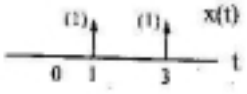
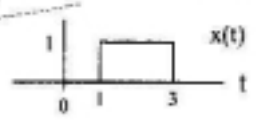
سیستم LTI با پاسخ پله $s(t) = e^{-t}u(t)$ را در نظر بگیرید.

الف) پاسخ سیستم به ورودی های داده شده در 2 و 3 را بدست آورید.

(a)



(b)



$$y(t) = s(t)\Big|_{x(t)=u(t)} = e^{-t}u(t) \Rightarrow y(t) = h(t)\Big|_{x(t)=\delta(t)} = \frac{ds(t)}{dt}$$

$$= -e^{-t}u(t) + e^{-t}\delta(t)$$

$$h(t) = \delta(t) - e^{-t}u(t)$$

a) $x(t) = \delta(t-1) + \delta(t-3) \Rightarrow y(t) = h(t) * x(t) = h(t) * (\delta(t-1) + \delta(t-3))$

$$= h(t-1) + h(t-3)$$

$$= \delta(t-1) - e^{-(t-1)}u(t-1) + \delta(t-3) - e^{-(t-3)}u(t-3)$$

$$= \delta(t-1) + \delta(t-3) - e^{-(t-1)}u(t-1) - e^{-(t-3)}u(t-3)$$

b) $x(t) = u(t-1) - u(t-3)$

$$y(t) = e^{-(t-1)}u(t-1) - e^{-(t-3)}u(t-3)$$

تکلیف ۱۲

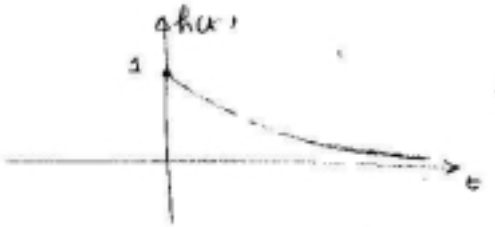
مسائل نمونه فصل دوم سیگنال ها و سیستم ها دانشگاه آزاد اسلامی- واحد تهران جنوب غفرانی

- سیستم LTI ، زمان پیوسته با پاسخ ضربه $h(t)=e^{-at}u(t)$, $a>0$ را در نظر بگیرید.

الف) پاسخ ضربه را به ازاء $a=2$ به دقت ترسیم کنید.

ب) پاسخ پله سیستم را به دست آورید، $S(t)$

ج) آیا سیستم علی است. چرا



د) $S(0)$ ، $S(\frac{1}{2})$ ، $S(1)$ را به دست آورید

$S(t)=h(t)*u(t)$

ت) $x(t-\lambda)\delta(\lambda)$ را به ازاء $t=\frac{1}{2}$ ، $t=-\frac{1}{2}$ ترسیم کنید، اگر $x(t)=u(t)$ باشد.

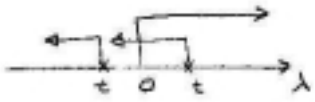
① $$S(t)\Big|_{x(t)=u(t)} = \int_{-\infty}^{t} h(\lambda)\,d\lambda = \int_{-\infty}^{t} e^{-a\lambda}u(\lambda)\,d\lambda \qquad \lambda\geq 0$$

$$\begin{cases} t<0 \Rightarrow S(t)=0 \\ t\geq 0 \Rightarrow S(t)=\int_{0}^{t} e^{-a\lambda}\,d\lambda = \frac{-1}{a}e^{-a\lambda}\Big|_{0}^{t} = \frac{1}{a}(1-e^{-at}) \end{cases}$$

$$\Rightarrow S(t)\Big|_{x(t)=u(t)} = \frac{1}{a}(1-e^{-at})u(t)$$

② $$S(t)=h(t)*u(t)=\int_{-\infty}^{\infty} e^{-a\lambda}u(\lambda)u(t-\lambda)\,d\lambda$$

$\lambda\geq 0$, $t-\lambda\geq 0$, $\lambda\leq t$



$t<0$, $*=0$

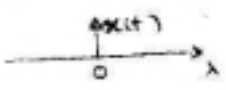
$t\geq 0$, $*=\int_{0}^{t} e^{-a\lambda}\,d\lambda = \frac{1}{a}(1-e^{-at})$

$$\Rightarrow S(t)=\frac{1}{a}(1-e^{-at})u(t)$$

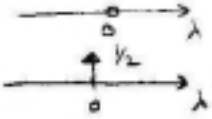
» $S(t)=\int_{-\infty}^{t} e^{-a\lambda}u(\lambda)\,d\lambda \Rightarrow$

$$\begin{cases} S(0)=\int_{-\infty}^{0} e^{-a\lambda}u(\lambda)\,d\lambda = 0 \\ S(\frac{1}{2})=\int_{-\infty}^{1/2} e^{-a\lambda}u(\lambda)\,d\lambda = \int_{0}^{1/2} e^{-a\lambda}\,d\lambda = \frac{+1}{a}(1-e^{-a/2}) \\ S(1)=\int_{-\infty}^{1} e^{-a\lambda}u(\lambda)\,d\lambda = \int_{0}^{1} e^{-a\lambda}\,d\lambda = \frac{1}{a}(1-e^{-a}) \end{cases}$$

ت) $x(t-\lambda)\delta(\lambda)=x(t)\delta(\lambda)$



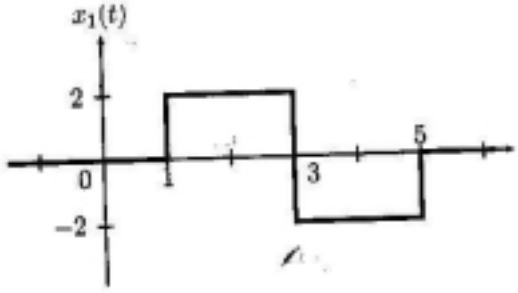
$$\rightarrow \begin{cases} x(t)=u(t), & t=\frac{-1}{2} \quad x(t)=0 \\ & t=\frac{1}{2} \quad x(t)=1 \end{cases}$$

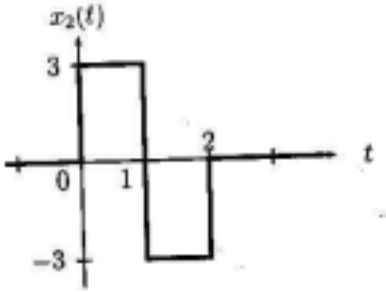


سیگنال $x_1(t)$ و $x_2(t)$ داده شده است.

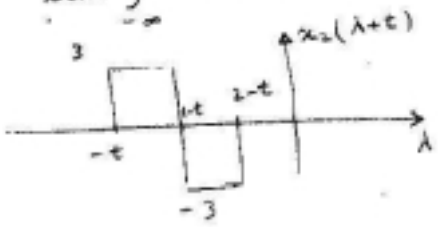
الف) $x(t) = x_1(t) * x_2(t)$ را به دست آورید.

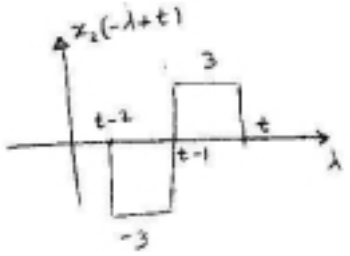
ج) $x_2(t)$ را برحسب $x_1(t)$ بیان کنید $x_2(t) = f\{x_1(t)\}$

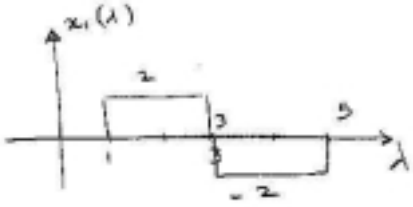




$$x(t) = \int_{-\infty}^{\infty} x_1(\lambda)\, x_2(t-\lambda)\, d\lambda$$







$t<1$ $\quad * = 0$

$1 \le t < 2$ $\quad * = \int_{1}^{t} 6\, d\lambda = 6(t-1)$

$2 \le t < 3$ $\quad * = \int_{1}^{t-1} -6\, d\lambda + \int_{t-1}^{t} 6\, d\lambda = 6\left(1-t+1+t-t+1\right) = 6(3-t)$

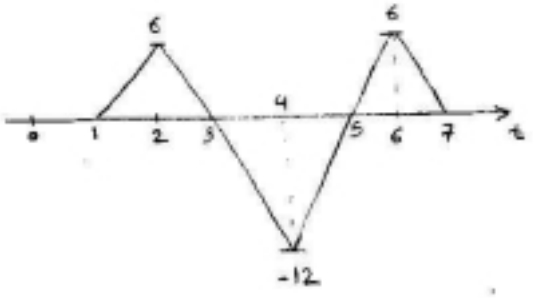
$3 \le t < 4$ $\quad * = \int_{t-2}^{t-1} -6\, d\lambda + \int_{t-1}^{3} 6\, d\lambda + \int_{3}^{t} -6\, d\lambda = 6(t-2-t+1+3-t+1+3-t) = 6(-2t+6)$

$4 \le t < 5$ $\quad * = \int_{t-2}^{3} -6\, d\lambda + \int_{3}^{t-1} 6\, d\lambda + \int_{t-1}^{t} -6\, d\lambda = 6\left(t-2-3+t-1-3+t-1-t\right) = 6(2t-10)$

$5 \le t < 6$ $\quad * = \int_{t-2}^{4} 6\, d\lambda + \int_{4}^{t-1} 6\, d\lambda + \int_{t-1}^{5} -6\, d\lambda = 6\left(4-t+2+t-1-4+t-1-5\right) = 6(t-5)$

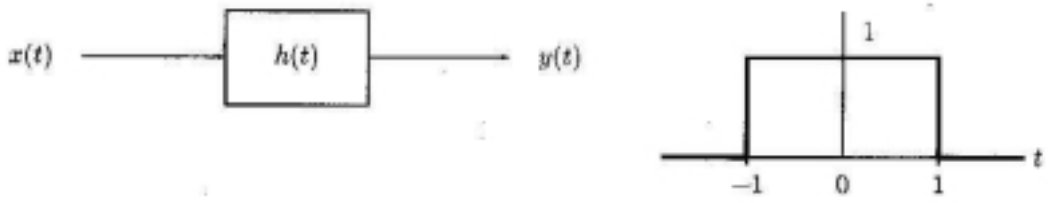
$6 \le t < 7$ $\quad * = \int_{t-2}^{5} 6\, d\lambda = 6(5-t+2) = 6(7-t)$

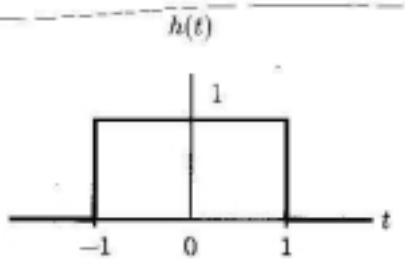
$t \ge 7$ $\quad * = 0$

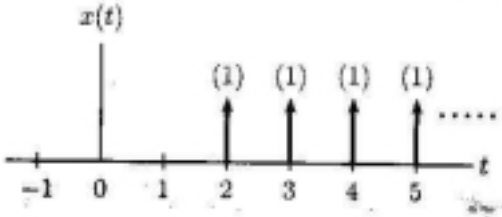


$x_2(t) = \frac{3}{2} x_1(2t+1)$ ✓✓

- سیستم LTI زمان پیوسته با پاسخ ضربه $h(t)$ داده شده است. ورودی قطار ضربه است که از $t=2$ شروع می‌شود. مطلوب است خروجی سیستم $y(t)$.

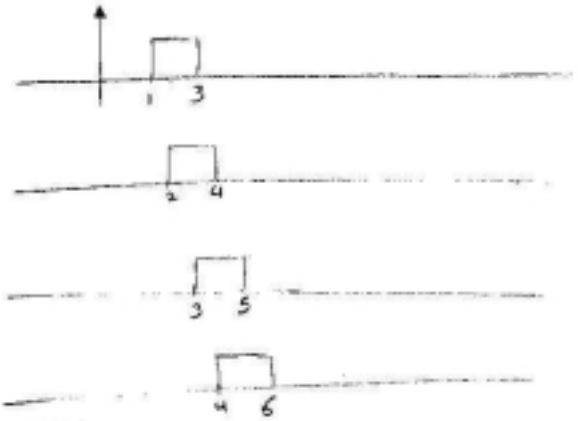


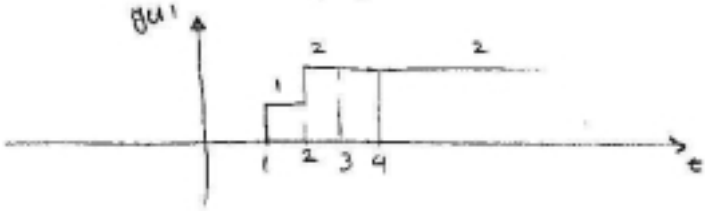




$x(t) = \delta(t-2) + \delta(t-3) + \delta(t-4) + \cdots$

$y(t) = x(t) * h(t) = h(t-2) + h(t-3) + h(t-4) + h(t-5) + \cdots$





$h(t) = u(t+1) - u(t-1)$

$g(t) = \big(u(t-1) - u(t-3)\big) + \big(u(t-2) - u(t-4)\big) + \big(u(t-3) - u(t-5)\big) + \big(u(t-4) - u(t-6)\big) + \cdots$

$y(t) = u(t-1) + u(t-2)$ ✓✓

- مطلوبست تعیین عبارت زیر:

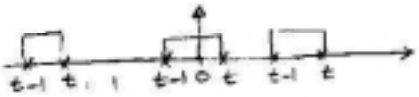
$$\int_{0}^{\infty} t^2 \delta(t-2)\,dt = 4 \int_{0}^{\infty} \delta(t-2)\,dt = 4$$

- رابطه بین ورودی و خروجی سیستم LTI داده شده است.

$$y(t) = \int_{t-1}^{t} x(\lambda)\,d\lambda$$

پاسخ ضربه سیستم را بدست آورید.

$$h(t) = \int_{t-1}^{t} \delta(\lambda)\,d\lambda$$



$t-1<0,\ t>0 \Rightarrow h(t)=1$

$o.w \Rightarrow h(t)=0$

$$\Rightarrow h(t)=\begin{cases} 1, & 0<t<1 \\ 0, & o.w \end{cases}$$

$$h(t) = u(t) - u(t-1)$$

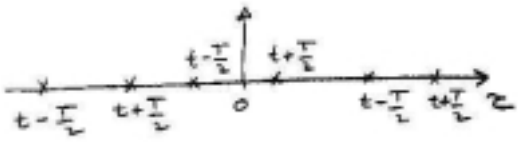
4. (10 marks) Consider a continuous-time LTI system described by

$$y(t) = T\{x(t)\} = \frac{1}{T}\int_{t-T/2}^{t+T/2} x(\tau)d\tau.$$

(a) Find and sketch the impulse response $h(t)$ of the system. , $T>0$

(b) Is the system causal?

$h(t)=\frac{1}{T}\int_{t-\frac{T}{2}}^{t+\frac{T}{2}} \delta(\tau)\, d\tau$



① $t+\frac{T}{2}<0$ $h(t)=0$

② $t-\frac{T}{2}\leq 0$ و $t+\frac{T}{2}\geq 0$ $\Rightarrow h(t)=\frac{1}{T}$

③ $t-\frac{T}{2}>0$ $h(t)=0$

$$\Rightarrow h(t)=\begin{cases}0 & t<-\frac{T}{2}\\ \frac{1}{T} & -\frac{T}{2}\leq t\leq\frac{T}{2}\\ 0 & t>\frac{T}{2}\end{cases} \qquad \Rightarrow h(t)=\begin{cases}0 & |t|>\frac{T}{2}\\ \frac{1}{T} & |t|\leq\frac{T}{2}\end{cases}$$

$h(t)=\frac{1}{T}\left(u(t+\frac{T}{2})-u(t-\frac{T}{2})\right)$

سیستم علی غیر