},$$

we have

$$\frac{n(K)}{n(K_{\text{avg}})} = \left(\frac{K}{K_{\text{avg}}}\right)^{1/2} e^{-(K-K_{\text{avg}})/kT} = \left(\frac{5.00\,\text{keV}}{1.94\,\text{keV}}\right)^{1/2} \exp\left(-\frac{5.00\,\text{keV} - 1.94\,\text{keV}}{(8.62\times10^{-8}\,\text{keV})(1.50\times10^{7}\,\text{K})}\right)$$
$$= 0.151.$$

35. The kinetic energy of each proton is

$$K = k_B T = (1.38\times10^{-23} \text{ J/K})(1.0\times10^{7} \text{ K}) = 1.38\times10^{-16} \text{ J}.$$

At the closest separation, r_{min}, all the kinetic energy is converted to potential energy:

$$K_{\text{tot}} = 2K = U = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r_{\text{min}}}.$$

Solving for r_{min}, we obtain

$$r_{\text{min}} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{2K} = \frac{(8.99\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\text{ C})^2}{2(1.38\times10^{-16}\,\text{J})} = 8.33\times10^{-13}\text{ m} \approx 1 \text{ pm}.$$

36. The energy released is

$$\begin{aligned} Q &= -\Delta mc^2 = -(m_{\text{He}} - m_{\text{H2}} - m_{\text{H1}})c^2 \\ &= -(3.016029 \text{ u} - 2.014102 \text{ u} - 1.007825 \text{ u})(931.5 \text{ MeV/u}) \\ &= 5.49 \text{ MeV}. \end{aligned}$$

37. (a) Let M be the mass of the Sun at time t and E be the energy radiated to that time. Then, the power output is

$$P = dE/dt = (dM/dt)c^2,$$

where $E = Mc^2$ is used. At the present time,

$$\frac{dM}{dt} = \frac{P}{c^2} = \frac{3.9\times10^{26}\,\text{W}}{\left(2.998\times10^{8}\text{ m/s}\right)^2} = 4.3\times10^{9}\text{ kg/s}.$$

(b) We assume the rate of mass loss remained constant. Then, the total mass loss is

$$\begin{aligned} \Delta M &= (dM/dt)\,\Delta t = (4.33 \times 10^{9} \text{ kg/s})\,(4.5 \times 10^{9} \text{ y})\,(3.156 \times 10^{7} \text{ s/y}) \\ &= 6.15 \times 10^{26} \text{ kg}. \end{aligned}$$

The fraction lost is

$$\frac{\Delta M}{M+\Delta M}=\frac{6.15\times 10^{26}\,\text{kg}}{2.0\times 10^{30}\,\text{kg}+6.15\times 10^{26}\,\text{kg}}=3.1\times 10^{-4}\,.$$

38. In Fig. 43-10, let Q_1 = 0.42 MeV, Q_2 = 1.02 MeV, Q_3 = 5.49 MeV, and Q_4 = 12.86 MeV. For the overall proton-proton cycle

$$\begin{aligned}Q&=2Q_1+2Q_2+2Q_3+Q_4\\&=2(0.42\ \text{MeV}+1.02\ \text{MeV}+5.49\ \text{MeV})+12.86\ \text{MeV}=26.7\ \text{MeV}.\end{aligned}$$

39. If M_{He} is the mass of an atom of helium and M_{C} is the mass of an atom of carbon, then the energy released in a single fusion event is

$$Q=\left(3M_{\text{He}}-M_{\text{C}}\right)c^2=[3(4.0026\ \text{u})-(12.0000\ \text{u})](931.5\ \text{MeV/u})=7.27\ \text{MeV}.$$

Note that $3M_{\text{He}}$ contains the mass of six electrons and so does M_{C}. The electron masses cancel and the mass difference calculated is the same as the mass difference of the nuclei.

40. (a) We are given the energy release per fusion (Q = 26.7 MeV = 4.28×10^{-12} J) and that four protons are consumed in each fusion event. To find how many sets of four protons are in the sample, we adapt Eq. 42-21:

$$N_{4p}=\frac{M_{\text{sam}}}{4M_{\text{H}}}N_{\text{A}}=\left(\frac{1000\,\text{g}}{4(1.0\,\text{g/mol})}\right)\left(6.02\times 10^{23}/\text{mol}\right)=1.5\times 10^{26}\ .$$

Multiplying this by Q gives the total energy released: 6.4×10^{14} J. It is not required that the answer be in SI units; we could have used MeV throughout (in which case the answer is 4.0×10^{27} MeV).

(b) The number of ^{235}U nuclei is

$$N_{235}=\left(\frac{1000\,\text{g}}{235\,\text{g/mol}}\right)\left(6.02\times 10^{23}/\text{mol}\right)=2.56\times 10^{24}\ .$$

If all the U-235 nuclei fission, the energy release (using the result of Eq. 43-6) is

$$N_{235}Q_{\text{fission}}=\left(2.56\times 10^{22}\right)\left(200\,\text{MeV}\right)=5.1\times 10^{26}\ \text{MeV}=8.2\times 10^{13}\ \text{J}\ .$$

We see that the fusion process (with regard to a unit mass of fuel) produces a larger amount of energy (despite the fact that the Q value per event is smaller).

41. Since the mass of a helium atom is

$$(4.00 \text{ u})(1.661 \times 10^{-27} \text{ kg/u}) = 6.64 \times 10^{-27} \text{ kg},$$

the number of helium nuclei originally in the star is

$$(4.6 \times 10^{32} \text{ kg})/(6.64 \times 10^{-27} \text{ kg}) = 6.92 \times 10^{58}.$$

Since each fusion event requires three helium nuclei, the number of fusion events that can take place is

$$N = 6.92 \times 10^{58}/3 = 2.31 \times 10^{58}.$$

If Q is the energy released in each event and t is the conversion time, then the power output is $P = NQ/t$ and

$$t = \frac{NQ}{P} = \frac{(2.31\times10^{58})(7.27\times10^{6} \text{ eV})(1.60\times10^{-19} \text{ J/eV})}{5.3\times10^{30} \text{ W}} = 5.07\times10^{15} \text{ s}$$
$$= 1.6\times10^{8} \text{ y} .$$

42. We assume the neutrino has negligible mass. The photons, of course, are also taken to have zero mass.

$$\begin{aligned}
Q_1 &= (2m_p - m_2 - m_e)c^2 = \ 2(m_1 - m_e) - (m_2 - m_e) - m_e \ c^2 \\
&= \ 2(1.007825\,\text{u}) - 2.014102\,\text{u} - 2(0.0005486\,\text{u}) \ (931.5\,\text{MeV/u}) \\
&= 0.42\,\text{MeV} \\
Q_2 &= (m_2 + m_p - m_3)c^2 = (m_2 + m_p - m_3)c^2 \\
&= (2.014102\,\text{u}) + 1.007825\,\text{u} - 3.016029\,\text{u})(931.5\,\text{MeV/u}) \\
&= 5.49\ \text{MeV} \\
Q_3 &= (2m_3 - m_4 - 2m_p)c^2 = (2m_3 - m_4 - 2m_p)c^2 \\
&= \ 2(3.016029\,\text{u}) - 4.002603\,\text{u} - 2(1.007825\,\text{u}) \ (931.5\,\text{MeV/u}) \\
&= 12.86\,\text{MeV} .
\end{aligned}$$

43. (a) The energy released is

$$\begin{aligned}
Q &= (5m_{^2\text{H}} - m_{^3\text{He}} - m_{^4\text{He}} - m_{^1\text{H}} - 2m_n)c^2 \\
&= [5(2.014102\,\text{u}) - 3.016029\,\text{u} - 4.002603\,\text{u} - 1.007825\,\text{u} - 2(1.008665\,\text{u})](931.5\,\text{MeV/u}) \\
&= 24.9\,\text{MeV}.
\end{aligned}$$

(b) Assuming 30.0% of the deuterium undergoes fusion, the total energy released is

$$E = NQ = \left(\frac{0.300\,M}{5m_{^2\mathrm{H}}}\right)Q\ .$$

Thus, the rating is

$$\begin{aligned} R &= \frac{E}{2.6\times10^{28}\ \mathrm{MeV/megaton\,TNT}} \\ &= \frac{(0.300)(500\,\mathrm{kg})(24.9\,\mathrm{MeV})}{5(2.0\,\mathrm{u})(1.66\times10^{-27}\ \mathrm{kg/u})(2.6\times10^{28}\ \mathrm{MeV/megaton\,TNT})} \\ &= 8.65\,\mathrm{megaton\,TNT}\ . \end{aligned}$$

44. The mass of the hydrogen in the Sun's core is $m_{\mathrm{H}} = 0.35(\tfrac{1}{8}\,M_{\mathrm{Sun}})$. The time it takes for the hydrogen to be entirely consumed is

$$t = \frac{M_{\mathrm{H}}}{dm/dt} = \frac{(0.35)(\tfrac{1}{8})(2.0\times10^{30}\ \mathrm{kg})}{(6.2\times10^{11}\ \mathrm{kg/s})(3.15\times10^{7}\ \mathrm{s/y})} = 5\times10^{9}\ \mathrm{y}\ .$$

45. (a) Since two neutrinos are produced per proton-proton cycle (see Eq. 43-10 or Fig. 43-10), the rate of neutrino production R_ν satisfies

$$R_\nu = \frac{2P}{Q} = \frac{2(3.9\times10^{26}\ \mathrm{W})}{(26.7\,\mathrm{MeV})(1.6\times10^{-13}\ \mathrm{J/MeV})} = 1.8\times10^{38}\ \mathrm{s}^{-1}\ .$$

(b) Let d_{es} be the Earth to Sun distance, and R be the radius of Earth (see Appendix C). Earth represents a small cross section in the "sky" as viewed by a fictitious observer on the Sun. The rate of neutrinos intercepted by that area (very small, relative to the area of the full "sky") is

$$R_{\nu,\mathrm{Earth}} = R_\nu\left(\frac{\pi R_e^2}{4\pi d_{es}^2}\right) = \frac{(1.8\times10^{38}\ \mathrm{s}^{-1})}{4}\left(\frac{6.4\times10^{6}\ \mathrm{m}}{1.5\times10^{11}\ \mathrm{m}}\right)^2 = 8.2\times10^{28}\ \mathrm{s}^{-1}\ .$$

46. (a) The products of the carbon cycle are $2e^{+} + 2\nu + {}^{4}\mathrm{He}$, the same as that of the proton-proton cycle (see Eq. 43-10). The difference in the number of photons is not significant.

(b) We have

$$\begin{aligned} Q_{\mathrm{carbon}} &= Q_1 + Q_2 + \cdots + Q_6 \\ &= (1.95\times1.19 + 7.55 + 7.30 + 1.73 + 4.97)\,\mathrm{MeV} \\ &= 24.7\,\mathrm{MeV} \end{aligned}$$

which is the same as that for the proton-proton cycle (once we subtract out the electron-positron annihilations; see Fig. 43-10):

$$Q_{p-p} = 26.7 \text{ MeV} - 2(1.02 \text{ MeV}) = 24.7 \text{ MeV}.$$

47. **THINK** The energy released by burning 1 kg of carbon is 3.3×10^7 J.

EXPRESS The mass of a carbon atom is $(12.0 \text{ u})(1.661 \times 10^{-27} \text{ kg/u}) = 1.99 \times 10^{-26}$ kg, so the number of carbon atoms in 1.00 kg of carbon is

$$(1.00 \text{ kg})/(1.99 \times 10^{-26} \text{ kg}) = 5.02 \times 10^{25}.$$

ANALYZE (a) The heat of combustion per atom is

$$(3.3 \times 10^7 \text{ J/kg})/(5.02 \times 10^{25} \text{ atom/kg}) = 6.58 \times 10^{-19} \text{ J/atom}.$$

This is 4.11 eV/atom.

(b) In each combustion event, two oxygen atoms combine with one carbon atom, so the total mass involved is 2(16.0 u) + (12.0 u) = 44 u. This is

$$(44 \text{ u})(1.661 \times 10^{-27} \text{ kg/u}) = 7.31 \times 10^{-26} \text{ kg}.$$

Each combustion event produces 6.58×10^{-19} J so the energy produced per unit mass of reactants is $(6.58 \times 10^{-19} \text{ J})/(7.31 \times 10^{-26} \text{ kg}) = 9.00 \times 10^6$ J/kg.

(c) If the Sun were composed of the appropriate mixture of carbon and oxygen, the number of combustion events that could occur before the Sun burns out would be

$$(2.0 \times 10^{30} \text{ kg})/(7.31 \times 10^{-26} \text{ kg}) = 2.74 \times 10^{55}.$$

The total energy released would be

$$E = (2.74 \times 10^{55})(6.58 \times 10^{-19} \text{ J}) = 1.80 \times 10^{37} \text{ J}.$$

If P is the power output of the Sun, the burn time would be

$$t = \frac{E}{P} = \frac{1.80\times10^{37}\text{ J}}{3.9\times10^{26}\text{ W}} = 4.62\times10^{10}\text{ s} = 1.46\times10^{3}\text{ y},$$

or $1.5\times10^3\text{ y}$, to two significant figures.

LEARN The Sun burns not coal but hydrogen via the proton-proton cycle in which the fusion of hydrogen nuclei into helium nuclei take place. The mechanism of thermonuclear fusion reactions allows the Sun to radiate energy at a rate of 3.9×10^{26} W for several billion years.

48. In Eq. 43-13,

$$Q = \left(2m_{^2\text{H}} - m_{^3\text{He}} - m_n\right)c^2 = \left[2\left(2.014102\,\text{u}\right) - 3.016049\,\text{u} - 1.008665\,\text{u}\right]\left(931.5\,\text{MeV/u}\right)$$
$$= 3.27\text{MeV} .$$

In Eq. 43-14,

$$Q = \left(2m_{^2\text{H}} - m_{^3\text{H}} - m_{^1\text{H}}\right)c^2 = \left[2\left(2.014102\,\text{u}\right) - 3.016049\,\text{u} - 1.007825\,\text{u}\right]\left(931.5\,\text{MeV/u}\right)$$
$$= 4.03\,\text{MeV} .$$

Finally, in Eq. 43-15,

$$\begin{aligned} Q &= \left(m_{^2\text{H}} + m_{^3\text{H}} - m_{^4\text{He}} - m_n\right)c^2 \\ &= \ 2.014102\,\text{u} + 3.016049\,\text{u} - 4.002603\,\text{u} - 1.008665\,\text{u}\ \left(931.5\,\text{MeV/u}\right) \\ &= 17.59\,\text{MeV} . \end{aligned}$$

49. Since 1.00 L of water has a mass of 1.00 kg, the mass of the heavy water in 1.00 L is 0.0150×10^{-2} kg = 1.50×10^{-4} kg. Since a heavy water molecule contains one oxygen atom, one hydrogen atom and one deuterium atom, its mass is

$$\begin{aligned} (16.0\text{ u} + 1.00\text{ u} + 2.00\text{ u}) = 19.0\text{ u} &= (19.0\text{ u})(1.661 \times 10^{-27}\text{ kg/u}) \\ &= 3.16 \times 10^{-26}\text{ kg}. \end{aligned}$$

The number of heavy water molecules in a liter of water is

$$(1.50 \times 10^{-4}\text{ kg})/(3.16 \times 10^{-26}\text{ kg}) = 4.75 \times 10^{21}.$$

Since each fusion event requires two deuterium nuclei, the number of fusion events that can occur is $N = 4.75 \times 10^{21}/2 = 2.38 \times 10^{21}$. Each event releases energy

$$Q = (3.27 \times 10^{6}\text{ eV})(1.60 \times 10^{-19}\text{ J/eV}) = 5.23 \times 10^{-13}\text{ J}.$$

Since all events take place in a day, which is 8.64×10^4 s, the power output is

$$P = \frac{NQ}{t} = \frac{\left(2.38 \times 10^{21}\right)\left(5.23 \times 10^{-13}\text{ J}\right)}{8.64 \times 10^{4}\text{ s}} = 1.44 \times 10^{4}\text{ W} = 14.4\,\text{kW} .$$

50. (a) From $E = NQ = (M_{\text{sam}}/4m_p)Q$ we get the energy per kilogram of hydrogen consumed:

$$\frac{E}{M_{\rm sam}} = \frac{Q}{4m_p} = \frac{(26.2\,\text{MeV})(1.60\times10^{-13}\,\text{J/MeV})}{4(1.67\times10^{-27}\,\text{kg})} = 6.3\times10^{14}\,\text{J/kg}\,.$$

(b) Keeping in mind that a watt is a joule per second, the rate is

$$\frac{dm}{dt} = \frac{3.9\times10^{26}\,\text{W}}{6.3\times10^{14}\,\text{J/kg}} = 6.2\times10^{11}\,\text{kg/s}\,.$$

This agrees with the computation shown in Sample Problem — "Consumption rate of hydrogen in the Sun."

(c) From the Einstein relation $E = Mc^2$ we get $P = dE/dt = c^2 dM/dt$, or

$$\frac{dM}{dt} = \frac{P}{c^2} = \frac{3.9\times10^{26}\,\text{W}}{(3.0\times10^{8}\,\text{m/s})^2} = 4.3\times10^{9}\,\text{kg/s}\,.$$

(d) This finding, that $dm/dt > dM/dt$, is in large part due to the fact that, as the protons are consumed, their mass is mostly turned into alpha particles (helium), which remain in the Sun.

(e) The time to lose 0.10% of its total mass is

$$t = \frac{0.0010\,M}{dM/dt} = \frac{(0.0010)(2.0\times10^{30}\,\text{kg})}{(4.3\times10^{9}\,\text{kg/s})(3.15\times10^{7}\,\text{s/y})} = 1.5\times10^{10}\,\text{y}\,.$$

51. Since plutonium has $Z = 94$ and uranium has $Z = 92$, we see that (to conserve charge) two electrons must be emitted so that the nucleus can gain a $+2e$ charge. In the beta decay processes described in Chapter 42, electrons and neutrinos are emitted. The reaction series is as follows:

$$^{238}\text{U} + \text{n} \rightarrow {}^{239}\text{Np} + {}^{239}\text{U} + e + v$$
$$^{239}\text{Np} \rightarrow {}^{239}\text{Pu} + e + v$$

52. Conservation of energy gives $Q = K_\alpha + K_n$, and conservation of linear momentum (due to the assumption of negligible initial velocities) gives $|p_\alpha| = |p_n|$. We can write the classical formula for kinetic energy in terms of momentum:

$$K = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

which implies that $K_n = (m_\alpha/m_n)K_\alpha$.

(a) Consequently, conservation of energy and momentum allows us to solve for kinetic energy of the alpha particle, which results from the fusion:

$$K_\alpha = \frac{Q}{1+(m_\alpha/m_\text{n})} = \frac{17.59\,\text{MeV}}{1+(4.0015\,\text{u}/1.008665\,\text{u})} = 3.541\,\text{MeV}$$

where we have found the mass of the alpha particle by subtracting two electron masses from the ^{4}He mass (quoted several times in this Chapter 42).

(b) Then, $K_\text{n} = Q - K_\alpha$ yields 14.05 MeV for the neutron kinetic energy.

53. At T = 300 K, the average kinetic energy of the neutrons is (using Eq. 20-24)

$$K_\text{avg} = \frac{3}{2}KT = \frac{3}{2}(8.62\times10^{-5}\ \text{eV/K})(300\ \text{K}) \approx 0.04\ \text{eV}.$$

54. First, we figure out the mass of U-235 in the sample (assuming "3.0%" refers to the proportion by weight as opposed to proportion by number of atoms):

$$\begin{aligned} M_{\text{U}-235} &= (3.0\%)M_\text{sam}\left(\frac{(97\%)m_{238}+(3.0\%)m_{235}}{(97\%)m_{238}+(3.0\%)m_{235}+2m_{16}}\right) \\ &= (0.030)(1000\ \text{g})\left(\frac{0.97(238)+0.030(235)}{0.97(238)+0.030(235)+2(16.0)}\right) \\ &= 26.4\ \text{g}. \end{aligned}$$

Next, the number of ^{235}U nuclei is

$$N_{235} = \frac{(26.4\ \text{g})(6.02\times10^{23}/\text{mol})}{235\ \text{g/mol}} = 6.77\times10^{22}.$$

If all the U-235 nuclei fission, the energy release (using the result of Eq. 43-6) is

$$N_{235}Q_\text{fission} = (6.77\times10^{22})(200\ \text{MeV}) = 1.35\times10^{25}\ \text{MeV} = 2.17\times10^{12}\ \text{J}.$$

Keeping in mind that a watt is a joule per second, the time that this much energy can keep a 100-W lamp burning is found to be

$$t = \frac{2.17\times10^{12}\ \text{J}}{100\ \text{W}} = 2.17\times10^{10}\ \text{s} \approx 690\ \text{y}.$$

If we had instead used the Q = 208 MeV value from Sample Problem — "Q value in a fission of uranium-235," then our result would have been 715 y, which perhaps suggests that our result is meaningful to just one significant figure ("roughly 700 years").

55. (a) From $\rho_H = 0.35\rho = n_p m_p$, we get the proton number density n_p:

$$n_p = \frac{0.35\rho}{m_p} = \frac{(0.35)(1.5\times10^5 \text{ kg/m}^3)}{1.67\times10^{-27}\text{ kg}} = 3.1\times10^{31}\,\text{m}^{-3}.$$

(b) From Chapter 19 (see Eq. 19-9), we have

$$\frac{N}{V} = \frac{p}{kT} = \frac{1.01\times10^5 \text{ Pa}}{(1.38\times10^{-23}\text{ J/K})(273\text{K})} = 2.68\times10^{25}\text{ m}^{-3}$$

for an ideal gas under "standard conditions." Thus,

$$\frac{n_p}{(N/V)} = \frac{3.14\times10^{31}\text{ m}^{-3}}{2.44\times10^{25}\text{ m}^{-3}} = 1.2\times10^6\ .$$

56. (a) Rather than use $P(v)$ as it is written in Eq. 19-27, we use the more convenient nK expression given in Problem 43-34. The $n(K)$ expression can be derived from Eq. 19-27, but we do not show that derivation here. To find the most probable energy, we take the derivative of $n(K)$ and set the result equal to zero:

$$\left.\frac{dn(K)}{dK}\right|_{K=K_p} = \frac{1.13n}{(kT)^{3/2}}\left(\frac{1}{2K^{1/2}} - \frac{K^{3/2}}{kT}\right)e^{-K/kT}\bigg|_{K=K_p} = 0,$$

which gives $K_p = \frac{1}{2}kT.$ Specifically, for $T = 1.5\times10^7$ K we find

$$K_p = \frac{1}{2}kT = \frac{1}{2}(8.62\times10^{-5}\text{ eV/K})(1.5\times10^7\text{ K}) = 6.5\times10^2\text{ eV}$$

or 0.65 keV, in good agreement with Fig. 43-10.

(b) Equation 19-35 gives the most probable speed in terms of the molar mass M, and indicates its derivation. Since the mass m of the particle is related to M by the Avogadro constant, then using Eq. 19-7,

$$v_p = \sqrt{\frac{2RT}{M}} = \sqrt{\frac{2RT}{mN_A}} = \sqrt{\frac{2kT}{m}}\ .$$

With $T = 1.5\times10^7$ K and $m = 1.67\times10^{-27}$ kg, this yields $v_p = 5.0\times10^5$ m/s.

(c) The corresponding kinetic energy is

$$K_{v,p} = \frac{1}{2}mv_p^2 = \frac{1}{2}m\left(\sqrt{\frac{2kT}{m}}\right)^2 = kT$$

which is twice as large as that found in part (a). Thus, at T = 1.5 × 10^7 K we have $K_{v,p}$ = 1.3 keV, which is indicated in Fig. 43-10 by a single vertical line.

57. (a) The mass of each DT pellet is

$$m = \frac{4}{3}\pi r^3 \rho = \frac{4}{3}\pi (20\times10^{-6}\,\text{m})^3(200\text{ kg/m}^3) = 6.7\times10^{-12}\,\text{kg}$$

Since there are equal number of ${}^2\text{H}$ and ${}^3\text{H}$ present, we have

$$N_{{}^2\text{H}} = N_{{}^3\text{H}} = \frac{mN_A}{M_{{}^2\text{H}} + M_{{}^3\text{H}}} = \frac{(6.7\times10^{-12}\,\text{kg})(6.02\times10^{23})}{(0.020\text{ kg}) + (0.030\text{ kg})} = 8.07\times10^{14}$$

Each fusion reaction releases 17.59 MeV of energy, with 10% efficiency, the total energy released by the pellet is

$$E = (0.10)(8.07\times10^{14})(17.59\text{ MeV}) = 1.42\times10^{15}\text{ MeV} = 227\text{ J}$$

or about 230 J.

(b) Since 1.0 kg of TNT gives off 4.6 MJ, the TNT equivalent of the pellet is

$$m = \frac{227\text{ J}}{4.6\times10^6\text{ J}} = 4.93\times10^{-5}\text{ kg}.$$

(c) The power generated is

$$P = \left(\frac{dN}{dt}\right)E = (100\,/\,\text{s})(227\text{ J}) = 2.3\times10^4\text{ W}$$

58. (a) Equation 19-35 gives the most probable speed in terms of the molar mass M: $v_p = \sqrt{2RT/M}$. With T = 1 × 10^8 K and M = 2.0 ×10^{-3} kg/mol, this yields

$$v_p = \sqrt{\frac{2RT}{M}} = \sqrt{\frac{2(8.314\text{ J/mol}\cdot\text{K})(108\text{ K})}{2.0\times10^{-3}\text{ kg}}} = 9.1\times10^5\,\text{m/s}.$$

(b) The distance moved is $r = v_p\Delta t = (9.1\times10^5\,\text{m/s})(1\times10^{-12}\text{s}) = 9.1\times10^{-7}\,\text{m}$.

Chapter 44

1. By charge conservation, it is clear that reversing the sign of the pion means we must reverse the sign of the muon. In effect, we are replacing the charged particles by their antiparticles. Less obvious is the fact that we should now put a "bar" over the neutrino (something we should also have done for some of the reactions and decays discussed in Chapters 42 and 43, except that we had not yet learned about antiparticles, which are usually denoted with a "bar." The decay of the negative pion is $\pi^- \to \mu^- + \bar{\nu}$. A subscript can be added to the antineutrino to clarify what "type" it is.

2. Since the density of water is $\rho = 1000$ kg/m^3 = 1 kg/L, then the total mass of the pool is $\rho\mathcal{V} = 4.32 \times 10^5$ kg, where $\mathcal{V}$ is the given volume. Now, the fraction of that mass made up by the protons is 10/18 (by counting the protons versus total nucleons in a water molecule). Consequently, if we ignore the effects of neutron decay (neutrons can beta decay into protons) in the interest of making an order-of-magnitude calculation, then the number of particles susceptible to decay via this $T_{1/2} = 10^{32}$ y half-life is

$$N = \frac{(10/18)M_{\text{pool}}}{m_p} = \frac{(10/18)\left(4.32\times10^5\ \text{kg}\right)}{1.67\times10^{-27}\ \text{kg}} = 1.44\times10^{32}.$$

Using Eq. 42-20, we obtain

$$R = \frac{N\ln 2}{T_{1/2}} = \frac{\left(1.44\times10^{32}\right)\ln 2}{10^{32}\,\text{y}} \approx 1\,\text{decay/y}.$$

3. The total rest energy of the electron-positron pair is

$$E = m_e c^2 + m_e c^2 = 2m_e c^2 = 2(0.511\ \text{MeV}) = 1.022\ \text{MeV}.$$

With two gamma-ray photons produced in the annihilation process, the wavelength of each photon is (using $hc = 1240\ \text{eV}\cdot\text{nm}$)

$$\lambda = \frac{hc}{E/2} = \frac{1240\ \text{eV}\cdot\text{nm}}{0.511\times10^6\,\text{eV}} = 2.43\times10^{-3}\ \text{nm} = 2.43\ \text{pm}.$$

4. Conservation of momentum requires that the gamma ray particles move in opposite directions with momenta of the same magnitude. Since the magnitude p of the momentum of a gamma ray particle is related to its energy by $p = E/c$, the particles have the same energy E. Conservation of energy yields $m_\pi c^2 = 2E$, where m_π is the mass of a neutral pion. The rest energy of a neutral pion is $m_\pi c^2 = 135.0$ MeV, according to Table

44-4. Hence, E = (135.0 MeV)/2 = 67.5 MeV. We use $hc = 1240\,\text{eV}\cdot\text{nm}$ to obtain the wavelength of the gamma rays:

$$\lambda = \frac{1240\,\text{eV}\cdot\text{nm}}{67.5\times10^{6}\,\text{eV}} = 1.84\times10^{-5}\ \text{nm} = 18.4\ \text{fm}.$$

5. We establish a ratio, using Eq. 22-4 and Eq. 14-1:

$$\frac{F_{\text{gravity}}}{F_{\text{electric}}} = \frac{Gm_e^2/r^2}{ke^2/r^2} = \frac{4\pi\varepsilon_0 Gm_e^2}{e^2} = \frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(9.11\times10^{-31}\ \text{kg}\right)^2}{\left(9.0\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\ \text{C}\right)^2}$$
$$= 2.4\times10^{-43}.$$

Since $F_{\text{gravity}} \ll F_{\text{electric}}$, we can neglect the gravitational force acting between particles in a bubble chamber.

6. (a) Conservation of energy gives

$$Q = K_2 + K_3 = E_1 - E_2 - E_3$$

where E refers here to the *rest* energies (mc^2) instead of the total energies of the particles. Writing this as

$$K_2 + E_2 - E_1 = -(K_3 + E_3)$$

and squaring both sides yields

$$K_2^2 + 2K_2E_2 - 2K_2E_1 + \left(E_1 - E_2\right)^2 = K_3^2 + 2K_3E_3 + E_3^2.$$

Next, conservation of linear momentum (in a reference frame where particle 1 was at rest) gives $|p_2| = |p_3|$ (which implies $(p_2c)^2 = (p_3c)^2$). Therefore, Eq. 37-54 leads to

$$K_2^2 + 2K_2E_2 = K_3^2 + 2K_3E_3$$

which we subtract from the above expression to obtain

$$-2K_2E_1 + \left(E_1 - E_2\right)^2 = E_3^2.$$

This is now straightforward to solve for K_2 and yields the result stated in the problem.

(b) Setting $E_3 = 0$ in

$$K_2 = \frac{1}{2E_1}\ \left(E_1 - E_2\right)^2 - E_3^2$$

and using the rest energy values given in Table 44-1 readily gives the same result for K_μ as computed in Sample Problem – "Momentum and kinetic energy in a pion decay."

7. Table 44-4 gives the rest energy of each pion as 139.6 MeV. The magnitude of the momentum of each pion is p_π = (358.3 MeV)/c. We use the relativistic relationship between energy and momentum (Eq. 37-54) to find the total energy of each pion:

$$E_\pi = \sqrt{(p_\pi c)^2 + (m_\pi c^2)^2} = \sqrt{(358.3\text{ MeV})^2 + (139.6\text{ Mev})} = 384.5\text{ MeV}.$$

Conservation of energy yields

$$m_\rho c^2 = 2E_\pi = 2(384.5\text{ MeV}) = 769\text{ MeV}.$$

8. (a) In SI units, the kinetic energy of the positive tau particle is

$$K = (2200\text{ MeV})(1.6 \times 10^{-13}\text{ J/MeV}) = 3.52 \times 10^{-10}\text{ J}.$$

Similarly, $mc^2 = 2.85 \times 10^{-10}$ J for the positive tau. Equation 37-54 leads to the relativistic momentum:

$$p = \frac{1}{c}\sqrt{K^2 + 2Kmc^2} = \frac{1}{2.998\times10^8\text{ m/s}}\sqrt{\left(3.52\times10^{-10}\text{ J}\right)^2 + 2\left(3.52\times10^{-10}\text{ J}\right)\left(2.85\times10^{-10}\text{ J}\right)}$$

which yields $p = 1.90 \times 10^{-18}$ kg·m/s.

(b) The radius should be calculated with the relativistic momentum:

$$r = \frac{\gamma mv}{|q|B} = \frac{p}{eB}$$

where we use the fact that the positive tau has charge $e = 1.6 \times 10^{-19}$ C. With B = 1.20 T, this yields r = 9.90 m.

9. From Eq. 37-48, the Lorentz factor would be

$$\gamma = \frac{E}{mc^2} = \frac{1.5\times10^6\text{ eV}}{20\text{ eV}} = 75000.$$

Solving Eq. 37-8 for the speed, we find

$$\gamma = \frac{1}{\sqrt{1-(v/c)^2}} \Rightarrow v = c\sqrt{1-\frac{1}{\gamma^2}}$$

which implies that the difference between v and c is

$$c - v = c\left(1 - \sqrt{1 - \frac{1}{\gamma^2}}\right) \approx c\left(1 - \left(1 - \frac{1}{2\gamma^2} + \cdots\right)\right)$$

where we use the binomial expansion (see Appendix E) in the last step. Therefore,

$$c - v \approx c\left(\frac{1}{2\gamma^2}\right) = (299792458\,\text{m/s})\,\left(\frac{1}{2(75000)^2}\right) = 0.0266\,\text{m/s} \approx 2.7\,\text{cm/s}\,.$$

10. From Eq. 37-52, the Lorentz factor is

$$\gamma = 1 + \frac{K}{mc^2} = 1 + \frac{80\text{ MeV}}{135\text{ MeV}} = 1.59.$$

Solving Eq. 37-8 for the speed, we find

$$\gamma = \frac{1}{\sqrt{1-(v/c)^2}} \;\Rightarrow\; v = c\sqrt{1 - \frac{1}{\gamma^2}}$$

which yields v = 0.778c or v = 2.33 × 10^8 m/s. Now, in the reference frame of the laboratory, the lifetime of the pion is not the given τ value but is “dilated.” Using Eq. 37-9, the time in the lab is

$$t = \gamma\tau = (1.59)\left(8.3\times10^{-17}\text{ s}\right) = 1.3\times10^{-16}\text{ s}.$$

Finally, using Eq. 37-10, we find the distance in the lab to be

$$x = vt = \left(2.33\times10^8\text{ m/s}\right)\,\left(1.3\times10^{-16}\text{ s}\right) = 3.1\times10^{-8}\text{ m}.$$

11. **THINK** The conservation laws we shall examine are associated with energy, momentum, angular momentum, charge, baryon number, and the three lepton numbers.

EXPRESS In all particle interactions, the net lepton number for each family (L_e for electron, L_μ for muon, and L_τ for tau) is separately conserved. Conservation of baryon number implies that a process cannot occur if the net baryon number is changed.

ANALYZE (a) For the process $\mu^- \to e^- + \nu_\mu$, the rest energy of the muon is 105.7 MeV, the rest energy of the electron is 0.511 MeV, and the rest energy of the neutrino is zero. Thus, the total rest energy before the decay is greater than the total rest energy after. The excess energy can be carried away as the kinetic energies of the decay products and energy can be conserved. Momentum is conserved if the electron and neutrino move away from the decay in opposite directions with equal magnitudes of momenta. Since the orbital angular momentum is zero, we consider only spin angular momentum. All the particles have spin $\hbar/2$. The total angular momentum after the decay must be either $\hbar$ (if the spins are aligned) or zero (if the spins are anti-aligned). Since the spin before the decay is $\hbar/2$ angular momentum cannot be conserved. The muon has charge $-e$, the electron has charge $-e$, and the neutrino has charge zero, so the total charge before the decay is $-e$ and the total charge after is $-e$. Charge is conserved. All particles have baryon number zero, so baryon number is conserved. The muon lepton number of the muon is +1, the muon lepton number of the muon neutrino is +1, and the muon lepton number of the electron is 0. Muon lepton number is conserved. The electron lepton numbers of the muon and muon neutrino are 0 and the electron lepton number of the electron is +1. Electron lepton number is not conserved. The laws of conservation of angular momentum and electron lepton number are not obeyed and this decay does not occur.

(b) We analyze the decay $\mu^- \to e^+ + \nu_e + \bar{\nu}_\mu$ in the same way. We find that charge and the muon lepton number L_μ are not conserved.

(c) For the process $\mu^+ \to \pi^+ + \nu_\mu$, we find that energy cannot be conserved because the mass of muon is less than the mass of a pion. Also, muon lepton number L_μ is not conserved.

LEARN In all three processes considered, since the initial particle is stationary, the question associated with energy conservation amounts to asking whether the initial mass energy is sufficient to produce the mass energies and kinetic energies of the decayed products.

12. (a) Noting that there are two positive pions created (so, in effect, its decay products are doubled), then we count up the electrons, positrons, and neutrinos: $2\mathrm{e}^+ + \mathrm{e}^- + 5\nu + 4\bar{\nu}$.

(b) The final products are all leptons, so the baryon number of A_2^+ is zero. Both the pion and rho meson have integer-valued spins, so A_2^+ is a boson.

(c) A_2^+ is also a meson.

(d) As stated in (b), the baryon number of A_2^+ is zero.

13. The formula for T_z as it is usually written to include strange baryons is $T_z = q - (S + B)/2$. Also, we interpret the symbol q in the T_z formula in terms of elementary charge units; this is how q is listed in Table 44-3. In terms of charge q as we have used it in previous chapters, the formula is

$$T_z = \frac{q}{e} - \frac{1}{2}(B+S).$$

For instance, $T_z = +\frac{1}{2}$ for the proton (and the neutral Xi) and $T_z = -\frac{1}{2}$ for the neutron (and the negative Xi). The baryon number B is +1 for all the particles in Fig. 44-4(a). Rather than use a sloping axis as in Fig. 44-4 (there it is done for the q values), one reproduces (if one uses the "corrected" formula for T_z mentioned above) exactly the same pattern using regular rectangular axes (T_z values along the horizontal axis and Y values along the vertical) with the neutral lambda and sigma particles situated at the origin.

14. (a) From Eq. 37-50,

$$\begin{aligned} Q &= -\Delta mc^2 = (m_{\Sigma^+} + m_{K^+} - m_{\pi^+} - m_p)c^2 \\ &= 1189.4\,\text{MeV} + 493.7\,\text{MeV} - 139.6\,\text{MeV} - 938.3\,\text{MeV} \\ &= 605\,\text{MeV}. \end{aligned}$$

(b) Similarly,

$$\begin{aligned} Q &= -\Delta mc^2 = (m_{\Lambda^0} + m_{\pi^0} - m_{K^-} - m_p)c^2 \\ &= 1115.6\,\text{MeV} + 135.0\,\text{MeV} - 493.7\,\text{MeV} - 938.3\,\text{MeV} \\ &= -181\,\text{MeV}. \end{aligned}$$

15. (a) The lambda has a rest energy of 1115.6 MeV, the proton has a rest energy of 938.3 MeV, and the kaon has a rest energy of 493.7 MeV. The rest energy before the decay is less than the total rest energy after, so energy cannot be conserved. Momentum can be conserved. The lambda and proton each have spin $\hbar/2$ and the kaon has spin zero, so angular momentum can be conserved. The lambda has charge zero, the proton has charge +*e*, and the kaon has charge –*e*, so charge is conserved. The lambda and proton each have baryon number +1, and the kaon has baryon number zero, so baryon number is conserved. The lambda and kaon each have strangeness –1 and the proton has strangeness zero, so strangeness is conserved. Only energy cannot be conserved.

(b) The omega has a rest energy of 1680 MeV, the sigma has a rest energy of 1197.3 MeV, and the pion has a rest energy of 135 MeV. The rest energy before the decay is greater than the total rest energy after, so energy can be conserved. Momentum can be conserved. The omega and sigma each have spin $\hbar/2$ and the pion has spin zero, so angular momentum can be conserved. The omega has charge –*e*, the sigma has charge –*e*, and the pion has charge zero, so charge is conserved. The omega and sigma have baryon number +1 and the pion has baryon number 0, so baryon number is conserved. The omega has strangeness –3, the sigma has strangeness –1, and the pion has strangeness zero, so strangeness is not conserved.

(c) The kaon and proton can bring kinetic energy to the reaction, so energy can be conserved even though the total rest energy after the collision is greater than the total rest energy before. Momentum can be conserved. The proton and lambda each have spin $\hbar/2$ and the kaon and pion each have spin zero, so angular momentum can be conserved. The kaon has charge –*e*, the proton has charge +*e*, the lambda has charge zero, and the pion has charge +*e*, so charge is not conserved. The proton and lambda each have baryon number +1, and the kaon and pion each have baryon number zero; baryon number is conserved. The kaon has strangeness –1, the proton and pion each have strangeness zero, and the lambda has strangeness –1, so strangeness is conserved. Only charge is not conserved.

16. To examine the conservation laws associated with the proposed reaction $\mathrm{p}+\overline{\mathrm{p}} \to \Lambda^0 + \Sigma^+ + e^-$, we make use of particle properties found in Tables 44-3 and 44-4.

(a) With $q(\mathrm{p}) = +1, q(\overline{\mathrm{p}}) = -1, \quad q(\Lambda^0) = 0, \quad q(\Sigma^+) = +1,$ and $q(e^-) = -1$, we have $1+(-1) = 0+1+(-1)$. Thus, the process conserves charge.

(b) With $B(\mathrm{p}) = +1, B(\overline{\mathrm{p}}) = -1, \quad B(\Lambda^0) = 1, \quad B(\Sigma^+) = +1,$ and $B(e^-) = 0$, we have $1+(-1) \neq 1+1+0$. Thus, the process does not conserve baryon number.

(c) With $L_e(\mathrm{p}) = L_e(\overline{\mathrm{p}}) = 0, \quad L_e(\Lambda^0) = L_e(\Sigma^+) = 0,$ and $L_e(e^-) = 1$, we have $0+0 \neq 0+0+1$, so the process does not conserve electron lepton number.

(d) All the particles on either side of the reaction equation are fermions with $s = 1/2$. Therefore, $(1/2)+(1/2) \neq (1/2)+(1/2)+(1/2)$ and the process does not conserve spin angular momentum.

(e) With $S(\mathrm{p}) = S(\overline{\mathrm{p}}) = 0, \quad S(\Lambda^0) = 1, \quad S(\Sigma^+) = +1,$ and $S(e^-) = 0$, we have $0+0 \neq 1+1+0$, so the process does not conserve strangeness.

(f) The process does conserve muon lepton number since all the particles involved have muon lepton number of zero.

17. To examine the conservation laws associated with the proposed decay process $\Xi^- \to \pi^- + \mathrm{n} + \mathrm{K}^- + \mathrm{p}$, we make use of particle properties found in Tables 44-3 and 44-4.

(a) With $q(\Xi^-) = -1$, $q(\pi^-) = -1, q(\mathrm{n}) = 0, q(\mathrm{K}^-) = -1,$ and $q(\mathrm{p}) = +1,$ we have $-1 = -1+0+(-1)+1$. Thus, the process conserves charge.

(b) Since $B(\Xi^-) = +1$, $B(\pi^-) = 0, \quad B(\mathrm{n}) = +1, B(\mathrm{K}^-) = 0,$ and $B(\mathrm{p}) = +1,$ we have $+1 \neq 0+1+0+1 = 2$. Thus, the process does not conserve baryon number.

(c) Ξ^-, n and p are fermions with $s = 1/2$, while π^- and K^- are mesons with spin zero. Therefore, $+1/2 \neq 0+(1/2)+0+(1/2)$ and the process does not conserve spin angular momentum.

(d) Since $S(\Xi^-) = -2$, $S(\pi^-) = 0$, $S(\text{n}) = 0, S(\text{K}^-) = -1$, and $S(\text{p}) = 0$, we have $-2 \neq 0 + 0 + (-1) + 0$, so the process does not conserve strangeness.

18. (a) Referring to Tables 44-3 and 44-4, we find that the strangeness of K^0 is +1, while it is zero for both π^+ and π^-. Consequently, strangeness is not conserved in this decay; $K^0 \rightarrow \pi^+ + \pi^-$ does not proceed via the strong interaction.

(b) The strangeness of each side is –1, which implies that the decay is governed by the strong interaction.

(c) The strangeness or Λ^0 is –1 while that of p + π^- is zero, so the decay is not via the strong interaction.

(d) The strangeness of each side is –1; it proceeds via the strong interaction.

19. For purposes of deducing the properties of the antineutron, one may cancel a proton from each side of the reaction and write the equivalent reaction as $\pi^+ \rightarrow p + \bar{n}$.

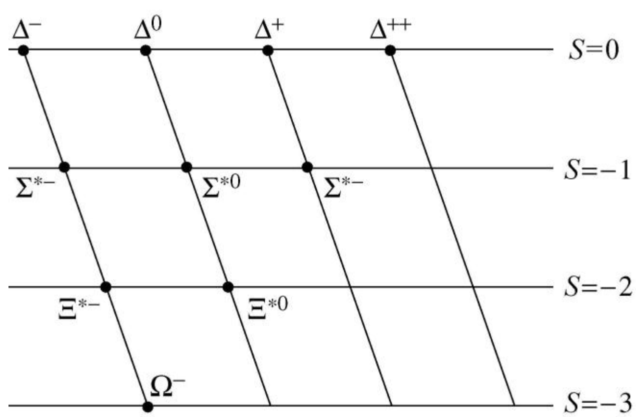
Particle properties can be found in Tables 44-3 and 44-4. The pion and proton each have charge +*e*, so the antineutron must be neutral. The pion has baryon number zero (it is a meson) and the proton has baryon number +1, so the baryon number of the antineutron must be –1. The pion and the proton each have strangeness zero, so the strangeness of the antineutron must also be zero. In summary, for the antineutron,

(a) $q = 0$,

(b) $B = -1$,

(c) and $S = 0$.

20. If we were to use regular rectangular axes, then this would appear as a right triangle. Using the sloping *q* axis as the problem suggests, it is similar to an "upside down" equilateral triangle as we show below.



The leftmost slanted line is for the –1 charge, and the rightmost slanted line is for the +2 charge.

21. (a) As far as the conservation laws are concerned, we may cancel a proton from each side of the reaction equation and write the reaction as $p \to \Lambda^0 + x$. Since the proton and the lambda each have a spin angular momentum of $\hbar/2$, the spin angular momentum of x must be either zero or $\hbar$. Since the proton has charge $+e$ and the lambda is neutral, x must have charge $+e$. Since the proton and the lambda each have a baryon number of +1, the baryon number of x is zero. Since the strangeness of the proton is zero and the strangeness of the lambda is –1, the strangeness of x is +1. We take the unknown particle to be a spin zero meson with a charge of $+e$ and a strangeness of +1. Look at Table 44-4 to identify it as a K^+ particle.

(b) Similar analysis tells us that x is a spin-$\frac{1}{2}$ antibaryon (B = –1) with charge and strangeness both zero. Inspection of Table 44-3 reveals that it is an antineutron.

(c) Here x is a spin-0 (or spin-1) meson with charge zero and strangeness +1. According to Table 44-4, it could be a K^0 particle.

22. Conservation of energy (see Eq. 37-47) leads to

$$\begin{aligned} K_f &= -\Delta mc^2 + K_i = (m_{\Sigma^-} - m_{\pi^-} - m_n)c^2 + K_i \\ &= 1197.3\,\text{MeV} - 139.6\,\text{MeV} - 939.6\,\text{MeV} + 220\,\text{MeV} \\ &= 338\,\text{MeV}. \end{aligned}$$

23. (a) From Eq. 37-50,

$$\begin{aligned} Q &= -\Delta mc^2 = (m_{\Lambda^0} - m_p - m_{\pi^-})c^2 \\ &= 1115.6\,\text{MeV} - 938.3\,\text{MeV} - 139.6\,\text{MeV} = 37.7\,\text{MeV}. \end{aligned}$$

(b) We use the formula obtained in Problem 44-6 (where it should be emphasized that E is used to mean the rest energy, not the total energy):

$$\begin{aligned} K_p &= \frac{1}{2E_\Lambda}\left(E_\Lambda - E_p\right)^2 - E_\pi^2 \\ &= \frac{(1115.6\,\text{MeV} - 938.3\,\text{MeV})^2 - (139.6\,\text{MeV})^2}{2(1115.6\,\text{MeV})} = 5.35\,\text{MeV}. \end{aligned}$$

(c) By conservation of energy,

$$K_{\pi^-} = Q - K_p = 37.7\,\text{MeV} - 5.35\,\text{MeV} = 32.4\,\text{MeV}.$$

24. From $\gamma = 1 + K/mc^2$ (see Eq. 37-52) and $v = \beta c = c\sqrt{1-\gamma^{-2}}$ (see Eq. 37-8), we get

$$v = c\sqrt{1-\left(1+\frac{K}{mc^2}\right)^{-2}}.$$

(a) Therefore, for the Σ^{*0} particle,

$$v = (2.9979\times10^8\ \text{m/s})\sqrt{1-\left(1+\frac{1000\,\text{MeV}}{1385\,\text{MeV}}\right)^{-2}} = 2.4406\times10^8\ \text{m/s}.$$

For Σ^0,

$$v' = (2.9979\times10^8\ \text{m/s})\sqrt{1-\left(1+\frac{1000\,\text{MeV}}{1192.5\,\text{MeV}}\right)^{-2}} = 2.5157\times10^8\ \text{m/s}.$$

Thus Σ^0 moves faster than Σ^{*0}.

(b) The speed difference is

$$\Delta v = v' - v = (2.5157 - 2.4406)(10^8\ \text{m/s}) = 7.51\times10^6\ \text{m/s}.$$

25. (a) We indicate the antiparticle nature of each quark with a "bar" over it. Thus, $\bar{\text{u}}\,\bar{\text{u}}\,\bar{\text{d}}$ represents an antiproton.

(b) Similarly, $\bar{\text{u}}\,\bar{\text{d}}\,\bar{\text{d}}$ represents an antineutron.

26. (a) The combination ddu has a total charge of $\left(-\frac{1}{3}-\frac{1}{3}+\frac{2}{3}\right)=0$, and a total strangeness of zero. From Table 44-3, we find it to be a neutron (n).

(b) For the combination uus, we have $Q = +\frac{2}{3}+\frac{2}{3}-\frac{1}{3} = 1$ and S = 0 + 0 – 1 = –1. This is the Σ^+ particle.

(c) For the quark composition ssd, we have $Q = -\frac{1}{3}-\frac{1}{3}-\frac{1}{3} = -1$ and S = – 1 – 1 + 0 = – 2. This is a Ξ^-.

27. The meson $\bar{\text{K}}^0$ is made up of a quark and an anti-quark, with net charge zero and strangeness $S = -1$. The quark with $S = -1$ is s. By charge neutrality condition, the anti-quark must be $\bar{\text{d}}$. Therefore, the constituents of $\bar{\text{K}}^0$ are s and $\bar{\text{d}}$.

28. (a) Using Table 44-3, we find q = 0 and S = –1 for this particle (also, B = 1, since that is true for all particles in that table). From Table 44-5, we see it must therefore contain a strange quark (which has charge –1/3), so the other two quarks must have charges to add

to zero. Assuming the others are among the lighter quarks (none of them being an anti-quark, since B = 1), then the quark composition is sud.

(b) The reasoning is very similar to that of part (a). The main difference is that this particle must have two strange quarks. Its quark combination turns out to be uss.

29. (a) The combination ssu has a total charge of $\left(-\frac{1}{3}-\frac{1}{3}+\frac{2}{3}\right)=0$, and a total strangeness of – 2. From Table 44-3, we find it to be the Ξ^0 particle.

(b) The combination dds has a total charge of $\left(-\frac{1}{3}-\frac{1}{3}-\frac{1}{3}\right)=-1$, and a total strangeness of –1. From Table 44-3, we find it to be the Σ^- particle.

30. **THINK** A baryon is made up of three quarks.

EXPRESS The quantum numbers of the up, down, and strange quarks are (see Table 44-5) as follows:

Particle	Charge q	Strangeness S	Baryon number B
Up (u)	+2/3	0	+1/3
Down (d)	–1/3	0	+1/3
Strange (s)	–1/3	–1	+1/3

ANALYZE (a) To obtain a strangeness of –2, two of them must be s quarks. Each of these has a charge of –e/3, so the sum of their charges is –2e/3. To obtain a total charge of e, the charge on the third quark must be 5e/3. There is no quark with this charge, so the particle cannot be constructed. In fact, such a particle has never been observed.

(b) Again the particle consists of three quarks (and no antiquarks). To obtain a strangeness of zero, none of them may be s quarks. We must find a combination of three u and d quarks with a total charge of 2e. The only such combination consists of three u quarks.

LEARN The baryon with three u quarks is Δ^{++}.

31. First, we find the speed of the receding galaxy from Eq. 37-31:

$$\beta=\frac{1-(f/f_0)^2}{1+(f/f_0)^2}=\frac{1-(\lambda_0/\lambda)^2}{1+(\lambda_0/\lambda)^2}$$
$$=\frac{1-(590.0\text{ nm}/602.0\text{ nm})^2}{1+(590.0\text{ nm}/602.0\text{ nm})^2}=0.02013$$

where we use $f = c/\lambda$ and $f_0 = c/\lambda_0$. Then from Eq. 44-19,

$$r = \frac{v}{H} = \frac{\beta c}{H} = \frac{\left(0.02013\right)\left(2.998\times10^{8}\ \text{m/s}\right)}{0.0218\ \text{m/s}\cdot\text{ly}} = 2.77\times10^{8}\ \text{ly}\ .$$

32. Since

$$\lambda = \lambda_0\sqrt{\frac{1+\beta}{1-\beta}} = 2\lambda_0 \quad \Rightarrow \quad \sqrt{\frac{1+\beta}{1-\beta}} = 2\,,$$

the speed of the receding galaxy is $v = \beta c = 3c/5$. Therefore, the distance to the galaxy when the light was emitted is

$$r = \frac{v}{H} = \frac{\beta c}{H} = \frac{(3/5)c}{H} = \frac{(0.60)(2.998\times10^{8}\ \text{m/s})}{0.0218\ \text{m/s}\cdot\text{ly}} = 8.3\times10^{9}\ \text{ly}\ .$$

33. We apply Eq. 37-36 for the Doppler shift in wavelength:

$$\frac{\Delta\lambda}{\lambda} = \frac{v}{c}$$

where v is the recessional speed of the galaxy. We use Hubble's law to find the recessional speed: $v = Hr$, where r is the distance to the galaxy and H is the Hubble constant $\left(21.8\times10^{-3}\,\frac{\text{m}}{\text{s}\cdot\text{ly}}\right)$. Thus,

$$v = \left(21.8\times10^{-3}\ \text{m/s}\cdot\text{ly}\right)\left(2.40\times10^{8}\ \text{ly}\right) = 5.23\times10^{6}\ \text{m/s}$$

and

$$\Delta\lambda = \frac{v}{c}\lambda = \left(\frac{5.23\times10^{6}\ \text{m/s}}{3.00\times10^{8}\ \text{m/s}}\right)(656.3\ \text{nm}) = 11.4\ \text{nm}\ .$$

Since the galaxy is receding, the observed wavelength is longer than the wavelength in the rest frame of the galaxy. Its value is

$$656.3\ \text{nm} + 11.4\ \text{nm} = 667.7\ \text{nm} \approx 668\ \text{nm}.$$

34. (a) Using Hubble's law given in Eq. 44-19, the speed of recession of the object is

$$v = Hr = \left(0.0218\,\text{m/s}\cdot\text{ly}\right)\left(1.5\times10^{4}\text{ly}\right) = 327\ \text{m/s}.$$

Therefore, the extra distance of separation one year from now would be

$$d = vt = (327\ \text{m/s})(365\ \text{d})(86400\ \text{s/d}) = 1.0\times10^{10}\ \text{m}.$$

(b) The speed of the object is $v = 327\text{ m/s} \approx 3.3\times10^2$ m/s.

35. Letting $v = Hr = c$, we obtain

$$r = \frac{c}{H} = \frac{3.0\times10^8\text{ m/s}}{0.0218\text{ m/s}\cdot\text{ly}} = 1.376\times10^{10}\text{ ly} \approx 1.4\times10^{10}\text{ ly}\,.$$

36. From $F_{\text{grav}} = GMm/r^2 = mv^2/r$ we find $M \propto v^2$. Thus, the mass of the Sun would be

$$M_s' = \left(\frac{v_{\text{Mercury}}}{v_{\text{Pluto}}}\right)^2 M_s = \left(\frac{47.9\text{ km/s}}{4.74\text{ km/s}}\right)^2 M_s = 102\,M_s\,.$$

37. (a) For the universal microwave background, Wien's law leads to

$$T = \frac{2898\,\mu\text{m}\cdot\text{K}}{\lambda_{\max}} = \frac{2898\text{ mm}\cdot\text{K}}{1.1\text{ mm}} = 2.6\text{ K}\,.$$

(b) At "decoupling" (when the universe became approximately "transparent"),

$$\lambda_{\max} = \frac{2898\,\mu\text{m}\cdot\text{K}}{T} = \frac{2898\,\mu\text{m}\cdot\text{K}}{2970\text{K}} = 0.976\ \mu\text{m} = 976\text{ nm.}$$

38. (a) We substitute $\lambda = (2898\ \mu\text{m}\cdot\text{K})/T$ into the expression:

$$E = hc/\lambda = (1240\text{ eV}\cdot\text{nm})/\lambda.$$

First, we convert units:

$$2898\ \mu\text{m}\cdot\text{K} = 2.898\times10^6\text{ nm}\cdot\text{K and } 1240\text{ eV}\cdot\text{nm} = 1.240\times10^{-3}\text{ MeV}\cdot\text{nm.}$$

Thus,

$$E = \frac{(1.240\times10^{-3}\text{ MeV}\cdot\text{nm})T}{2.898\times10^6\text{ nm}\cdot\text{K}} = (4.28\times10^{-10}\text{ MeV/K})T\,.$$

(b) The minimum energy required to create an electron-positron pair is twice the rest energy of an electron, or 2(0.511 MeV) = 1.022 MeV. Hence,

$$T = \frac{E}{4.28\times10^{-10}\text{ MeV/K}} = \frac{1.022\text{ MeV}}{4.28\times10^{-10}\text{ MeV/K}} = 2.39\times10^9\text{ K}\,.$$

39. (a) Letting $v(r) = Hr \le v_e = \sqrt{2GM/r}$, we get $M/r^3 \ge H^2/2G$. Thus,

$$\rho = \frac{M}{4\pi r^2/3} = \frac{3}{4\pi}\frac{M}{r^3} \geq \frac{3H^2}{8\pi G}.$$

(b) The density being expressed in H-atoms/m^3 is equivalent to expressing it in terms of $\rho_0 = m_{\rm H}/{\rm m}^3 = 1.67 \times 10^{-27}$ kg/m^3. Thus,

$$\rho = \frac{3H^2}{8\pi G\rho_0}\left(\text{H atoms/m}^3\right) = \frac{3\left(0.0218\,\text{m/s}\cdot\text{ly}\right)^2\left(1.00\,\text{ly}/9.460\times10^{15}\ \text{m}\right)^2\left(\text{H atoms/m}^3\right)}{8\pi\left(6.67\times10^{-11}\,\text{m}^3/\text{kg}\cdot\text{s}^2\right)\left(1.67\times10^{-27}\,\text{kg/m}^3\right)}$$
$$= 5.7\ \text{H atoms/m}^3.$$

40. (a) From $f = c/\lambda$ and Eq. 37-31, we get

$$\lambda_0 = \lambda\sqrt{\frac{1-\beta}{1+\beta}} = (\lambda_0 + \Delta\lambda)\sqrt{\frac{1-\beta}{1+\beta}}.$$

Dividing both sides by λ_0 leads to

$$1 = (1+z)\sqrt{\frac{1-\beta}{1+\beta}}$$

where $z = \Delta\lambda/\lambda_0$. We solve for β:

$$\beta = \frac{(1+z)^2 - 1}{(1+z)^2 + 1} = \frac{z^2 + 2z}{z^2 + 2z + 2}.$$

(b) Now $z = 4.43$, so

$$\beta = \frac{(4.43)^2 + 2(4.43)}{(4.43)^2 + 2(4.43) + 2} = 0.934\ .$$

(c) From Eq. 44-19,

$$r = \frac{v}{H} = \frac{\beta c}{H} = \frac{\left(0.934\right)\left(3.0\times10^8\ \text{m/s}\right)}{0.0218\,\text{m/s}\cdot\text{ly}} = 1.28\times10^{10}\ \text{ly}\ .$$

41. Using Eq. 39-33, the energy of the emitted photon is

$$E = E_3 - E_2 = -(13.6\ \text{eV})\left(\frac{1}{3^2} - \frac{1}{2^2}\right) = 1.89\ \text{eV}$$

and its wavelength is

$$\lambda_0 = \frac{hc}{E} = \frac{1240\ \text{eV}\cdot\ \text{nm}}{1.89\ \text{eV}} = 6.56\times10^{-7}\ \text{m}\ .$$

Given that the detected wavelength is $\lambda = 3.00\times10^{-3}$ m, we find

$$\frac{\lambda}{\lambda_0} = \frac{3.00\times10^{-3}\ \text{m}}{6.56\times10^{-7}\ \text{m}} = 4.57\times10^{3}.$$

42. (a) From Eq. 41-29, we know that $N_2/N_1 = e^{-\Delta E/kT}$. We solve for ΔE:

$$\Delta E = kT\ln\frac{N_1}{N_2} = \left(8.62\times10^{-5}\ \text{eV/K}\right)\left(2.7\text{K}\right)\ln\left(\frac{1-0.25}{0.25}\right)$$
$$= 2.56\times10^{-4}\,\text{eV} \approx 0.26\ \text{meV}.$$

(b) Using $hc = 1240\text{eV}\cdot\text{nm}$, we get

$$\lambda = \frac{hc}{\Delta E} = \frac{1240\text{eV}\cdot\text{nm}}{2.56\times10^{-4}\text{eV}} = 4.84\times10^{6}\,\text{nm} \approx 4.8\text{mm}.$$

43. **THINK** The radius of the orbit is still given by 1.50×10^{11} km, the original Earth-Sun distance.

EXPRESS The gravitational force on Earth is only due to the mass M within Earth's orbit. If r is the radius of the orbit, R is the radius of the new Sun, and M_S is the mass of the Sun, then

$$M = \left(\frac{r}{R}\right)^3 M_s = \left(\frac{1.50\times10^{11}\ \text{m}}{5.90\times10^{12}\ \text{m}}\right)^3\left(1.99\times10^{30}\ \text{kg}\right) = 3.27\times10^{25}\ \text{kg}.$$

The gravitational force on Earth is given by GMm/r^2, where m is the mass of Earth and G is the universal gravitational constant. Since the centripetal acceleration is given by v^2/r, where v is the speed of Earth, $GMm/r^2 = mv^2/r$ and

$$v = \sqrt{\frac{GM}{r}}.$$

ANALYZE (a) Substituting the values given, we obtain

$$v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{\left(6.67\times10^{-11}\ \text{m}^3/\text{s}^2\cdot\text{kg}\right)\left(3.27\times10^{25}\ \text{kg}\right)}{1.50\times10^{11}\ \text{m}}} = 1.21\times10^{2}\ \text{m/s}.$$

(b) The ratio of the speeds is

$$\frac{v}{v_0} = \frac{1.21\times10^{2}\ \text{m/s}}{2.98\times10^{4}\ \text{m/s}} = 0.00405.$$

(c) The period of revolution is

$$T = \frac{2\pi r}{v} = \frac{2\pi\left(1.50\times10^{11}\ \text{m}\right)}{1.21\times10^{2}\ \text{m/s}} = 7.82\times10^{9}\ \text{s} = 247\ \text{y}\ .$$

LEARN An alternative ways to calculate the speed ratio and the periods are as follows. Since $v \sim \sqrt{M}$, the ratio of the speeds can be obtained as

$$\frac{v}{v_0} = \sqrt{\frac{M}{M_S}} = \left(\frac{r}{R}\right)^{3/2} = \left(\frac{1.50\times10^{11}\ \text{m}}{5.90\times10^{12}\ \text{m}}\right)^{3/2} = 0.00405.$$

In addition, since $T \sim 1/v \sim 1/\sqrt{M}$, we have

$$T = T_0\sqrt{\frac{M_S}{M}} = T_0\left(\frac{R}{r}\right)^{3/2} = (1\ \text{y})\left(\frac{5.90\times10^{12}\ \text{m}}{1.50\times10^{11}\ \text{m}}\right)^{3/2} = 247\ \text{y}.$$

44. (a) The mass of the portion of the galaxy within the radius r from its center is given by $M' = (r/R)^3 M$. Thus, from $GM'm/r^2 = mv^2/r$ (where m is the mass of the star) we get

$$v = \sqrt{\frac{GM'}{r}} = \sqrt{\frac{GM}{r}\left(\frac{r}{R}\right)^3} = r\sqrt{\frac{GM}{R^3}}\ .$$

(b) In the case where $M' = M$, we have

$$T = \frac{2\pi r}{v} = 2\pi r\sqrt{\frac{r}{GM}} = \frac{2\pi r^{3/2}}{\sqrt{GM}}\ .$$

45. **THINK** A meson is made up of a quark and an antiquark.

EXPRESS Only the strange quark has nonzero strangeness; an s quark has strangeness $S = -1$ and charge $q = -1/3$, while an $\bar{\text{s}}$ quark has strangeness $S = +1$ and charge $q = +1/3$.

ANALYZE (a) In order to obtain $S = -1$ we need to combine s with some non-strange antiquark (which would have the negative of the quantum numbers listed in Table 44-5). The difficulty is that the charge of the strange quark is –1/3, which means that (to obtain a total charge of +1) the antiquark would have to have a charge of $+\frac{4}{3}$. Clearly, there are no such antiquarks in our list. Thus, a meson with $S = -1$ and $q = +1$ cannot be formed with the quarks/antiquarks of Table 44-5.

(b) Similarly, one can show that, since no quark has $q = -\frac{4}{3}$, there cannot be a meson with $S = +1$ and $q = -1$.

LEARN Quarks and antiquarks can be combined to form baryons and mesons, but not all combinations are allowed because of the constraint from the quantum numbers.

46. Assuming the line passes through the origin, its slope is $0.40c/(5.3 \times 10^9 \text{ ly})$. Then,

$$T = \frac{1}{H} = \frac{1}{\text{slope}} = \frac{5.3\times 10^9 \text{ ly}}{0.40\,c} = \frac{5.3\times 10^9 \text{ y}}{0.40} \approx 13\times 10^9 \text{ y} .$$

47. **THINK** Pair annihilation is a process in which a particle and its antiparticle collide and annihilate each other.

EXPRESS The energy released would be twice the rest energy of Earth, or $E = 2M_E c^2$.

ANALYZE The mass of the Earth is $M_E = 5.98 \times 10^{24}$ kg (found in Appendix C). Thus, the energy released is

$$E = 2M_E c^2 = 2(5.98 \times 10^{24} \text{ kg})(2.998 \times 10^8 \text{ m/s})^2 = 1.08 \times 10^{42} \text{ J}.$$

LEARN As in the case of annihilation between an electron and a positron, the total energy of the Earth and the anti-Earth after the annihilation would appear as electromagnetic radiation.

48. We note from track 1, and the quantum numbers of the original particle (A), that positively charged particles move in counterclockwise curved paths, and — by inference — negatively charged ones move along clockwise arcs. This immediately shows that tracks 1, 2, 4, 6, and 7 belong to positively charged particles, and tracks 5, 8 and 9 belong to negatively charged ones. Looking at the fictitious particles in the table (and noting that each appears in the cloud chamber once [or not at all]), we see that this observation (about charged particle motion) greatly narrows the possibilities:

$$\text{tracks } 2, 4, 6, 7, \ \leftrightarrow \ \text{particles } C, F, H, J$$
$$\text{tracks } 5, 8, 9 \ \leftrightarrow \ \text{particles } D, E, G$$

This tells us, too, that the particle that does not appear at all is either B or I (since only one neutral particle "appears"). By charge conservation, tracks 2, 4 and 6 are made by particles with a single unit of positive charge (note that track 5 is made by one with a single unit of negative charge), which implies (by elimination) that track 7 is made by particle H. This is confirmed by examining charge conservation at the end-point of track 6. Having exhausted the charge-related information, we turn now to the fictitious quantum numbers. Consider the vertex where tracks 2, 3, and 4 meet (the Whimsy number is listed here as a subscript):

$$\text{tracks } 2,4 \;\leftrightarrow\; \text{particles } C_2, F_0, J_{-6}$$
$$\text{tracks } 3 \;\leftrightarrow\; \text{particle } B_4 \text{ or } I_6$$

The requirement that the Whimsy quantum number of the particle making track 4 must equal the sum of the Whimsy values for the particles making tracks 2 and 3 places a powerful constraint (see the subscripts above). A fairly quick trial and error procedure leads to the assignments: particle *F* makes track 4, and particles *J* and *I* make tracks 2 and 3, respectively. Particle *B*, then, is irrelevant to this set of events. By elimination, the particle making track 6 (the only positively charged particle not yet assigned) must be *C*. At the vertex defined by

$$A \to F + C + (\text{track } 5)_{-} \;,$$

where the charge of that particle is indicated by the subscript, we see that Cuteness number conservation requires that the particle making track 5 has Cuteness = –1, so this must be particle *G*. We have only one decision remaining:

$$\text{tracks } 8, 9, \;\leftrightarrow\; \text{particles } D, E$$

Re-reading the problem, one finds that the particle making track 8 must be particle *D* since it is the one with seriousness = 0. Consequently, the particle making track 9 must be *E*.

Thus, we have the following:

(a) Particle *A* is for track 1.

(b) Particle *J* is for track 2.

(c) Particle *I* is for track 3.

(d) Particle *F* is for track 4.

(e) Particle *G* is for track 5.

(f) Particle *C* is for track 6.

(g) Particle *H* is for track 7.

(h) Particle *D* is for track 8.

(i) Particle *E* is for track 9.

49. (a) We use the relativistic relationship between speed and momentum:

$$p = \gamma mv = \frac{mv}{\sqrt{1-(v/c)^2}} \ ,$$

which we solve for the speed *v*:

$$\frac{v}{c} = \sqrt{1 - \frac{1}{\left(pc/mc^2\right)^2 + 1}} \ .$$

For an antiproton mc^2 = 938.3 MeV and *pc* = 1.19 GeV = 1190 MeV, so

$$v = c\sqrt{1 - \frac{1}{(1190\,\text{MeV}/938.3\,\text{MeV})^2 + 1}} = 0.785c \ .$$

(b) For the negative pion mc^2 = 193.6 MeV, and *pc* is the same. Therefore,

$$v = c\sqrt{1 - \frac{1}{(1190\,\text{MeV}/193.6\,\text{MeV})^2 + 1}} = 0.993c \ .$$

(c) Since the speed of the antiprotons is about 0.78*c* but not over 0.79*c*, an antiproton will trigger C2.

(d) Since the speed of the negative pions exceeds 0.79*c*, a negative pion will trigger C1.

(e) We use $\Delta t = d/v$, where *d* = 12 m. For an antiproton

$$\Delta t = \frac{1}{0.785\left(2.998\times 10^8 \text{ m/s}\right)} = 5.1\times 10^{-8}\,\text{s} = 51\text{ns}.$$

(f) For a negative pion

$$\Delta t = \frac{12\,\text{m}}{0.993\left(2.998\times 10^8 \text{ m/s}\right)} = 4.0\times 10^{-8}\text{ s} = 40\,\text{ns} \ .$$

50. (a) Eq. 44-14 conserves charge since both the proton and the positron have $q = +e$ (and the neutrino is uncharged).

(b) Energy conservation is not violated since $m_pc^2 > m_ec^2 + m_\nu c^2$.

(c) We are free to view the decay from the rest frame of the proton. Both the positron and the neutrino are able to carry momentum, and so long as they travel in opposite directions with appropriate values of *p* (so that $\sum \vec{p} = 0$) then linear momentum is conserved.

(d) If we examine the spin angular momenta, there does seem to be a violation of angular momentum conservation (Eq. 44-14 shows a spin-one-half particle decaying into two spin-one-half particles).

51. (a) During the time interval Δt, the light emitted from galaxy A has traveled a distance $c\Delta t$. Meanwhile, the distance between Earth and the galaxy has expanded from r to $r' = r + r\alpha\,\Delta t$. Let $c\Delta t = r' = r + r\alpha\Delta t$, which leads to

$$\Delta t = \frac{r}{c - r\alpha} .$$

(b) The detected wavelength λ' is longer than λ by $\lambda\alpha\Delta t$ due to the expansion of the universe: $\lambda' = \lambda + \lambda\alpha\Delta t$. Thus,

$$\frac{\Delta\lambda}{\lambda} = \frac{\lambda' - \lambda}{\lambda} = \alpha\Delta t = \frac{\alpha r}{c - \alpha r} .$$

(c) We use the binomial expansion formula (see Appendix E):

$$(1 \pm x)^n = 1 \pm \frac{nx}{1!} + \frac{n(n-1)x^2}{2!} + \cdots \quad (x^2 < 1)$$

to obtain

$$\frac{\Delta\lambda}{\lambda} = \frac{\alpha r}{c - \alpha r} = \frac{\alpha r}{c}\left(1 - \frac{\alpha r}{c}\right)^{-1} = \frac{\alpha r}{c}\left[1 + \frac{-1}{1!}\left(-\frac{\alpha r}{c}\right) + \frac{(-1)(-2)}{2!}\left(-\frac{\alpha r}{c}\right)^2 + \cdots\right]$$
$$\approx \frac{\alpha r}{c} + \left(\frac{\alpha r}{c}\right)^2 + \left(\frac{\alpha r}{c}\right)^3 .$$

(d) When only the first term in the expansion for $\Delta\lambda/\lambda$ is retained we have

$$\frac{\Delta\lambda}{\lambda} \approx \frac{\alpha r}{c} .$$

(e) We set

$$\frac{\Delta\lambda}{\lambda} = \frac{v}{c} = \frac{Hr}{c}$$

and compare with the result of part (d) to obtain $\alpha = H$.

(f) We use the formula $\Delta\lambda/\lambda = \alpha r/(c - \alpha r)$ to solve for r:

$$r = \frac{c(\Delta\lambda/\lambda)}{\alpha(1 + \Delta\lambda/\lambda)} = \frac{(2.998\times10^8\ \text{m/s})(0.050)}{(0.0218\ \text{m/s}\cdot\text{ly})(1 + 0.050)} = 6.548\times10^8\,\text{ly} \approx 6.5\times10^8\,\text{ly}.$$

(g) From the result of part (a),

$$\Delta t=\frac{r}{c-\alpha r}=\frac{\left(6.5\times10^{8}\,\text{ly}\right)\left(9.46\times10^{15}\ \text{m/ly}\right)}{2.998\times10^{8}\ \text{m/s}-\left(0.0218\ \text{m/s}\cdot\text{ly}\right)\left(6.5\times10^{8}\,\text{ly}\right)}=2.17\times10^{16}\,\text{s},$$

which is equivalent to 6.9×10^8 y.

(h) Letting $r = c\Delta t$, we solve for Δt:

$$\Delta t=\frac{r}{c}=\frac{6.5\times10^{8}\,\text{ly}}{c}=6.5\times10^{8}\,\text{y}.$$

(i) The distance is given by

$$r=c\Delta t=c\left(6.9\times10^{8}\,\text{y}\right)=6.9\times10^{8}\,\text{ly}.$$

(j) From the result of part (f),

$$r_B=\frac{c\left(\Delta\lambda/\lambda\right)}{\alpha\left(1+\Delta\lambda/\lambda\right)}=\frac{\left(2.998\times10^{8}\ \text{m/s}\right)\left(0.080\right)}{\left(0.0218\ \text{mm/s}\cdot\text{ly}\right)\left(1+0.080\right)}=1.018\times10^{9}\,\text{ly}\approx1.0\times10^{9}\,\text{ly}.$$

(k) From the formula obtained in part (a),

$$\Delta t_{\text{B}}=\frac{r_{\text{B}}}{c-r_{\text{B}}\alpha}=\frac{\left(1.0\times10^{9}\ \text{ly}\right)\left(9.46\times10^{15}\ \text{m/ly}\right)}{2.998\times10^{8}\ \text{m/s}-\left(1.0\times10^{9}\ \text{ly}\right)\left(0.0218\,\text{m/s}\cdot\text{ly}\right)}=3.4\times10^{16}\ \text{s},$$

which is equivalent to 1.1×10^9 y.

(l) At the present time, the separation between the two galaxies A and B is given by $r_{\text{now}}=c\Delta t_{\text{B}}-c\Delta t_{\text{A}}$. Since $r_{\text{now}} = r_{\text{then}} + r_{\text{then}}\alpha\Delta t$, we get

$$r_{\text{then}}=\frac{r_{\text{now}}}{1+\alpha\Delta t}=3.9\times10^{8}\,\text{ly}.$$

52. Using Table 44-1, the difference in mass between the muon and the pion is

$$\Delta m=\left(139.6\ \text{MeV}/c^{2}-105.7\ \text{MeV}/c^{2}\right)=\frac{(33.9\ \text{MeV})\left(1.60\times10^{-13}\ \ \text{J/MeV}\right)}{\left(2.998\times10^{8}\ \text{m/s}\right)^{2}}$$

$$=6.03\times10^{-29}\ \text{kg}.$$

53. (a) The quark composition for Σ^- is dss.

(b) The quark composition for $\overline{\Sigma}^-$ is $\overline{\text{d}}\overline{\text{s}}\,\overline{\text{s}}$.

54. The speed of the electron is relativistic, so we first calculated the Lorentz factor:

$$\gamma = 1 + \frac{K}{mc^2} = 1 + \frac{2.5\text{ MeV}}{0.511\text{ MeV}} = 5.892$$

The total energy carried by the electron or the positron is

$$E = \gamma mc^2 = (5.892)(0.511\text{ MeV}) = 3.011\text{ MeV} = 4.82\times10^{-13}\text{ J}$$

The corresponding frequency of the photons produced is

$$f = \frac{E}{h} = \frac{4.82\times10^{-13}\text{ J}}{6.626\times10^{-34}\text{ J}\cdot\text{s}} = 7.3\times10^{20}\text{ Hz}\,.$$