فصل هفتم

استنتاج فازی

Fuzzy Inference

shavandi@sharif.edu

Fuzzy Rules and Implication

- **Fuzzy if-then Rules**

  A fuzzy rule generally assumes the form

  R: If x is A, *then y is B.*

  where A and B are linguistic values defined by fuzzy sets on universe of discourse X and Y, respectively.

- The rule is also called a "fuzzy implication" or fuzzy conditional statement. The part *"x is A" is called* the "antecedent" or "premise", while *"y is B" is called the "consequence*“ or "conclusion".

- In general, the antecedent and consequence are represented by the form of linguistic variables.

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Fuzzy Rules and Implication

- **Fuzzy if-then Rules**

  Before we employ fuzzy if-then rules to model and analyze a system, first we have to formalize what is meant by the expression:

  R: "If *x is A then y is B*",

  which is sometimes abbreviated as

  $$R: A \to B$$

- In essence, the expression describes a relation between two variables *x* and *y*.

- *This suggests that a fuzzy rule can be defined as a binary relation* R on the product space X x Y.

Fuzzy Rules and Implication

- **Fuzzy Implications**

Based on the interpretations of the Cartesian product and various t-norm and t-conorm operators, a number of qualified methods can be formulated to calculate the fuzzy relation

$$R = A \to B$$

R can be viewed as a fuzzy set with a two-dimensional membership function

$$\mu_R(x, y) = f(\mu_A(x), \mu_B(y))$$

where the function f, called the "fuzzy implication function", performs the task of transforming the membership degrees of x in A and y in B into those of (x, y) in A × B. We introduce here two well known fuzzy implication functions.

Fuzzy Rules and Implication

Fuzzy Implications

(1) Min operation rule of fuzzy implication [Mamdani]. It interprets the fuzzy implication as the minimum operation.

$$R_C = A \times B = \int_{X\times Y} \mu_A(x) \wedge \mu_B(y) / (x, y)$$

where $\wedge$ is the min operator

(2) Product operation rule of fuzzy implication [Larsen]. It implements the implication by the product operation.

$$R_P = A \times B = \int_{X\times Y} \mu_A(x) \bullet \mu_B(y) / (x, y)$$

where $\bullet$ is the algebraic product operator

Fuzzy Rules and Implication

Example of Fuzzy Implications

Example 9.3 There is a fuzzy rule in the following.

If temperature is high, then humidity is fairly high.

It is a fuzzy rule and a fuzzy relation. We want to determine the membership function of the rule. Let T and H be universe of discourse of temperature and humidity, respectively, and let's define variables $t \in T$ and $h \in H$. We represent the fuzzy terms "high" and "fairly high" by A and B respectively:

$$A = \text{"high"}, \quad A \subseteq T$$

$$B = \text{"fairly high"}, \quad B \subseteq H$$

Fuzzy Rules and Implication

- **Example of Fuzzy Implications**

Table 9.6. Membership of A in T (temperature)

t	20	30	40
$\mu_A(t)$	0.1	0.5	0.9

Table 9.7. Membership degrees of B in H (humidity)

h	20	50	70	90
$\mu_B(h)$	0.2	0.6	0.7	1

Fuzzy Rules and Implication

- **Example of Fuzzy Implications**

then the above rule can be rewritten as

R(t, h): If t is A, then h is B.

In the rule (relation), we can find two predicate propositions:

R(t): t is A

R(h): h is B

the rule becomes

$$R(t, h): R(t) \to R(h)$$

if we know membership functions of A and B, we can determine $R_{(t,h)} = A \times B$ by using the fuzzy implication function where $R_{(t,h)} \subseteq T \times H$. Assume membership functions $\mu_A(t)$ and $\mu_B(h)$ are given in (Tables 9.6 , 9.7) respectively.

Fuzzy Rules and Implication

Example of Fuzzy Implications

In order to get the relation for the implication in the above fuzzy rule, we have to select an implication function between A and B. For simplicity, let's take the min operation of Mamdani in the previous section.

$$R_C(t, h) = A \times B$$
$$= \int \mu_A(t) \wedge \mu_B(h) / (t, h)$$

when we apply the min operation on the Cartesian product A × B, we obtain the relation R_C as shown in (Table 9.8.) This membership of R_C represents the fuzzy rule. Note that $\mu_{R_C}(20, 50) = 0.1$ is obtained by the min between $\mu_A(20) = 0.1$ and $\mu_B(50) = 0.6$. Similarly, $\mu_{R_C}(30, 20) = 0.2$ from $\mu_A(30) = 0.5$ and $\mu_B(20) = 0.2$. □

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Fuzzy Rules and Implication

Example of Fuzzy Implications

Example 9.4 Now suppose, we want to get information about the humidity when there is the following premise about the temperature.

"Temperature is fairly high"

This fact is rewritten as

R(t): "t is A′" where A′ = "fairly high"

where the fuzzy term $A' \subseteq T$ is defined in (Table 9.9)

Table 9.8. Membership of rule $R_C = A \times B$

t \ h	20	50	70	90
20	0.1	0.1	0.1	0.1
30	0.2	0.5	0.5	0.5
40	0.2	0.6	0.7	0.9

Table 9.9. Membership function of A′ in T (temperature)

t	20	30	40
$\mu_{A'}(t)$	0.01	0.25	0.81

Fuzzy Rules and Implication

Example of Fuzzy Implications

As we can see, A′ is not same with A and thus we apply the fuzzy inference method of generalized modus ponens. We use the composition rule of inference with the max-min composition.

$$R(h) = R(t) \circ R_C(t, h)$$

where R(t) is in (Table 9.9.) and $R_C(t, h)$ in (Table 9.8.)

If we denote the result of the inference as B′, B′ is the information about humidity when "temperature is fairly high" (Table 9.10). □

Table 9.10. Result of fuzzy inference

h	20	50	70	90
$\mu_{B'}(h)$	0.2	0.6	0.7	0.81

Inference Mechanism

Decomposition of Rule Base

- When we model a knowledge system, it is often represented by the form of "fuzzy rule base".
- The fuzzy rule base consists of fuzzy if-then rules.
- In many cases, the fuzzy reasoning on the fuzzy rule base is based on one level forward data-driven inference (GNP: generalized modus ponens).
- The rule base has' the form of a MIMO (multiple input multiple output) system.

$$R = \{R^1_{MIMO}, R^2_{MIMO}, \ldots, R^n_{MIMO}\}$$

where R^i_{MIMO} represents the rule:

If x is A_i and y is B_i, then z_1 is C_i, …, z_q is D_i,

Inference Mechanism

Decomposition of Rule Base

The antecedent of R^i_{MIMO} forms a fuzzy set $A_i \times \ldots \times B_i$ in the "product space" $U \times \ldots \times V$. The consequence is the "union" of q independent control actions ($z_1 + z_2 + \ldots + z_q$). Thus the ith rule R^i_{MIMO} may be represented as a fuzzy implication.

$$R^i_{MIMO} : (A_i \times \ldots \times B_i) \rightarrow (z_1 + \ldots + z_q)$$

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Inference Mechanism

Decomposition of Rule Base

From the above statement, it follows that the rule base R may be represented as the union

$$R = \{\bigcup_{i=1}^{n} R^i_{MIMO}\}$$

$$= \{\bigcup_{i=1}^{n} [(A_i \times \cdots \times B_i) \rightarrow (z_1 + \cdots + z_q)]\}$$

$$= \{\bigcup_{i=1}^{n} [(A_i \times \cdots \times B_i) \rightarrow z_1], \bigcup_{i=1}^{n} [(A_i \times \cdots \times B_i) \rightarrow z_2], \ldots, \bigcup_{i=1}^{n} [(A_i \times \cdots \times B_i) \rightarrow z_q]\}$$

$$= \{\bigcup_{k=1}^{q} \bigcup_{i=1}^{n} [(A_i \times \cdots \times B_i) \rightarrow z_k]\}$$

$$= \{\bigcup_{k=1}^{q} RB^k_{MISO}\} \quad \text{where } RB^k_{MISO} = \bigcup_{i=1}^{n} [(A_i \times \cdots \times B_i) \rightarrow z_i]$$

$$= \{RB^1_{MISO}, RB^2_{MISO}, \cdots, RB^k_{MISO}, \cdots, RB^q_{MISO}\}$$

shavandi@sharif.edu

Inference Mechanism

- **Decomposition of Rule Base**

In effect, the rule base R is composed of a set of sub-rule-bases RB^k_{MISO} where k = 1, 2, … , q. The sub-rule-base RB^k_{MISO} has "multiple input" variables and a "single control" variable. Therefore the general rule structure of a MIMO fuzzy system can be represented as a collection of MISO fuzzy systems.

$$R = \{RB^1_{MISO}, RB^2_{MISO}, \cdots, RB^k_{MISO}, \cdots, RB^q_{MISO}\}$$

where RB^k_{MISO} represents the rule:

If x is A_i and … , and y is B_i then z_k is C_i, for i = 1, 2, … , n

shavandi@sharif.edu

Inference Mechanism

- **Two-Input/Single-Output Rule Base**

For simplicity, let's consider the general form of MISO fuzzy control rules in the case of two-input/single-output systems.

Input: u is A′ and v is B′

R_1: if u is A_1 and v is B_1 then is w is C_1

else R_2: if u is A_2 and v is B_2 then is w is C_2

…

…

else R_n: if u is A_n and v is B_n then is w is C_n

consequence: w is C′

Inference Mechanism

- **Two-Input/Single-Output Rule Base**

where u, v, and w are linguistic variables representing the process state variables and the control variables, respectively. A_i, B_i, and C_i are linguistic values of the linguistic variables u, v, and w in the universe of discourse U, V, and W respectively for i=1, 2, … , n.

The fuzzy control rule

R_i: If u is A_i and v is B_i then w is C_i

is implemented as a fuzzy implication relation R_i and is defined as

R_i: (A_i and B_i) $\rightarrow C_i$ or

$$\mu_{R_i} = \mu_{(A_i \text{ and } B_i \rightarrow C_i)}(u, v, w)$$

$$= [\mu_{A_i}(u) \text{ and } \mu_{B_i}(v)] \rightarrow \mu_{C_i}(w)$$

where "A_i and B_i" is a fuzzy set $A_i \times B_i$ in $U \times V$.

R_i: (A_i and B_i) $\rightarrow C_i$ is a fuzzy implication relation in $U \times V \times W$, and $\rightarrow$ denotes a fuzzy implication function.

Inference Mechanism

- **Compositional Rule of Inference**

Lemma 1 (For 1 singleton input, result C' is obtained from C and matching degree α_1, Fig 9.1)

When a fuzzy rule R_1 and singleton input u_0 are given

R_1: If u is A then w is C,

Or R_1: $A \rightarrow C$

The inference result C' is defined by the membership function $\mu_{C'}(w)$

$\mu_{C'}(w) = \alpha_1 \wedge \mu_C(w)$ for R_C (Mamdani implication)

$\mu_{C'}(w) = \alpha_1 \bullet \mu_C(w)$ for R_P (Larsen implication)

where $\alpha_1 = \mu_A(u_0)$

Inference Mechanism

- Compositional Rule of Inference

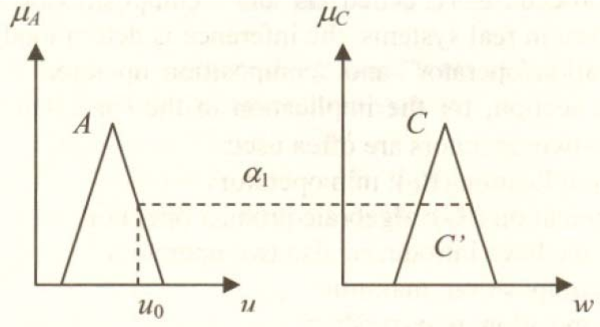


Fig. 9.1. Graphical representation of Lemma 1 with R_C
(When a singleton input is given, C' is obtained from C and α_1)

Inference Mechanism

- Compositional Rule of Inference

Lemma 2: (For 1 fuzzy input, result C' is obtained from C and matching degree α_1, Fig 9.2)

When a fuzzy rule R_1: $A \rightarrow C$ and input A' are given, the inference result C' is defined by the membership function $\mu_{C'}$.

$$\mu_{C'}(w) = \alpha_1 \wedge \mu_C(w) \quad \text{for } R_C$$
$$\mu_{C'}(w) = \alpha_1 \cdot \mu_C(w) \quad \text{for } R_P$$
$$\text{where } \alpha_1 = \max_u [\mu_{A'}(u) \wedge \mu_A(u)]$$

Inference Mechanism

- **Compositional Rule of Inference**

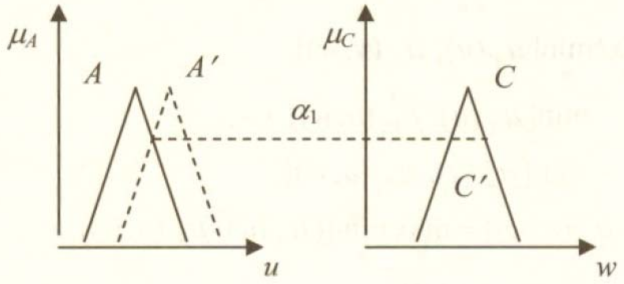


Fig. 9.2. Graphical representation of Lemma 2 with R_C
(When a fuzzy set input is given, C' is obtained from C and α_1)

Inference Mechanism

- **Fuzzy Inference with Rule Base**
- In this section, we generalize the properties of compositional rule of inference discussed in the previous section to the case such as

$$R: \bigcup_{i=1}^{n} R_i$$

$$R_i: A_i \rightarrow C_i$$

Lemma 3 (Total result C' is an aggregation of individual results C'_i, Fig 9.3)

The result of inference C is an aggregation of result C'_i derived from individual rules.

$$C' = A' \circ \bigcup_{i=1}^{n} R_i = \bigcup_{i=1}^{n} A' \circ R_i = \bigcup_{i=1}^{n} C'_i$$

Inference Mechanism

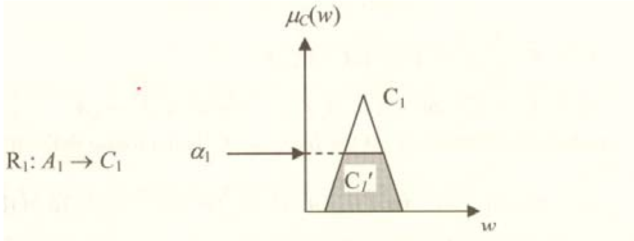
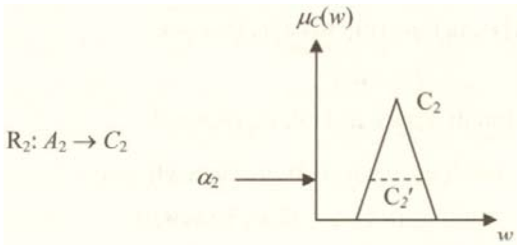


Fig. 9.3. Lemma 3 (Total result C' is a union of individual result C_i')



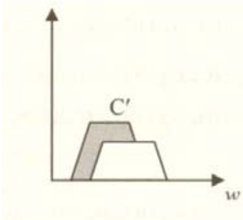


Fig. 9.3. (cont')

Inference Mechanism

- **Fuzzy Inference with Rule Base**

Now, we generalize Lemma 3 to the case of multiple input variables such as

$$R: \bigcup_{i=1}^{n} R_i$$

$$R_i: A_i \text{ and } B_i \to C_i$$

Colorally of Lemma 3: (Lemma 3 in the case of multiple inputs)
The result of inference C is an aggregation of result C_i' derived from individual rules.

$$C' = (A', B') \circ \bigcup_{i=1}^{n} R_i = \bigcup_{i=1}^{n} (A', B') \circ R_i = \bigcup_{i=1}^{n} C_i'$$

shavandi@sharif.edu

Inference Mechanism

- **Fuzzy Inference with Rule Base**

Lemma 4: (R_i: $(A_i \times B_i \to C_i)$ consists of R_i^1: $(A_i \to C_i)$ and R_i^2: $(B_i \to C_i)$, Fig 9.4)

When there is a rule R_i with two inputs variables A_i and B_i, the inference result C_i' is obtained from individual inferences of R_i^1: $(A_i \to C_i)$ and R_i^2: $(B_i \to C_i)$.

$$\begin{aligned}
C_i' &= (A', B') \circ (A_i \text{ and } B_i \to C_i) \\
&= [A' \circ (A_i \to C_i)] \cap [B' \circ (B_i \to C_i)] \qquad \text{if } \mu_{A_i \times B_i} = \mu_{A_i} \wedge \mu_{B_i} \quad (\text{for } R_C) \\
&= [A' \circ R_i^1] \cap [B' \circ R_i^2] \qquad \text{where } R_i^1 = A_i \to C_i \text{ and } R_i^2 = B_i \to C_i \\
&= C_i^1 \cap C_i^2 \qquad \text{where } C_i^1 = A' \circ R_i^1 \text{ and } C_i^2 = A' \circ R_i^2
\end{aligned}$$

$$\begin{aligned}
C_i' &= (A', B') \circ (A_i \text{ and } B_i \to C_i) \\
&= [A' \circ (A_i \to C_i)] \bullet [B' \circ (B_i \to C_i)] \qquad \text{if } \mu_{A_i \times B_i} = \mu_{A_i} \bullet \mu_{B_i} \quad (\text{for } R_P)
\end{aligned}$$

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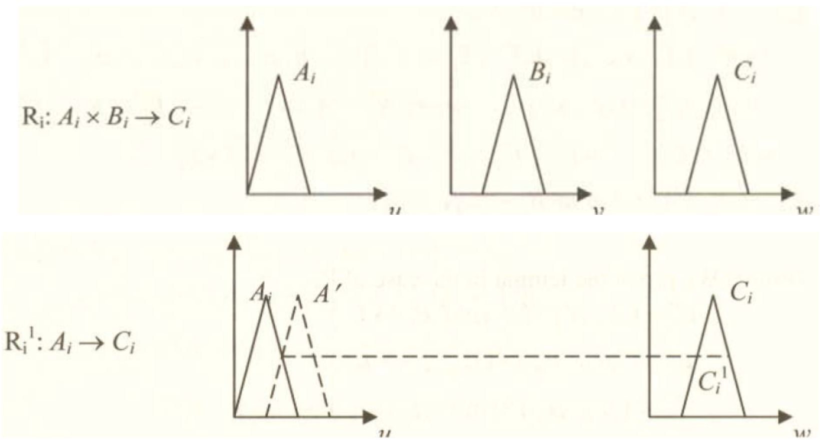
Inference Mechanism

- **Fuzzy Inference with Rule Base**



 shavandi@sharif.edu

Inference Mechanism

- **Fuzzy Inference with Rule Base**

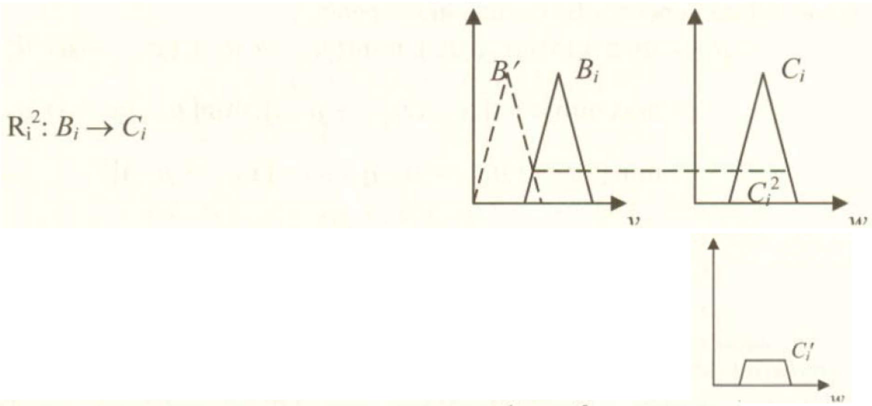


Fig. 9.4. Lemma 4 (Rule R_i can be decomposed into R_i^1 and R_i^2 and the result C_i' of R_i is an intersection of the results C_i^1 and C_i^2 of R_i^1 and R_i^2, respectively.)

 shavandi@sharif.edu

Inference Mechanism

- **Fuzzy Inference with Rule Base**

Lemma 5: (For singleton input, C_i' is determined by the minimum matching degree of A_i and B_i, Fig 9.5)

If the inputs are fuzzy singletons, namely, A′ = u0, B′ = v0, the matching degree αi is the minimum value between .$\mu_{Ai}(u_0)$ and $\mu_{Bi}(v_0)$ from the lemma 1, the inference result can be derived by employing Mamdani's minimum operation rule R_C and Larsen's product operation rule R_P for the implication.

$$\mu_{C_i'}(w) = \alpha_i \wedge \mu_{C_i}(w) \quad for\ R_C$$

$$\mu_{C_i'}(w) = \alpha_i \bullet \mu_{C_i}(w) \quad for\ R_P$$

$$where\ \alpha_i = \mu_{A_i}(u_0) \wedge \mu_{B_i}(v_0) = \min[\mu_{A_i}(u_0), \mu_{B_i}(v_0)]$$

Inference Mechanism

- Fuzzy Inference with Rule Base

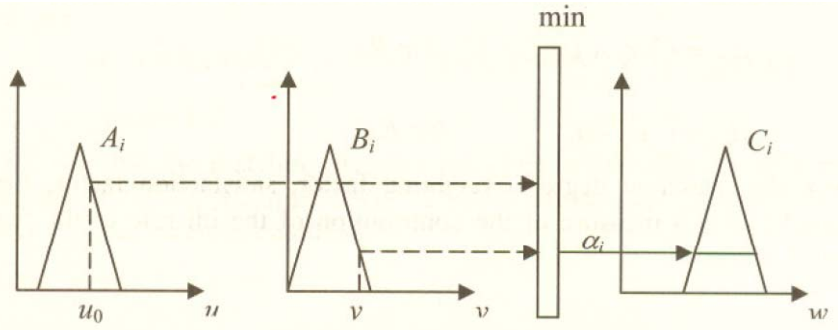


Fig. 9.5. Lemma 5 (α_i is the minimum matching degree between $A_i(u_0)$ and $B_i(v_0)$.)

Inference Mechanism

- Fuzzy Inference with Rule Base

Lemma 6: (For fuzzy input, C_i' is determined by the minimum matching degree of (A' and A_i) and (B' and B_i), Fig 9.6)

If the inputs are given as fuzzy sets A' and B', the matching degree α_i is determined by the minimum between (A' and A_i) and (B' and B_i). From the lemma 2, the results can be derived by employing the min operation for R_C and the product operation for R_P.

$$\mu_{C_i'}(w) = \alpha_i \wedge \mu_{C_i}(w) \qquad for\ R_C$$

$$\mu_{C_i'}(w) = \alpha_i \bullet \mu_{C_i}(w) \qquad for\ R_P$$

$$where\ \alpha_i = \min[\max_u(\mu_{A'}(u) \wedge \mu_{A_i}(u)), \max_v(\mu_{B'}(v) \wedge \mu_{B_i}(v))]$$

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Inference Mechanism

- **Fuzzy Inference with Rule Base**

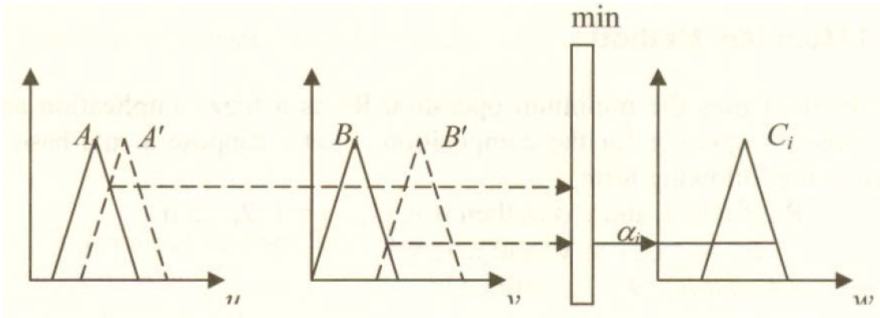

Fig. 9.6. Lemma 6 (α_i is the minimum matching degree between (A' and A_i) and (B' and B_i).)

shavandi@sharif.edu

Inference Methods

- Based upon the previous lemmas, now we develop inference methods.

- **Mamdani Method**
- **Larsen Method**
- **TSK Method**

Inference Methods: Mamdani Method

This method uses the minimum operation R_C as a fuzzy implication and the max-min operator for the composition. Let's suppose a rule base is given in the following form.

R_i: if u is A_i and v is B_i then w is C_i, i = 1, 2, …, n

for $u \in U$, $v \in V$, and $w \in W$.

then, $R_i = (A_i \text{ and } B_i) \rightarrow C_i$ is defined by

$$\mu_{R_i} = \mu_{(A_i \text{ and } B_i \rightarrow C_i)}(u, v, w)$$

(1) When input data are singleton $u = u_0$, $v = v_0$

$$\mu_{C_i'}(w) = [\mu_{A_i}(u_0) \text{ and } \mu_{B_i}(v_0)] \rightarrow \mu_{C_i}(w)$$

The Mamdani method uses the minimum operation ($\wedge$) for the fuzzy implication ($\rightarrow$). From lemma 5,

$$\mu_{C_i'}(w) = \alpha_i \wedge \mu_{C_i}(w)$$

$$\text{where } \alpha_i = \mu_{A_i}(u_0) \wedge \mu_{B_i}(v_0)$$

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Inference Methods: Mamdani Method

From the previous Lemma 3, we know the membership function μ_C of the inferred consequence C is given by the aggregated result derived from individual control rules. Thus, when there are two rules R_1 and R_2,

$$\mu_{C'}(w) = \mu_{C_1'} \vee \mu_{C_2'}$$

$$= [\alpha_1 \wedge \mu_{C_1}(w)] \vee [\alpha_2 \wedge \mu_{C_2}(w)]$$

The procedure of Mamdani fuzzy inference when the inputs are given as singletons is represented in (Fig 9.7).

Therefore in general, from Lemma 3,

$$\mu_{C'}(w) = \bigvee_{i=1}^{n} [\alpha_i \wedge \mu_{C_i}(w)] = \bigvee_{i=1}^{n} \mu_{C_i'}(w)$$

$$C' = \bigcup_{i=1}^{n} C_i'$$

Inference Methods: Mamdani Method

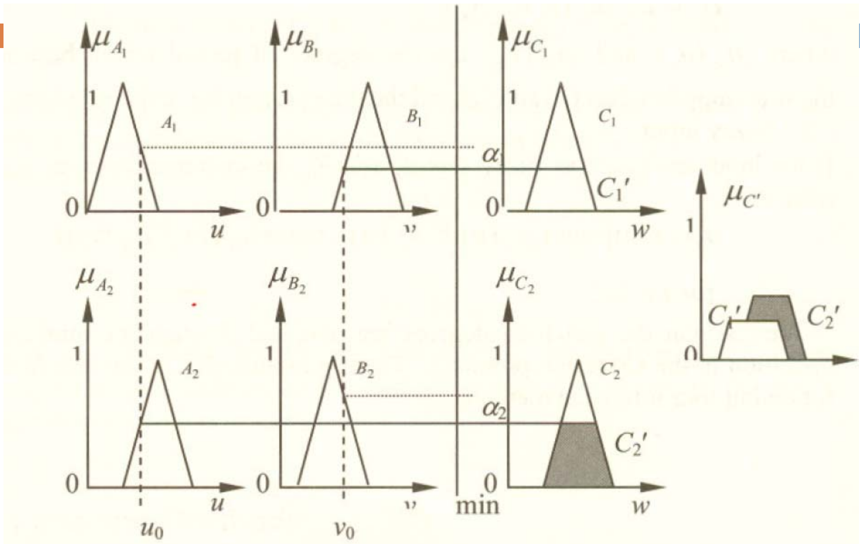

Fig. 9.7. Graphical representation of Mamdani method with singleton input

Inference Methods: Mamdani Method

(2) When input data are fuzzy sets, A' and B'

From Lemma 6,

$$\mu_{C_i'}(w) = \alpha_i \wedge \mu_{C_i}(w)$$

$$where\ \alpha_i = \min[\max_u(\mu_{A'}(u) \wedge \mu_{A_i}(u)), \max_v(\mu_{B'}(v) \wedge \mu_{B_i}(v))]$$

From Lemma 3, we have the aggregated result

$$\mu_{C'}(w) = \bigvee_{i=1}^{n}[\alpha_i \wedge \mu_{C_i}(w)] = \bigvee_{i=1}^{n} \mu_{C_i'}(w)$$

$$C' = \bigcup_{i=1}^{n} C_i'$$

The graphical interpretation of this inference is given in Fig 9.8.

The result C' is a fuzzy set and thus if we want to obtain a deterministic control action, a defuzzification method is used which will be discussed in the next chapter.

Inference Methods: Mamdani Method

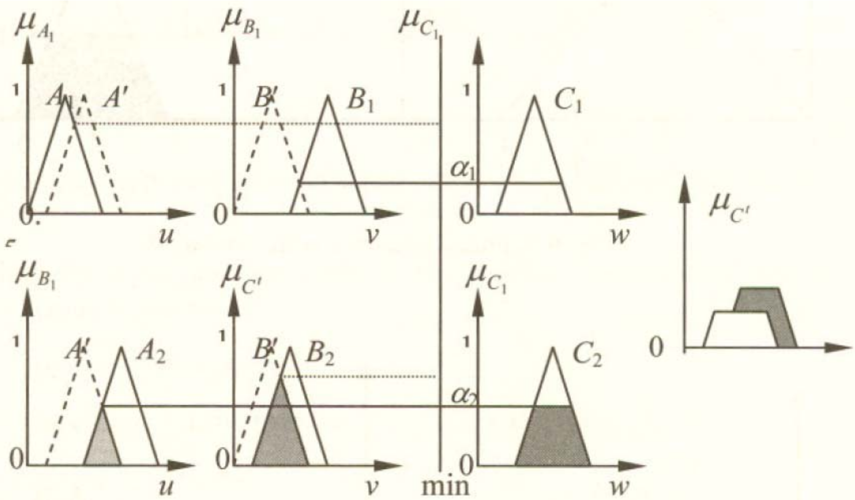

Fig. 9.8. Graphical interpretation of Mamdani method with fuzzy set input

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Inference Methods: Mamdani Method

Example 9.5 There is a fuzzy rulebase including one rule such as :

R: If u is A then v is B

where A=(0, 2, 4) and B=(3, 4, 5) are triangular fuzzy sets.

If an input is given as singleton value u_0=3, how can we calculate the output B′ using the Mamdani method?

Inference Methods: Mamdani Method

Example 9.5

We can see the matching degree between A and u_0 is α=0.5 Therefore the output B′ is obtained by the intersection between B and α=0.5. That is, B′ is expressed by the lower area of 0.5 in B(Fig.9.9).

Now, consider the case that input is given as a triangular set A′=(0, 1, 2) . That is,

$$\mu_{A'}(x) = x \text{ for } 0 \leq x \leq 1$$
$$= -x+2 \text{ for } 1 \leq x \leq 2$$
$$= 0 \quad \text{otherwise}$$

We can obtain the mathching degree α=2/3 and then B′ is the lower part of 2/3 in B (Fig 9.10). □

Inference Methods: Mamdani Method

Example 9.5

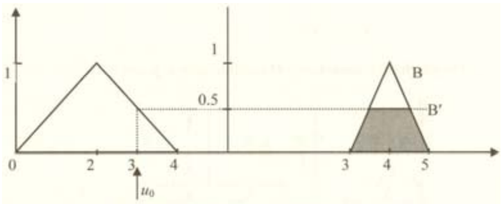

Fig. 9.9. Fuzzy inference with input u_0=3

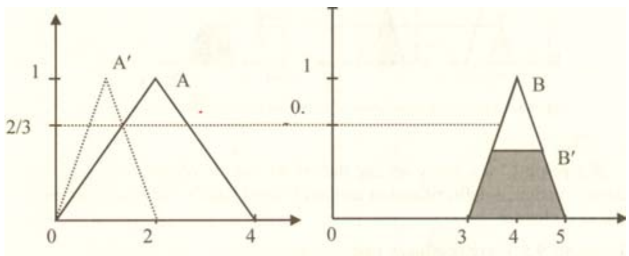

Fig. 9.10. Fuzzy inference with input A′=(0, 1, 2).

Inference Methods: Larsen Method

This method uses the product operator R_P for the fuzzy implication and the max-product operator for the composition. For the following rule base,

R_i: if u is A_i and v is B_i then w is C_i, i = 1, 2, … , n

then

$R_i = (A_i \text{ and } B_i) \rightarrow C_i$ is defined by

$$\mu_{R_i} = \mu_{(A_i \text{ and } B_i \rightarrow C_i)}(u, v, w)$$

(1) When the singleton input data are given as $u = u_0$, $v = v_0$, from Lemma 5 we have

$$\mu_{C_i'}(w) = [\mu_{A_i}(u_0) \text{ and } \mu_{B_i}(v_0)] \rightarrow \mu_{C_i}(w)$$
$$= [\mu_{A_i}(u_0) \wedge \mu_{B_i}(v_0)] \bullet \mu_{C_i}(w)$$
$$= \alpha_i \bullet \mu_{C_i}(w) \quad \text{where } \alpha_i = \mu_{A_i}(u_0) \wedge \mu_{B_i}(v_0)$$

Inference Methods: Larsen Method

From Lemma 3,

$$\mu_{C'}(w) = \bigvee_{i=1}^{n} [\alpha_i \bullet \mu_{C_i}(w)] = \bigvee_{i=1}^{n} \mu_{C_i'}(w)$$

$$C' = \bigcup_{i=1}^{n} C_i'$$

The graphical representation of this method with singleton input is given in (Fig 9.11

shavandi@sharif.edu

Inference Methods: Larsen Method

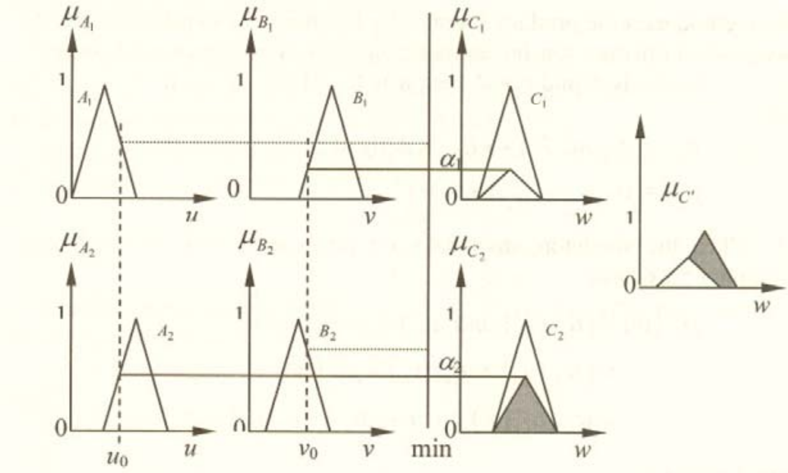

Fig. 9.11. Graphical representation of Larsen method with singleton input

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shavandi@sharif.edu

Inference Methods: Larsen Method

(2) When the input data are given as the form of fuzzy sets A' and B', from Lemma 6, we know

$$\mu_{C_i'}(w) = \alpha_i \bullet \mu_{C_i}(w)$$

$$where\ \alpha_i = \min[\max_u(\mu_{A'}(u) \wedge \mu_{A_i}(u)), \max_v(\mu_{B'}(v) \wedge \mu_{B_i}(v))$$

From Lemma 3, we have

$$\mu_{C'}(w) = \bigvee_{i=1}^{n}[\alpha_i \bullet \mu_{C_i}(w)] = \bigvee_{i=1}^{n} \mu_{C_i'}(w)$$

$$C' = \bigcup_{i=1}^{n} C_i'$$

The graphical interpretation of this inference is shown in (Fig 9.10.)

Sharif University of Technology
Industrial Engineering Dept.

shavandi@sharif.edu

Inference Methods: Larsen Method

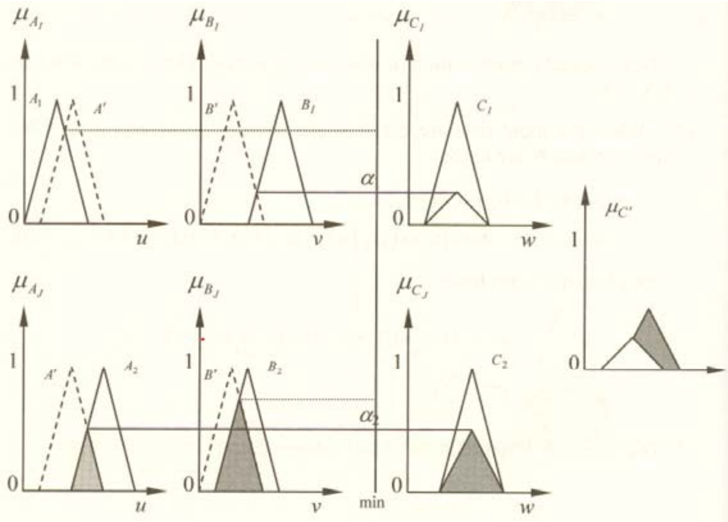


Fig. 9.12. Graphical representation of Larsen method with fuzzy set input

shavandi@sharif.edu

Inference Methods: Larsen Method

Example 9.6 There is a fuzzy rule

R : if u is A and v B then w is C

where A=(0, 2, 4), B=(3, 4, 5) and C=(3, 4, 5)

i) Find inference result C′ when input is u_0 =3, v_0=4 by using Larsen method

ii) Find inference result C′ when input is A=(0, 1, 2) and B=(2, 3, 4). The solutions are illustrated in (Figs 9.13 , 9.14), respectively.

Inference Methods: Larsen Method

Example 9.6

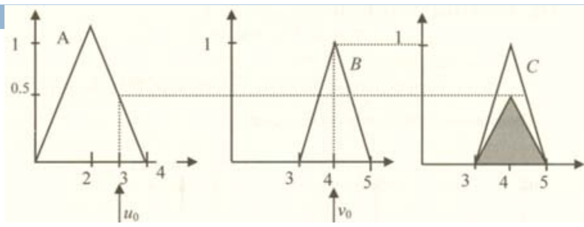


Fig. 9.13. Larsen method with input u_0=3, v_0=4

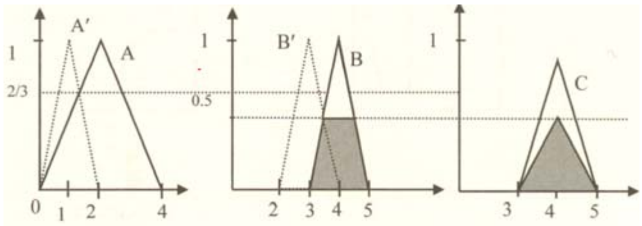


Fig. 9.14. Larsen method with input A′=(0, 1, 2), B′=(2, 3, 4).

Inference Methods: TSK Method

This method was proposed by Takagi, Sugeno, and Kang. A typical fuzzy rule in this model has the form

If u is A and v is B then $w = f(u, v)$,

A and B are fuzzy sets in the antecedent while $w = f(u, v)$ is a crisp function in the consequent. Usually $f(u, v)$ is a polynomial in the input variable u and v, and thus this method works when inputs are given as singleton values (Fig 9.16).

For simplicity, assume we have two fuzzy rules as follows.

R_1: if u is A_1 and v is B_1 then $w = f_1(u, v) = p_1u + q_1v + r_1$

R_2: if u is A_1 and v is B_1 then $w = f_2(u, v) = p_2u + q_2v + r_2$

where p_1, p_2, q_1, and q_2 are constant.

The inferred value of the control action from the first rule is $f_1(u_0, v_0)$ where u_0 and v_0 are singleton inputs, and α_1 is the matching degree. The inferred value from the second is $f_2(u, v)$ with the matching degree α_2. The matching degrees are obtained like in the previous methods.

Inference Methods: TSK Method

$$\alpha_i = \mu_{A_i}(u_0) \wedge \mu_{B_i}(v_0)$$

They are all crisp values. The aggregated result is given by the weighted average.

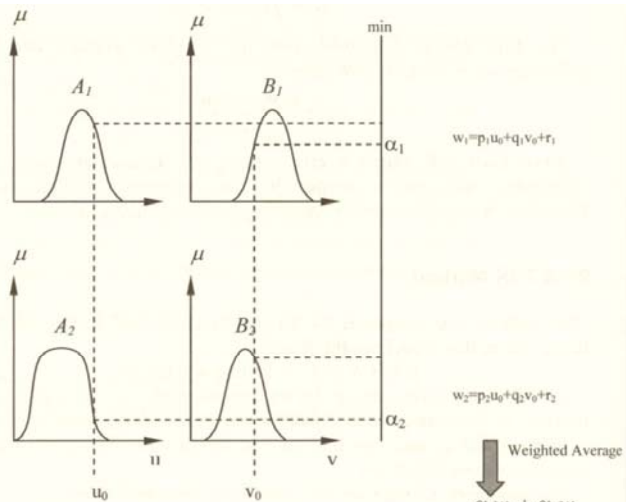
$$w_0 = \frac{\alpha_1 f_1(u_0, v_0) + \alpha_2 f_2(u_0, v_0)}{\alpha_1 + \alpha_2}$$

$$= \frac{\alpha_1 w_1 + \alpha_2 w_2}{\alpha_1 + \alpha_2}$$

This method also saves the defuzzification time because the final result w_0 is a crisp value.

Fuzzy Sets. Chapter 6- Fuzzy Logic shavandi@sharif.edu

Inference Methods: TSK Method



$$w_0 = \frac{\alpha_1 w_1 + \alpha_2 w_2}{\alpha_1 + \alpha_2}$$

Fig. 9.16. Graphical representation of TSK method